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Circuit Theory

U. A. Bakshi
A. V. Bakshi

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1 Basic Circuit Analysis and Network Reduction Techniques

1.1 Introduction

The Ohm's law can be successfully applied to the various circuits to analyse them. But in practice, the circuits may consist of one or more sources of e.m.f. and number of electrical parameters, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements alongwith various sources of energy give rise to complicated electrical circuits, generally referred as **networks**. The terms circuit and network are used synonymously in the electrical literature.

The **network analysis** or **circuit analysis** means to find a current through or voltage across any branch of network or circuit.

In circuit analysis, many a times it is necessary to reduce the complicated electrical network to simple form. In this chapter, some network simplification techniques such as source transformation, application of Kirchhoff's laws and star-delta transformations are discussed. These techniques are basic techniques which are further useful to apply various network theorems to the complicated electrical networks.

1.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1.2.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network.** Such a network is shown in the Fig. 1.1.

1.2.2 Network Element

Any individual circuit element with two terminals which can be connected to other circuit element, is called a **network element.**

Network elements can be either active elements or passive elements. Active elements are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements. Passive elements are the elements which either store energy or dissipate energy in the form of heat. Resistor, inductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

1.2.3 Branch

A part of the network which connects the various points of the network with one another is called **a branch.** In the Fig. 1.1, AB, BC, CD, DA, DE, CF and EF are the various branches. A branch may consist more than one element.

1.2.4 Junction Point

A point where three or more branches meet is called a **junction point.** Point D and C are the junction points in the network shown in the Fig. 1.1.

1.2.5 Node

A point at which two or more elements are joined together is called **node.** The junction points are also the nodes of the network. In the network shown in the Fig. 1.1, A, B, C, D, E and F are the nodes of the network.

1.2.6 Mesh (or Loop)

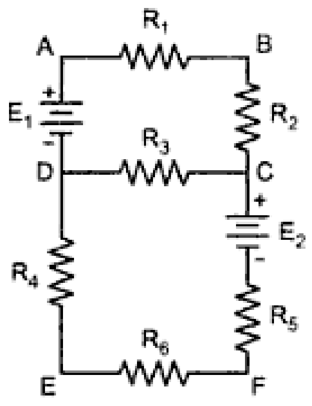


Fig. 1.1 An electrical network

Mesh (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any node twice. In the Fig. 1.1 paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc. are the loops of the network.

In this chapter, the analysis of d.c. circuits consisting of pure resistors and d.c. sources is included.

1.3 Classification of Electrical Networks

The behaviour of the entire network depends on the behaviour and characteristics of its elements. Based on such characteristics electrical network can be classified as below :

i) Linear network : A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network.** The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

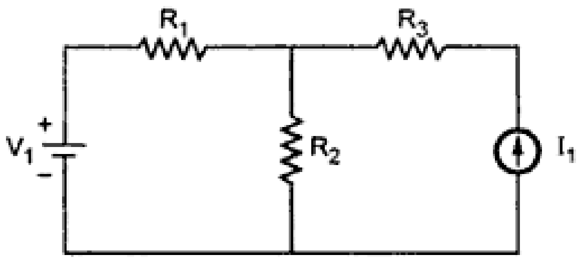
ii) Non linear network : A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network**. The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

iii) Bilateral network : A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network.** Network consisting only resistances is good example of bilateral network.

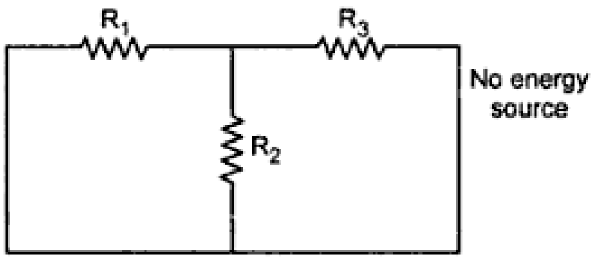
iv) Unilateral network : A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network.** Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

v) Active network : A circuit which contains at least one source of energy is called **active.** An energy source may be a voltage or current source.

vi) Passive network : A circuit which contains no energy source is called **passive circuit.** This is shown in the Fig. 1.2.



(a) Active network



(b) Passive network

Fig. 1.2

vii) Lumped network : A network in which all the network elements are physically separable is known as **lumped network.** Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

viii) Distributed network : A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network.** The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, any where in the circuit.

The classification of networks can be shown as,

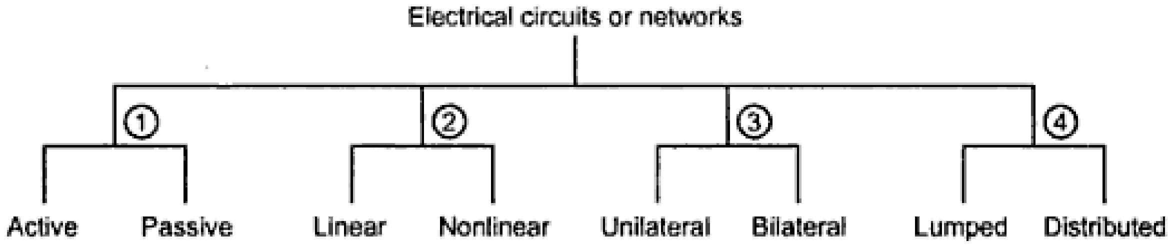


Fig. 1.3 Classification of networks

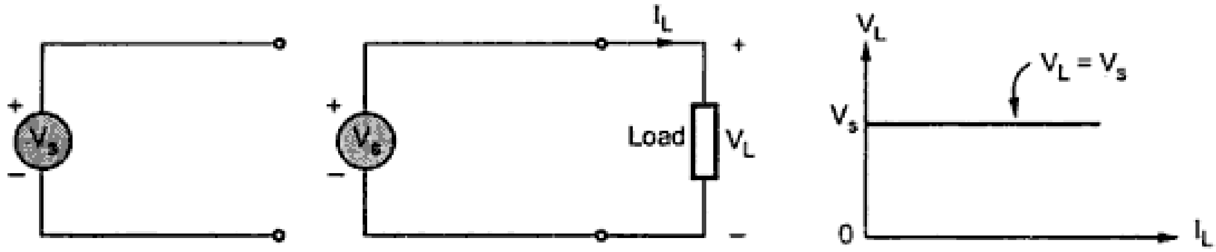
1.4 Energy Sources

There are basically two types of energy sources ; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

1.4.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 1.4 (a). This is connected to the load as shown in Fig. 1.4 (b). At any time the value of voltage at load terminals remains same. This is indicated by V- I characteristics shown in the Fig. 1.4 (c).

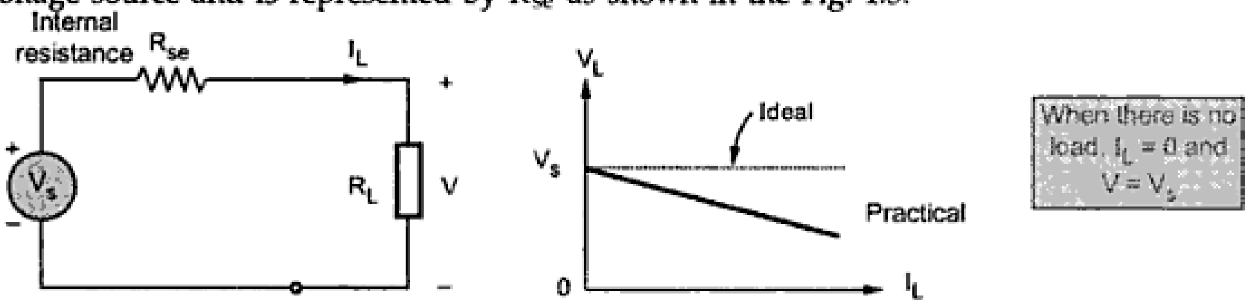


(a) Symbol (b) Circuit (c) Characteristics

Fig. 1.4 Ideal voltage source

Practical voltage source :

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 1.5.



(a) Circuit (b) Characteristics

Fig. 1.5 Practical voltage source

Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$\boxed{V_L = -(R_{se})\, I_L + V_s = V_s - I_L\, R_{se}}$$

Key Point : *For ideal voltage source,*

$$R_{se} = 0$$

Voltage sources are further classified as follows,

i) Time invariant sources :

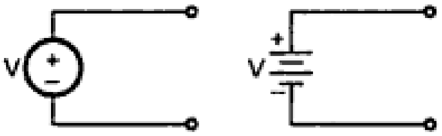


Fig. 1.6 (a) D.C. source

The sources in which voltage is not varying with time are known as **time invariant voltage sources** or **D.C. sources.** These are denoted by capital letters. Such a source is represented in the Fig. 1.6 (a).

ii) Time variant sources :

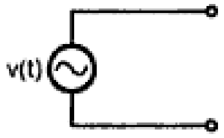


Fig. 1.6 (b) A.C. source

The sources in which voltage is varying with time are known as **time variant voltage sources** or **A.C. sources.** These are denoted by small letters. This is shown in the Fig. 1.6 (b).

1.4.2 Current Source

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 1.7 (a). This is connected to the load as shown in the Fig. 1.7 (b). At any time, the value of the current flowing through load I_L is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 1.7 (c).

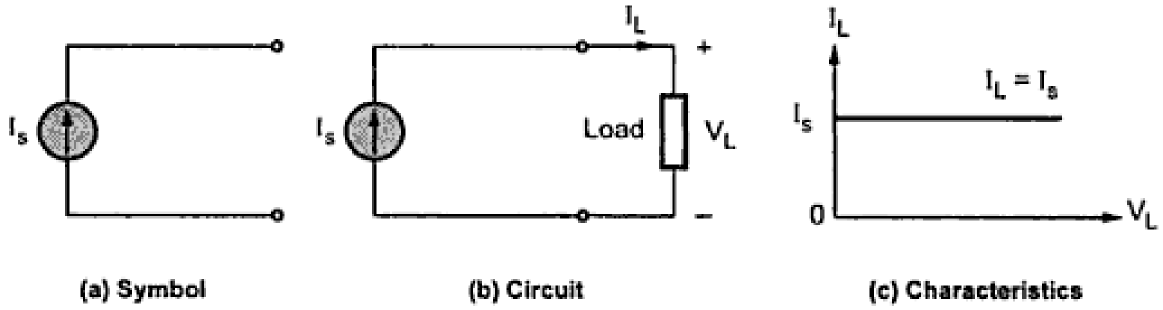
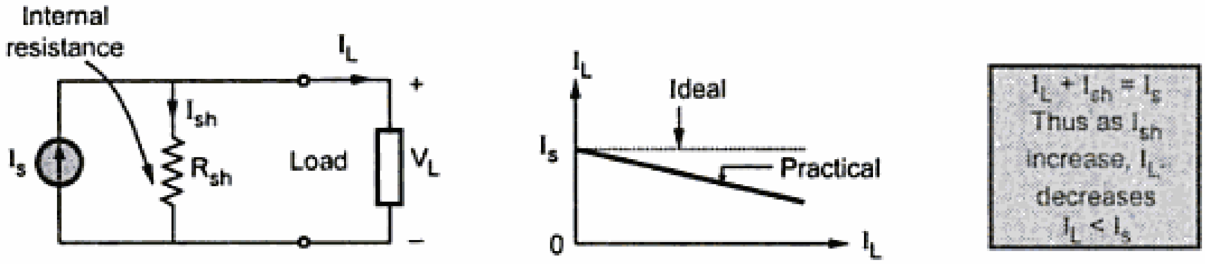


Fig. 1.7 Ideal current source

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 1.8.



(a) Circuit (b) Characteristic

Fig. 1.8 Practical current source

Because of R_{sh}, current through its terminals decreases slightly with increase in voltage at its terminals.

Key Point : *For ideal current source,* $R_{sh} = \infty$.

Similar to voltage sources, current sources are classified as follows :

i) Time invariant sources :

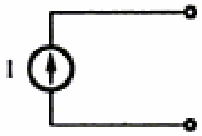


Fig. 1.9 (a) D.C. source

The sources in which current is not varying with time are known as **time invariant current sources** or **D.C. sources**. These are denoted by capital letters.

Such a current source is represented in the Fig. 1.9 (a).

ii) Time variant sources :

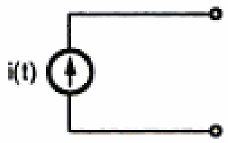


Fig. 1.9 (b) A.C. source

The sources in which current is varying with time are known as **time variant current sources** or **A.C. sources**. These are denoted by small letters.

Such a source is represented in the Fig. 1.9 (b).

The sources which are discussed above are independent sources because these sources does not depend on other voltages or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside.

1.4.3 Dependent Sources

Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 1.10 and further classified as,

i) Voltage dependent voltage source : It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS.** It is shown in the Fig. 1.10 (a).

ii) Current dependent current source : It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. 1.10 (b).

iii) Current dependent voltage source : It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. 1.10 (c).

iv) Voltage dependent current source : It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 1.10 (d).

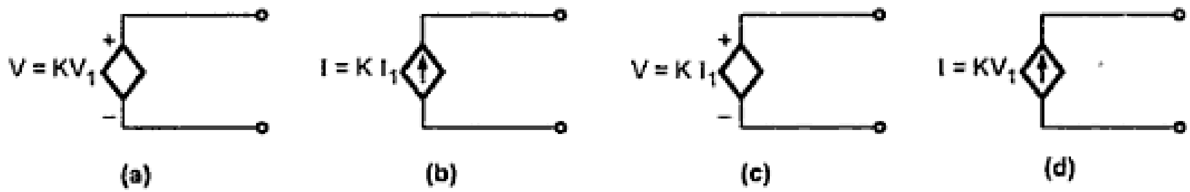


Fig. 1.10 Types of dependent sources

K is constant and V_1 and I_1 are the voltage and current respectively, present elsewhere in the given circuit. The dependent sources are also known as **controlled sources.**

1.4.4 Regulation and Loading of Sources

It is seen that practically the output voltage of the voltage source decreases as load current increases. The allowable drop in voltage is specified interms of parameter called **regulation** of source. It is defined as,

$$\boxed{\%\ \text{Regulation} = \frac{\text{No load voltage} - \text{Full load voltage}}{\text{Full load voltage}} \times 100 = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100}$$

On no load, V_{NL} is same as the rated value of voltage source. While V_{FL} is terminal voltage on full load.

> **Key Point :** *Lesser the regulation, better is the performance of the source. Ideal value of regulation is zero.*

If the source is loaded in such a way that the load voltage falls below specified full load value and the regulation is higher than that specified for the source then the source is said to be loaded. This is **loading of sources.**

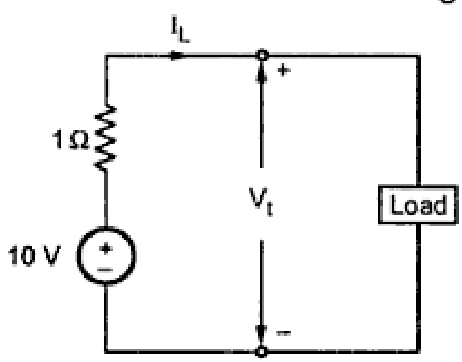


Fig. 1.11 Loaded source

For example : Consider the source of 10 V shown having internal resistance of 1Ω, in the Fig.1.11. The terminal voltage is,

$$V_t = I_L R_L$$

$$I_L = \frac{10}{R_L + 1} \quad \text{... (1)}$$

On no load, $R_L = \infty$ and $V_t = V = V_{NL} = 10$ V

Let the specified full load current be 1 A i.e. $I_L = I_{FL} = 1$ A.

On full load,

$$R_L = \frac{10}{I_L} - 1 = 9\ \Omega \qquad \text{... from (1)}$$

$$\therefore \quad V_{FL} = I_{FL} \times R_L = 1 \times 9 = 9\ \text{V}$$

$$\therefore \quad \%\ \text{Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 = \frac{10-9}{9} \times 100 = 11.11\ \%$$

If now R_L is changed to 5Ω then,

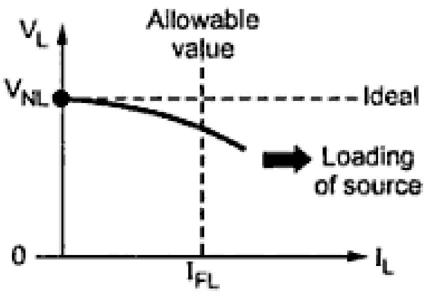


Fig. 1.12 Loading of source

$$I_L = \frac{10}{5+1} = 1.667\ \text{A}$$

$$\therefore \quad V_L = I_L\ R_L = 1.667 \times 5 = 8.335\ \text{V}$$

$$\therefore \%\ \text{Regulation} = \frac{10 - 8.335}{8.335} \times 100 = 19.97\ \%$$

This worsens the regulation and the source is said to be loaded. Thus for any $I_L > 1$ A there is loading of source which is shown in the Fig. 1.12.

1.5 Ohm's Law

This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit. **Dr. Ohm** in 1827 discovered a law called **Ohm's Law.** It states,

Ohm's Law : *The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.*

Mathematically, $\boxed{I \propto \frac{V}{R}}$

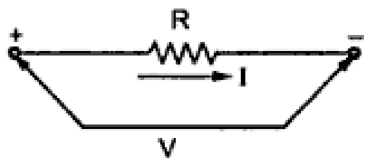


Fig. 1.13 Ohm's law

Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 1.13.

Now $I = \frac{V}{R}$

The unit of potential difference is defined in such a way that the constant of proportionality is unity.

Ohm's law is,

$I = \frac{V}{R}$	amperes
$V = I\,R$	volts
$\frac{V}{I} = \text{Constant} = R$	ohms

The Ohm's law can be defined as,

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

Key Point: *Ohm's law can be applied either to the entire circuit or to the part of a circuit. If it is applied to entire circuit, the voltage across the entire circuit and resistance of the entire circuit should be taken into account. If the Ohm's law is applied to the part of a circuit, then the resistance of that part and potential across that part should be used.*

1.5.1 Limitations of Ohm's Law

The limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.

2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = k\,I^m \quad \text{where k, m are constants.}$$

1.6 Linear Passive Parameters (R, L and C)

The three basic linear passive elements used in various circuits are,

1) Resistance 2) Inductance 3) Capacitance

1.6.1 Resistance and its V-I Relationship

Fig. 1.14

It is the property of the material by which it opposes the flow of current through it. The resistance of element is denoted by the symbol 'R'. Resistance is measured in ohms (Ω).

The relation between voltage and current is given by Ohm's law.

$$\therefore \quad \boxed{V = R \cdot I}$$

Resistor dissipates energy in the form of heat. So power absorbed by the resistor is given by,

$$\therefore \quad \boxed{P = VI = (I \cdot R)\, I = I^2R = \frac{V^2}{R} \text{ watts}}$$

Resistance converts amount of energy into heat during time t, is given by,

$$\therefore \quad \boxed{W = P\, dt = \int_0^t I^2 R\, dt = I^2 \cdot R \cdot t = V \cdot I \cdot t \quad \text{joules}}$$

1.6.2 Inductance and its V-I Relationship

Fig. 1.15

Inductance is the element in which energy is stored in the form of **electromagnetic** field. The inductance is denoted by 'L' and it is measured in Henry (H).

For inductance, the voltage is proportional to the rate of change of current.

$$\therefore v \propto \frac{di}{dt}$$

$$\therefore \quad \boxed{v = L\frac{di(t)}{dt}}$$

Assuming that initially zero current flows through the inductance, if a current i is made to flow through a coil, the energy stored in time interval is given by,

$$W = \int_0^t v \cdot i\, dt = \int_0^t \left(L\frac{di}{dt}\right) i\, dt = L\int_0^t i \cdot di$$

$$\therefore \quad \boxed{W = \frac{1}{2}\, i^2 L \quad \text{joules}} \qquad \text{... Energy stored in an inductor.}$$

1.6.3 Capacitance and its V-I Relationship

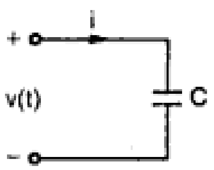


Fig. 1.16

An element in which energy is stored in the form of **electrostatic** field is known as capacitance. The capacitance is denoted by 'C' and it is measured in Farads (F).

For capacitor, the voltage is proportional to the charge.

$$\therefore \qquad v \propto q$$

$$\therefore \quad v \propto \int_{-\infty}^{t} i\, dt$$

$$\therefore \quad v = \frac{1}{C}\int i\, dt$$

$$\therefore \quad \boxed{i = C\frac{dv(t)}{dt}}$$

With zero initial voltage across capacitor, if the current i flows for time t spent, the energy supplied to capacitor will be,

$$W = \int_0^t v.i\, dt = \int_0^t v.C\frac{dv}{dt}dt = C\int_0^V v\, dv$$

$$\therefore \quad \boxed{W = \frac{1}{2}Cv^2 \text{ joules}} \quad \text{... Energy stored in a capacitor.}$$

Key Point : ***While calculating energy stored in the capacitor, V must be voltage across capacitor.***

Summarizing the behaviour of the three basic elements, we can write,

Element	Voltage across element if current is known	Current through element if voltage is known
R	$v = iR$	$i = \frac{v}{R}$
L	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\, dt$
C	$v = \frac{1}{C}\int i\, dt$	$i = C\frac{dv}{dt}$

Table 1.1 Behaviour of circuit elements

Example 1.1 *A 0.5 μF capacitor has voltage waveform v(t) (Fig. 1.17) .Plot i(t) as function of t.* **(April/May-2005)**

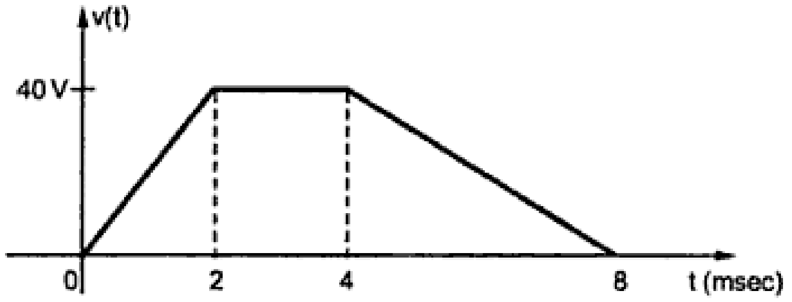


Fig. 1.17

Solution : For $0 < t < 2$, v(t) is a ramp of slope $\frac{40}{2} = 20$.

$$\therefore \quad v(t) = 20\,t \qquad \text{... for } 0 < t < 2$$

$$\therefore \quad i(t) = C\frac{dv(t)}{dt} = 0.5\times10^{-6}\times20 = 1\times10^{-5}\,\text{A} = \mathbf{10\ \mu A} \qquad \text{... for } 0 < t < 2$$

For $2 < t < 4$, v(t) is constant.

$$\therefore \quad v(t) = 40\ \text{V} \qquad \text{... for } 2 < t < 4$$

$$\therefore \quad i(t) = C\frac{dv(t)}{dt} = 0.5\times10^{-6}\times0 = \mathbf{0\ A} \qquad \text{... for } 2 < t < 4$$

For $4 < t < 8$, v(t) is a ramp with slope $= \frac{0-40}{8-4} = -10$

$$\therefore \quad v(t) = -10\,t + 80 \qquad \text{... for } 4 < t < 8$$

$$\therefore \quad i(t) = C\frac{dv(t)}{dt} = 0.5\times10^{-6}\times(-10) = \mathbf{-5\ \mu A} \qquad \text{... for } 4 < t < 8$$

The current waveform is shown in the Fig. 1.17(a).

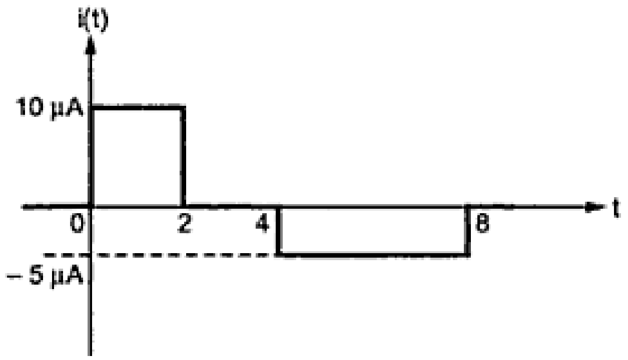


Fig. 1.17(a)

Note : For $4 < t < 8$, the equation of the line is $y = mt + C$ where m = slope = –10. It gives equation $y = -10\,t + C$. Now point (8, 0) is on the line so substituting in the equation,

$$0 = -10\times8+C \qquad \text{i.e. } C = 80$$

Thus equation of line is $y = -10\,t + 80$ where $y = i(t)$, as used above.

1.7 Analysis of Series Circuits

A series circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade connection**. There is only one path for the flow of current.

1.7.1 Resistors in Series

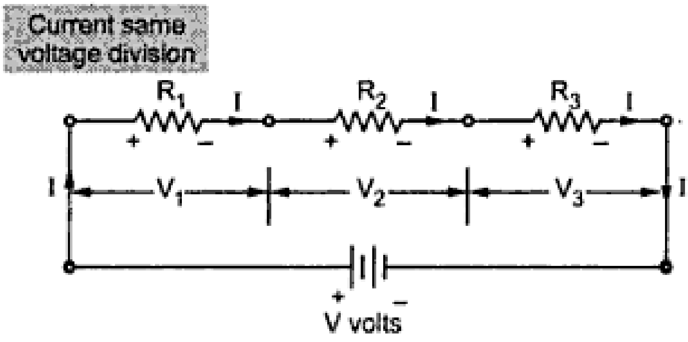


Fig. 1.18 A series circuit

Consider the resistances shown in the Fig. 1.18.

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. e.g. the chain of small lights, used for the decoration purposes is good example of series combination.

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively.

Then, $$V = V_1 + V_2 + V_3$$

Now according to Ohm's law, $$V_1 = I R_1, \quad V_2 = I R_2, \quad V_3 = I R_3$$

Current through all of them is same i.e. I

$$\therefore \quad V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I\, R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistances in series, $$\boxed{R = R_1 + R_2 + R_3 + \ldots\ldots + R_n}$$

1.7.1.1 Characteristics of Series Circuits

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots.. + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances.

i.e $R > R_1,\ R > R_2,\ \ldots.. R > R_n$

1.7.2 Inductors in Series

Consider the Fig. 1.19 (a). Two inductors L_1 and L_2 are connected in series. The currents flowing through L_1 and L_2 are i_1 and i_2 while voltages developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in the Fig. 1.19 (b).

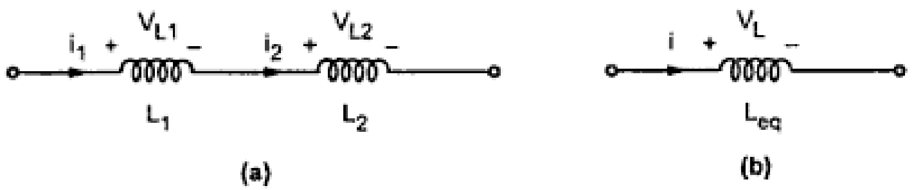


Fig. 1.19 Inductors in series

We have, $V_{L1} = L_1 \dfrac{di_1}{dt}$ and $V_{L2} = L_2 \dfrac{di_2}{dt}$ while $V_L = L_{eq} \dfrac{di}{dt}$

For series combination,

$$i = i_1 = i_2$$

and $$V_L = V_{L1} + V_{L2}$$

$$\therefore \quad L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$\therefore \quad L_{eq} \frac{di}{dt} = (L_1 + L_2) \frac{di}{dt}$$

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2}$$

That means, equivalent inductance of the series combination of two inductances is the sum of inductances connected in series.

Key Point : *The total equivalent inductance of the series circuit is sum of the inductances connected in series.*

For n inductances in series,

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n}$$

1.7.3 Capacitors in Series

Consider the Fig. 1.20 (a). Two capacitors C_1 and C_2 are connected in series. The currents flowing through and voltages developed across C_1 and C_2 are i_1, i_2 and V_{C1} and V_{C2} respectively. The equivalent circuit is shown in the Fig. 1.20 (b).

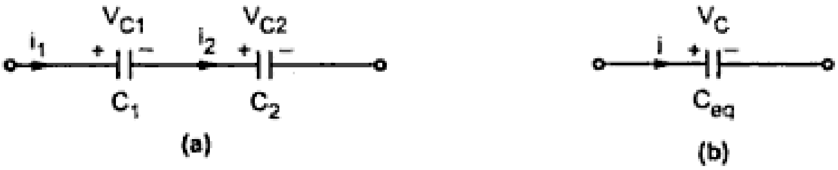


Fig. 1.20 Capacitors in series

We have, $V_{C1} = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt$, $V_{C2} = \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$ while $V = \frac{1}{C_{eq}}\int_{-\infty}^{t} i dt$

For series combination,

$$i = i_1 = i_2 \quad \text{and}$$

$$V_C = V_{C1} + V_{C2}$$

$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt + \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$$

But $\quad i = i_1 = i_2$

$$\therefore \quad \frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \left(\frac{1}{C_1}+\frac{1}{C_2}\right)\int_{-\infty}^{t} i\, dt$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}$$

$$\therefore \quad \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

That means, reciprocal of equivalent capacitor of the series combination is the sum of the reciprocal of individual capacitances.

Key Point : *The reciprocal of the total equivalent capacitor of the series combination is the sum of the reciprocols of the individual capacitors, connected in series.*

For n capacitors in series,

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\frac{1}{C_n}}$$

Example 1.2 : *Two capacitances C_1 and C_2 of values of 10 μF and 5 μF, respectively are connected in series. What is the equivalent capacitance of the combination ?* **[April/May-2003]**

Solution : $C_1 = 10\ \mu F$ and $C_2 = 5\ \mu F$

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{10\times 10^{-6} \times 5\times 10^{-6}}{10\times 10^{-6} + 5\times 10^{-6}} = 3.333 \times 10^{-6}$$

$$= 3.33\ \mu F$$

1.8 Analysis of Parallel Circuits

The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

1.8.1 Resistors in Parallel

Voltage same current division

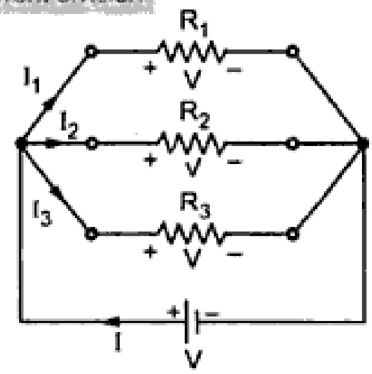


Fig. 1.21 A parallel circuit

Consider a parallel circuit shown in the Fig. 1.21.

In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V '.

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. While the voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1,\quad V = I_2 R_2,\quad V = I_3 R_3$$

$$I_1 = \frac{V}{R_1},\quad I_2 = \frac{V}{R_2},\quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \qquad \dots (1)$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}} \quad \dots (2)$$

Where R_{eq} = Total or **equivalent resistance** of the circuit

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

Where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\dots\dots+\frac{1}{R_n}$$

Conductance (G) :

It is known that, $\frac{1}{R} = G$ (conductance) hence,

$$\therefore \quad \boxed{G = G_1+G_2+G_3+\dots\dots+G_n} \quad \dots \text{For parallel circuit.}$$

Important result :

Now if $n = 2$, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}$$

$$\boxed{R = \frac{R_1 R_2}{R_1+R_2}}$$

This formula is directly used hereafter, for two resistances in parallel.

1.8.1.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1+I_2+I_3+\dots\dots+I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4) The equivalent resistance is the smallest of all the resistances.

$$R < R_1,\ R<R_2, \dots\dots, R<R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

Key Point : *The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.*

1.8.2 Inductors in Parallel

Consider the Fig. 1.22 (a). Two inductors L_1 and L_2 are connected in parallel. The currents flowing through L_1 and L_2 are i_1 and i_2 respectively. The voltage developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in Fig. 1.22 (b).

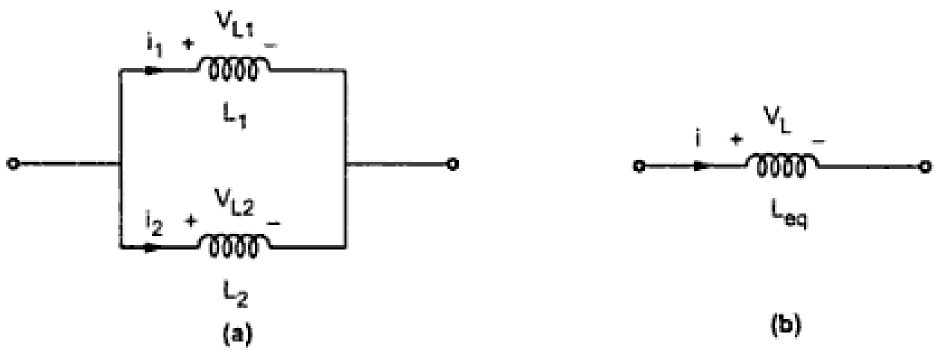


Fig. 1.22 Inductors in parallel

For inductor we have,

$$i_1 = \frac{1}{L_1}\int_{-\infty}^{t} V_{L1}\,dt, \qquad i_2 = \frac{1}{L_2}\int_{-\infty}^{t} V_{L2}\,dt, \quad \text{while} \quad i = \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\,dt$$

For parallel combination,

$$V_L = V_{L1} = V_{L2} \quad \text{and}$$

$$i = i_1 + i_2$$

$$\therefore \quad \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\,dt = \frac{1}{L_1}\int_{-\infty}^{t} V_L\,dt + \frac{1}{L_2}\int_{-\infty}^{t} V_L\,dt = \left(\frac{1}{L_1}+\frac{1}{L_2}\right)\int_{-\infty}^{t} V_L\,dt$$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

That means, reciprocal of equivalent inductance of the parallel combination is the sum of reciprocals of the individual inductances.

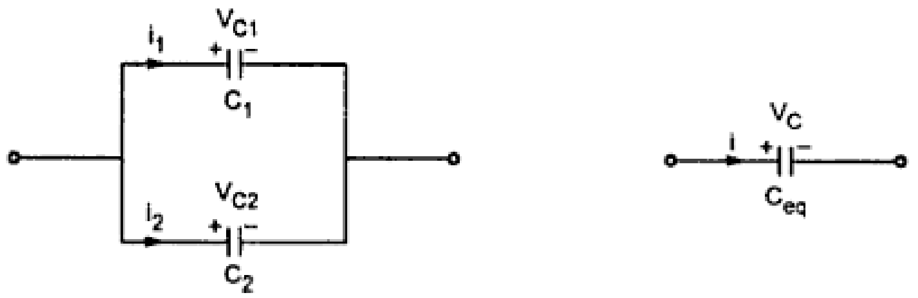
For n inductances in parallel,

$$\therefore \quad \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n}}$$

1.8.3 Capacitors in Parallel

Consider the Fig. 1.23 (a). Two capacitors C_1 and C_2 are connected in parallel. The currents flowing through C_1 and C_2 are i_1 and i_2 respectively and voltages developed across C_1, C_2 are V_{C1} and V_{C2} respectively.

The equivalent circuit is shown in the Fig. 1.23 (b).



(a) (b)

Fig. 1.23 Capacitors in parallel

For capacitor we have, $i_1 = C_1 \frac{dV_{C1}}{dt}$, $i_2 = C_2 \frac{dV_{C2}}{dt}$, while $i = C_{eq} \frac{dV_C}{dt}$

For parallel combination,

$$V_{C1} = V_{C2} = V_C \quad \text{and}$$

$$i = i_1 + i_2$$

$$C_{eq} \frac{dV_C}{dt} = C_1 \frac{dV_{C1}}{dt} + C_2 \frac{dV_{C2}}{dt}$$

$$\therefore \quad C_{eq} \frac{dV_C}{dt} = (C_1 + C_2) \frac{dV_C}{dt}$$

$$\therefore \quad C_{eq} = C_1 + C_2$$

That means, equivalent capacitance of the parallel combination of the capacitances is the sum of the individual capacitances connected in series.

For n capacitors in parallel,

$$\therefore \quad \boxed{C_{eq} = C_1 + C_2 + \ldots + C_n}$$

The Table 1.2 gives the equivalent at 'n' basic elements is series.

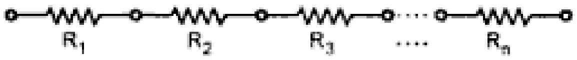
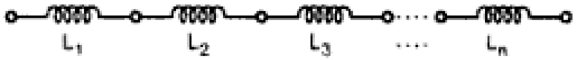
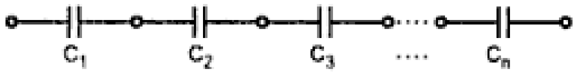
Element	Equivalent
'n' Resistances in series 	$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$
'n' Inductors in series 	$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n$
'n' Capacitors in series 	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

Table 1.2 Series combinations of elements

The Table 1.3 gives the equivalent of 'n' basic elements in parallel.

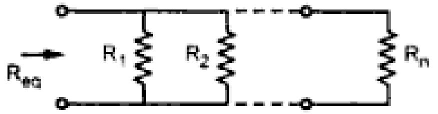
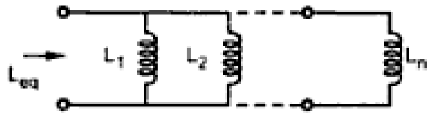
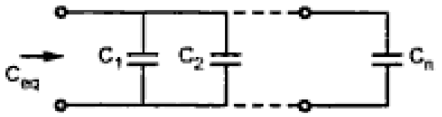
Element	Equivalent
'n' Resistances in parallel 	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$
'n' Inductors in parallel 	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
'n' Capacitors in parallel 	$C_{eq} = C_1 + C_2 + \ldots + C_n$

Table 1.3 Parallel combinations of elements

Key Point : *The current through series combination remains same and voltage gets divided while in parallel combination voltage across combination remains same and current gets divided.*

Example 1.3 : *Find the equivalent resistance between the two points A and B shown in the Fig. 1.24.*

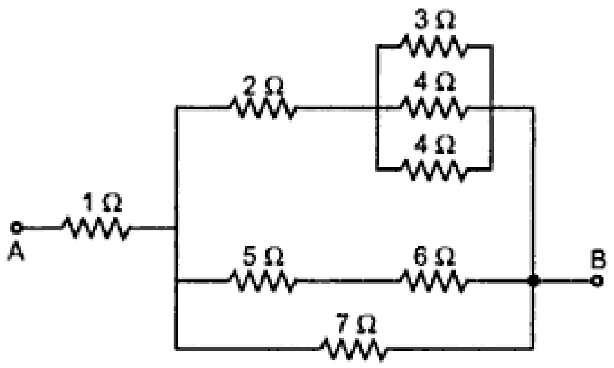


Fig. 1.24

Solution : Identify combinations of series and parallel resistances.

The resistances 5 Ω and 6 Ω are in series, as going to carry same current.

So equivalent resistance is 5 + 6 = 11 Ω

While the resistances 3 Ω , 4 Ω, and 4 Ω are in parallel, as voltage across them same but current divides.

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$

∴ $R = \frac{12}{10} = 1.2\ \Omega$

Replacing these combinations redraw the figure as shown in the Fig. 1.25 (a).

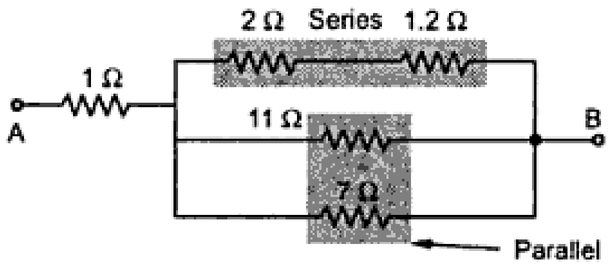


Fig. 1.25 (a)

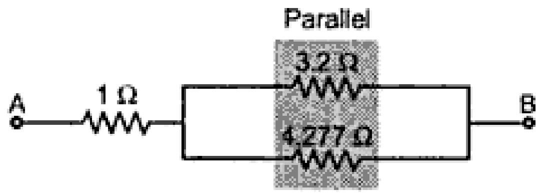


Fig. 1.25 (b)

Now again 2.2 Ω and 2 Ω are in series so equivalent resistance is 2 + 1.2 = 3.2 Ω while 11 Ω and 7 Ω are in parallel.

Using formula $\frac{R_1 R_2}{R_1 + R_2}$ equivalent resistance is $\frac{11 \times 7}{11+7} = \frac{77}{18} = 4.277\ \Omega$.

Replacing the respective combinations redraw the circuit as shown in the Fig. 1.25 (b).

Now 3.2 and 4.277 are in parallel.

$\therefore$ Replacing them by $\dfrac{3.2 \times 4.277}{3.2 + 4.277} = 1.8304\ \Omega$

$$\therefore \qquad R_{AB.} = 1 + 1.8304 = \mathbf{2.8304}\ \Omega$$

1.9 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the network plays an important role.

1.9.1 Short Circuit

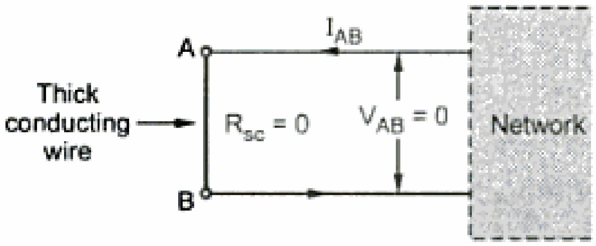


Fig. 1.26 Short circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire, the two points are said to be short circuited. **The resistance of such short circuit is zero.**

The part of the network, which is short circuited is shown in the Fig. 1.26. The points A and B are short circuited. The resistance of the branch AB is $R_{sc} = 0\ \Omega$.

The curent I_{AB} is flowing through the short circuited path.

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0\ V$$

Key Point : *Thus, voltage across short circuit is always zero though current flows through the short circuited path.*

1.9.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

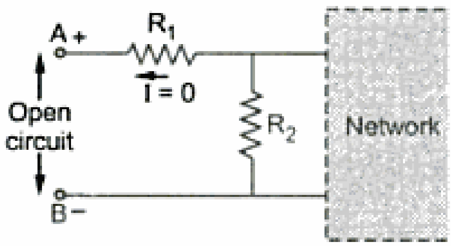


Fig. 1.27 Open circuit

As there is no direct connection in an open circuit, **the resistance of the open circuit is ∞.**

The part of the network which is open circuited is shown in the Fig. 1.27. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{oc} = \infty\ \Omega$.

There exists a voltage across the points AB called open circuit voltage, V_{AB} but $R_{oc} = \infty\ \Omega$.

According to Ohm's law,

$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0\ A$$

Key Point : *Thus, current through open circuit is always zero though there exists a voltage across open circuited terminals.*

1.9.3 Redundant Branches and Combinations

The redundant means excessive and unwanted.

Key Point : *If in a circuit there are branches or combinations of elements which do not carry any current then such branches and combinations are called redundant from circuit point of view.*

The redundant branches and combinations can be removed and these branches do not affect the performance of the circuit.

The two important situations of **redundancy** which may exist in practical circuits are,

Situation 1 : Any branch or combination across which there exists a short circuit, becomes redundant as it does not carry any current.

If in a network, there exists a direct short circuit across a resistance or the combination of resistances then that resistance or the entire combination of resistances becomes **inactive** from the circuit point of view. Such a combination is redundant from circuit point of view.

To understand this, consider the combination of resistances and a short circuit as shown in the Fig. 1.28 (a) and (b).

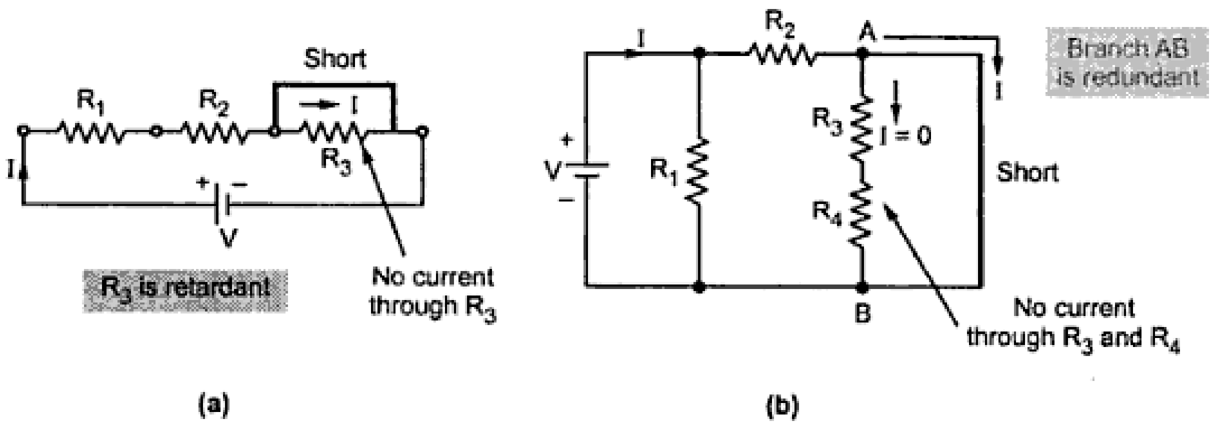


Fig. 1.28 Redundant branches

In Fig. 1.28 (a), there is short circuit across R_3. The current always prefers low resistance path hence entire current I passes through short circuit and hence resistance R_3 becomes redundant from the circuit point of view.

In Fig. 1.28 (b), there is short circuit across combination of R_3 and R_4. The entire current flows through short circuit across R_3 and R_4 and no current can flow through combination of R_3 and R_4. Thus that combination becomes meaningless from the circuit point of view. **Such combinations can be eliminated while analysing the circuit.**

Situation 2 : If there is open circuit in a branch or combination, it can not carry any current and becomes redundant.

In Fig. 1.28(c) as there exists open circuit in branch BC, the branch BC and CD can not carry any current and are become redundant from circuit point of view.

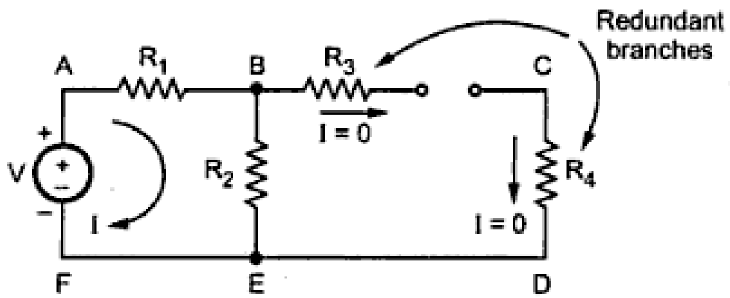


Fig. 1.28(c) Redundant branches due to open circuit

1.10 Voltage Division in Series Circuit of Resistors

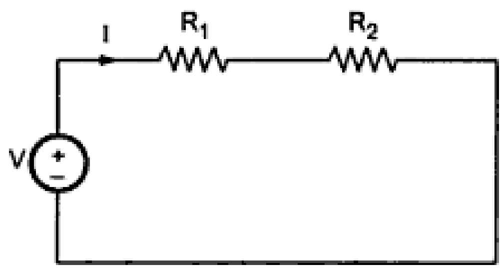


Fig. 1.29

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts.

As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I. Then applying KVL, we get,

$$V = I R_1 + I R_2$$

$$\therefore \quad I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of voltage drops V_{R1} and V_{R2} across R_1 and R_2 respectively.

$$\therefore \quad V_{R1} = I \cdot R_1$$

$$\therefore \quad \boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

Similarly, $\quad V_{R2} = I \cdot R_2$

$$\therefore \qquad \boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So this circuit is a **voltage divider circuit.**

Key Point : *So in general, voltage drop across any resistor, or combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage.*

1.11 Current Division in Parallel Circuit of Resistors

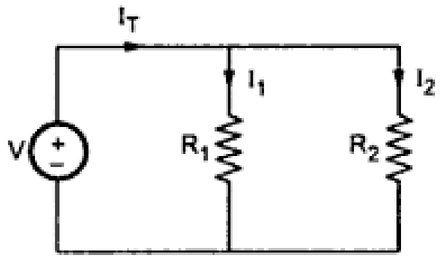


Fig. 1.30

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.

Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

$$\therefore \qquad I_T = I_1 + I_2$$

$$\text{But} \qquad I_1 = \frac{V}{R_1}, \qquad I_2 = \frac{V}{R_2}$$

$$\text{i.e.} \qquad V = I_1 R_1 = I_2 R_2$$

$$\therefore \qquad I_1 = I_2 \left(\frac{R_2}{R_1}\right)$$

Substituting value of I_1 in I_T,

$$\therefore \qquad I_T = I_2 \left(\frac{R_2}{R_1}\right) + I_2 = I_2 \left[\frac{R_2}{R_1} + 1\right] = I_2 \left[\frac{R_1 + R_2}{R_1}\right]$$

$$\therefore \qquad \boxed{I_2 = \left[\frac{R_1}{R_1 + R_2}\right] I_T}$$

$$\text{Now} \qquad I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1 + R_2}\right] I_T$$

$$\therefore \qquad I_1 = \left[\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right] I_T$$

$$\therefore \qquad \boxed{I_1 = \left[\frac{R_2}{R_1 + R_2}\right] I_T}$$

Key Point : *In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.*

Example 1.4 : *Find the voltage across the three resistances shown in the Fig. 1.31.*

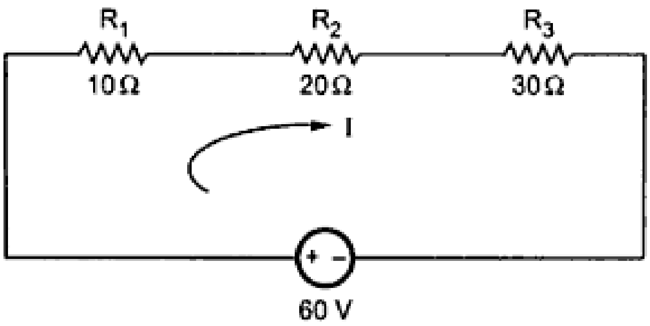


Fig. 1.31

Solution :

$$I = \frac{V}{R_1+R_2+R_3} \quad \text{... Series circuit}$$

$$= \frac{60}{10+20+30} = 1 \text{ A}$$

$$\therefore \quad V_{R1} = IR_1 = \frac{V \times R_1}{R_1+R_2+R_3} = 1 \times 10 = \mathbf{10\ V}$$

$$\therefore \quad V_{R2} = IR_2 = \frac{V \times R_2}{R_1+R_2+R_3} = 1 \times 20 = \mathbf{20\ V}$$

$$\text{and} \quad V_{R3} = IR_3 = \frac{V \times R_3}{R_1+R_2+R_3} = 1 \times 30 = \mathbf{30\ V}$$

Key Point : *It can be seen that voltage across any resistance of series circuit is ratio of that resistance to the total resistance, multiplied by the source voltage.*

Example 1.5 : *Find the magnitudes of total current, current through R_1 and R_2 if,*

$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.

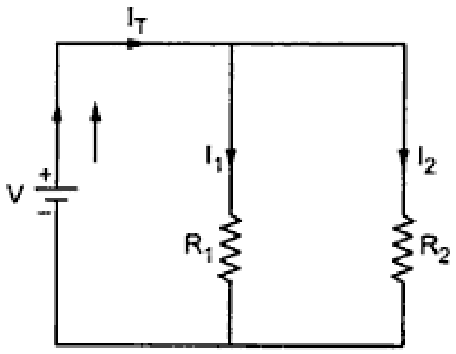


Fig. 1.32

Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{10 + 20} = 6.67 \ \Omega$$

$$\therefore \quad I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = \mathbf{7.5 \ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T\left(\frac{R_2}{R_1 + R_2}\right) = 7.5 \times \left(\frac{20}{10+20}\right)$$

$$= \mathbf{5 \ A}$$

and

$$I_2 = I_T\left(\frac{R_1}{R_1 + R_2}\right) = 7.5 \times \left(\frac{10}{10+20}\right)$$

$$= \mathbf{2.5 \ A}$$

It can be verified that $I_T = I_1 + I_2$

1.12 Source Transformation

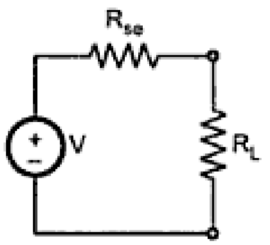


Fig. 1.33 (a) Voltage source

Consider a practical voltage source shown in the Fig. 1.33 (a) having internal resistance R_{se}, connected to the load having resistance R_L.

Now we can replace voltage source by equivalent current source.

Key Point : *The two sources are said to be* ***equivalent****, if they supply equal load current to the load, with same load connected across its terminals*

The current delivered in above case by voltage source is,

$$I = \frac{V}{(R_{se} + R_L)}, \quad R_{se} \text{ and } R_L \text{ in series} \quad \ldots(1)$$

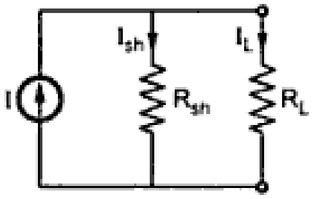


Fig. 1.33 (b) Current source

If it is to be replaced by a current source then load current must be $\frac{V}{(R_{se} + R_L)}$

Consider an equivalent current source shown in the Fig. 1.33 (b).

The total current is ' I '.

Both the resistances will take current proportional to their values.

From the current division in parallel circuit we can write,

$$I_L = I \times \frac{R_{sh}}{(R_{sh} + R_L)} \qquad ...(2)$$

Now this I_L and $\frac{V}{R_{se} + R_L}$ must be same, so equating (1) and (2),

$$\therefore \qquad \frac{V}{R_{se} + R_L} = \frac{I \times R_{sh}}{R_{sh} + R_L}$$

Let internal resistance be, $R_{se} = R_{sh} = R$ say.

Then, $V = I \times R_{sh} = I \times R$

or $I = \frac{V}{R_{sh}}$

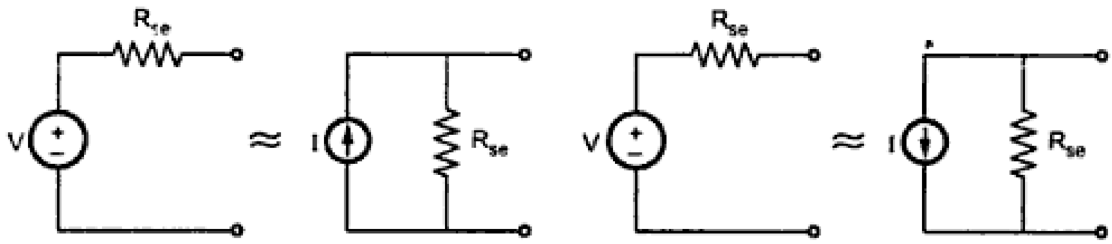
$$\therefore \qquad \boxed{I = \frac{V}{R} = \frac{V}{R_{se}}}$$

Key Point : *If voltage source is converted to current source, then current source* $I = \frac{V}{R_{se}}$ *with parallel internal resistance equal to* R_{se}.

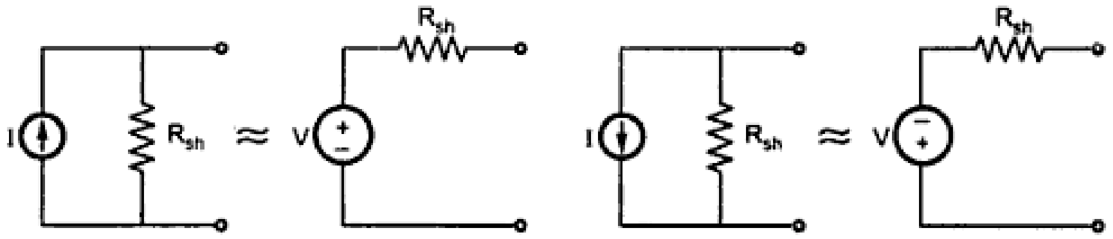
Key Point : *If current source is converted to voltage source, then voltage source* $V = I\,R_{sh}$ *with series internal resistance equal to* R_{sh}.

The direction of current of equivalent current source is always from – ve to + ve, **internal to the source.** While converting current source to voltage source, polarities of voltage is always as +ve terminal at top of arrow and –ve terminal at bottom of arrow, as direction of current is from –ve to +ve, internal to the source. **This ensures that current flows from positive to negative terminal in the external circuit.**

Note the directions of transformed sources, shown in the Fig. 1.34 (a), (b), (c) and (d).



(a) $I = \frac{V}{R_{se}}$ (b) $I = \frac{V}{R_{se}}$

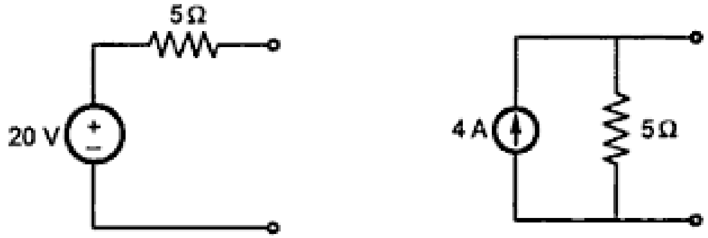


(c) $V = I \times R_{sh}$ (d) $V = I \times R_{sh}$

Fig. 1.34 Source transformation

Example 1.6 : *Transform a voltage source of 20 volts with an internal resistance of 5 Ω to a current source.*

Solution: Refer to the Fig. 1.35 (a).



(a) (b)

Fig. 1.35

Then current of current source is, $I = \frac{V}{R_{se}} = \frac{20}{5} = 4$ A with internal parallel resistance same as R_{se}.

∴ Equivalent current source is as shown in the Fig. 1.35 (b).

Example 1.7 : *Convert into a voltage source.* **[April/May-2004]**

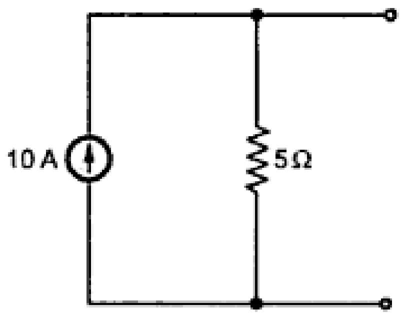


Fig. 1.36

Solution :

$$I = 10\text{ A}, \quad R = 5\ \Omega$$

$$\therefore \quad V = I \times R = 10 \times 5 = 50\text{ V}$$

The polarities are positive at top and negative at bottom of the arrow. Thus the voltage source is as shown in the Fig. 1.37.

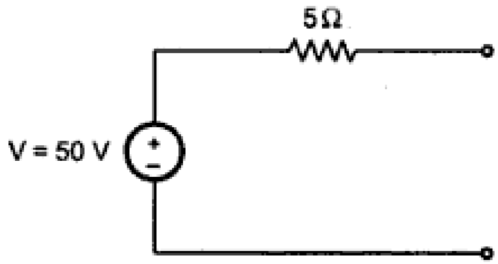


Fig. 1.37

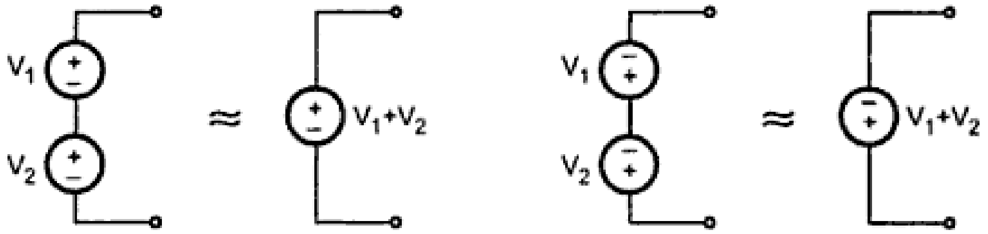
1.13 Combinations of Sources

In a network consisting of many sources, series and parallel combinations of sources exist. If such combinations are replaced by the equivalent source then the network simplification becomes much more easy. Let us consider such series and parallel combinations of energy sources.

1.13.1 Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources.

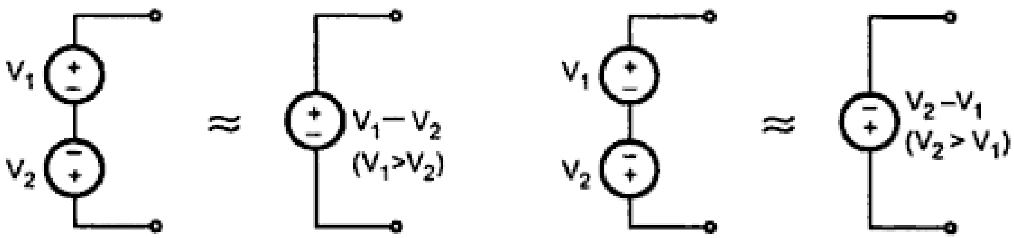
Consider the two sources as shown in the Fig. 1.38.



(a) **Fig. 1.38** (b)

Thus if the polarities of the two sources are same then the equivalent single source is the addition of the two sources with polarities same as that of the two sources.

Consider the two sources as shown in the Fig. 1.39.



(a) **Fig. 1.39** (b)

Thus if the polarities of the two sources are different then the equivalent single source is the difference between the two voltage sources. The polarities of such source is same as that of the greater of the two sources.

Key Point : *The voltage sources to be connected in series must have same current ratings though their voltage ratings may be same or different.*

The technique can be used to reduce the series combination of more than two voltage sources connected in series.

1.13.2 Voltage Sources in Parallel

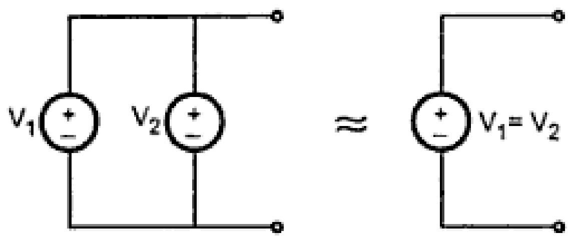


Fig. 1.40

Consider the two voltage sources in parallel as shown in the Fig. 1.40.

The equivalent single source has a value same as V_1 and V_2.

It must be noted that at the terminals open circuit voltage provided by each source must be equal as the sources are in parallel.

Key Point : *Hence the voltage sources to be connected in parallel must have same voltage ratings though their current rating may be same or different.*

1.13.3 Current Sources in Series

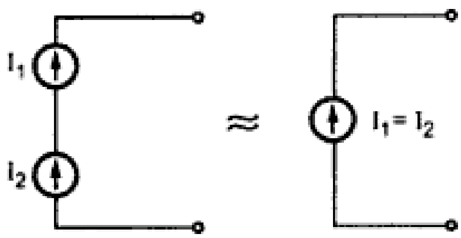


Fig. 1.41

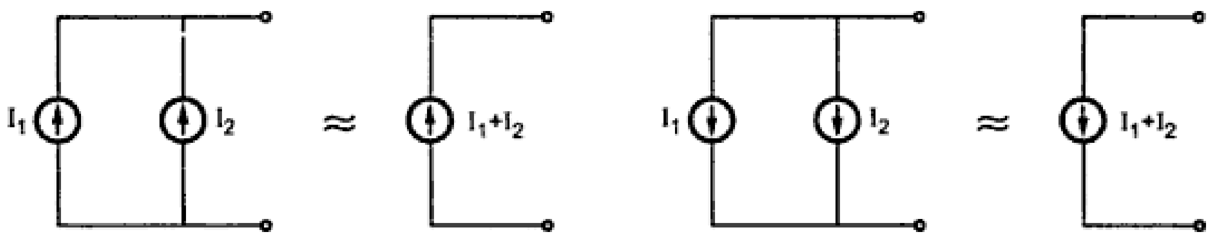
Consider the two current sources in series as shown in the Fig. 1.41.

The equivalent single source has a value same as I_1 and I_2.

Key Point : *The current through series circuit is always same hence it must be noted that the current sources to be connected in series must have same current ratings though their voltage ratings may be same or different.*

1.13.4 Current Sources in Parallel

Consider the two current sources in parallel as shown in the Fig. 1.42.



(a) Fig. 1.42 (b)

Thus if the directions of the currents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources.

Consider the two current sources with opposite directions connected in parallel as shown in the Fig. 1.43.

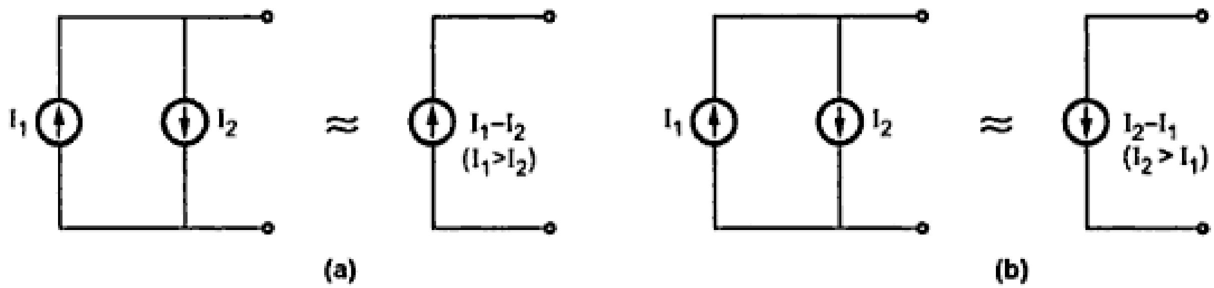


Fig. 1.43

Thus if the directions of the two sources are different then the equivalent single source has a direction same as greater of the two sources with a value equal to the difference between the two sources.

Key Point : *The current sources to be connected in parallel must have same voltage ratings though their current ratings may be same or different.*

1.14 Kirchhoff's Laws

In 1847, a German physicist, **Kirchhoff,** formulated two fundamental laws of electricity. These laws are of tremendous importance from network simplification point of view.

1.14.1 Kirchhoff's Current Law (KCL)

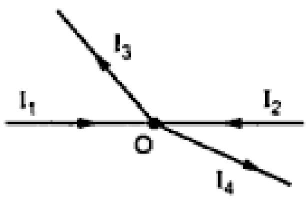


Fig. 1.44 Junction point

Consider a junction point in a complex network as shown in the Fig. 1.44.

At this junction point if $I_1 = 2$ A, $I_2 = 4$ A and $I_3 = 1$ A then to determine I_4 we write, total current entering is $2 + 4 = 6$ A while total current leaving is $1 + I_4$ A

And hence, $I_4 = 5$ A.

This analysis of currents entering and leaving is nothing but the application of Kirchhoff's Current Law. The law can be stated as,

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

The word algebraic means considering the signs of various currents.

$$\boxed{\sum I \quad \text{at junction point} = 0}$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

e.g. Refer to Fig. 1.44, currents I_1 and I_2 are positive while I_3 and I_4 are negative.

Applying KCL, $\sum I$ at junction O $= 0$

$$I_1 + I_2 - I_3 - I_4 = 0 \text{ i.e. } I_1 + I_2 = I_3 + I_4$$

The law is very helpful in network simplification.

1.14.2 Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.fs in the path"

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\boxed{\text{Around a closed path } \sum V = 0}$$

(a) Fig. 1.45 (b)

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

This law is very useful in the loop analysis of the network.

1.14.3 Sign Conventions to be Followed while Applying KVL

When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.

In the Fig. 1.45 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.

In the Fig. 1.45 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Once all such polarities are marked in the given circuit, we can apply KVL to any closed path in the circuit.

Now while tracing a closed path, if we go from − ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise.**

For example, if the branch AB is traced from A to B then the drop across it must be considered as rise and must be taken as + I R while writing the equations.

While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop**.

For example, in the Fig. 1.45 (a) only, if the branch is traced from B to A then it should be taken as negative, as – I R while writing the equations.

Similarly in the Fig. 1.45 (b), if branch is traced from A to B then there is a voltage drop and term must be written negative as – I R while writing the equation. If the branch is traced from B to A, it becomes a rise in voltage and term must be written positive as + I R while writing the equation.

Key Point :

1) ***Potential rise*** *i.e. travelling from negative to positively marked terminal, must be considered as* ***Positive.***
2) ***Potential drop*** *i.e. travelling from positive to negatively marked terminal, must be considered as* ***Negative.***
3) *While tracing a closed path, select any one direction clockwise or anticlockwise. This selection is totally independent of the directions of currents and voltages of various branches of that closed path.*

1.14.4 Application of KVL to a Closed Path

Consider a closed path of a complex network with various branch currents assumed as shown in the Fig. 1.46 (a).

As the loop is assumed to be a part of complex network, the branch currents are assumed to be different from each other.

Due to these currents the various voltage drops taken place across various resistances are marked as shown in the Fig. 1.46 (b).

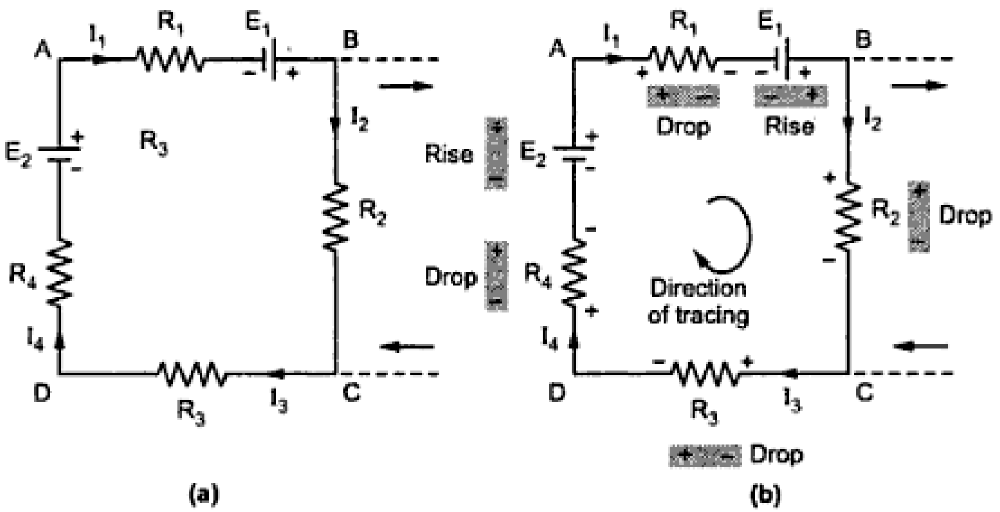


Fig. 1.46 (a), (b) Closed loop of a complex network

The polarity of voltage drop along the current direction is to be marked as positive (+) to negative (–).

Let us trace this closed path in clockwise direction i.e. **A-B-C-D-A.**

Across R_1 there is voltage drop $I_1 R_1$ and as getting traced from +ve to –ve, it is drop and must be taken as negative while applying KVL.

Battery E_1 is getting traced from negative to positive i.e. it is a rise hence must be considered as positive.

Across R_2 there is a voltage drop $I_2 R_2$ and as getting traced from +ve to –ve, it is drop and must be taken negative.

Across R_3 there is a drop $I_3 R_3$ and as getting traced from +ve to –ve, it is drop and must be taken as negative.

Across R_4 there is drop $I_4 R_4$ and as getting traced from +ve to –ve, it is drop must be taken as negative.

Battery E_2 is getting traced from –ve to +ve, it is rise and must be taken as positive.

∴ We can write an equation by using KVL around this closed path as,

$-I_1 R_1 + E_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 + E_2 = 0$ ... Required KVL equation.

i.e. $E_1 + E_2 = I_1 R_1 + I_2 R_2 + I_3 R_3 + I_4 R_4$

If we trace the closed loop in opposite direction i.e. along **A-D-C-B-A** and follow the same sign convention, the resulting equation will be same as what we have obtained above.

Key Point : *So while applying KVL, direction in which loop is to be traced is not important but following the sign convention is most important.*

The same sign convention is followed in this book to solve the problems.

1.14.5 Steps to Apply Kirchhoff's Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1 : Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

Step 2 : Mark all the branch currents with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

Assumed directions may be wrong, in such case answer of such current will be mathematically negative which indicates the correct direction of the current. A particular current leaving a particular source has some magnitude, then same magnitude of current should enter that source after travelling through various branches of the network.

Step 3 : Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

Step 4 : Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any previous equation.

> **Key Point** : *KVL must be applied to sufficient number of loops such that each element of the network is included atleast once in any of the equations.*

Step 5 : Solve the simultaneous equations for the unknown currents. From these currents unknown voltages and power consumption in different resistances can be calculated.

What to do if current source exists ?

> **Key Point** : *If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, the loops should not be considered involving current source. The loop equations must be written to those loops which do not include any current source. This is because drop across current source is unknown.*

For example, consider the circuit shown in the Fig. 1.47. The current distribution is completed interms of current source value. Then KVL must be applied to the loop bcdeb, which does not include current source. **The loop abefa should not be used for KVL application**, as it includes current source. Its effect is already considered at the time of current distribution.

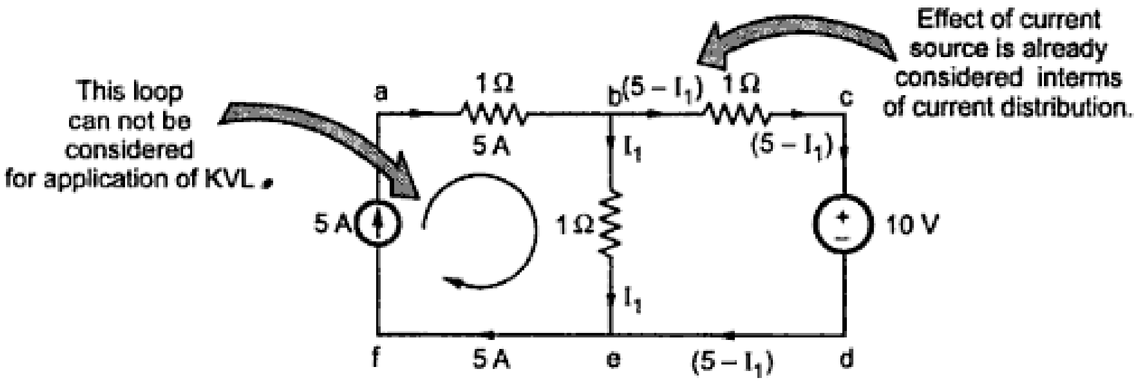


Fig. 1.47

1.15 Cramer's Rule

If the network is complex, the number of equations i.e. unknowns increases. In such case, the solution of simultaneous equations can be obtained by **Cramer's Rule** for determinants.

Let us assume that set of simultaneous equations obtained is, as follows :

$$\therefore \quad \boxed{\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n &= C_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n &= C_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots + a_{nn}x_n &= C_n \end{aligned}}$$

Where $C_1, C_2, \ldots\ldots\ldots\ldots C_n$ are constants.

Then Cramer's rule says that form a system determinant Δ or D as,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D$$

Then obtain the subdeterminants D_j by replacing j^{th} column of Δ by the column of constants existing on right hand side of equations i.e. $C_1, C_2, \ldots C_n$;

$$D_1 = \begin{vmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_2 & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad D_2 = \begin{vmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{vmatrix}$$

and
$$D_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ \vdots & & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & C_n \end{vmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D}, \; X_2 = \frac{D_2}{D}, \ldots\ldots\ldots\ldots, X_n = \frac{D_n}{D}$$

Where $D_1, D_2, \ldots, D_n$ and D are values of the respective determinants.

Example 1.8 : *Apply Kirchhoff's current law and voltage law to the circuit shown in the Fig. 1.48.*

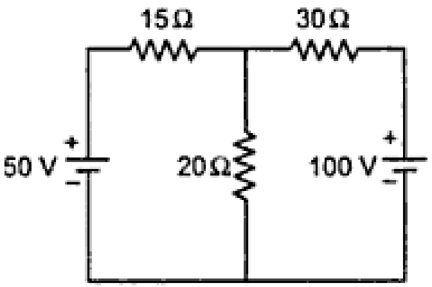


Fig. 1.48

Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative ?

If yes, explain the significance of the negative sign.

Solution : Application of Kirchhoff's law :

Step 1 and 2 : Draw the circuit with all the values which are same as the given network. Mark all the branch currents starting from +ve of any of the source, say +ve of 50 V source.

Step 3 : Mark all the polarities for different voltages across the resistances. This is combined with step 2 shown in the network below in Fig. 1.48 (a).

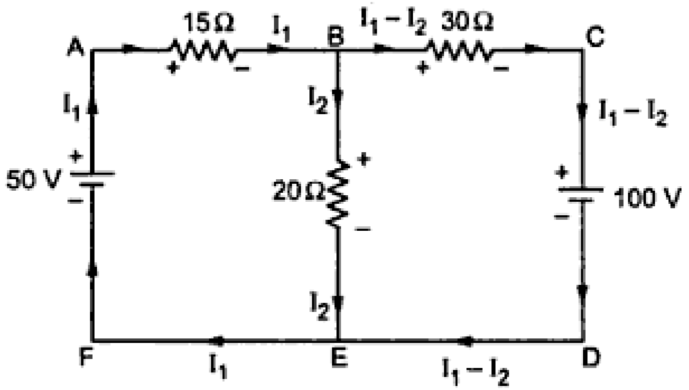


Fig. 1.48 (a)

Step 4 : Apply KVL to different loops.

Loop 1 : A-B-E-F-A.,

$$-15\, I_1 - 20\, I_2 + 50 = 0 \qquad \text{... (1)}$$

Loop 2 : B-C-D-E-D,

$$-30\,(I_1 - I_2) - 100 + 20\, I_2 = 0 \qquad \text{... (2)}$$

Rewriting all the equations, taking constants on one side.

$$15\, I_1 + 20\, I_2 = 50 \quad \text{...(1)} \quad \text{and} \quad -30\, I_1 + 50\, I_2 = 100 \quad \text{...(3)}$$

Apply Cramer's rule, $D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$

Calculating D_1, $D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500$

$$I_1 = \frac{D_1}{D} = \frac{500}{1350} = \mathbf{0.37\ A}$$

Calculating D_2, $D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000$

$$I_2 = \frac{D_2}{D} = \frac{3000}{1350} = \mathbf{2.22\ A}$$

For I_1 and I_2, as answer is positive, assumed direction is correct.

$\therefore$ For I_1 answer is 0.37 A. For I_2 answer is 2.22 A.

$$I_1 - I_2 = 0.37 - 2.22$$

$$= -\mathbf{1.85\ A}$$

Negative sign indicates assumed direction is wrong.

i.e. $I_1 - I_2 = \mathbf{1.85\ A}$ flowing in opposite direction to that of the assumed direction.

1.16 A.C. Fundamentals

Let us discuss in brief, the fundamentals of alternating circuits consisting of various alternating current and voltage sources, resistances alongwith inductive and capacitive reactances.

An alternating quantity is the one which changes periodically both in magnitude and direction. Practically a purely sinusoidal waveform is accepted as a standard alternating waveform for alternating voltages and currents, due to its advantages.

Let us consider the waveform of an alternating quantity which is purely sinusoidal as shown in the Fig. 1.49.

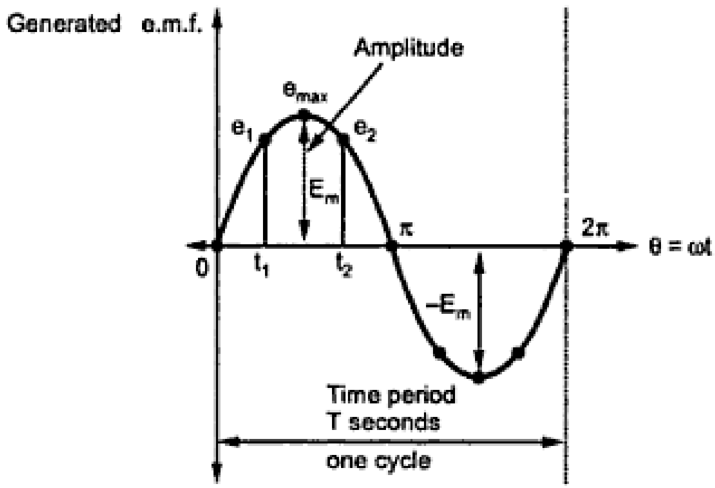


Fig. 1.49 Waveform of an alternating e.m.f.

1.16.1 Instantaneous Value

The value of an alternating quantity at a particular instant is known as its **instantaneous value.** e.g. e_1 and e_2 are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. 1.49.

1.16.2 Waveform

The graph of instantaneous values of an alternating quantity plotted against time in called its **waveform.**

1.16.3 Cycle

Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**

A **cycle** can also be defined as that interval of time during which a complete set of non-repeating events or wave form variations occur (containing positive as well as negative loops). One such cycle of the alternating quantity is shown in the Fig. 1.49.

Key Point: *One cycle corresponds to 2π radians or $360°$.*

1.16.4 Time Period (T)

The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds. After every T seconds, the cycle of an alternating quantity repeats. This is shown in the Fig. 1.49.

1.16.5 Frequency (f)

The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by f and it is measured in **cycles / second** which is known as **Hertz,** denoted as **Hz.** As time period T is time for one cycle i.e. seconds / cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

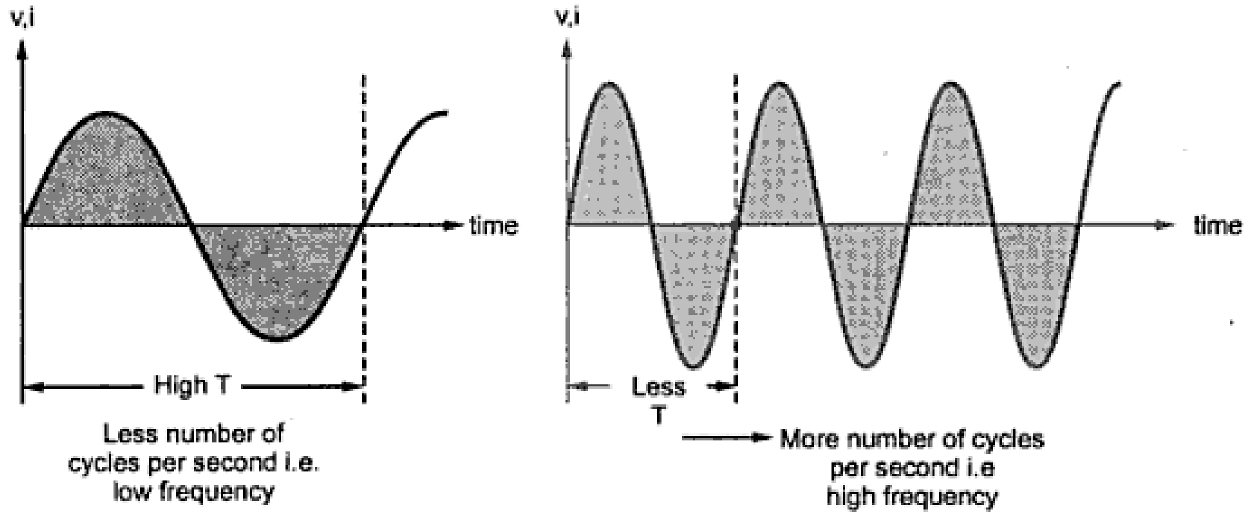


Fig. 1.50 Relation between T and f

$$\boxed{f = \frac{1}{T} \quad \text{Hz}}$$

As time period increases, frequency decreases while as time period decreases, frequency increases. This is shown in the Fig. 1.50.

In our nation, standard frequency of alternating voltages and currents is 50 Hz.

1.16.6 Amplitude

The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

Thus E_m is called peak value of the voltage while I_m is called peak value of the current.

1.16.7 Angular Frequency (ω)

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2 π radians, the angular frequency can be expressed as (2 π × cycles/sec.) It is denoted by ' ω ' and its unit is radians/second. Now, cycles/ sec. means frequency. Hence the relation between frequency 'f' and angular frequency ' ω ' is,

$$\boxed{\omega = 2\pi f \quad \text{radians/sec.} \quad \text{or} \quad \omega = \frac{2\pi}{T} \quad \text{radians/sec.}}$$

1.16.8 Equation of an Alternating Quantity

As alternating quantity is sinusoidal in nature, its equation is expressed using sin θ where θ is angle expressed in radians. Hence alternating voltage is expressed as,

$$\boxed{e = E_m \sin\theta}$$

While alternating current is expressed as,

$$\boxed{i = I_m \sin\theta}$$

This equation gives instantaneous values of an alternating quantity, at any time t.

Now $\theta = \omega t$ in radians

Thus various forms of the equation of an alternating quantity are,

$$e = E_m \sin(\omega t) = E_m \sin(2\pi f t) = E_m \sin\left(\frac{2\pi}{T} t\right)$$

and

$$i = I_m \sin(\omega t) = I_m \sin(2\pi f t) = I_m \sin\left(\frac{2\pi}{T} t\right)$$

Important Note : *In all the above equations, the angle θ is expressed in* ***radians****. Hence, while calculating the instantaneous value of the e.m.f., it is necessary to calculate the sine of the angle expressed in radians.*

Key Point: *Mode of the calculator should be converted to radians, to calculate the sine of the angle expressed in radians, before substituting in any of the above equations.*

In practice the alternating quantities are expressed interms of their r.m.s. values. The relation between r.m.s value and the maximum value is,

$$\boxed{V_{r.m.s} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{r.m.s} = \frac{I_m}{\sqrt{2}}}$$

The r.m.s values are denoted by the capital letters as V or I.

1.17 Phasor Representation of an Alternating Quantity

In the analysis of a.c. circuits, it is very difficult to deal with alternating quantities in terms of their waveforms and mathematical equations. The job of adding, subtracting, etc. of the two alternating quantities is tedious and time consuming in terms of their mathematical equations. Hence, it is necessary to study a method which gives an easier way of representing an alternating quantity. Such a representation is called **phasor representation** of an alternating quantity.

The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in this method. The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a **phasor.**

Key Point: *The phasors are assumed to be rotated in anticlockwise direction.*

One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.

Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. 1.51. If the projections of this phasor on

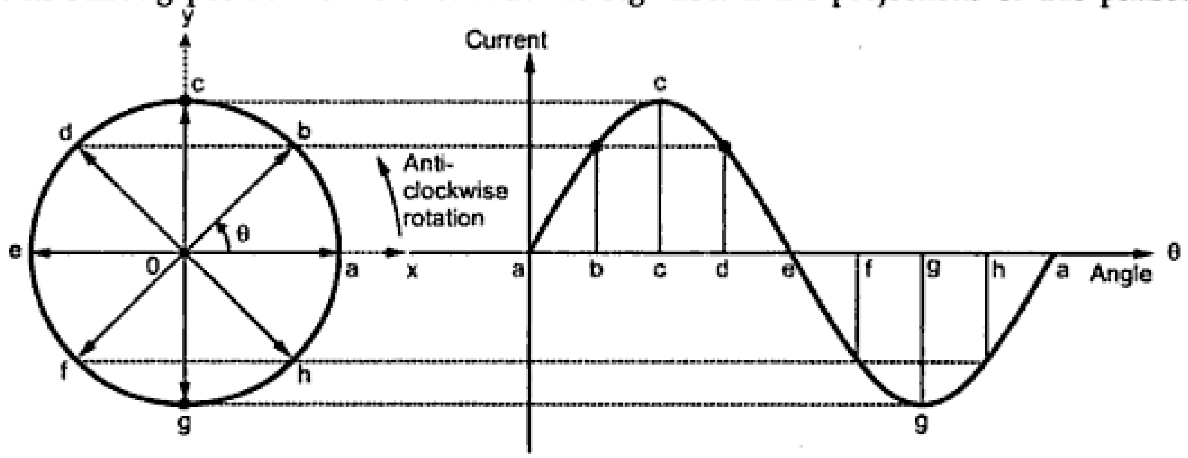


Fig. 1.51 Phasor representation of an alternating quantity

Y-axis are plotted against the angle turned through ' θ ', (or time as $\theta = \omega t$), we get a sine waveform.

Consider the various positions shown in the Fig. 1.51.

1. At point 'a', the Y-axis projection is zero. The instantaneous value of the current is also zero.
2. At point 'b', the Y-axis projection is [l (ob) sin θ)]. The length of the phasor is equal to the maximum value of an alternating quantity. So, instantaneous value of the current at this position is $I = I_m \sin \theta$, represented in the waveform.
3. At point 'c', the Y-axis projection 'oc' represents entire length of the phasor i.e. instantaneous value equal to the maximum value of current I_m.
4. Similarly, at point d, the Y-axis projection becomes $I_m \sin \theta$ which is the instantaneous value of the current at that instant.
5. At point 'e', the Y-axis projection is zero and instantaneous value of the current is zero at this instant.
6. Similarly, at points f, g, h the Y-axis projections give us instantaneous values of the current at the respective instants and when plotted, give us negative half cycle of the alternating quantity.

Thus, if the length of the phasor is taken equal to the maximum value of the alternating quantity, then its rotation in space at any instant is such that the length of its projection on the Y–axis gives the instantaneous value of the alternating quantity at that particular instant. The angular velocity 'ω ' in an anticlockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle i.e. $\theta = \omega t$,

Where $\omega = 2 \pi f$ rad/sec

Points to Remember :

In practice, the alternating quantities are represented by their r.m.s. values. Hence, the length of the phasor represents r.m.s. value of the alternating quantity. In such case, projection on Y-axis does not give directly the instantaneous value but as $I_m = \sqrt{2}\, I_{r.m.s.}$, the projection on Y-axis must be multiplied by $\sqrt{2}$ to get an instantaneous value of that alternating quantity.

Phasors are always assumed to be rotated in **anticlockwise** direction.

Two alternating quantities of same frequencies can be represented on same phasor diagram.

Key Point: *If frequencies of the two quantities are different, then such quantities cannot be represented on the same phasor diagram.*

1.18 Concept of Phase of an Alternating Quantity

In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant. It is represented in terms of angle θ in radians or degrees, measured from certain reference. Thus, phase can be defined as,

Phase : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

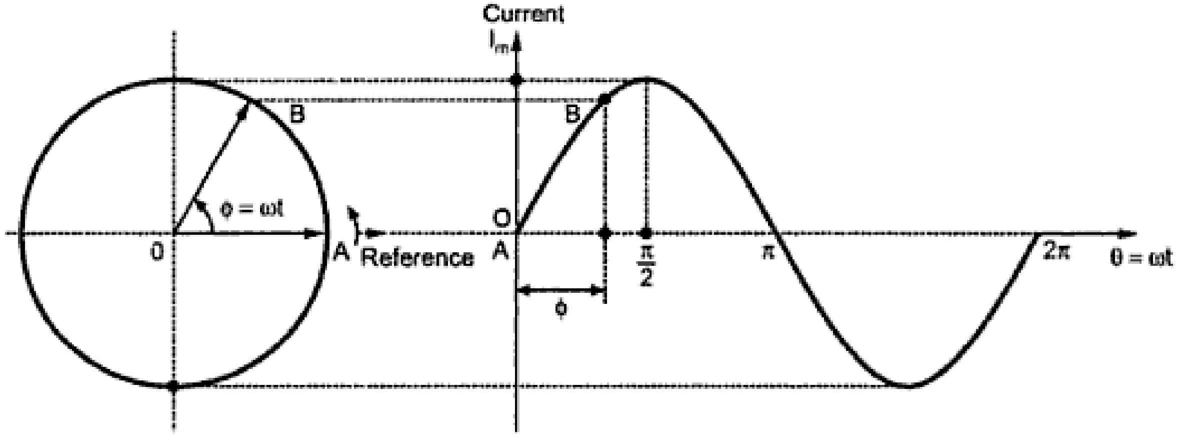


Fig. 1.52 Concept of phase

Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. 1.53 at the instant A is $\phi = 0°$. While the phase of the current at the instant B is the angle ϕ through which the phasor has travelled, measured from the reference axis i.e. X-axis.

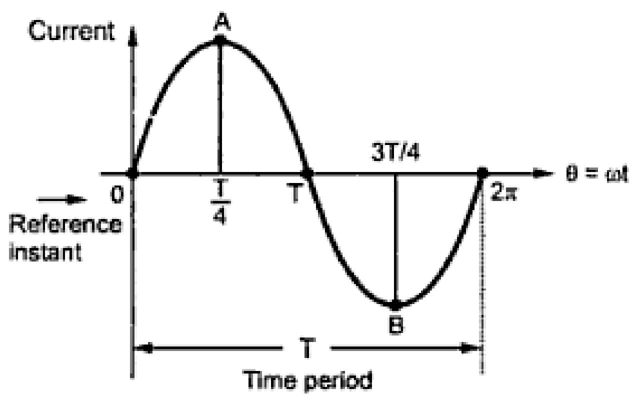


Fig. 1.53

In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0°$ to $360°$.

Another way of defining the phase is in terms of a time period T. The phase of an alternating quantity at any particular instant is the fraction of the time period (T) through which the quantity is advanced from the reference instant.

Consider alternating quantity represented in the Fig. 1.53. As per above definition, the phase of quantity at instant A is $\frac{T}{4}$, while phase at instant B is $\frac{3T}{4}$. Generally, the phase is expressed in terms of angle ϕ which varies from 0 to 2 π radians and measured with respect to positive x-axis direction.

In terms of phase the equation of alternating quantity can be modified as,

$$\boxed{e = E_m \sin(\omega t \pm \phi)}$$

Where ϕ = Phase of the alternating quantity.

Let us consider three cases;

Case 1 : $\phi = 0°$

When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig. 1.54 (a).

Case 2 : Positive phase ϕ

When phase of an alternating quantity is positive it means that quantity has some positive instantaneous value at t = 0. This is shown in the Fig. 1.54 (b).

Case 3 : Negative phase ϕ

When phase of an alternating quantity is negative it means that quantity has some negative instantaneous value at t = 0. This is shown in the Fig. 1.54 (c).

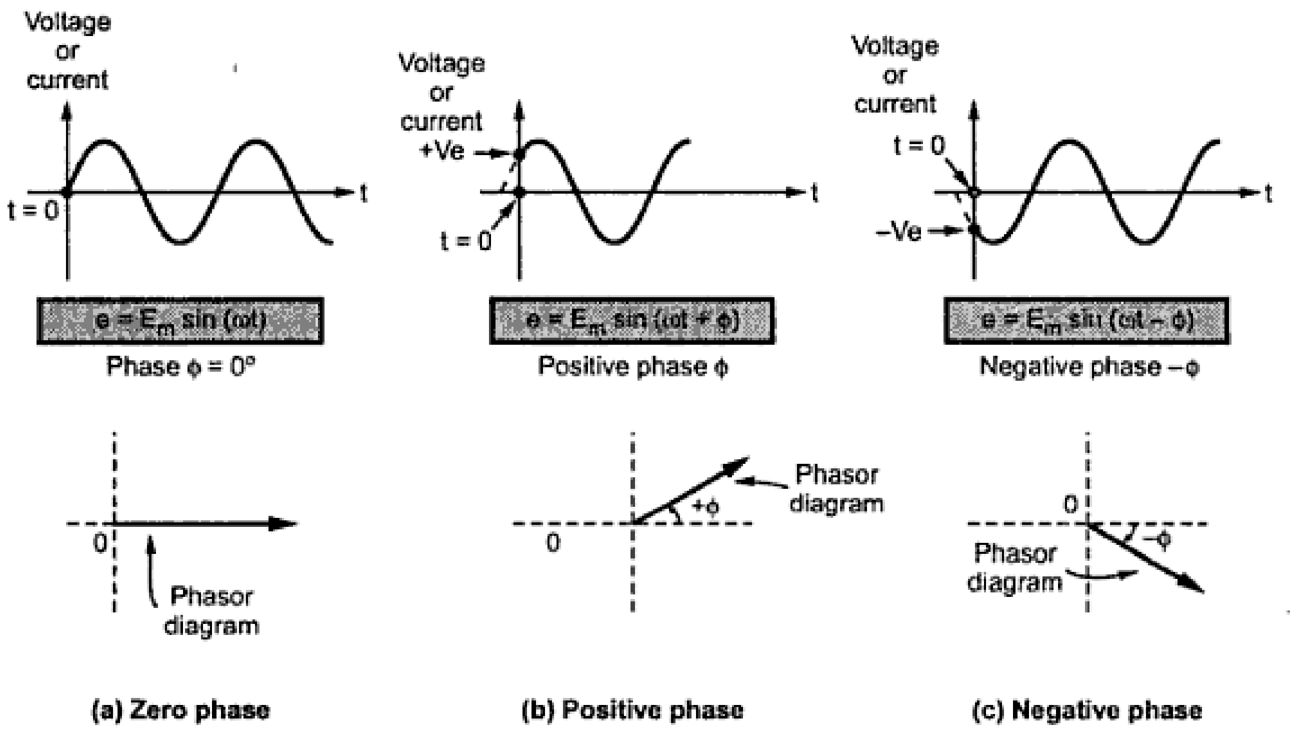


Fig. 1.54 Concept of phase

1. The **phase** is measured with respect to **reference direction** i.e. positive x axis direction.
2. The **phase** measured in **anticlockwise** direction is **positive** while the **phase** measured in **clockwise** direction is **negative.**

1.18.1 Phase Difference

Consider the two alternating quantities having same frequency f Hz having different maximum values.

$$e = E_m \sin(\omega t)$$

and $$i = I_m \sin(\omega t)$$

where $$E_m > I_m$$

The phasor representation and waveforms of both the quantities are shown in the Fig. 1.55.

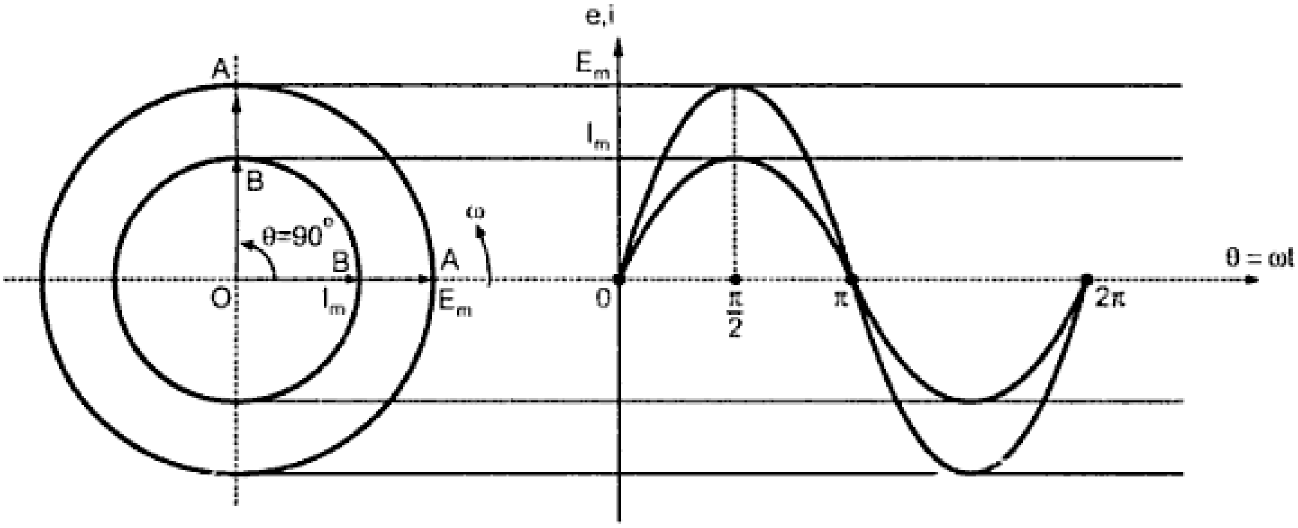


Fig. 1.55

The phasors $OA = E_m$

and $OB = I_m$

After $\theta = \frac{\pi}{2}$ radians, the OA phasor achieves its maximum E_m while at the same instant, the OB phasor achieves its maximum I_m. As the frequency of both is same, the angular velocity ω of both is also the same. So, they rotate together in synchronism.

So, at any instant, we can say that the phase of voltage e will be same as phase of i. Thus, the angle travelled by both within a particular time is always the same. So, the difference between the phases of the two quantities is zero at any instant. **The difference between the phases of the two alternating quantities is called the phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.

Key Point : *When such **phase difference** between the two alternating quantities is **zero**, the two quantities are said to be **in phase**.*

The two alternating quantities having same frequency, reaching maximum positive and negative values and zero values at the same time are said to be in phase. **Their amplitudes may be different.**

In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value, at the same instant, the other is having some negative value or positive value.

Such two quantities are said to have phase difference between them. If there is difference between the phases (angles) of the two quantities, expressed in degrees or radians at any particular instant, then as both rotate with same speed, this difference remains same at all the instants.

Consider an e.m.f. having maximum value E_m and current having maximum value I_m. Now, when e.m.f. 'e' is at its zero value, the current 'i' has some negative value as shown in the Fig. 1.56.

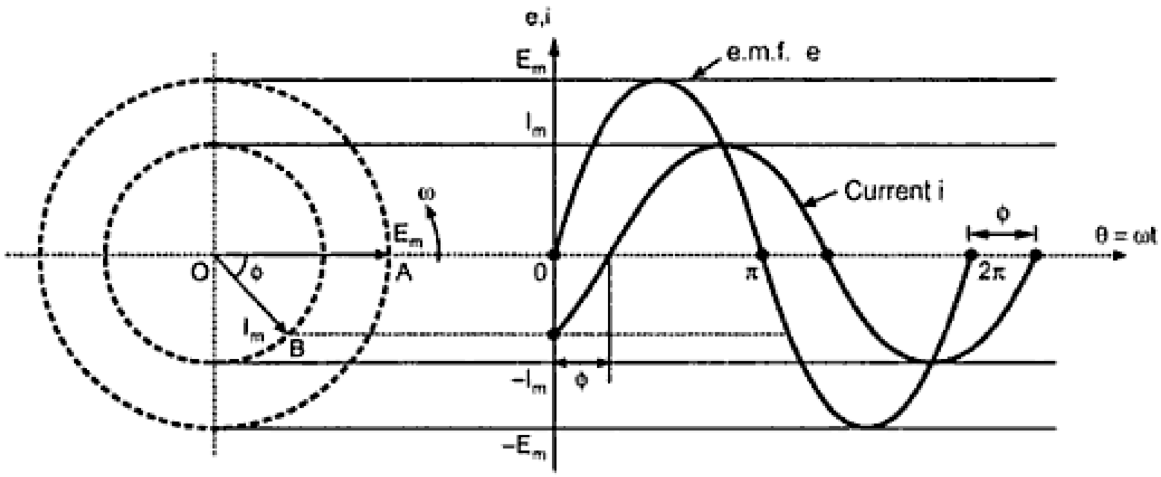


Fig. 1.56 Concept of phase difference (Lag)

Thus, there exists a phase difference ϕ between the two phasors. Now, as the two are rotating in anticlockwise direction, we can say that current is falling back with respect to voltage, at all the times by angle ϕ. This is called **lagging phase difference.** The current i is said to lag the voltage e by angle ϕ. The current i achieves its maxima, zero values ϕ angle later than the corresponding maximum, zero values of voltage.

The equations of the two quantities are written as,

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin (\omega t - \phi)$$

'i' is said to lag 'e' by angle ϕ.

It is possible in practice that the current 'i' may have some positive value when voltage 'e' is zero. This is shown in the Fig. 1.57.

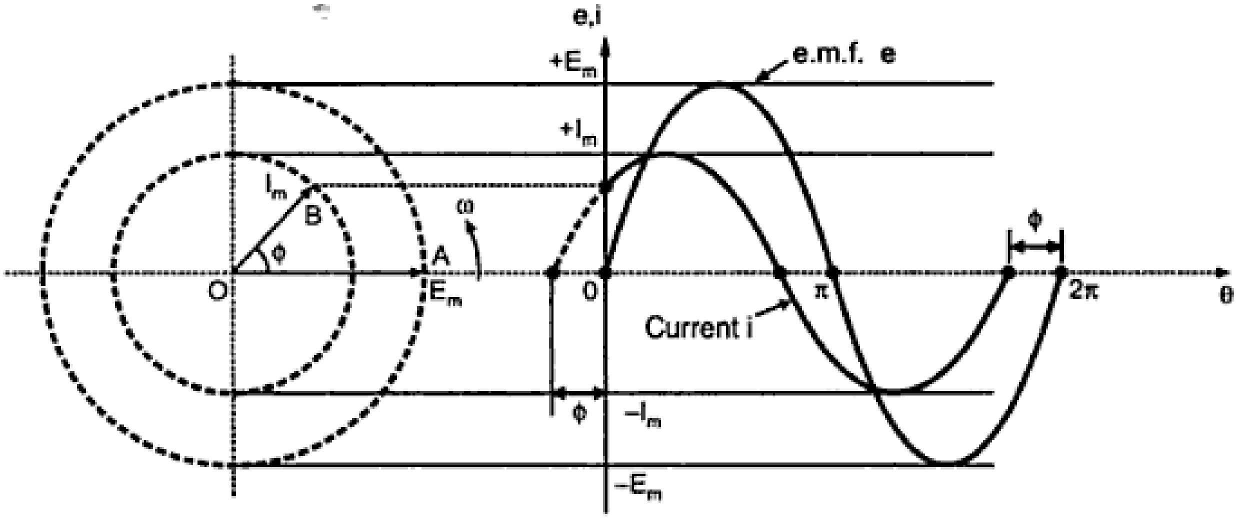


Fig. 1.57 Concept of phase difference (Lead)

It can be seen that there exists a phase difference of ϕ angle between the two. But in this case, current 'i' is ahead of voltage 'e', as both are rotating in anticlockwise direction with same speed. Thus, current is said to be leading with respect to voltage and the phase difference is called **leading phase difference.** The current i achieves its maximum, zero values ϕ angle before than the corresponding maximum, zero values of the voltage. At all instants, current i is going to remain ahead of voltage 'e' by angle 'ϕ'.

The equations of such two quantities are written as

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin (\omega t + \phi)$$

'i' is said to lead 'e' by angle ϕ.

Key Point : *Thus, related to the phase difference, it can be remembered that a plus (+) sign of angle indicates lead where as a minus (–) sign of angle indicates lag with respect to the reference.*

1.18.2 Phasor Diagram

The diagram in which different alternating quantities of the same frequency, sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as **phasor diagram.**

The phasors are rotating in anticlockwise direction with an angular velocity of $\omega = 2\pi f$ rad/sec. Hence, all phasors have a particular fixed position with respect to each other.

Key Point : *Hence, phasor diagram can be considered as a still picture of these phasors at a particular instant.*

To clear this point, consider two alternating quantities in phase with each other.

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin \omega t$$

At any instant, phase difference between them is zero i.e. angle difference between the two phasors is zero. Hence, the phasor diagram for such case drawn at different instants will be alike giving us the same information that two quantities are in phase. The phasor diagram drawn at different instants are shown in the Fig. 1.58.

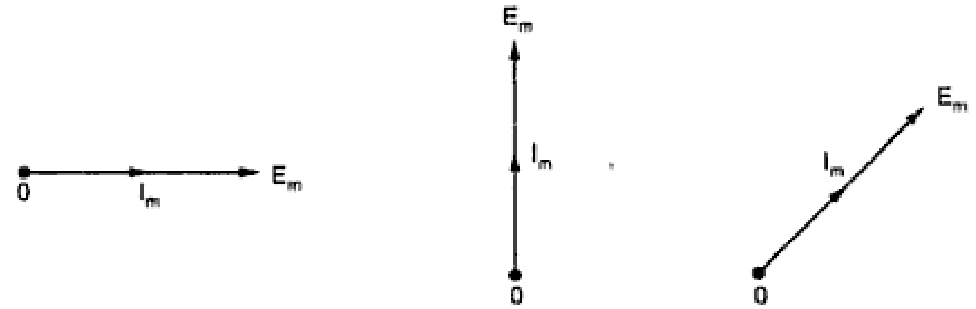


Fig. 1.58 Same phasor diagram at different instants

Consider another example where current i is lagging voltage e by angle ϕ.
So, difference between the angles of the phasors representing the two quantities is angle ϕ.

$$e = E_m \sin \omega t$$

and $$i = I_m \sin (\omega t - \phi)$$

The phasor diagram for such case, at various instants will be same, as shown in the Fig. 1.59 (a), (b) and (c).

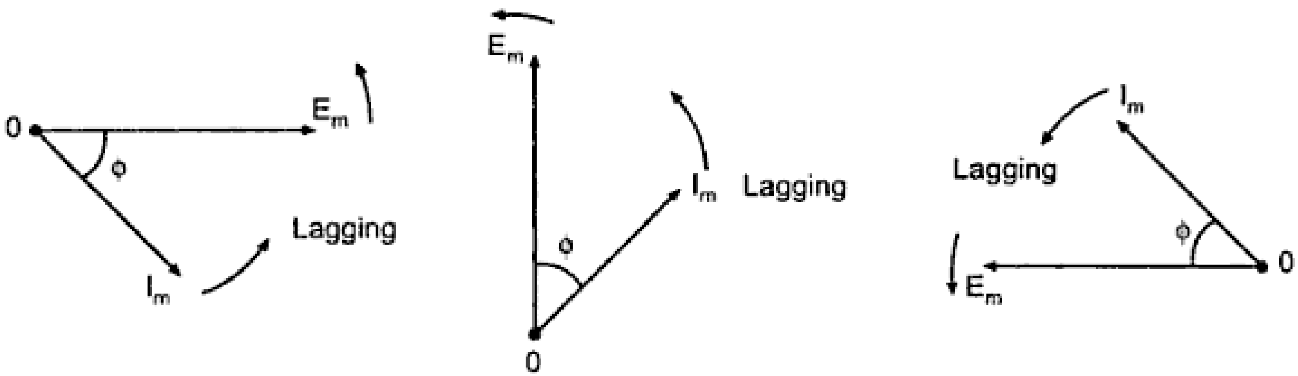


Fig. 1.59

The phasor diagram drawn at any instant gives the same information.

Key Point : *Remember that the lagging and leading word is relative to the reference. In the above case, if we take current as reference, we have to say that the voltage leads current by angle ϕ. The direction of rotation of phasors is always anticlockwise.*

Important Points Regarding Phasor Diagram :

1) As phasor diagram can be drawn at any instant, X and Y axis are not included in it. But, generally, the reference phasor chosen is shown along the positive X axis direction and at that instant other phasors are shown. This is just from convenience point of view. **The individual phase of an alternating quantity is always referred with respect to the positive x-axis direction.**

2) There may be more than two quantities represented in phasor diagram. Some of them may be current and some may be voltages or any other alternating quantities like flux, etc. The frequency of all of them must be the same.

3) Generally, length of phasor is drawn equal to r.m.s. value of an alternating quantity, rather than maximum value.

4) The phasors which are ahead, in anticlockwise direction, with respect to reference phasor are said to be leading with respect to reference and phasors behind are said to be lagging.

5) Different arrow heads may be used to differentiate phasors drawn for different alternating quantities like current, voltage, flux, etc.

Example 1.9 : *Two sinusoidal currents are given by,*

$i_1 = 10 \sin(\omega t + \pi/3)$ *and*

$i_2 = 15 \sin(\omega t - \pi/4)$

Calculate the phase difference between them in degrees.

Solution : The phase of current i_1 is $\pi/3$ radians i.e. 60° while the phase of the current i_2 is $-\pi/4$ radians i.e. $-45°$. This is shown in the Fig. 1.60.

Hence the phase difference between the two is,

$$\phi = \theta_1 - \theta_2 = 60° - (-45°) = 105°$$

And i_2 lags i_1.

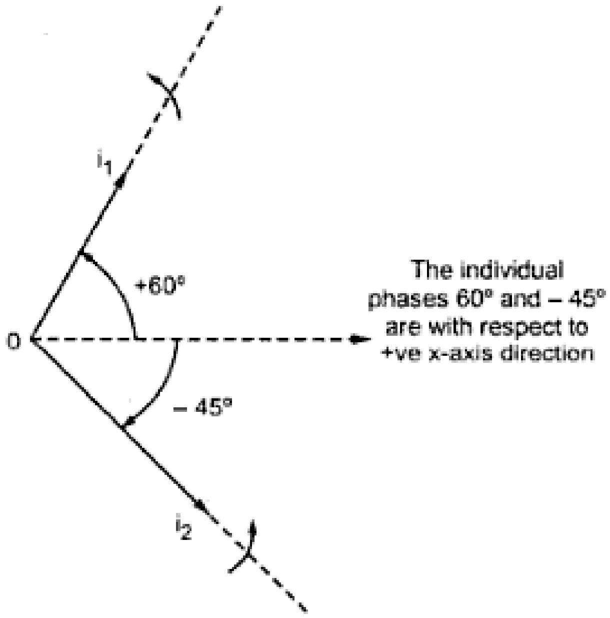


Fig. 1.60

1.19 Mathematical Representation of Phasor

Any phasor can be represented mathematically in two ways,

1) Polar co-ordinate system and 2) Rectangular co-ordinate system

Let $i = I_m \sin(\omega t + \phi)$

The phase of current i is ϕ. The phase is always with respect the x-axis as shown in the Fig. 1.61.

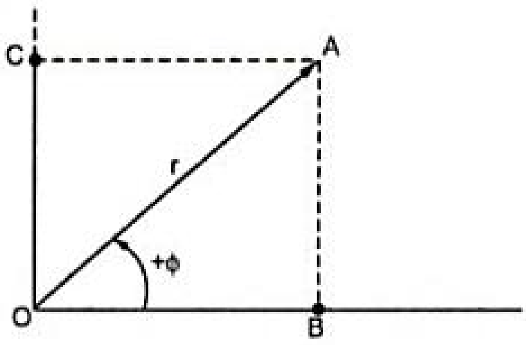


Fig. 1.61

$l(OA) = I_m = r$ and phase = $+\phi$

In polar system, the phasor is represented as $r \angle \pm \phi$. So current i above, is represented as $I_m \angle + \phi$ in polar system.

In rectangular system, the phasor is divided into x and y components i.e. real and imaginary components as $x \pm j\, y$. The current i above is represented as,

$I_m \cos\phi + j\, i_m \sin\phi$ in rectangular system.

Polar system,	$r \angle \pm \phi$ while	Rectangular system, $x \pm j\, y$
and	$x = r\cos\phi$,	$y = r\sin\phi$
while	$r = \sqrt{x^2+y^2}$,	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

...(1)

...(2)

The equations (1) and (2) can be used to convert rectangular form to polar or vice versa.

Such rectangular to polar and polar to rectangular conversion is often required in phasor mathematical operations like addition, subtraction, multiplication and division.

Key Point : *Instead of using above relations, students can use the polar to rectangular ($P \to R$) and rectangular to polar ($R \to P$) functions available on calculator for the required conversions.*

As the graphical method is time consuming which includes plotting the phasors to the scale, generally, analytical method is used. Also, graphical method may give certain error which will vary from person to person depending upon the skills of plotting the phasors. The answer by analytical method is always accurate.

Very Important :

The polar form of an alternating quantity can be easily obtained from its equation or phase as,

$$e = E_m \sin(\omega t \pm \phi) \quad \text{then}$$

$$E \text{ in polar} = E \angle \pm\phi \quad \text{where } E = \text{r.m.s. value}$$

Example 1.10 : *Write the polar form of the voltage given by,*
$V = 100 \sin(100\,\pi\, t + \pi/6)$ V

Obtain its rectangular form.

Solution : $V_m = 100$ V and $\phi = +\frac{\pi}{6}$ rad $= +30°$, $V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 70.7106$ V

$\therefore$ In polar form = $70.7106 \angle +30^\circ$ V

$\therefore$ Rectangular form = $61.2371 + j\ 35.3553$ V

Key Point: *The r.m.s. vlaue of an alternating quantity exists in its polar form and not in rectangular form. Thus to find r.m.s. value of an alternating quantity express it in polar form.*

Example 1.11 : *Find r.m.s. value and phase of the current $I = 25 + j\ 40$ A.*

Solution : The r.m.s. value is not 25 or 40 as it exists in polar form.

Converting it to polar form,

$$I = 47.1699\angle 57.99^\circ \text{ A} = I_{r.m.s.} \angle \phi \text{ A}$$

$\therefore$ r.m.s. value of current = **47.1699 A**

Phase = **57.99°**

Key Point: *To obtain phase, express the equation in sine form if given in cosine as,*

If $e = E_m \cos(\omega t)$

then $e = E_m \sin(\omega t + 90)$ as $\sin(90 + \theta) = \cos\theta$

Thus the phase is 90° and not zero.

In general,	$e =$	$E_m \cos(\omega t \pm \phi)$
then	$e =$	$E_m \sin(\omega t + 90 \pm \phi)$
$\therefore$	The phase =	$90 \pm \phi$

Example 1.12 : *A voltage is defined as $-E_m \cos \omega t$. Express it in polar form.*

Solution : To express a voltage in polar form express it in the form, $e = E_m \sin \omega t$

Now $e = -E_m \cos \omega t = -E_m \sin\left(\omega t + \frac{\pi}{2}\right)$ as $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$

$= E_m \sin\left(\omega t + \frac{3\pi}{2}\right)$ as $\sin(\pi + \theta) = -\sin\theta$

Now it can be expressed in polar form as,

$$e = E_m \angle +\frac{3\pi}{2} \text{ rad} = E_m \angle +270^\circ \text{ V}$$

But + 270° phase is nothing but − 90°

$\therefore$ $e = E_m \angle -90^\circ$ V

1.20 Multiplication and Division of Phasors

In the last section, the addition and subtraction of phasors is discussed, which is to be carried out using rectangular form of phasors. But the rectangular form is not suitable to perform multiplication and division of phasors. Hence multiplication and division **must be performed** using **polar form** of the phasors.

Let P and Q be the two phasors such that,

$$P = x_1 + jy_1 \quad \text{and} \quad Q = x_2 + jy_2$$

To obtain the multiplication $P \times Q$ both must be expressed in polar form

$$\therefore \quad P = r_1 \angle \phi_1 \quad \text{and} \quad Q = r_2 \angle \phi_2$$

Then
$$\boxed{P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2] = [r_1 \times r_2] \angle \phi_1 + \phi_2}$$

Key Point : *Thus in multiplication of complex numbers in polar form, the magnitudes get multiplied while their angles get added.*

The result then can be expressed back to rectangular form, if required. Now consider the division of the phasors P and Q.

$$\boxed{\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2}$$

Key Point : *Thus in division of complex numbers in polar form, the magnitudes get divided while their angles get subtracted.*

Note : For converting polar to rectangular form, the students can use the function $P \leftrightarrow R$ on calculators without using basic conversion expressions. Similarly for rectangular to polar conversion, the students can use the function $R \leftrightarrow P$ on calculators without using basic conversion expressions.

Remember :

While addition and subtraction, use rectangular form.

While multiplication and division, use polar form.

1.21 Impedance

In the alternating circuits alongwith the resistances, inductances and capacitances also play an important role.

The inductances are represented by inductive reactances in a.c. circuits. An inductive reactance is the ohmic representation of an inductance denoted as X_L and given by,

$$\boxed{X_L = \omega L = 2\pi f L \quad \Omega}$$

The capacitances are represented by capacitive reactances in a.c. circuits. A capacitive reactance is the ohmic representation of a capacitance denoted as X_C and given by,

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \Omega}$$

The combination of R, X_L and X_C present in the circuit is called an **impedance** of the circuit. The impedance is denoted by letter Z. But the behaviour of R, L and C is different from each other in a.c. circuits hence R, X_L and X_C cannot be algebraically added to find total impedance of the circuit.

Let us summarize the behaviour of R, L and C in the tabular form.

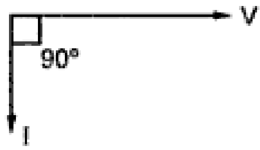
Parameter	Characteristics	Impedance in rectangular form	Impedance in polar form	Phasor diagram
Pure resistance R	V and I are inphase	$Z = R + j0$	$Z = R \angle 0°$	
Pure inductance L	I lags V by 90°	$Z = 0 + j X_L$	$Z = X_L \angle +90°$	
Pure capacitance C	I leads V by 90°	$Z = 0 - j X_C$	$Z = X_C \angle -90°$	

Table 1.4

Inductive reactances are represented by positive sign + X_L in the impedance while capacitive reactances are represented by negative sign − X_C in the impedance.

Thus for **R - L series circuit**, the impedance is represented as,

$$Z = R + j X_L = |Z| \angle \theta° \Omega$$

$$\text{where} \quad |Z| = \sqrt{R^2 + (X_L)^2} \quad \text{and } \theta = \tan^{-1} \frac{X_L}{R}$$

In such circuit, **current lags voltage by angle** θ.

For **R-C series circuit**, the impedance is represented as,

$$Z = R - j X_C = |Z| \angle \theta°$$

$$\text{where} \quad |Z| = \sqrt{R^2 + (X_C)^2} \quad \text{and } \theta = \tan^{-1} \left[\frac{-X_C}{R}\right]$$

In this case θ is negative and **current leads voltage** by angle θ.

Example 1.13 : *Find out the resistance and inductance or capacitance of the given impedances if frequency is 50 Hz. i) 25 ∠ 45° Ω ii) 6 + j 8 Ω iii) 8 – j 10 Ω*

Solution : i) $Z = 25 \angle 45° \ \Omega$

Converting to rectangular form,

$$Z = 17.68 + j\,17.68 \ \Omega$$

Comparing with $Z = R + j X_L \ \Omega$

$$R = 17.68 \ \Omega \quad \text{and} \quad X_L = 17.68 \ \Omega$$

Now $X_L = 2\pi f L$

$$\therefore L = \frac{X_L}{2\pi f} = \frac{17.68}{2\pi \times 50} = 0.0562 \text{ H}$$

ii) $Z = 6 + j\,8 \ \Omega$

Comparing with $Z = R + j X_L \ \Omega$

$$\therefore R = 6 \ \Omega \quad \text{and} \quad X_L = 8 \ \Omega$$

Now $$L = \frac{X_L}{2\pi f} = \frac{8}{2\pi \times 50} = 0.0254 \text{ H}$$

iii) $Z = 8 - j\,10 \ \Omega$

Comparing with, $Z = R - j X_C$

$$\therefore R = 8 \ \Omega \quad \text{and} \quad X_C = 10 \ \Omega$$

Now $$X_C = \frac{1}{2\pi f C}$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 10} = 3.18 \times 10^{-4} \text{F} = 318 \ \mu\text{F}$$

1.22 Power Factor

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor.

It is also defined as the ratio of resistance to the impedance. It is denoted as $\cos\phi$.

$$\boxed{\cos\phi = \text{p.f.} = \frac{R}{Z}}$$

For pure L and C, $\phi = 90°$ hence the p.f. is zero.

For other combinations, the p.f. is defined as lagging or leading i.e. whether the resultant current lags or leads the supply voltage.

1.23 Power

The power in a.c. circuit is given by,

$$\boxed{P = V\, I \cos \phi}$$

where $\cos \phi$ = Power factor of the circuit.

Now pure inductance and capacitance does not consume any power as $\cos \phi = 0$ for such circuits. Only resistance consumes power. Hence power also can be obtained as,

$$P = I^2 R$$

where R = Resistive part of the equivalent impedance of the circuit.

Key Point: *While calculating the equivalent impedance, it should be noted that while adding the impedances in series, the impedances must be expressed in rectangular form while multiplication and division of impedances must be carried out by expressing the impedances in the polar form.*

1.24 Series R-L-C Circuit

The series R-L-C circuit is shown in the Fig. 1.62.

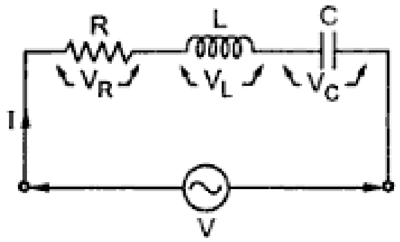


Fig. 1.62

The impedance is given by,

$$Z = R + j\,X_L - j\,X_C$$

$$= R + j\,(X_L - X_C)$$

If $X_L > X_C$ then resultant impedance is inductive and current I lags voltage V. If $X_L < X_C$ then resultant impedance is capacitive and current I leads voltage V.

If $X_L = X_C$ then circuit becomes purely resistive and I is in phase with V.

The voltage drops across the various elements are denoted as V_R, V_L and V_C. The magnitudes of these drops are given by,

$$V_R = |I| R, \quad V_L = |I|\, X_L \quad \text{and} \quad V_C = |I|\, X_C$$

Note that algebraic sum of V_R, V_L and V_C is not V as these three voltages are not in phase. But the **vector addition** of V_R, V_L and V_C is the supply voltage V.

➠ **Example 1.14 :** *The network shown in the Fig. 1.63 is operating in a sinusoidal steady state. Find voltage across capacitor, resistor and inductor.*

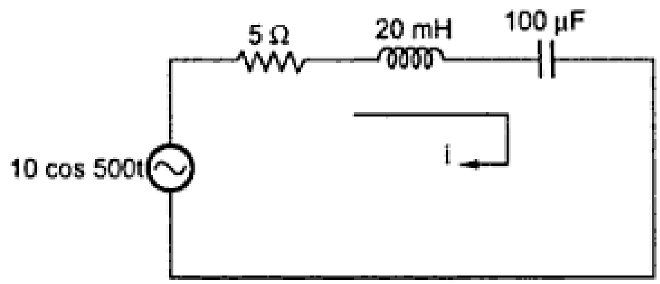


Fig. 1.63

Solution : The applied voltage is 10 cos (500 t). Expressing it as,

$$V = 10 \sin(500\,t + 90°) \;\ldots \text{ as } \sin(90+\theta) = \cos\theta$$

Compare with, $V = V_m \sin(\omega t + \theta)$

$$\therefore \omega = 500 \text{ rad/sec}, \quad V_m = 10 \text{ V}, \quad \theta = 90°$$

$$\therefore V(\text{RMS}) = \frac{V_m}{\sqrt{2}} = 7.071 \text{ V}$$

$$\therefore V = 7.071 \angle 90° \text{ V}$$

Now $R = 5\ \Omega,\ L = 20 \text{ mH},\ C = 100\ \mu\text{F}$

$$\therefore X_L = \omega L = 10\ \Omega$$

and $X_C = \dfrac{1}{\omega C} = 20\ \Omega$

$$\therefore Z_T = R + j(X_L - X_C) = 5 + j(10 - 20)$$

$$= 5 - j\,10\ \Omega = 11.1803 \angle -63.43°\ \Omega$$

$$\therefore i = \frac{V}{Z_T} = \frac{7.071\angle 90°}{11.1803\angle -63.43°} = 0.6324 \angle 153.43° \text{ A}$$

Hence voltages across the various elements are,

$$V_R = |i| \times R = 0.6324 \times 5 = \mathbf{3.162\ V}$$

$$V_L = |i| \times X_L = 0.6324 \times 10 = \mathbf{6.324\ V}$$

$$V_C = |i| \times X_C = 0.6324 \times 20 = \mathbf{12.648\ V}$$

Sr. No.	Circuit	Impedance (Z)		ϕ	p.f. $\cos\phi$	Remark
		Polar	Rectangular			
1.	Pure R	$R \angle 0° \Omega$	$R + j0 \Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90° \Omega$	$0 + j X_L \Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90° \Omega$	$0 - j X_C \Omega$	− 90°	0	Zero leading
4.	Series RL	$\lvert Z\rvert \angle +\phi° \Omega$	$R + j X_L \Omega$	$0° \angle \phi \angle 90°$	$\cos\phi$	Lagging
5.	Series RC	$\lvert Z\rvert \angle -\phi° \Omega$	$R - j X_C \Omega$	$-90° \angle \phi \angle 0$	$\cos\phi$	Leading
6.	Series RLC	$\lvert Z\rvert \angle \pm \phi° \Omega$	$R + j X \Omega$ $X = X_L - X_C$	ϕ	$\cos\phi$	$X_L > X_C$ Lagging $X_L < X_C$ Leading $X_L = X_C$ Unity

Table 1.5 Summary of R, L and C series circuits

1.25 A.C. Parallel Circuit

A parallel circuit is one in which two or more impedances are connected in parallel across the supply voltage. Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.

The Fig. 1.64 shows a parallel circuit consisting of three impedances connected in parallel across an a.c. supply of V volts.

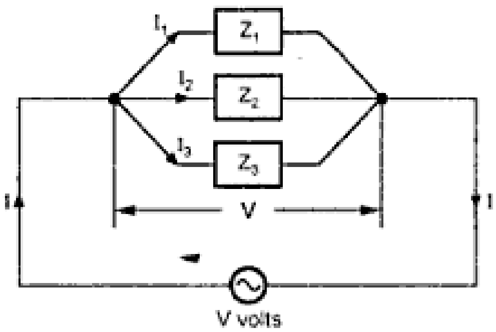


Fig. 1.64 A.C. parallel circuit

Key Point: *The voltage across all the impedances is same as supply voltage of V volts.*

The current taken by each impedance is different.

Applying Kirchhoff's law, $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$ ... (Phasor addition)

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z}_1} + \frac{\bar{V}}{\bar{Z}_2} + \frac{\bar{V}}{\bar{Z}_3}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

Where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

1.25.1 Current Division for Parallel Impedances

If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents.

$$\boxed{\begin{aligned} \bar{I}_1 &= \bar{I}_T \times \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} \\ \bar{I}_2 &= \bar{I}_T \times \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2} \end{aligned}}$$

1.25.2 Concept of Admittance

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit **siemens or mho.**

Now, current equation for the circuit shown in the Fig. 1.65 is,

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

$$\bar{I} = \bar{V} \times \left(\frac{1}{\bar{Z}_1}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_2}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

where **Y** is the admittance of the total circuit. **The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series,** as shown in the Fig. 1.65.

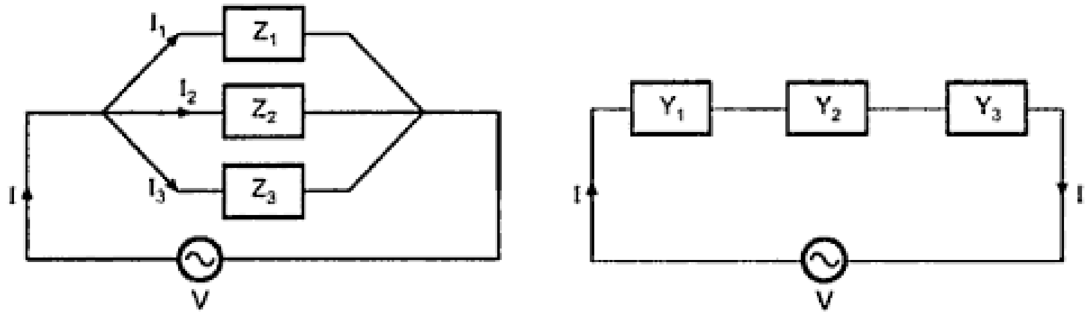


Fig. 1.65 Equivalent parallel circuit using admittances

1.25.3 Components of Admittance

Consider an impedance given as,

$$Z = R \pm j X$$

Positive sign for inductive and negative for capacitive circuit.

Admittance $\quad Y = \dfrac{1}{Z} = \dfrac{1}{R \pm j X}$

Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2+X^2}\right) \mp j\left(\frac{X}{R^2+X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$\therefore$ $\boxed{Y = G \mp jB}$

In the above expression, $G = \text{Conductance} = \frac{R}{Z^2}$

and $B = \text{Susceptance} = \frac{X}{Z^2}$

1.25.4 Conductance (G)

It is defined as the ratio of the resistance to the square of the impedance. It is measured in the unit **siemens.**

1.25.5 Susceptance (B)

It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit **siemens.**

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

Note : The sign convention for the reactance and the susceptance are opposite to each other.

Remember,

$$Y = G + jB = |Y| \angle \phi \text{ siemens or mho}$$

$$|Y| = \sqrt{G^2 + B^2}, \ \phi = \tan^{-1}\frac{B}{G}$$

B is negative if inductive and B is positive if capacitive.

Key Point: *Impedances in parallel get converted to admittances in series while impedances in series get converted to admittances in parallel.*

Example 1.15 : *Calculate the equivalent impedance of the network as viewed through the terminals A-B shown in the Fig. 1.66. If an alternating voltage of 150 ∠ 0° V is connected across A-B, calculate the current drawn from the source. Hence calculate the power consumed.*

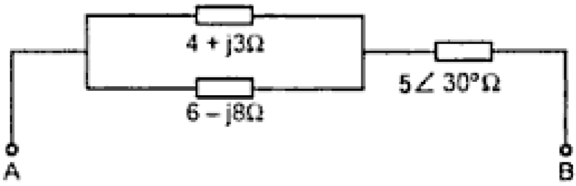


Fig. 1.66

Solution : Let

$$Z_1 = 4+3j\ \Omega = 5\angle 36.86^\circ\ \Omega$$

$$Z_2 = 6-j8\ \Omega = 10\angle -53.13^\circ\ \Omega$$

$$Z_3 = 5\angle 30^\circ\ \Omega = 4.33+j\,2.5\ \Omega$$

Now Z_1 and Z_2 are in parallel,

$$\therefore (Z_1 \parallel Z_2) = \frac{Z_1 Z_2}{Z_1+Z_2}$$

$Z_1 Z_2$ must be in polar form while $(Z_1 + Z_2)$ in rectangular form

$$\therefore (Z_1 \parallel Z_2) = \frac{5\angle 36.86^\circ \times 10\angle -53.13^\circ}{(4+j3+6-j8)} = \frac{50\angle -16.27^\circ}{(10-j5)}$$

$$= \frac{50\angle -16.27^\circ}{11.18\angle -26.56^\circ} = 4.4721\angle -16.27^\circ + 26.56^\circ$$

$$= 4.4721\angle +10.29^\circ = 4.4+j\,0.8\ \Omega$$

Now $Z_{AB} = (Z_1 \parallel Z_2) + Z_3$ as in series

$$= 4.4+j\,0.8+4.33+j\,2.5 = 8.73+j\,3.3\ \Omega = 9.332\angle +20.706^\circ\ \Omega$$

This is the equivalent impedance.

$$V = 150\angle 0^\circ$$

Hence the circuit becomes as shown in the Fig. 1.67 (a).

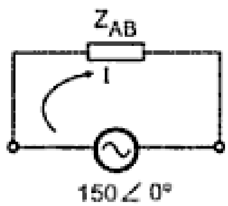


Figure 1.67 (a)

$$I = \frac{V}{Z_{AB}} = \frac{150\angle 0^\circ}{9.332\angle 20.706^\circ} = 16.07\angle -20.706^\circ\ \text{A}$$

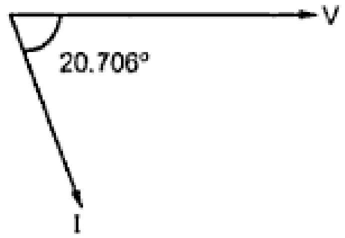


Fig. 1.67 (b)

Thus the current is 16.07 A, lagging voltage by angle 20.706°. The phasor diagram is shown in the Fig. 1.67 (b).

The power consumed can be calculated as,

$$P = V\,I\cos\phi = 150 \times 16.07 \times \cos(-20.706°)$$

$$= \mathbf{2254.798\ W}$$

Alternatively the power can be obtained as,

$$P = I^2 \times (R_{AB})$$

$$\text{Now}\quad Z_{AB} = 8.73 + j\,3.3\ \Omega$$

$$\therefore\ R_{AB} = 8.73\ \Omega$$

$$\therefore\ P = (16.07)^2 \times 8.73 = \mathbf{2254.4\ W}$$

This is because inductive part j 3.3 of an equivalent impedance does not consume any power.

Example 1.16 : *Two impedance $Z_1 = 5 - j\ 13.1\ \Omega$ and $Z_1 = 8.57 + j\ 6.42\ \Omega$ are connected in parallel across a voltage of (100 + j200) volts.*
Estimate :-
i) Branch currents in complex form ii) Total power consumed,
Draw a neat phasor diagram showing voltage, branch currents and all phase angles.

Solution : The circuit is shown in the Fig. 1.68.

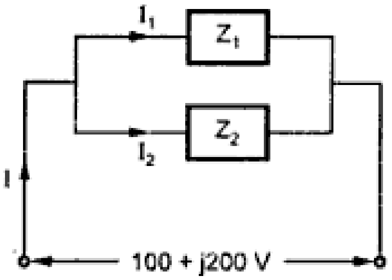


Fig. 1.68

$$V = 100 + j\,200 = 223.607\ \angle\ 63.43°\ V$$

$$Z_1 = 5 - j\,13.1 = 14.021\ \angle -69.109°\ \Omega$$

$$Z_2 = 8.57 + j\,6.42 = 10.71\ \angle +36.83°\ \Omega$$

i) $$I_1 = \frac{V}{Z_1} = \frac{223.607\angle 63.43°}{14.021\angle 69.109°}$$

$$= 15.948\ \angle\ 132.539°\ A$$

$$= \mathbf{-10.782 + j\,11.75\ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{223.607\angle 63.43°}{10.71\angle +36.83°} = 20.878\ \angle\ 26.6°\ A = \mathbf{18.668 + j\ 9.3483\ A}$$

$$\therefore\quad I_T = \bar{I}_1 + \bar{I}_2 = -10.782 + j\,11.75 + 18.668 + j\,9.3483$$

$= 7.886 + j\ 21.0983$ A

$= \mathbf{22.5239 \angle 69.5^\circ}$ **A**

$\therefore \quad \phi_T$ = Angle between V and I_T

$= 69.5 - 63.43 = \mathbf{6.075^\circ}$ **leading**

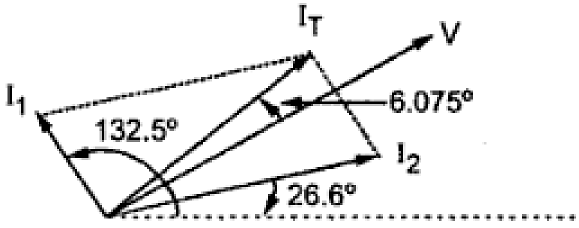


Fig. 1.69

$\therefore \quad P_T = V\ I_T \cos\phi_T = 223.607 \times 22.5239 \times \cos(6.075) = \mathbf{5008.212\ W}$

The phasor diagram is shown in the Fig. 1.69.

1.26 Star and Delta Connection of Resistances

In the complicated networks involving large number of resistances, Kirchhoff's laws give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case application of Star-Delta or Delta-Star transformation, considerably reduces the complexity of the network and brings the network into a very simple form. This reduces the number of unknowns and hence network can be analysed very quickly for the required result. These transformations allow us to replace three star connected resistances of the network, by equivalent delta connected resistances, without affecting currents in other branches and vice-versa.

Let us see what is star connection ?

If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in Star.

The Fig. 1.70 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 1.70 (a) and (b) are exactly identical. The Fig. 1.70 (b) can be redrawn as Fig. 1.70 (a) or vice-versa, in the circuit from simplification point of view.

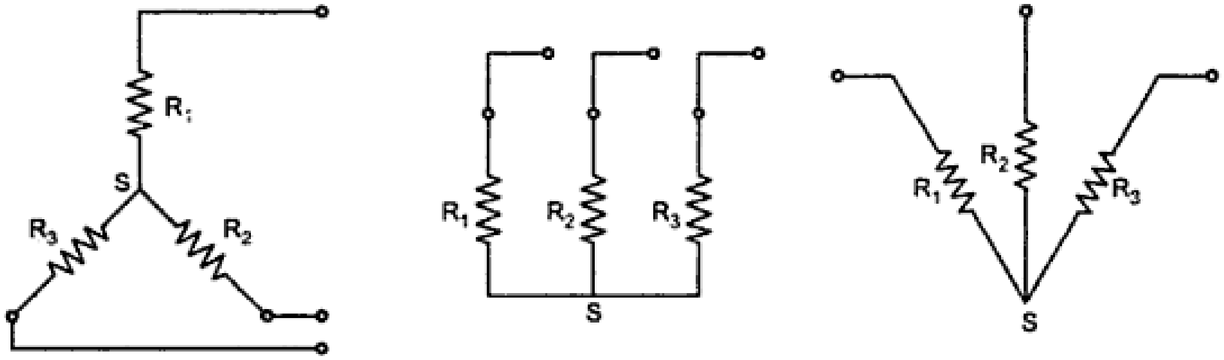


Fig. 1.70 Star connection of three resistances

Let us see what is delta connection ?

If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in Delta.

Key Point: *Delta connection always forms a loop, closed path.*

The Fig. 1.71 (a) and (b) show delta connection of three resistances. The Fig. 1.71 (a) and (b) are exactly identical.

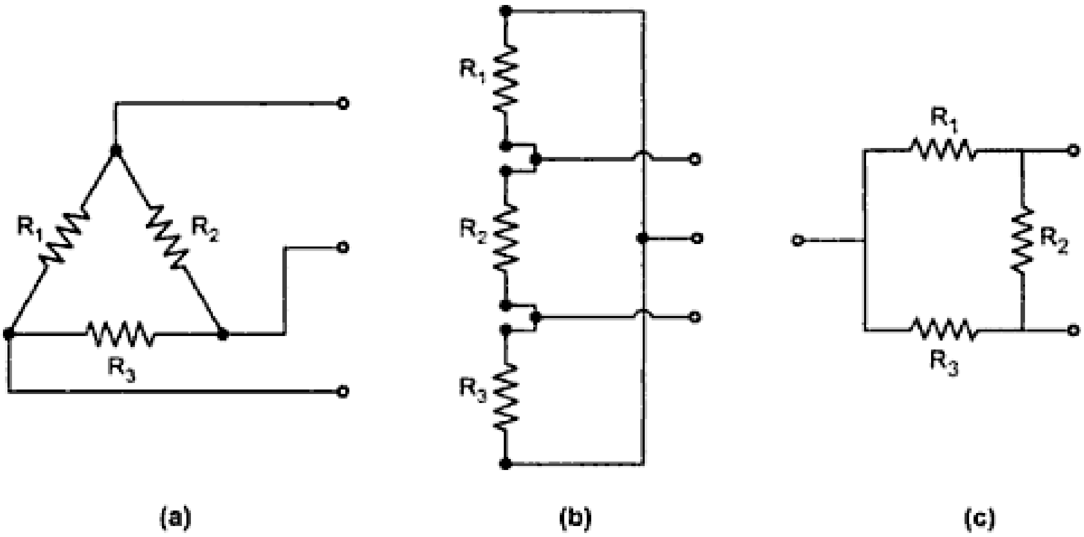
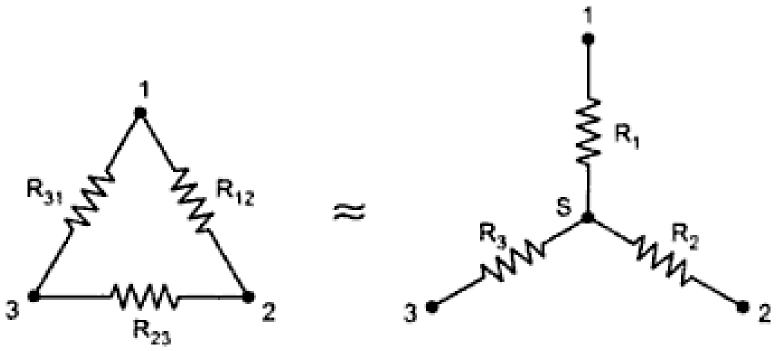


Fig. 1.71 Delta connection of three resistances

1.26.1 Delta-Star Transformation



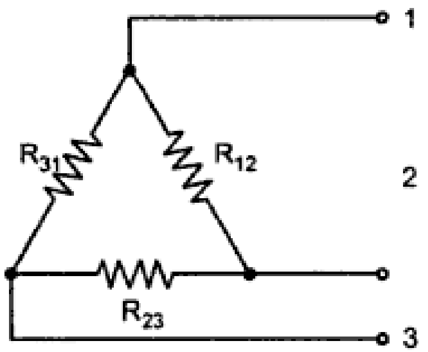
Given Delta **Fig. 1.72** **Equivalent Star**

Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta as shown in the Fig. 1.72. The terminals between which these are connected in Delta are named as 1, 2 and 3.

Now it is always possible to replace these Delta connected resistances by three equivalent Star connected resistances R_1, R_2, R_3 between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 1.72 which is called **equivalent Star of Delta connected resistances.**

Key Point: *Now to call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.*

Let us analyse Delta connection first, shown in the Fig. 1.72 (a).



(a) Given Delta

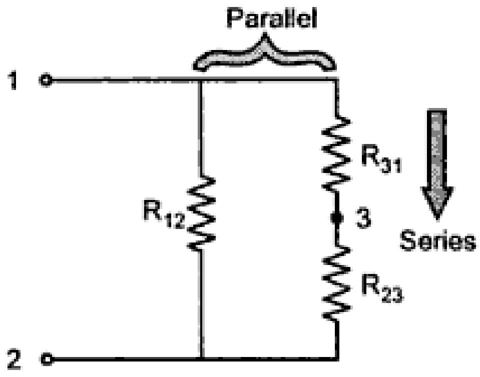


Fig. 1.72 (b) Equivalent between 1 and 2

Now consider the terminals (1) and (2). Let us find equivalent resistance between (1) and (2). We can redraw the network as viewed from the terminals (1) and (2), without considering terminal (3). This is shown in the Fig. 1.72(b).

Now terminal '3' we are not considering, so between terminals (1) and (2) we get the combination as,

R_{12} parallel with $(R_{31}+R_{23})$ as R_{31} and R_{23} are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} \qquad \text{...(a)}$$

[using $\frac{R_1 R_2}{R_1+R_2}$ for parallel combination]

Now consider the same two terminals of equivalent Star connection shown in the Fig. 1.73.

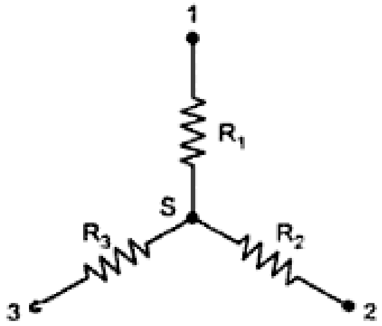


Fig. 1.73 Star connection

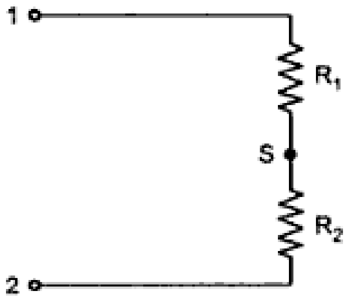


Fig. 1.74 Equivalent between 1 and 2

Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).

And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 1.74.

$\therefore$ Between (1) and (2) the resistance is $= R_1 + R_2$ ... (b)

$$R_2 = \frac{R_{12}\ R_{23}}{R_{12} + R_{23} + R_{31}} \quad \ldots(h)$$

$$R_3 = \frac{R_{23}\ R_{31}}{R_{12} + R_{23} + R_{31}} \quad \ldots(i)$$

Now multiply (g) and (h), (h) and (i), (i) and (g) to get following three equations.

$$R_1R_2 = \frac{R_{12}^2\ R_{31}\ R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \ldots(j)$$

$$\therefore \quad R_2R_3 = \frac{R_{23}^2\ R_{12}\ R_{31}}{(R_{12} + R_{23} + R_{31})^2} \quad \ldots(k)$$

$$R_3R_1 = \frac{R_{31}^2\ R_{12}\ R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \ldots(l)$$

Now add (j) ,(k) and (*l*)

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2R_{31}R_{23} + R_{23}^2R_{12}R_{31} + R_{31}^2R_{12}\ R_{23}}{(R_{12} + R_{23} + R_{31})^2}$$

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{31}R_{23}(R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{31}\ R_{23}}{R_{12} + R_{23} + R_{31}}$$

But $\dfrac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} = R_1$ From equation (g)

$\therefore$ Substituting in above in R.H.S. we get,

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = R_1\ R_{23}$$

$$\therefore \quad \boxed{R_{23} = R_2 + R_3 + \frac{R_2\ R_3}{R_1}}$$

Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1\ R_2}{R_3}}$$

and

$$\boxed{R_{31} = R_3 + R_1 + \frac{R_1\ R_3}{R_2}}$$

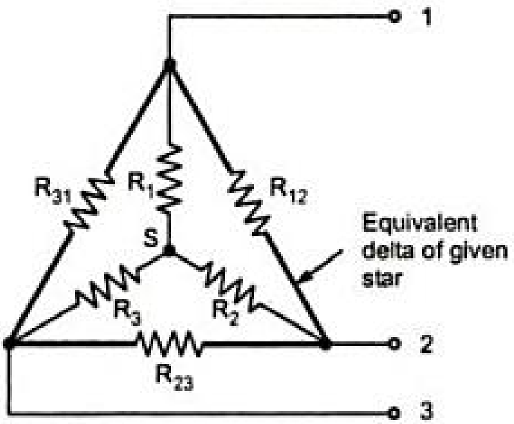


Fig. 1.77 Star and equivalent Delta

Easy way of remembering the result :

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

So if we want equivalent delta resistance between terminals (3) and (1), then take sum of the two resistances connected between same two terminals (3) and (1) and star point respectively i.e. terminal (3) to star point R_3 and terminal (1) to star point i.e. R_1. Then to this sum of R_1 and R_3, add the term which is the product of the same two resistances i.e. R_1 and R_3 divided by the third star resistance which is R_2.

$\therefore$ We can write, $R_{31} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$ which is same as derived above.

Result for equal resistances in star and delta :

If all resistances in a Delta connection have same magnitude say R, then its equivalent Star will contain,

$$R_1 = R_2 = R_3 = \frac{R \times R}{R+R+R} = \frac{R}{3}$$

i.e. **equivalent Star contains three equal resistances, each of magnitude one third the magnitude of the resistances connected in Delta.**

If all three resistances in a Star connection are of same magnitude say R, then its equivalent Delta contains all resistances of same magnitude of ,

$$R_{12} = R_{31} = R_{23} = R + R + \frac{R \times R}{R} = 3R$$

i.e. **equivalent delta contains three resistances each of magnitude thrice the magnitude of resistances connected in Star.**

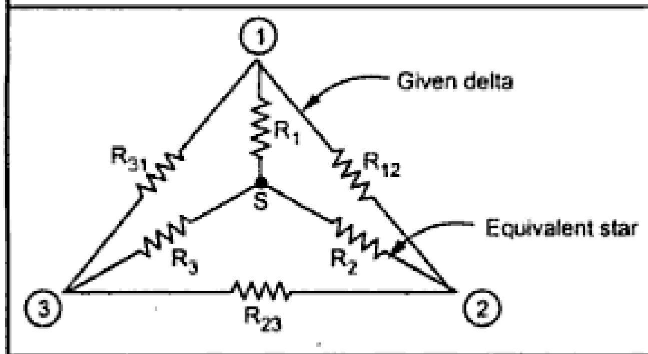
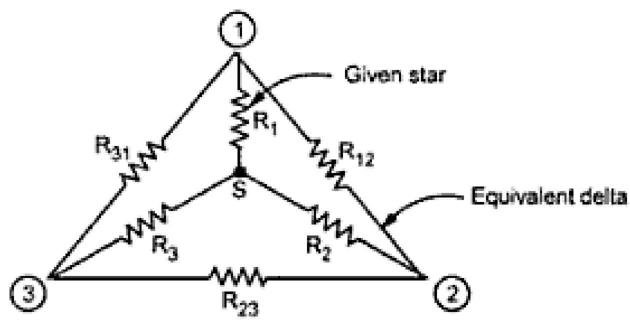
Delta-Star	Star-Delta
	
$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$
$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$	$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$
$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$

Table 1.6 Star-Delta and Delta-Star Transformations

Example 1.17 : *Convert the given Delta in the Fig. 1.78 into equivalent Star.*

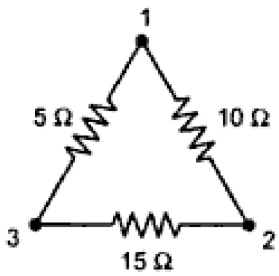


Fig. 1.78

Solution : Its equivalent star is as shown in the Fig. 1.79.

Where

$$R_1 = \frac{10\times 5}{5+10+15} = 1.67\ \Omega$$

$$R_2 = \frac{15\times 10}{5+10+15} = 5\ \Omega$$

$$R_3 = \frac{5\times 15}{5+10+15} = 2.5\ \Omega$$

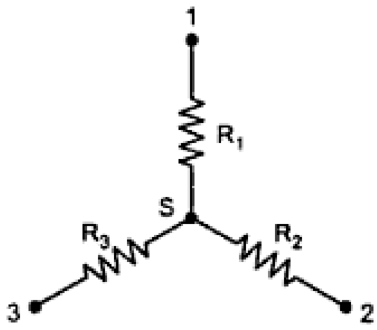


Fig. 1.79

Remember that the above expressions can be applied to the alternating circuits where resistances will be replaced by the impedances.

Example 1.18 : *Convert the given star into an equivalent delta.*

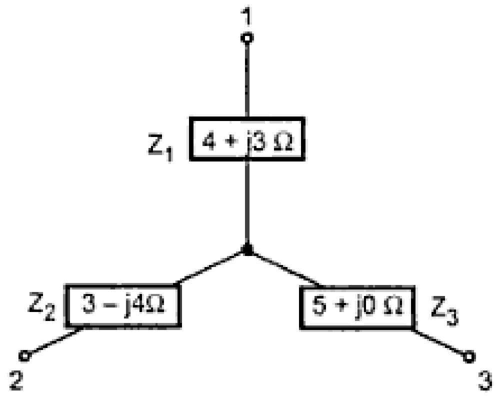


Fig. 1.80

Solution : For star to delta conversion,

$$Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} = 4 + j3 + 3 - j4 + \frac{(4+j3)(3-j4)}{(5+j0)}$$

$$= 7 - j + \frac{5\angle 36.86° \times 5\angle -53.13°}{5\angle 0°} = 7 - j + 5\angle -16.27°$$

$$= 7 - j + 4.8 - j\,1.4 = \mathbf{11.8 - j\,2.4\ \Omega}$$

Similarly Z_{23} and Z_{31} can be obtained. The equivalent delta is shown in the Fig. 1.81.

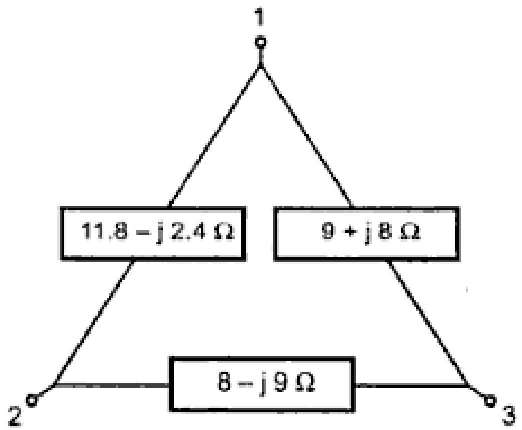


Fig. 1.81

Key Point : *Use rectangular system for addition or subtraction while polar for multiplication or division.*

Examples with Solutions

Example 1.19 : *Find equivalent resistance between points A-B.*

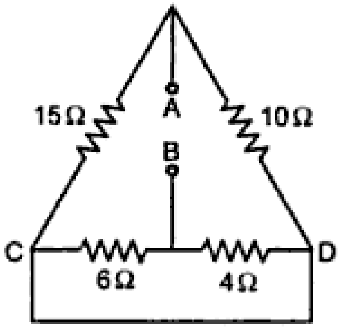


Fig. 1.82

Solution : Redraw the circuit,

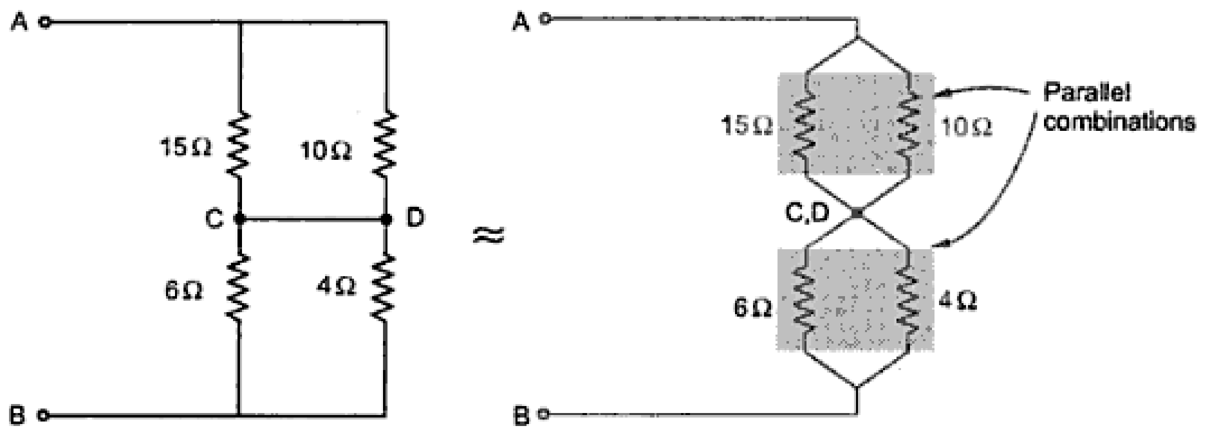


Fig. 1.83 (a)

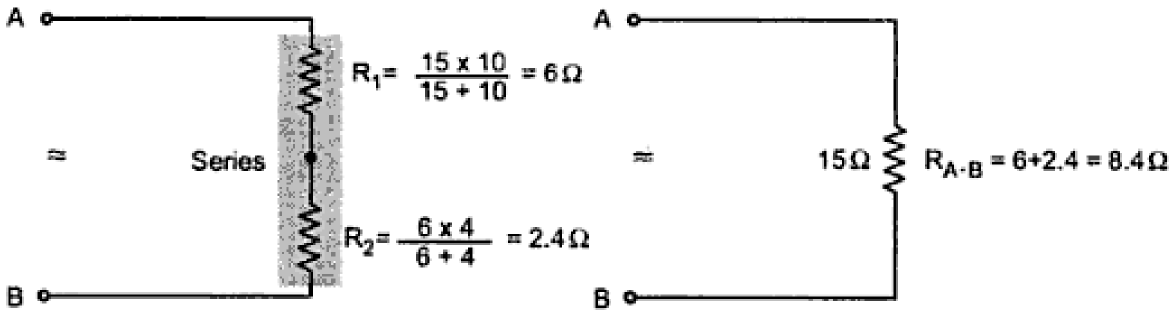


Fig. 1.83 (b)

$$\therefore \quad R_{AB} = 8.4\ \Omega$$

Example 1.20 : *Calculate the effective resistance between points A and B in the given circuit in Fig. 1.84.*

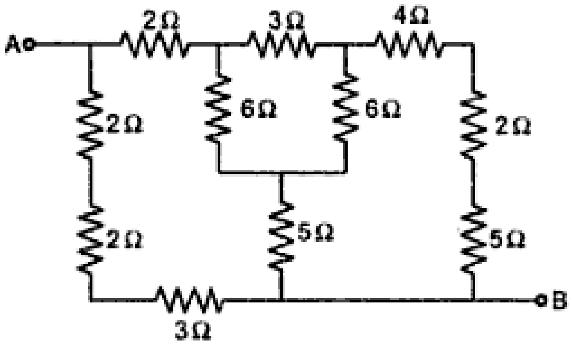


Fig. 1.84

Solution : The resistances 2, 2 and 3 are in series while the resistances 4, 2, and 5 are in series.

$\therefore \quad 2 + 2 + 3 = 7\ \Omega$

and $\quad 4 + 2 + 5 = 11\ \Omega$

The circuit becomes as shown in Fig. 1.85 (a).

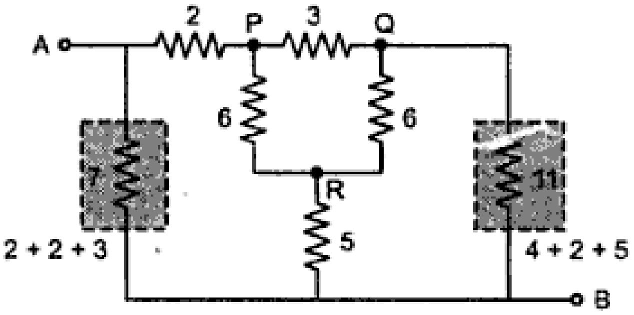


Fig. 1.85 (a)

Converting Δ PQR to equivalent star,

$$R_{PN} = \frac{6 \times 3}{6+3+6} = 1.2\ \Omega$$

$$R_{PN} = \frac{6 \times 6}{6+3+6} = 2.4\ \Omega$$

$$R_{QN} = \frac{6 \times 3}{6+3+6} = 1.2\ \Omega$$

Hence the circuit becomes as shown in the Fig. 1.85 (b).

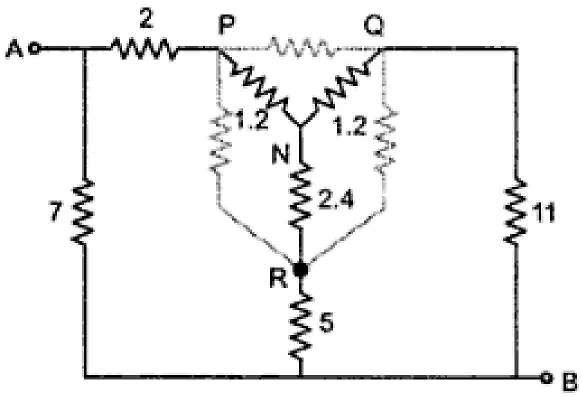


Fig. 1.85 (b)

The resistances 2 and 1.2 are in series.

1.2 and 11 are in series.

5 and 2.4 are in series.

∴ Circuit becomes after simplification as shown in the Fig. 1.85 (c).

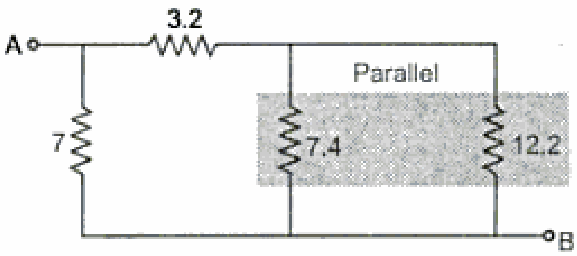
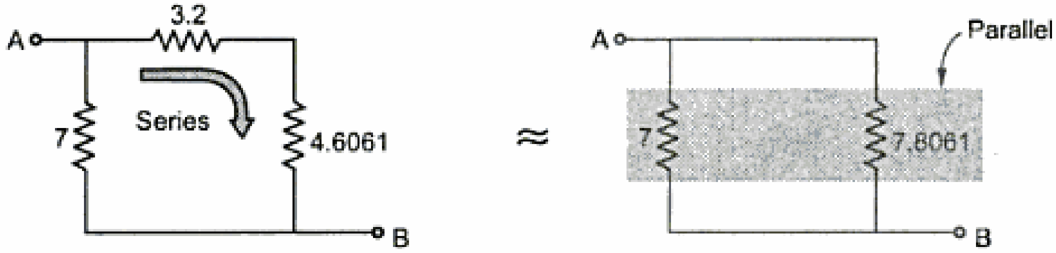


Fig. 1.85 (c)

The resistances 7.4 and 12.2 are in parallel.

$$\therefore \quad 7.4 \parallel 12.2 = \frac{7.4 \times 12.2}{7.4 + 12.2} = 4.6061\ \Omega$$

So circuit becomes,



(d) **Fig. 1.85** **(e)**

Now the two resistances are in parallel.

$$\therefore \quad R_{AB} = \frac{7 \times 7.8061}{7 + 7.8061} = 3.69\ \Omega$$

➠ **Example 1.21 :** *Find the current in the branch A - B in the d.c. circuit shown in the Fig. 1.86, using Kirchhoff's laws.*

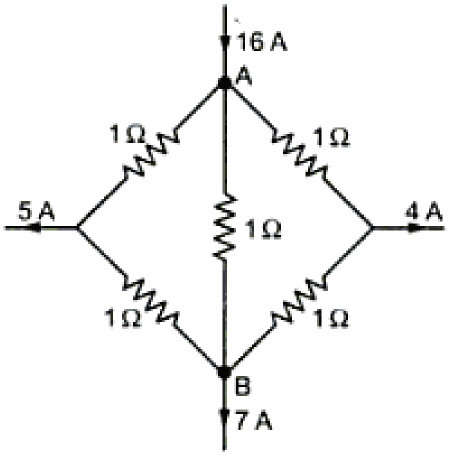


Fig. 1.86

Solution : The various branch currents are shown in the Fig. 1.86 (a).

Applying KVL to loop ADBA

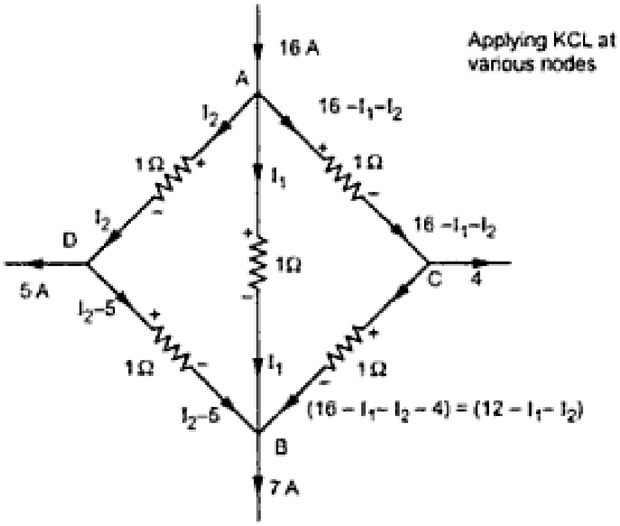


Fig. 1.86 (a)

$$-I_2 - (I_2 - 5) + I_1 = 0$$

$$\therefore \qquad I_1 - 2I_2 = -5 \qquad \ldots (1)$$

Applying KVL to the loop ACBA,

$$-(16 - I_1 - I_2) - (12 - I_1 - I_2) + I_1 = 0$$

$$\therefore -16 + I_1 + I_2 - 12 + I_1 + I_2 + I_1 = 0$$

$$\therefore 3I_1 + 2I_2 = 28 \qquad \ldots (2)$$

Add (1) and (2),

$$4I_1 = 23$$

$$\therefore \qquad I_1 = 5.75 \text{ A}$$

... This is the current through branch AB.

Example 1.22 : *Determine the equivalent resistance between the terminals A and B for the circuit shown in Fig. 1.87.*

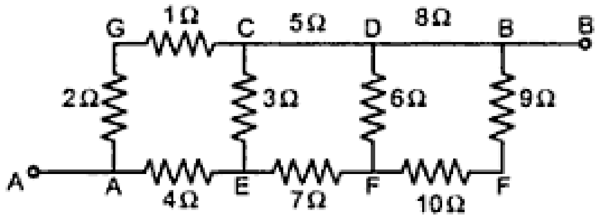


Fig. 1.87

Solution : In Fig. 1.87, the resistances 2 Ω and 1 Ω are in series and resistances 10 Ω and 9 Ω are in series.

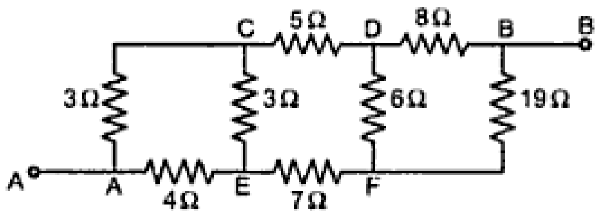


Fig. 1.87 (a)

Converting delta AEC to star

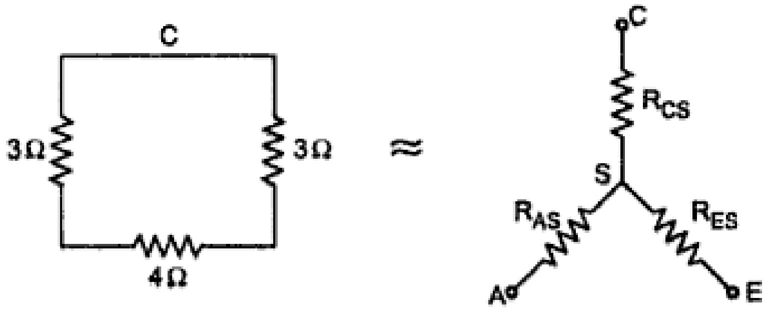


Fig. 1.87 (b)

$$\therefore \quad R_{CS} = \frac{3 \times 3}{3+4+3} = 0.9\ \Omega$$

$$\therefore \quad R_{AS} = \frac{4 \times 3}{3+4+3} = 1.2\ \Omega$$

$$\therefore \quad R_{ES} = \frac{3 \times 4}{3+4+3} = 1.2\ \Omega$$

Converting Delta DBF to star

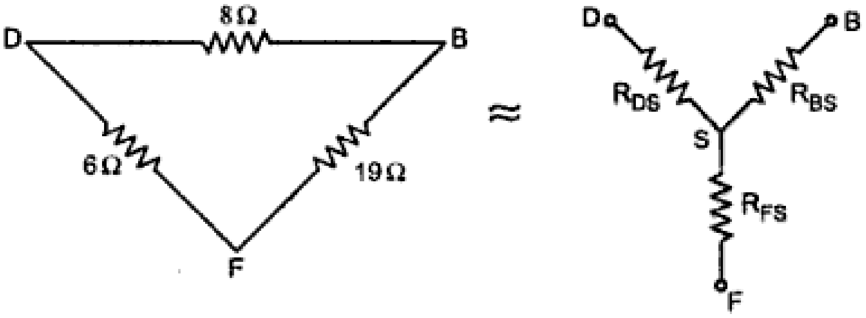


Fig. 1.87 (c)

$$\therefore \quad R_{DS} = \frac{6 \times 8}{6+8+19} = 1.4545\ \Omega$$

$$\therefore \quad R_{BS} = \frac{8 \times 19}{6+8+19} = 4.606\ \Omega$$

$$\therefore \quad R_{FS} = \frac{6 \times 19}{6+8+19} = 3.4545\ \Omega$$

$$\therefore \quad R_{AB} = 1.2 + (7.3545 \parallel 11.6545) + 4.606$$

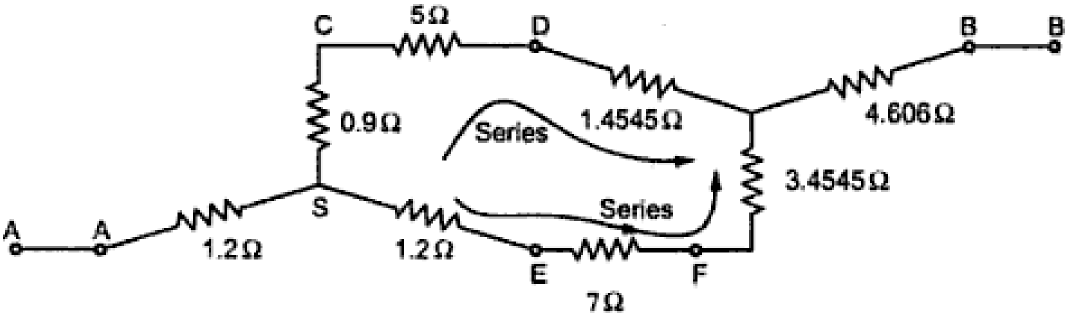


Fig. 1.87 (d)

$$= 1.2 + \frac{7.3545 \times 11.6545}{7.3545 + 11.6545} + 4.606 = 10.315\ \Omega$$

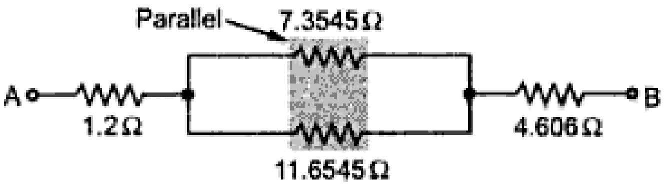


Fig. 1.87 (e)

Example 1.23 : *For the circuit shown in the Fig. 1.88, write KCL and KVL equation and solve to find currents I_1 and I_2.*

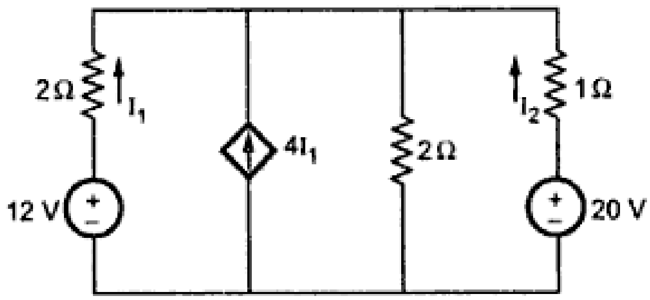


Fig. 1.88

Solution : Consider the various branch currents and node voltages as shown in the Fig. 1..88 (a).

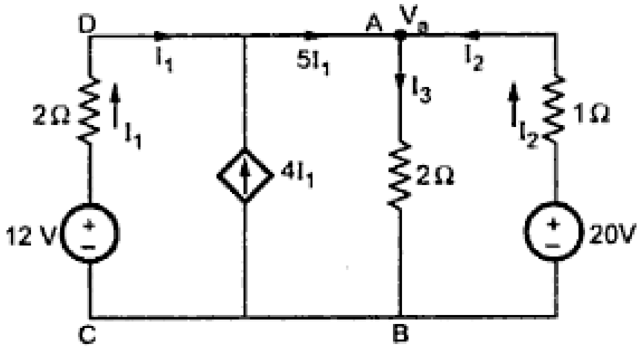


Fig. 1.88 (a)

Applying KCL at node A,

$$5\ I_1 - I_3 + I_2 = 0$$

Now $$I_3 = \frac{V_a}{2} \text{ and}$$

$$I_2 = \frac{20 - V_a}{1}$$

Note the direction of I_2 which is coming towards the node hence the 20 V source is at higher potential than V_a forcing I_2 towards node A from the base.

$$\therefore \quad 5I_1 - \frac{V_a}{2} + \frac{20 - V_a}{1} = 0$$

$$\therefore \quad 5I_1 = 1.5\ V_a - 20$$

$$\therefore \quad I_1 = 0.3\ V_a - 4 \quad \ldots (1)$$

Applying KVL to the loop ABCDA,

$$-2\ I_3 + 12 - 2I_1 = 0$$

$$-2\times\frac{V_a}{2}+12-2I_1 = 0$$

$$\therefore \quad V_a + 2I_1 = 12 \quad \text{... (2)}$$

Putting I_1 from (1) into (2),

$$V_a + 0.6\,V_a - 8 = 12$$

$$\therefore \quad 1.6\,V_a = 20$$

$$\therefore \quad V_a = 12.5\text{ V}$$

$$\therefore \quad I_1 = -0.25\text{ A i.e. } \mathbf{0.25\text{ A}}\downarrow$$

and $$I_2 = \frac{20-V_a}{1} = \mathbf{7.5\text{ A}}\uparrow$$

Example 1.24 : *Using successive source transformation, find the voltage across 1 Ω resistor between 'a' and 'b' shown in the Fig. 1.89.*

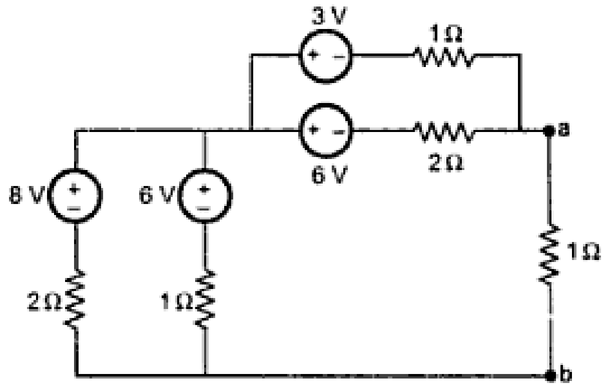


Fig. 1.89

Solution : Converting all the voltage sources to the current sources,

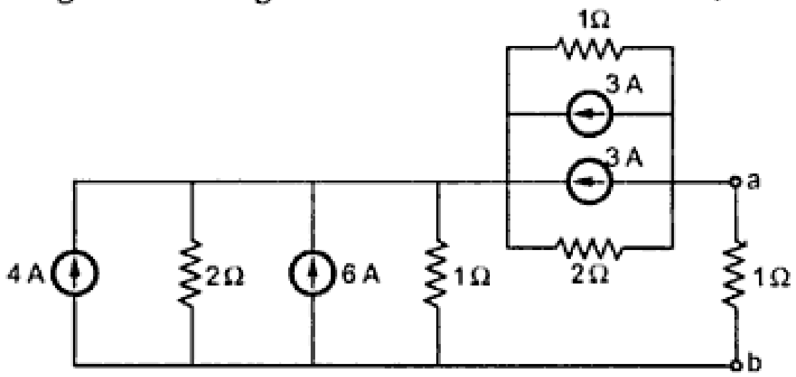


Fig. 1.89 (a)

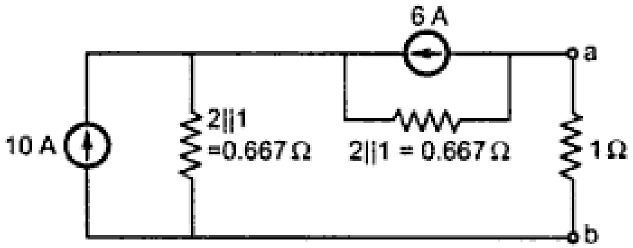


Fig. 1.89 (b)

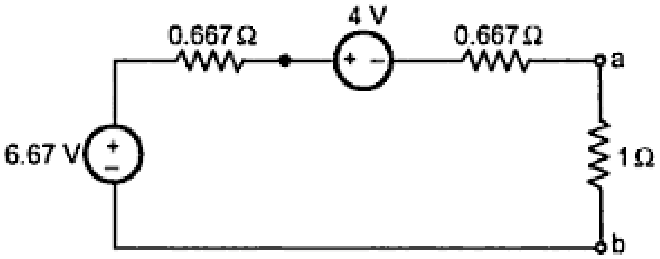


Fig. 1.89 (c)

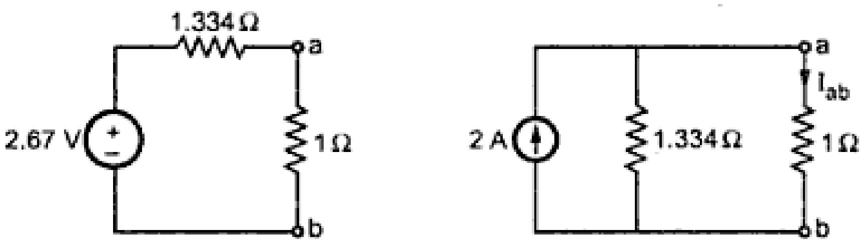


Fig. 1.89 (d) & (e)

$$\therefore \quad I_{ab} = \frac{2 \times 1.334}{(1.334+1)} = 1.143 \text{ A} \qquad \text{... Current division rule.}$$

$$\therefore \quad V_{ab} = I_{ab} \times 1\,\Omega = 1.143 \text{ V with 'a' positive.}$$

Example 1.25 : *Using source transformation, find V_1 and V_2 shown in the circuit.*

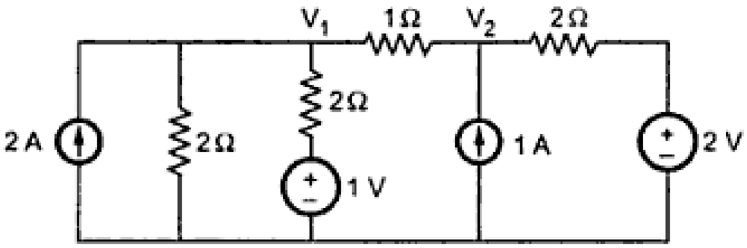


Fig. 1.90

Solution : Converting all voltage sources to current sources,

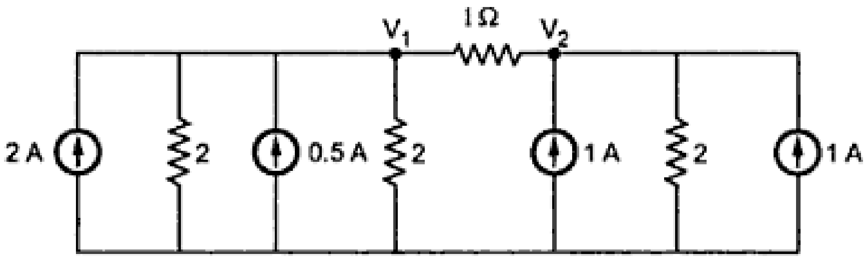


Fig. 1.90 (a)

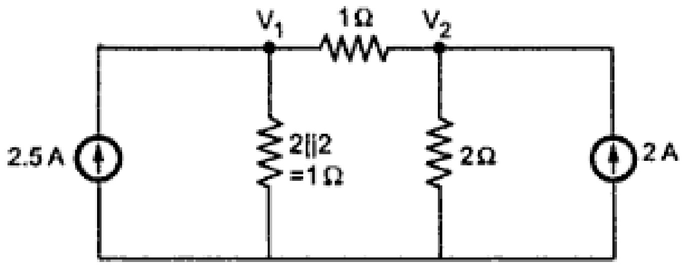


Fig. 1.90 (b)

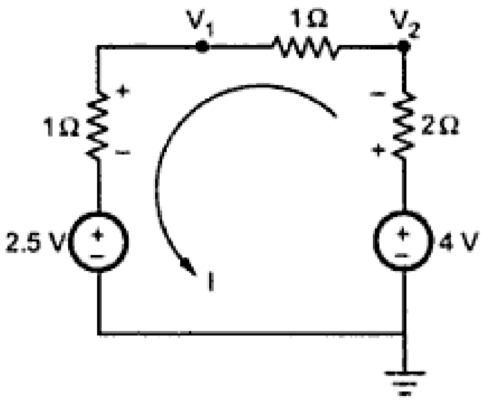


Fig. 1.90 (c)

Converting current sources to voltage sources,

Applying KVL,

$$-2I - I - I - 2.5 + 4 = 0$$

$\therefore \qquad -4I = -1.5$

$\therefore \qquad I = 0.375\ \text{A}$

$\therefore$ Drop across $2\,\Omega = 2 \times 0.375$

$= 0.75\ \text{V}$

and Drop across $1\,\Omega = 1 \times 0.375$

$= 0.375\ \text{V}$

$\therefore \qquad V_1 = 0.375 + 2.5$

$=$ **2.875 V +ve with respect to base.**

and $\qquad V_2 = -0.75 + 4$

$=$ **3.25 V +ve with respect to base.**

Example 1.26 : *For the circuit shown in the Fig. 1.91, use source transformations to determine current I.*

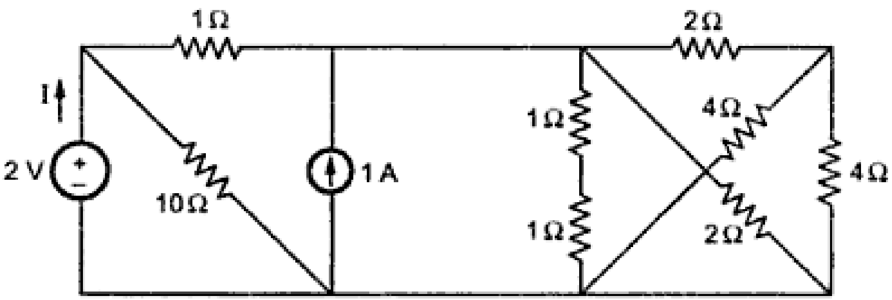


Fig. 1.91

Solution : Redrawing the given network we get,

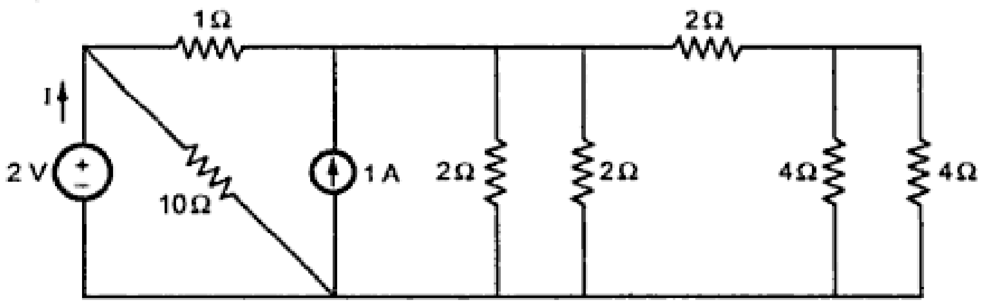


Fig. 1.91 (a)

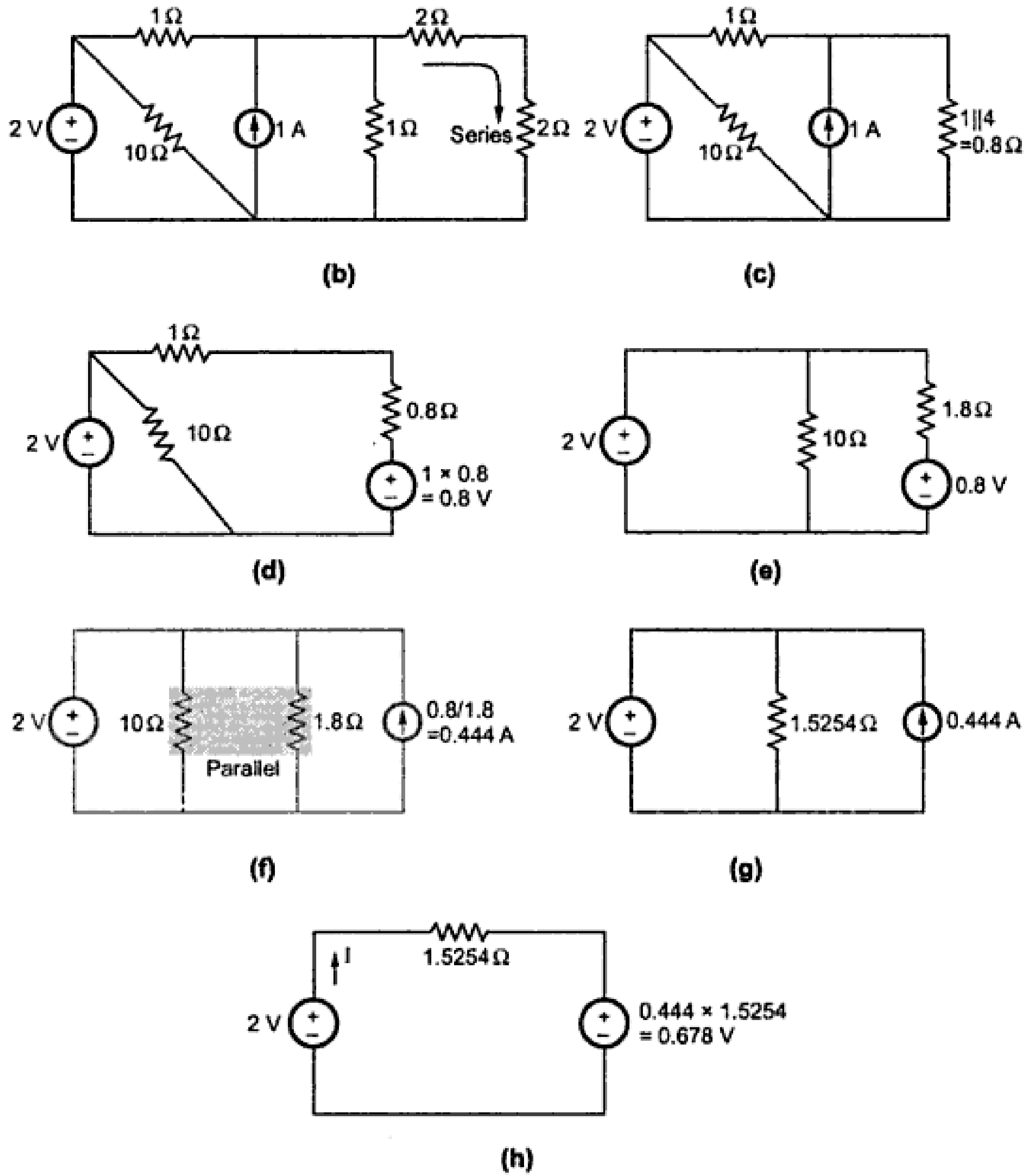


Fig. 1.91

Applying KVL,

$$-1.5254 \times I - 0.678 + 2 = 0$$

$$\therefore \quad I = \frac{2-0.678}{1.5254} = 0.8667 \text{ A}$$

Example 1.27 : *A Wheatstone bridge consists of the following resistances. Branches AB = 3 Ω, BC = 6 Ω, CD = 12 Ω and DA = 10 Ω.*

A 2 V cell is connected between points B and D and galvanometer of resistance 20 Ω is connected between A and C.

Find the current through galvanometer by Kirchhoff's laws.

Solution : Step 1 : Circuit diagram from given information.

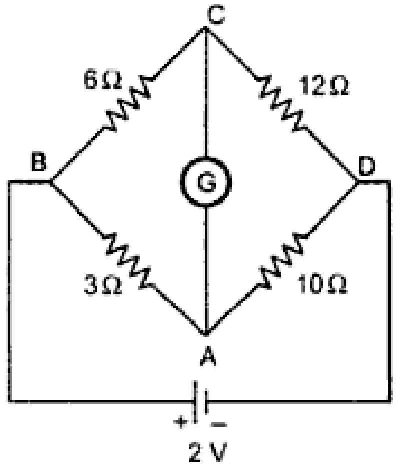


Fig. 1.92

Step 2 and 3 : Mark all the branch currents applying KCL at various codes and then mark the polarities of various voltages due to these assumed branch currents.

Step 4 : Apply KVL to various loops.

Loop 1 : Loop A-D-B-A (through cell)

$$-10\,(I_1 - I_2 + I_3) + 2 - 3\,(I_1 - I_2) = 0$$

$$\therefore \quad -13\,I_1 - 13\,I_2 - 10\,I_3 = -2 \qquad \dots (1)$$

Loop 2 : Loop A-D-C-A

$$-10\,(I_1 - I_2 + I_3) + 12\,(I_2 - I_3) - 20\,I_3 = 0$$

$$-10\,I_1 - 22\,I_2 - 42\,I_3 = 0 \qquad \dots (2)$$

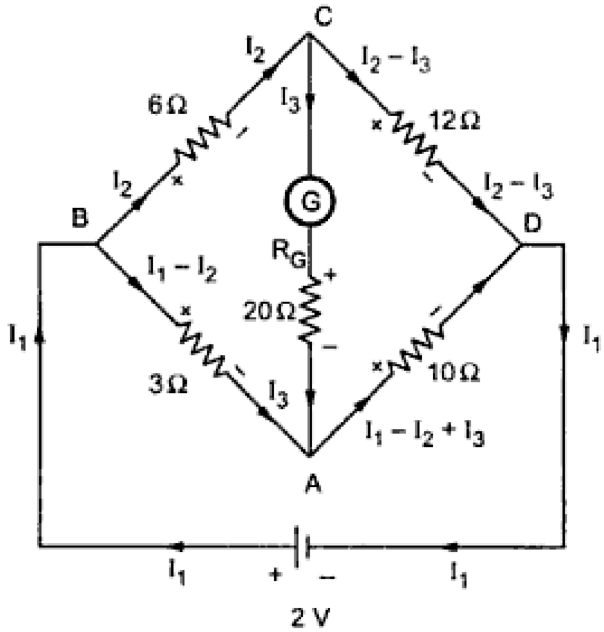


Fig. 1.92 (a)

Loop 3 : Loop A-B-C-A

$$+3(I_1 - I_2) - 6I_2 - 20I_3 = 0$$

$$\therefore \quad 3I_1 - 9I_2 - 20I_3 = 0 \qquad \dots (3)$$

We want current through galvanometer i.e. I_3. Applying Cramer's rule

$$D = \begin{vmatrix} -13 & 13 & -10 \\ -10 & 22 & -42 \\ 3 & -9 & -20 \end{vmatrix}$$

$$\therefore \quad D = 6156,$$

$$D_3 = \begin{vmatrix} -13 & 13 & -2 \\ -10 & 22 & 0 \\ 3 & -9 & 0 \end{vmatrix}$$

$$D_3 = -48$$

$$I_3 = \frac{D_3}{D} = \frac{-48}{6156} = -0.00779 \text{ A}$$

$\therefore$ **Current through galvanometer is 7.79 mA but negative sign indicates its direction is from A to C.**

Example 1.28 : *Two batteries A and B having e.m.fs of 209 V and 211 V having internal resistance 0.3 Ω and 0.8 Ω respectively are to be charged from a d.c. source of 225 V. If for that purpose they were connected in parallel and resistance of 4 Ω was connected between the supply and batteries to limit charging current, find*

i) Magnitude and direction of current through each battery.

ii) Power delivered by source.

Solution : The circuit diagram is as shown in Fig. 1.93.

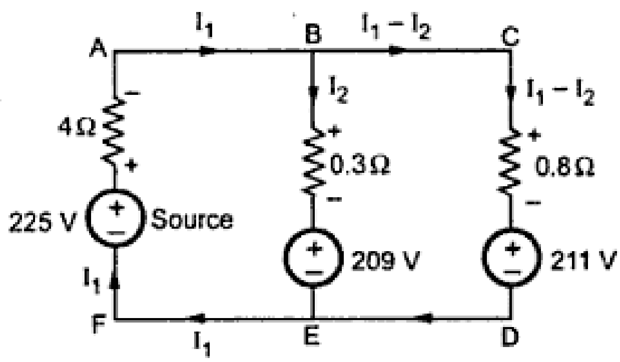


Fig. 1.93

We can use branch current method. Show the branch currents and polarities.

Apply KVL to different loops.

Loop 1 : Loop A-B-E-F-A

$$-0.3I_2 - 209 + 225 - 4I_1 = 0$$

i.e. $4I_1 + 0.3I_2 = 16 \qquad \dots (1)$

Loop 2 : Loop A-B-C-D-E-F-A,

i.e. $\quad -0.8(I_1 - I_2) - 211 + 225 - 4I_1 = 0$

i.e. $\quad -4.8I_1 - 0.8I_2 + 14 = 0 \qquad \dots (2)$

i.e. $4.8\,I_1 - 0.8\,I_2 = 14$

Solving equations (1) and (2) simultaneously,

$$I_1 = 3.663 \text{ A}$$

$$I_2 = 4.482 \text{ A}$$

$\therefore$ $$I_1 - I_2 = -0.8183 \text{ A}$$

i.e. it is in opposite direction to what is assured.

i) Magnitude of current through source = **3.663 A** ↑

Magnitude of current through battery A = **4.482 A** ↓

Magnitude of current through battery B = **0.8183 A** ↑

ii) Power delivered by,

$$\text{Source} = 225 \times 3.663 = \mathbf{824.175 \text{ watts}}$$

Example 1.29 : *Find the equivalent resistance between B and C.* **(April/May - 2004)**

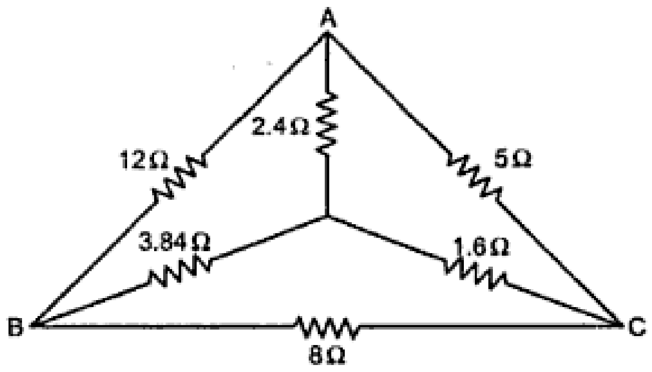


Fig. 1.94

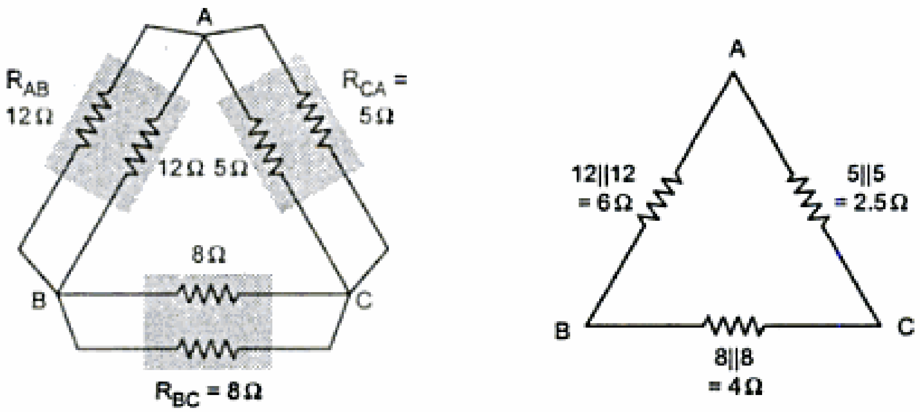
Solution : Converting internal star to delta,

$$R_{AB} = 3.84 + 2.4 + \frac{3.84 \times 2.4}{1.6} = 12 \ \Omega$$

$$R_{BC} = 3.84 + 1.6 + \frac{3.84 \times 1.6}{2.4} = 8 \ \Omega$$

$$R_{CA} = 2.4 + 1.6 + \frac{2.4 \times 1.6}{3.84} = 5 \ \Omega$$

The circuit reduces as shown in the Fig. 1.94.



Shaded : Parallel combinations

(a) (b)

Fig. 1.94

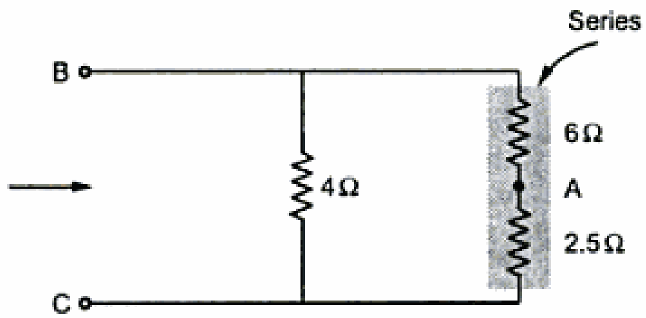


Fig. 1.95

Rearranging to calculate R_{BC},

$$\therefore R_{BC} = (4) \,||\, (6 + 2.5)$$

$$= \frac{4 \times 8.5}{4 + 8.5}$$

$$= \mathbf{2.72\ \Omega}$$

Example 1.30 : *Determine the value of R in the circuit when the current is zero in the branch CD.* **(April/May-2004)**

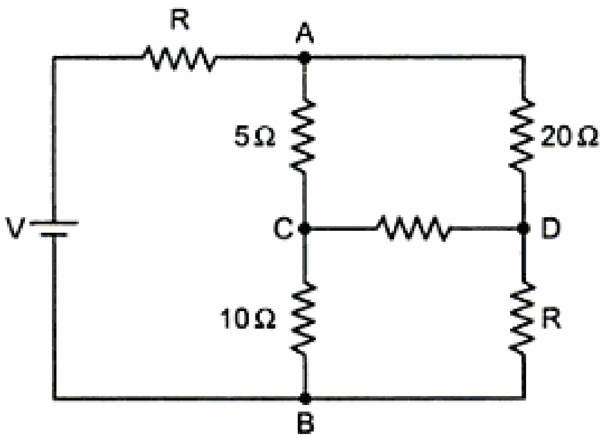


Fig. 1.96

Solution : Use Kirchhoff's laws. The various branch currents are shown in the Fig. 1.96(a).

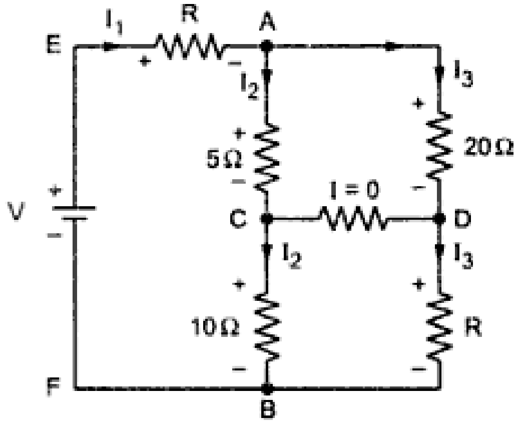


Fig. 1.96 (a)

Current through branch CD is zero.

Apply KVL to EACBFE,

$$-5\,I_2 - 10\,I_2 + V - I_1 R = 0$$

$$\therefore RI_1 + 15I_2 = V \qquad \text{... (1)}$$

Apply KVL to EADBFE,

$$-I_1 R - 20I_3 - RI_3 + V = 0$$

$$\therefore I_1 R + (20+R)\,I_3 = V \qquad \text{... (2)}$$

and $I_1 = I_2 + I_3$ ... KCL at node A.

i.e. $I_1 - I_2 - I_3 = 0$ ... (3)

$$D = \begin{vmatrix} R & 15 & 0 \\ R & 0 & 20+R \\ 1 & -1 & -1 \end{vmatrix} = 15\,(20 + R) + 15\,R + R\,(20 + R)$$

$$= R^2 + 60R + 300$$

$$D_2 = \begin{vmatrix} R & V & 0 \\ R & V & 20+R \\ 1 & 0 & -1 \end{vmatrix} = -\,RV + V\,(20 + R) + RV$$

$$= V\,(20 + R)$$

$$D_3 = \begin{vmatrix} R & 15 & V \\ R & 0 & V \\ 1 & -1 & 0 \end{vmatrix} = 15\,V - RV + RV$$

$$= 15\,V$$

$$\therefore \qquad I_2 = \frac{D_2}{D} = \frac{V\,(20+R)}{R^2 + 50R + 300}$$

$$\text{and} \qquad I_3 = \frac{D_3}{D} = \frac{15\,V}{R^2 + 50R + 300}$$

To have current through CD zero, the points C and D are **equipotential** i.e. drop across 5 Ω due to I_2 must be same as drop across 20 Ω due to I_3.

$$\therefore \qquad 5\,I_2 = 20\,I_3$$

$$\therefore \qquad 5\times\frac{V\,(20+R)}{R^2 + 50R + 300} = \frac{20\times 15\,V}{R^2 + 50R + 300}$$

$$\therefore \qquad 100 + 5\,R = 300$$

$$\therefore \qquad \mathbf{R} = \mathbf{40\ \Omega}$$

Example 1.31 : *Using source transformation, find the power delivered by the 50 V voltage source.* **(April/May 2004)**

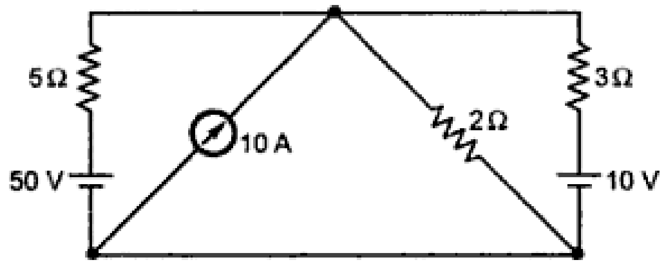


Fig. 1.97

Solution : Converting 10 V voltage source to current source and drawing a network,

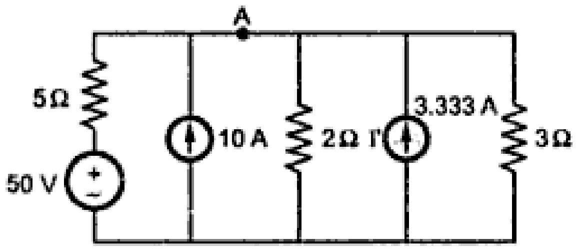


Fig. 1.97 (a)

$$I' = \frac{10}{3} = 3.333 \text{ A}$$

Adding two current sources, we get single current source of value (10 + 3.333) A i.e. 13.333 A.

Find equivalent resistance for parallel connection of 2 Ω and 3 Ω i.e.

$$R' = \frac{2\times 3}{(2+3)} = \frac{6}{5} = 1.2\ \Omega$$ and redrawing a network as shown in the Fig. 1.97 (b),

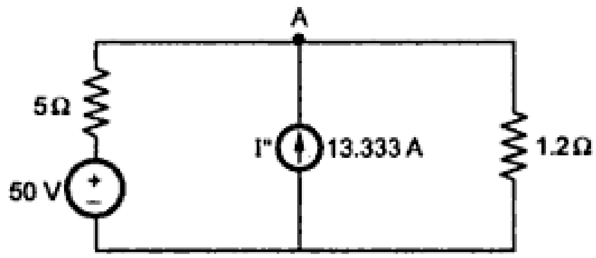


Fig. 1.97 (b)

Now, converting current source back to voltage source,

$$V' = I^{"} \cdot R = (13.333)\ (1.2) = 15.996 \text{ volts}$$

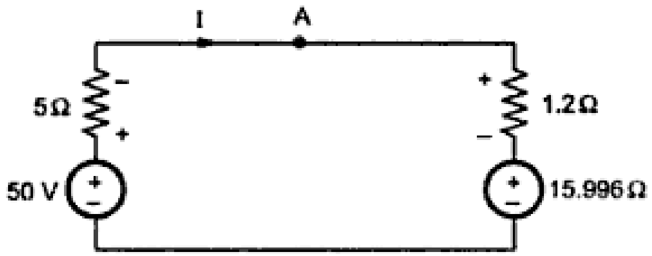


Fig. 1.97 (c)

Applying KVL,

$$-1.2\ I - 15.996 + 50 - 5\ I = 0$$

$$\therefore \quad I\ (1.2 + 5) = (50 - 15.996)$$

$$\therefore \quad I = \frac{(50-15.996)}{(5+1.2)}$$

$$= 5.484\ A$$

∴ Power supplied by 50 V source,

$$P = (50)\ (5.484)$$

$$= \mathbf{274.2\ watts}$$

Example 1.32 : *Calculate the current through 6 Ω resistance of the given network by application of Kirchhoff's laws.* **[April/May-2004]**

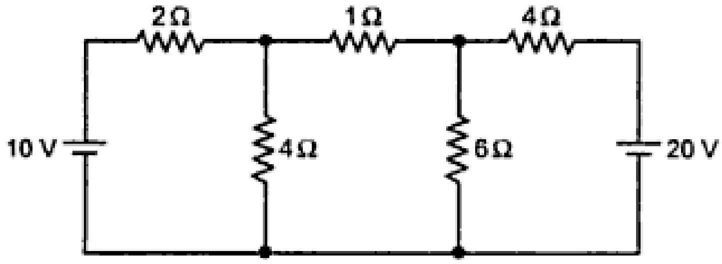


Fig. 1.98

Solution : Mark all the **branch currents** starting from positive of 10 V source. Mark all the polarities for different voltages across the resistances.

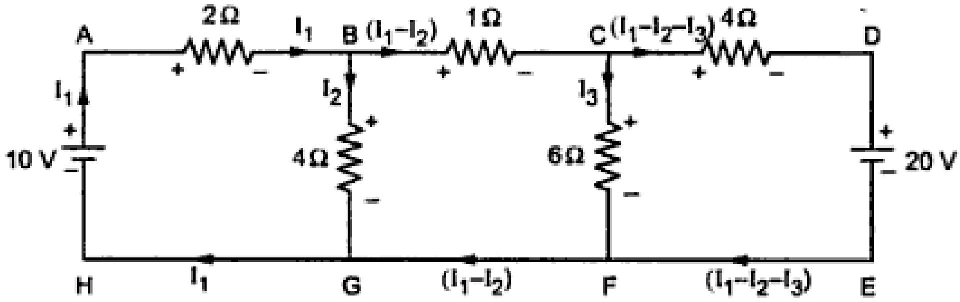


Fig. 1.98 (a)

Now applying KVL to different loops,

Loop 1 : A-B-G-H-A $\quad -2\ I_1 - 4\ I_2 + 10 = 0$ ... (1)

Loop 2 : B-C-F-G-B $\quad -1\ (I_1 - I_2) - 6\ I_3 + 4\ I_2 = 0$ ... (2)

Loop 3 : C-D-E-F-G $\quad -4(I_1 - I_2 - I_3) - 20 + 6I_3 = 0$ ... (3)

Rewriting above equations by taking constants on one side,

$$2I_1 + 4I_2 = 10$$
$$-I_1 + 5I_2 - 6I_3 = 0$$
$$-4I_1 + 4I_2 + 10I_3 = 20$$

Applying Cramer's Rule, $\quad D = \begin{vmatrix} 2 & 4 & 0 \\ -1 & 5 & -6 \\ -4 & 4 & 10 \end{vmatrix} = 284$

Now we are interested in finding current through 6 Ω i.e. I_3.

$$\therefore \quad D_3 = \begin{vmatrix} 2 & 4 & 10 \\ -1 & 5 & 0 \\ -4 & 4 & 20 \end{vmatrix} = 440$$

$$\therefore \quad I_3 = \frac{D_3}{D} = \frac{440}{284} = \mathbf{1.5492\ A}$$

As the answer is positive, assumed direction of I_3 is correct. So current through 6 Ω resistance is 1.5492 A flowing from C to F.

Example 1.33 : *Calculate the resistance R_{ab} when all the resistance values are equal to 1 Ω for the circuit in Fig. 1.99.* **(April/May-2005)**

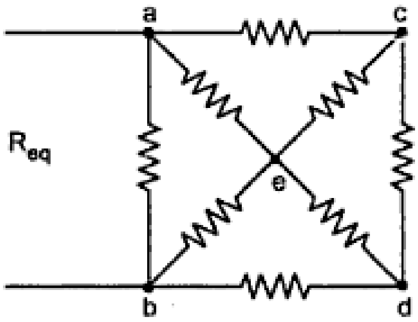


Fig. 1.99

Solution : Converting deltas aeb and ced to equivalent star,

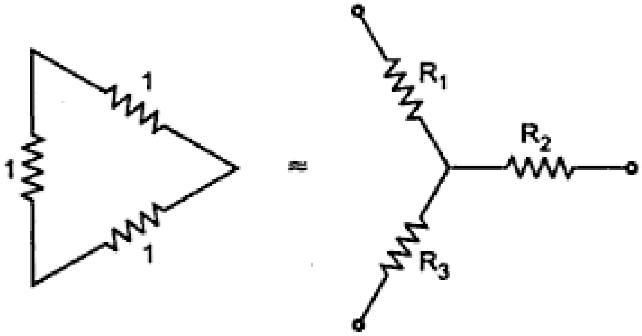


Fig. 1.99(a)

$$R_1 = R_2 = R_3 = \frac{1\times1}{1+1+1} = \frac{1}{3}\ \Omega$$

The circuit reduces as shown in the Fig. 1.99 (b)

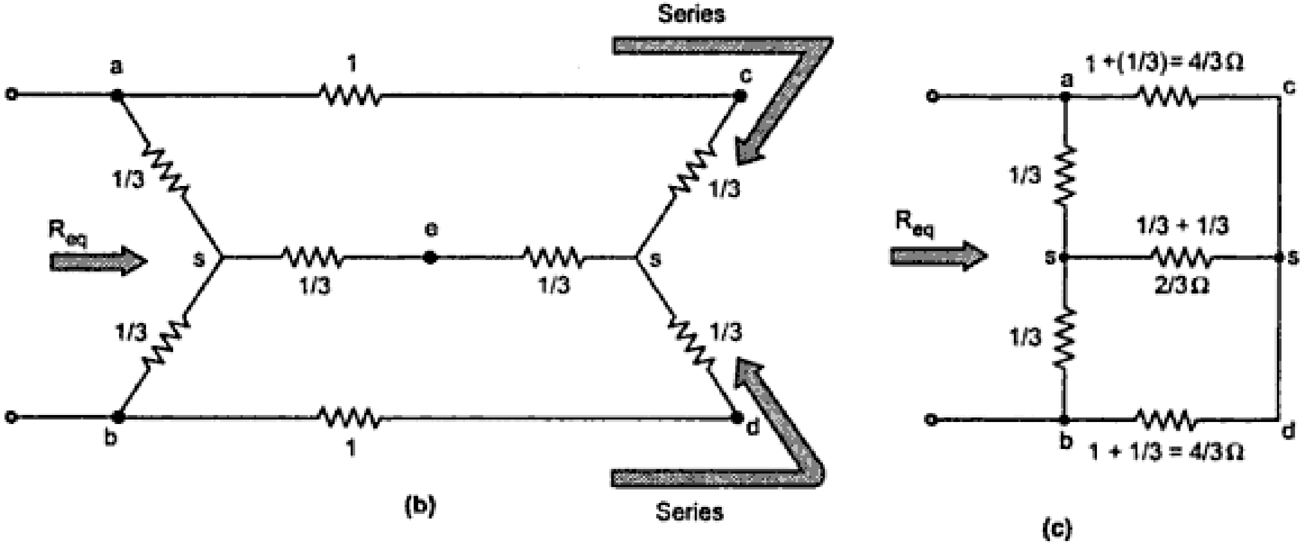


Fig. 1.99

Converting sbds delta into star, we get

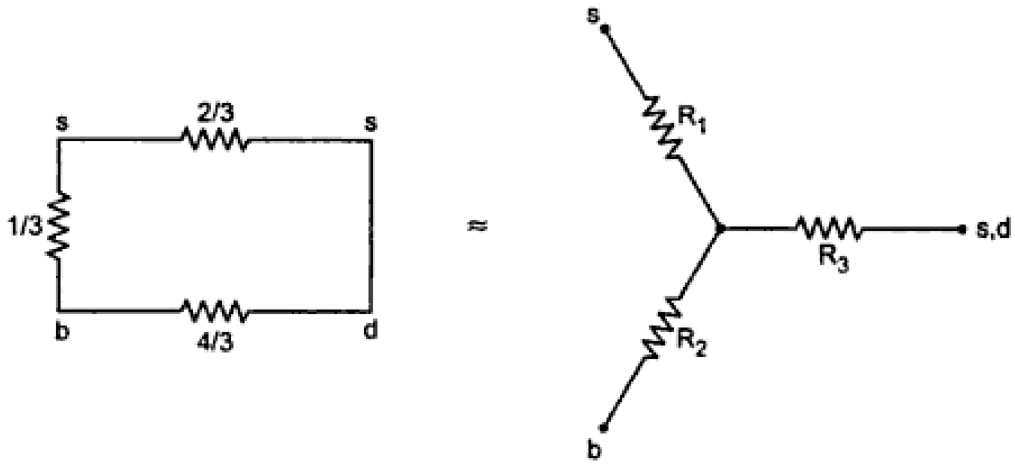


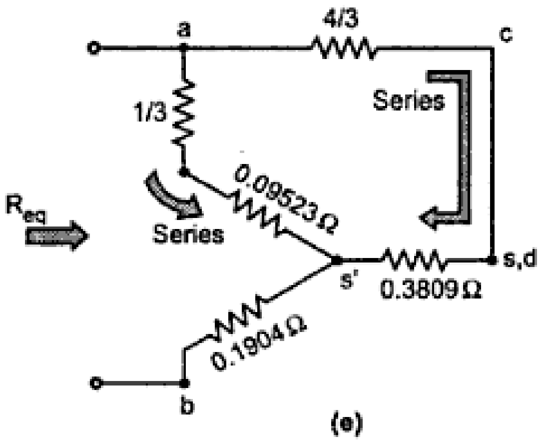
Fig. 1.99 (d)

$$R_1 = \frac{\frac{2}{3}\times\frac{1}{3}}{\frac{2}{3}+\frac{1}{3}+\frac{4}{3}} = 0.09523\ \Omega$$

$$R_2 = \frac{\frac{1}{3}\times\frac{4}{3}}{\frac{2}{3}+\frac{1}{3}+\frac{4}{3}} = 0.1904\ \Omega$$

$$R_3 = \frac{\frac{2}{3}\times\frac{4}{3}}{\frac{2}{3}+\frac{1}{3}+\frac{4}{3}} = 0.3809\ \Omega$$

The circuit reduces as shown in the Fig. 1.99 (e)



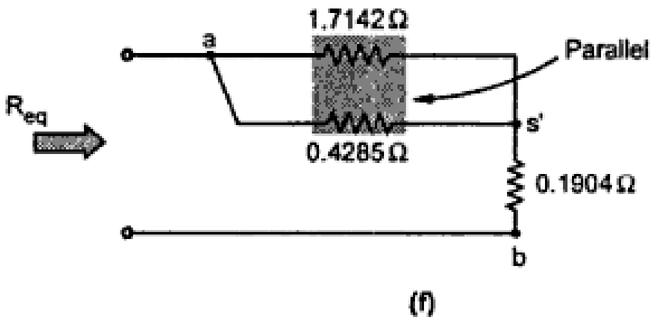


Fig. 1.99

$$\therefore \qquad R_{eq} = (1.7142 \| 0.4285) + 0.1904 = \mathbf{0.5332\ \Omega}$$

Example 1.34 : *An impedance consumes a power of 60 watts and takes a current of 10 – j 8 A when connected to a source of 'a + j 5' V. Find 'a' and circuit elements.*

Solution : The given circuit is shown in the Fig. 1.100.

$$Z = \frac{V}{I} = \frac{a+j5}{10-j8}$$

$$\therefore \qquad R + jx = \frac{a+j5}{10-j8}$$

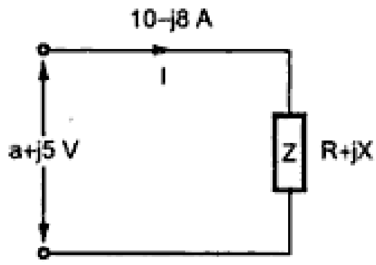


Fig. 1.100

Rationalising right hand side,

$$\therefore \qquad R + jx = \frac{(a+j5)(10+j8)}{(10-j8)(10+j8)}$$

$$= \frac{(a+j5)(10+j8)}{(100) - (j^2\ 64)} = \frac{(10a-40)+j(50+8a)}{164}$$

$$\therefore \qquad R + jx = \frac{10a-40}{164} + j\,\frac{50+8a}{164} \qquad \ldots (1)$$

Equating real and imaginary parts,

$$R = \frac{10\,a-40}{164} \qquad \ldots (2)$$

and

$$X = \frac{50+8\,a}{164} \qquad \ldots (3)$$

The power consumed by impedance Z is,

$$P = (|I|)^2 \times R$$

$$\therefore \quad 60 = \left(\sqrt{10^2 + 8^2}\right)^2 \times R$$

$$\therefore \quad 60 = 164\ R$$

$$\therefore \quad R = 0.366\ \Omega \qquad \dots (4)$$

Substituting in (2),

$$0.366 = \frac{10\,a - 40}{164}$$

$$\therefore \quad 10\ a = 40 + 0.366 \times 164$$

$$\therefore \quad a = \mathbf{10} \qquad \dots (5)$$

Substituting in (3), $X = \dfrac{50+80}{164} = \mathbf{0.7926\ \Omega}$

Hence the voltage applied is 10 +j 5 volts while the impedance is **(0.366 + j0.7926) Ω.**

Example 1.35 : *The network shown in the Fig. 1.101 consists of two star connected circuits in parallel. Obtain the single delta connected equivalent.*

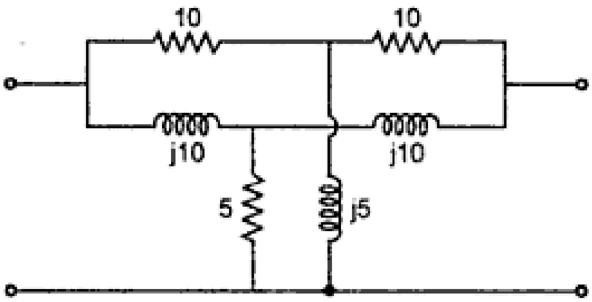


Fig. 1.101

Solution : Converting both the stars to delta,

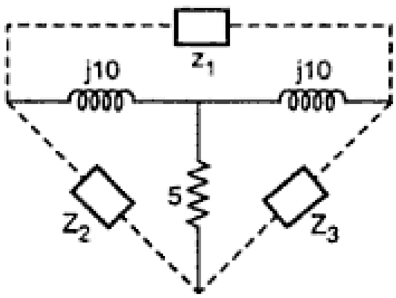


Fig. 1.101 (a)

$$Z_1 = j10 + j10 + \frac{j10 \times j10}{5} = -20 + j20$$

$$Z_2 = j10 + 5 + \frac{j10 \times 5}{j10} = 10 + j10$$

$$Z_3 = j10 + 5 + \frac{j10 \times 5}{j10} = 10 + j10$$

$$Z_1' = 10 + 10 + \frac{10 \times 10}{j5} = 20 - j20 \qquad \dots \frac{1}{j} = -j$$

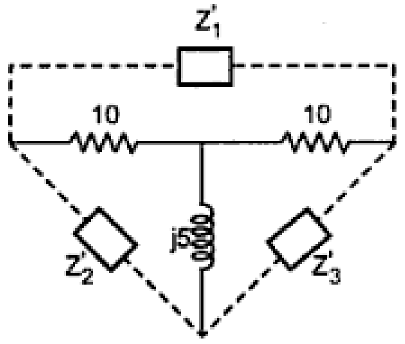


Fig. 1.101 (b)

$$Z_2' = 10 + j5 + \frac{10 \times j5}{10} = 10 + j10$$

$$Z_3' = 10 + j5 + \frac{10 \times j5}{10} = 10 + j10$$

Connecting both the deltas we get the circuit as shown in the Fig. 1.101 (c).

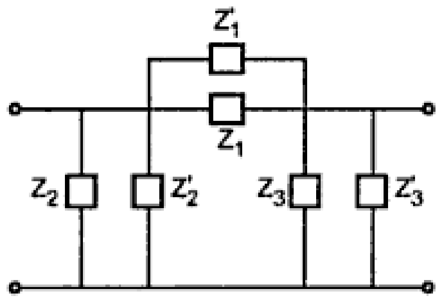


Fig. 1.101 (c)

$$Z_1 \parallel Z_1' = \frac{(-20+j20)(20-j20)}{-20+j20+20-j20}$$

$$= \infty \text{ open circuit}$$

$$Z_2 \parallel Z_2' = (10 + j10) \parallel (10 + j10)$$

$$= 5 + j5$$

$$Z_3 \parallel Z_3' = 5 + j5$$

So effective single delta connected circuit is as shown in the Fig. 1.101 (d).

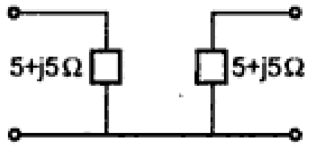


Fig 1.101 (d)

Example 1.36 : *Obtain the delta connected equivalent of the network shown in Fig.1.102.*

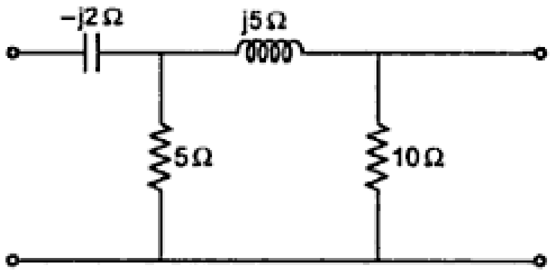
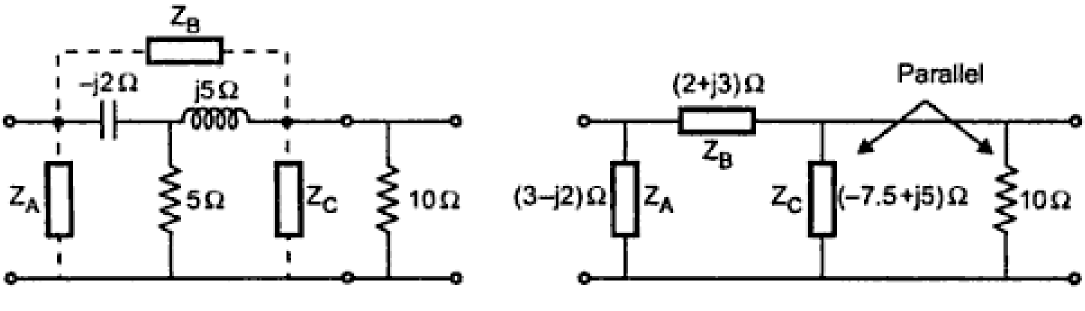


Fig. 1.102

Solution : Consider a star network formed with – j2 Ω, j5 Ω and 5 Ω as shown in Fig. 1.102 (a).



(a) (b)

Fig. 1.102

Converting star network into its equivalent delta network as follows.

$$Z_A = -j2 + 5 + \frac{(-j2)\,(5)}{j5} = -j2 + 5 - 2 = (3 - j2)\ \Omega$$

$$Z_B = -j2 + j5 + \frac{(-j2)\,(j5)}{5} = -j2 + j5 - 2 = (2 - j3)\ \Omega$$

$$Z_C = j5 + 5 + \frac{(j5)\,(5)}{-j2} = j5 + 5 - 12.5 = (-7.5 + j5)\ \Omega$$

Replacing star network by its equivalent delta network as shown in the Fig.1.102(b). It is clear that Z_C and 10 Ω in parallel. Hence we can write,

$$Z'_C = Z_C \parallel 10 = \frac{(-7.5+j\,5)\,(10)}{-7.5+j\,5+10} = \frac{(-7.5+j\,5)\,(10)}{2.5+j\,5}$$

$$= \frac{(9.0138\angle 146.3^\circ)(10\angle 0^\circ)}{(5.5901\angle 63.43^\circ)} = 16.1245\angle 82.87^\circ$$

$$= (2 + j16)\ \Omega$$

Hence delta connected equivalent of the network is as shown in the Fig. 1.102 (c).

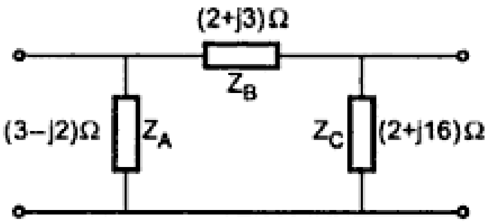


Fig. 1.102 (c)

11. *Find the equivalent resistance as viewed through the terminals,*

    i) B and C *ii) A and N*

(Ans. : i) 1.33 Ω ii) 0.77 Ω)

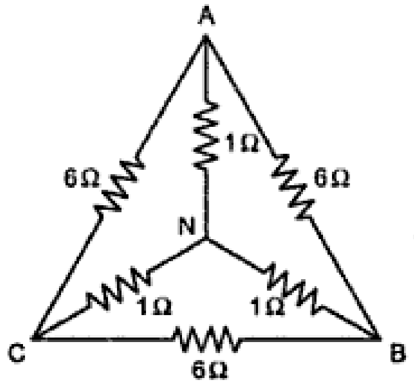


Fig. 1.107

12. *Determine current in branch A- B by Kirchhoff's laws.*

(Ans. : I_{A-B} = 0.5 A)

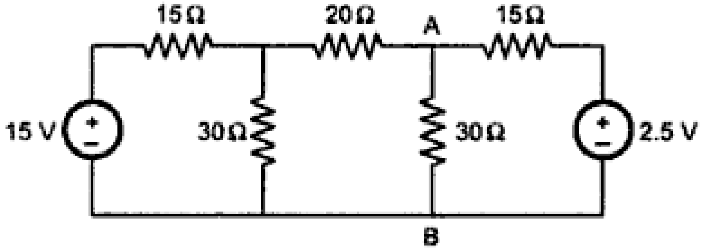


Fig. 1.108

13. *In the following circuit, determine*

    i) I_1, I_2 *and* I_3, *ii) Value of R and* *iii) Value of* E_X

(Ans. : I_1 = 0.3 A, I_2 = 1.5 A, E_x = 174 V, I_3 = 0.7 A, R = 71.4 Ω)

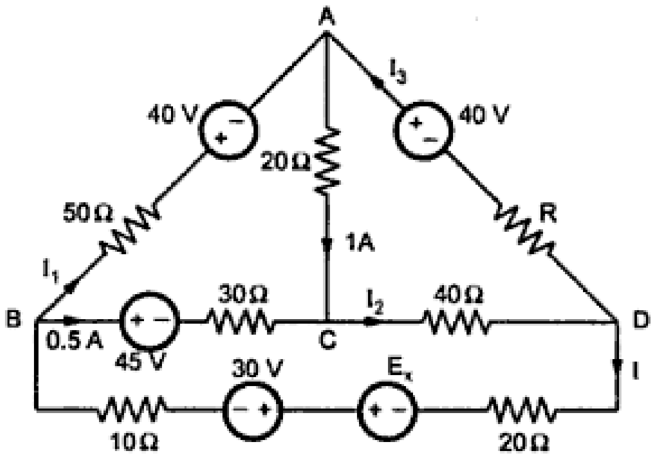


Fig. 1.109

14. *Reduce the network, using source transformation across the terminals A-B.*

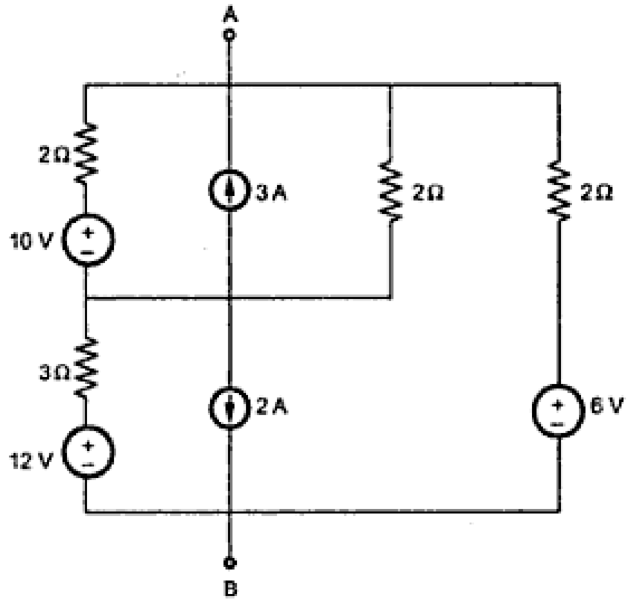


Fig. 1.110

15. *Derive the relationship to express three star connected resistances into equivalent delta.*
16. *Derive the relationship to express three delta connected resistances into equivalent star.*
17. *Two voltmeters A and B, having resistances of 5.2 kΩ and 15 kΩ respectively are connected in series across 240 V supply. What is the reading on each voltmeter ?* **(Ans. : 61.78 V, 178.21 V)**
18. *Two resistances 15 Ω and 20 Ω are connected in parallel. A resistance of 12 Ω is connected in series with the combination . A voltage of 120 V is applied across the entire circuit. Find the current in each resistance, voltage across 12 Ω resistance and power consumed in all the resistances.*

(Ans. : 3.33 A, 2.5 A, 70 V)

19. *A resistance R is connected in series with a parallel circuit comprising two resistances of 12 and 8 Ω. The total power dissipated in the circuit is 700 watts when the applied voltage is 200 V. Calculate the value of R.* **(Ans. : 52.3428 Ω)**
20. *In the series parallel circuit shown in the Fig. 1.111, find the*

    i) voltage drop across the 4 Ω resistance *ii) the supply voltage V*

(Ans. : 45 V, 140 V)

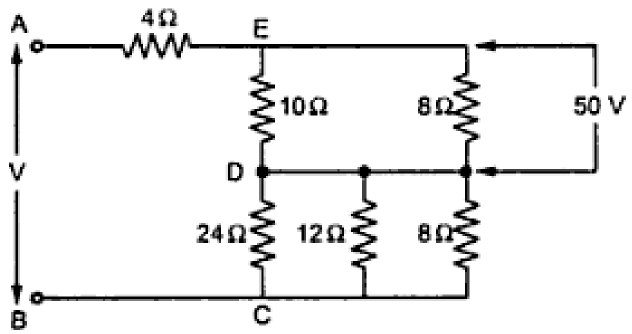


Fig. 1.111

21. *Find the current in all the branches of the network shown in the Fig. 1.112.*

(Ans. : 39 A, 21 A, 39 A, 81 A, 11 A, 41 A)

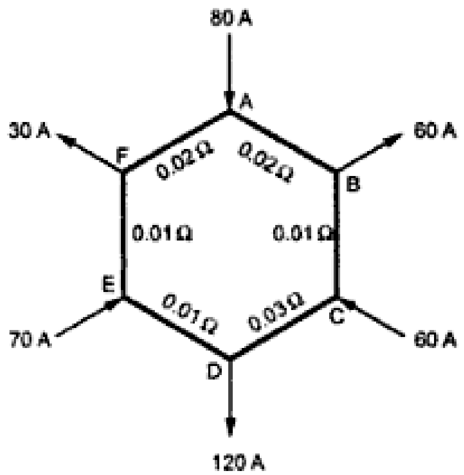


Fig. 1.112

22. *If the total power dissipated in the circuit shown in the Fig. 1.113 is 18 watts, find the value of R and current through it.*

(Ans. : 12 Ω, 0.6 A)

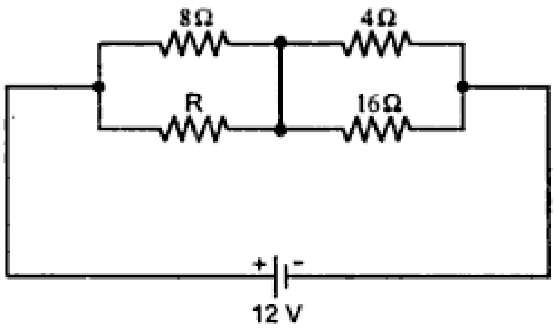


Fig. 1.113

23. *The current in the 6 Ω resistance of the network shown in the Fig. 1.114 is 2 A. Determine the currents in all the other resistances and the supply voltage V.*

(Ans. : 1.5 A, 2.5 A, 1 A, 46 V)

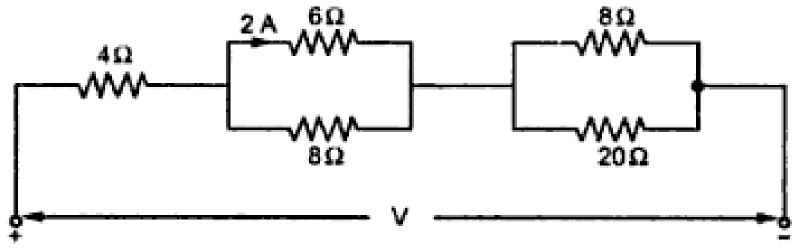


Fig. 1.114

24. *A particular battery when loaded by a resistance of 50 Ω gives the terminal voltage of 48.6 V. If the load resistance is increased to 100 Ω, the terminal voltage is observed to be 49.2 V.*

Determine, i) E.M.F. of battery

ii) Internal resistance of battery

25. *A resistance of 10 Ω is connected in series with the two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken will be 1.5 A from 20 V supply applied ?* **(Ans. : 6 Ω)**

26. *Two storage batteries A and B are connected in parallel to supply a load of 0.3 Ω. The open circuit e.m.f. of A is 11.7 V and that of B is 12.3 V. The internal resistances are 0.06 Ω and 0.05 Ω respectively. Find the current supplied to the load.* **(Ans. : 36.778 A)**

27. *A network ABCD is made up as follows :*

    AB has a cell of 2 V and negligible resistance, with the positive terminal connected to A; BC is a resistor of 25 Ω ; CD is a resistor of 100 Ω ; DA is a battery of 4 V and negligible resistance with positive terminal connected to D; A.C. is a milliammeter of resistance 10 Ω. Calculate the reading on the milliammeter. **(Ans. :26.67 mA)**

28. *Find the equivalent impedance across the terminals A-B.*

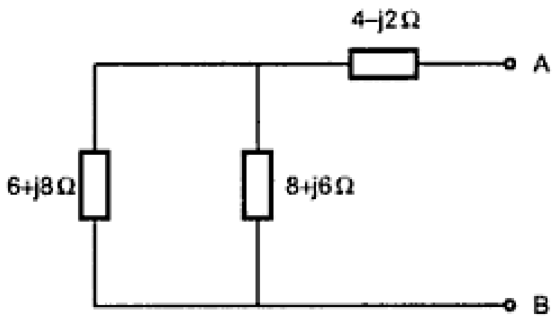


Fig. 1.115

(Ans. : 7.57 + j 1.57 Ω)

29. *Find the single voltage source equivalent across the terminals A-B. Hence obtain the single current source equivalent also.*

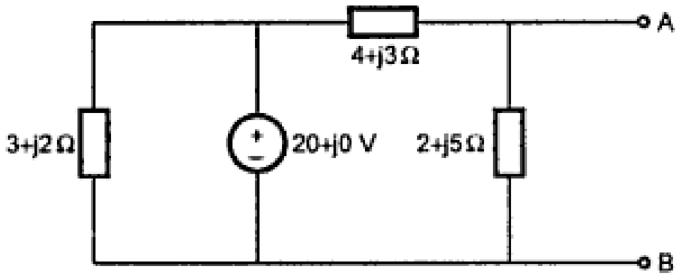


Fig. 1.116

(Ans. : V = 10.78 ∠15.06° V, Z = 1.66 + j 2.12 Ω, I = 4 ∠ – 36.86° A)

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Technical Publications Pune
1, Amit Residency, 412 Shaniwar Peth, Pune - 411030, M.S., India.
Telefax : +91 (020) 24495496/97, Email : technical@vtubooks.com
Visit us at : www.vtubooks.com