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Basic Electrical Engineering

U. A. Bakshi
V. U. Bakshi

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Table of Contents

1 Fundamentals of Electricity

1.1 Introduction

The study of an electrical engineering involves the analysis of the energy transfer from one form to another or from one point to another. So before beginning the actual study of an electrical engineering, it is necessary to discuss the fundamental ideas about the basic elements of an electrical engineering like electromotive force, current, resistance etc. The electricity is related with number of other types of systems like mechanical, thermal etc. There involves the transfer of different forms of energy into electrical or otherwise. To analyse such transfer, it is necessary to revise the S.I. units of measurement of different quantities like work, power, energy etc. in various systems.

This chapter explains the concept of basic electrical parameters alongwith the effect of temperature on resistance. The chapter involves the discussion of the characteristics of series and parallel circuits. At the end, the chapter includes the revision of units in different systems and their inter relations.

1.2 The Structure of Matter

In the understanding of fundamentals of electricity, the knowledge of the structure of matter plays an important role. The matter which occupies the space may be soild, liquid or gaseous. The molecules and atoms, of which all substances are composed are not at all elemental, but are themselves made up of simpler entities. We know this because we, up to certain extent, are successful in breaking atoms and studying the resulting products. For instance, such particles are obtained by causing ultraviolet light to fall on cold metal surfaces, such particles are spontaneously ejected from the radioactive elements. So these particles are obtained from many different substances under such widely varying conditions. It is believed that such particles are one of the elemental constituents of all matter, called **electrons.**

Infact, according to the **modern electron theory,** atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron,** the **proton** and the **electron.** The proton is defined as positively charged while the electron is defined as negatively charged. The neutron is uncharged i.e. neutral in nature possessing no charge. The mass of neutron and proton is same while the electron is very light, almost

1/1840th the mass of the neutron and proton. The following table gives information about these three particles.

Fundamental particles of matter	Symbol	Nature of charge possessed	Mass in kg.
Neutron	n	0	1.675×10^{-27}
Proton	p+	+	1.675×10^{-27}
Electron	e^-	–	9.107×10^{-31}

Table 1.1

1.2.1 Structure of an Atom

All of the protons and neutrons are bound together into a compact nucleus. **Nucleus** may be thought of as a central sun, about which electrons revolve in a particular fashion. This structure surrounding the nucleus is referred as the **electron cloud.**

In the normal atom the number of protons equal to the number of electrons. An atom as a whole is electrically neutral. The electrons are arranged in different **orbits**. The nucleus exerts a force of attraction on the revolving electrons and hold them together. All these different orbits are called **shells** and possess certain energy. Hence these are also called **energy shells** or **quanta**. The orbit which is closest to the nucleus is always under the tremendous force of attraction while the orbit which is farthest from the nucleus is under very weak force of attraction.

> **Key Point** : *The electron or the electrons revolving in farthest orbit are hence loosely held to the nucleus. Such a shell is called the valence shell. And such electrons are called* ***valence electrons.***

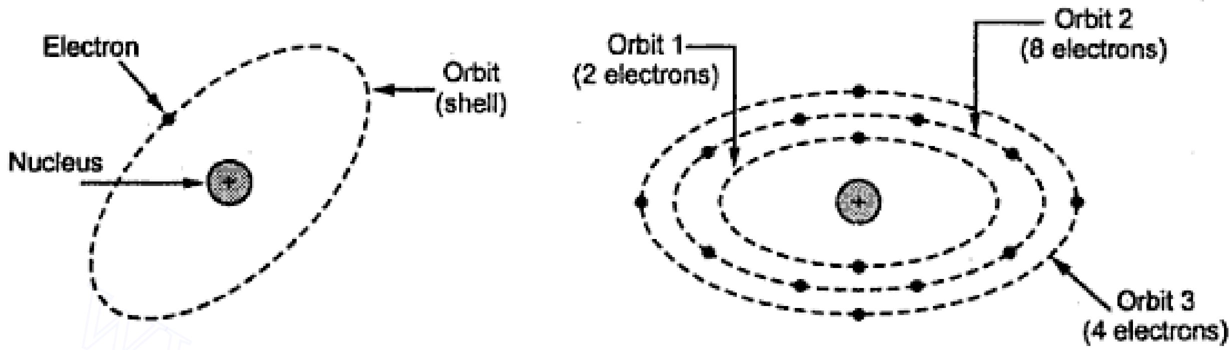
In some atoms such valence electrons are so loosely bound to the nucleus that at room temperature the additional energy imparted to the valence electrons causes them to escape from the shell and exist as **free electrons**. Such free electrons are basically responsible for the flow of electric current through metals.

> **Key Point** : *More the number of free electrons, better is the metal for the conduction of the current. For example, copper has 8.5×10^{28} free electrons per cubic meter and hence it is a good conductor of electricity.*

The electrons which are revolving round the nucleus, not revolve in a single orbit. Each orbit consists of fixed number of electrons. In general, an orbit can contain a maximum of $2n^2$ electrons where n is the number of orbit. So first orbit or shell can occupy maximum of 2×1^2 i.e. 2 electrons while the second shell can occupy maximum of 2×2^2 i.e. 8 electrons and so on. The exception to this rule is that the valence shell can occupy maximum 8 electrons irrespective of its number. Let us see the structure of two different atoms.

1) Hydrogen : This atom consists of one proton and one electron revolving around the nucleus. This is the simplest atom. This is shown in the Fig. 1.1 (a). The dot represents an electron while nucleus is represented by a circle with the positive sign inside it.

2) Silicon : This atom consists of 14 electrons. These revolve around the nucleus in three orbits. The first orbit has maximum 2 electrons, the second has maximum 8 electrons and the third orbit has remaining 4 electrons. This is shown in the Fig. 1.1 (b).



(a) Hydrogen atom **(b) Silicon atom**

Fig. 1.1

The 4 electrons located in the farthest shell are loosely held by the nucleus and generally available as free electrons. If by any means some of the electrons are removed, the negative charge of that atom decreases while positively charged protons remain same. The resultant charge on the atom remains more positive in nature and such element is called **positively charged.** While if by any means the electrons are added, then the total negative charge increases than positive and such element is called **negatively charged.**

1.3 Concept of Charge

In all the atoms, there exists number of electrons which are very loosely bound to its nucleus. Such electrons are free to wonder about, through the space under the influence of specific forces. Now when such electrons are removed from an atom it becomes positively charged. This is because of loosing negatively charged particles i.e. electrons from it. As against this, if excess electrons are added to the atom it becomes negatively charged.

Key Point: *Thus total deficiency or addition of excess electrons in an atom is called its charge and the element is said to be charged.*

The following table shows the different particles and charge possessed by them.

Particle	Charge possessed in Coulomb	Nature
Neutron	0	Neutral
Proton	1.602×10^{-19}	Positive
Electron	1.602×10^{-19}	Negative

Table 1.2

1.3.1 Unit of Charge

As seen from the Table 1.2 that the charge possessed by the electron is very very small hence it is not convenient to take it as the unit of charge.

The unit of the measurement of the charge is **Coulomb**.

The charge on one electron is 1.602×10^{-19}, so one coulomb charge is defined as the charge possessed by total number of ($1/\ 1.602 \times 10^{-19}$) electrons i.e. 6.24×10^{18} number of electrons.

Thus, $\boxed{1 \text{ coulomb } = \text{ charge on } 6.24 \times 10^{18} \text{ electrons}}$

From the above discussion it is clear that if an element has a positive charge of one coulomb then that element has a deficiency of 6.24×10^{18} number of electrons.

Key Point : *Thus, addition or removal of electrons causes the change in the nature of the charge possessed by the element.*

1.4 Concept of Electromotive Force and Current

It has been mentioned earlier that the free electrons are responsible for the flow of electric current. Let us see how it happens.

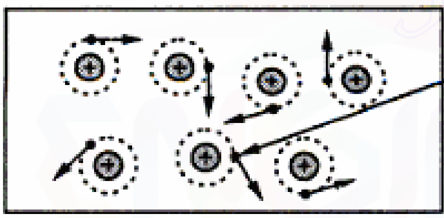


Fig. 1.2 Inside the piece of a conductor

To understand this, first we will see the enlarged view of the inside of a piece of a conductor. A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions as shown in the Fig. 1.2.

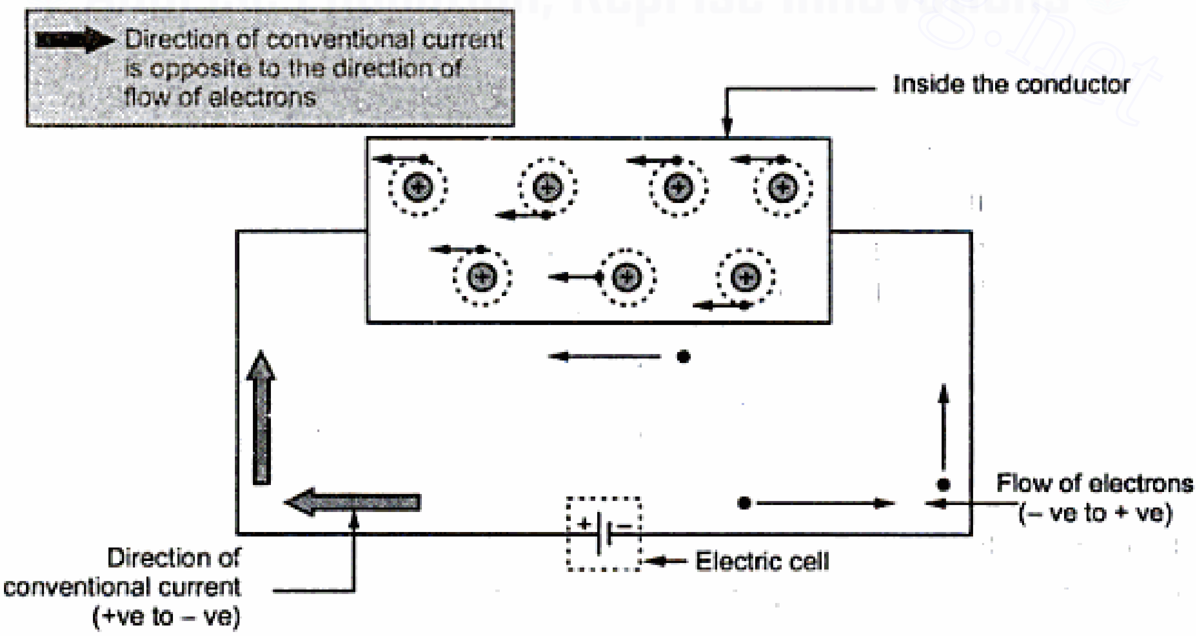


Fig. 1.3 The flow of current

The small electrical effort, externally applied to such conductor makes all such free electrons to drift along the metal in a definite particular direction. This direction depends on how the external electrical effort is applied to the conductor. Such an electrical effort may be an electrical cell, connected across the two ends of a conductor. Such physical phenomenon is represented in the Fig. 1.3.

Key Point: *An electrical effort required to drift the free electrons in one particular direction, in a conductor is called* **Electromotive Force** *(e.m.f.)*

The metal consists of particles which are charged. The like charges repel while unlike charges attract each other. But as external electric effort is applied, the free electrons as are negatively charged, get attracted by positive of the cell connected. And this is the reason why electrons get aligned in one particular direction under the influence of an electromotive force.

Key Point : *The electric effort i.e. e.m.f. is maintained across the positive and negative electrodes of the cell, due to the chemical action inside the solution contained in the cell.*

Atoms, when they loose or gain electrons, become charged accordingly and are called **ions**. Now when free electron gets dragged towards positive from an atom it becomes positively charged ion. Such positive ion drags a free electron from the next atom. This process repeats from atom to atom along the conductor. So there is flow of electrons from negative to positive of the cell, externally through the conductor across which the cell is connected. This movement of electrons is called an **Electric Current**.

The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This is called **direction of conventional current.**

Key Point: *Direction of conventional current is from positive to negative terminal while direction of flow of electrons is always from negative to positive terminal, through the external circuit across which the e.m.f. is applied.*

We are going to follow direction of the conventional current throughout this book. i.e. from positive to negative terminal, of the battery through the external circuit.

1.5 Relation between Charge and Current

The current is flow of electrons. Thus current can be measured by measuring how many electrons are passing through material per second. This can be expressed in terms of the charge carried by those electrons in the material per second. So the **flow of charge per unit time** is used to quantify an electric current.

Key Point: *So current can be defined as rate of flow of charge in an electric circuit or in any medium in which charges are subjected to an external electric field.*

The charge is indicated by Q coulombs while current is indicated by I. The unit for the current is **Amperes** which is nothing but coulombs/sec. Hence mathematically we can write the relation between the charge (Q) and the electric current (I) as,

$$\boxed{I = \frac{Q}{t} \quad \text{Amperes}}$$

Where I = average current flowing

Q = total charge transferred

t = time required for transfer of charge.

Definition of 1 Ampere : *A current of 1 Ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.*

Now 1 coulomb is 6.24×10^{18} number of electrons. So 1 ampere current flow means flow of 6.24×10^{18} electrons per second across a section taken any where in the circuit.

$$\boxed{1 \text{ Ampere current} = \text{Flow of } 6.24 \times 10^{8} \text{ electrons per second}}$$

1.6 Concept of Electric Potential and Potential Difference

When two similarly charged particles are brought near, they try to repel each other while dissimilar charges attract each other. This means, every charged particle has a tendency to do work.

> **Key Point :** *This ability of a charged particle to do the work is called its* ***electric potential****. The unit of electric potential is* ***volt****.*

The electric potential at a point due to a charge is one volt if one joule of work is done in bringing a unit positive charge i.e. positive charge of one coulomb from infinity to that point.

Mathematically it is expressed as,

$$\boxed{\text{Electrical Potential} = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}}$$

Let us define now the potential difference.

It is well known that, flow of water is always from higher level to lower level, flow of heat is always from a body at higher temperature to a body at lower temperature. Such a level difference which causes flow of water, heat and so on, also exists in electric circuits. In electric circuits flow of current is always from higher electric potential to lower electric potential. So we can define potential difference as below :

> **Key Point:** *The difference between the electric potentials at any two given points in a circuit is known as Potential Difference (p.d.). This is also called voltage between the two points and measured in volts. The symbol for voltage is V.*

For example, let the electric potential of a charged particle A is say V_1 while the electric potential of a charged particle B is say V_2. Then the potential difference between the two particles A and B is $V_1 - V_2$. If $V_1 - V_2$ is positive we say that A is at higher potential than B while if $V_1 - V_2$ is negative we say that B is at higher potential than A.

Key Point: *The potential difference between the two points is one volt if one joule of work is done in displacing unit charge (1 coulomb) from a point of lower potential to a point of higher potential.*

Consider two points having potential difference of V volts between them, as shown in the Fig. 1.4. The point A is at higher potential than B. As per the definition of volt, the V joules of work is to be performed to move unit charge from point B to point A.

Fig. 1.4

Thus, when such two points, which are at different potentials are joined together with the help of wire, the electric current flows from higher potential to lower potential i.e. the electrons start flowing from lower potential to higher potential. Hence, to maintain the flow of electrons i.e. flow of electric current, there must exist a potential difference between the two points.

Key Point: *No current can flow if the potential difference between the two points is zero.*

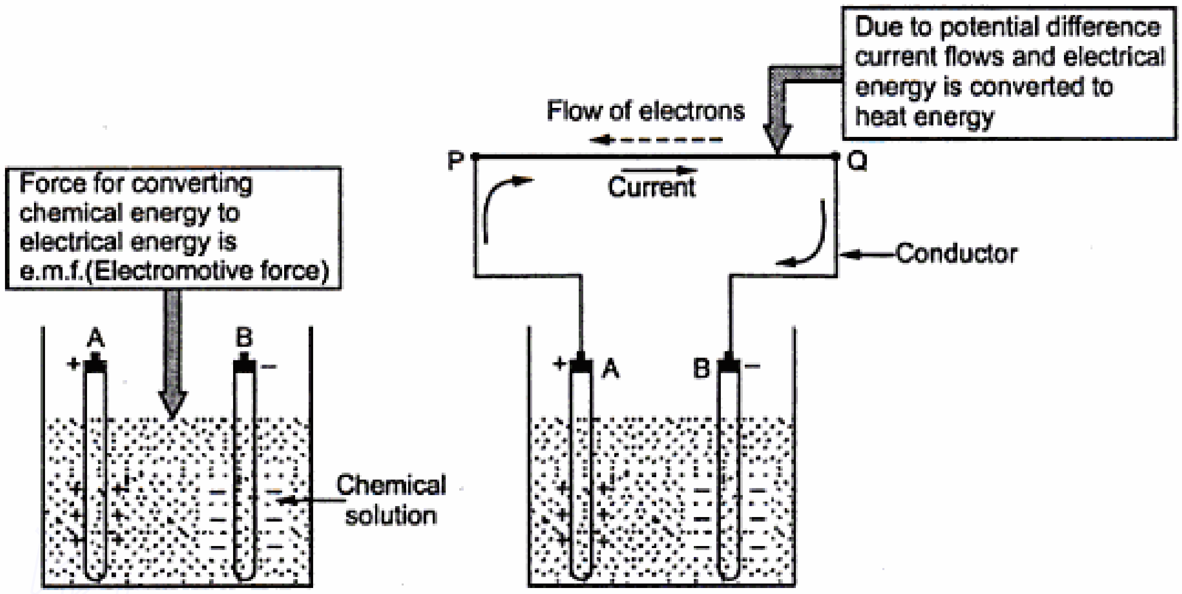
1.7 Electromotive Force and Potential Difference

Earlier we have seen the concept of e.m.f. The e.m.f. is that force which causes the flow of electrons i.e. flow of current in the given circuit. Let us understand its meaning more clearly.

Consider a simple cell shown in Fig. 1.5 (a). Due to the chemical reaction in the solution the terminal 'A' has acquired positive charge while terminal 'B' has acquired negative charge.

If now a piece of conductor is connected between the terminals A and B then flow of electrons starts through it. This is nothing but the flow of current through the conductor. This is shown in the Fig. 1.5 (b). The electrons will flow from terminal B to A and hence direction of current is from A to B i.e. positive to negative as shown.

One may think that once the positive charge on terminal A gets neutralised due to the electrons, then flow of electrons will stop. Both the terminals may get neutralised after some time. But this does not happen practically. This is because chemical reaction in the solution maintains terminal A positively charged and terminal B as negatively charged. This maintains the flow of current. The chemical reaction converts chemical energy into electric energy which maintains flow of electrons.



(a) Cell **Fig. 1.5** **(b) Current due to a cell**

> **Key Point :** *This force which requires to keep electrons in motion i.e. to convert chemical or any other form of energy into electric energy is known as Electromotive Force (e.m.f.) denoted by E. The unit of e.m.f. is volt and unless and untill there is some e.m.f. present in the circuit, a continuous flow of current is not possible.*

Consider two points P and Q as shown in the Fig. 1.5 (b), then the current is flowing from point P to point Q. This means there exists a potential difference between the points P and Q. This potential difference is called voltage denoted as V and measured in volts.

In other words we can explain the difference between e.m.f. and p.d. as below. In the cell two energy transformations are taking place simultaneously. The one is chemical energy because of solution in cell is getting converted to electrical energy which is basic cause for flow of electrons and hence current. The second is when current flows, the piece of metal gets heated up i.e. electrical energy is getting converted to heat energy, due to flow of current.

In the first transformation electrical energy is generated from other form of energy. The force involved in such transformation is electromotive force. When current flows, due to which metal gets heated up i.e. due to existence of potential difference between two points, voltage is existing. And in such case electrical energy gets converted to other form of energy. The force involved in such transformation is nothing but the potential difference or voltage. Both e.m.f. and potential difference are in generally referred as voltage.

1.8 Resistance

The current in the electrical circuit not only depends on e.m.f. or p.d. but also on the circuit parameters. For example if lamp is connected in a circuit, current gets affected and lamp filament becomes hot radiating light. But if contact at one end is loose, current decreases but sparking occurs at loose contact making it hot. If two lamps are connected one after the other, brightness obtained is less than that obtained by a single lamp. These examples show that current, flow of electrons depends on the circuit parameters and not only the e.m.f. alone.

> **Key Point** : *This property of an electric circuit tending to prevent the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.*

The concept of resistance is analogous to the friction involved in the mechanical motion. Every metal has a tendency to oppose the flow of current. Higher the availability of the free electrons, lesser will be the opposition to the flow of current. The conductor due to the high number of free electrons offer less resistance to the flow of current. The opposition to the flow of current and conversion of electrical energy into heat energy can be explained with the help of atomic structure as below.

When the flow of electrons is established in the metal, the ions get formed which are charged particles as discussed earlier. Now free electrons are moving in specific direction when connected to external source of e.m.f. So such ions always become obstruction for the flowing electrons. So there is collision between ions and free flowing electrons. This not only reduces the speed of electrons but also produces the heat. The effect of this is nothing but the reduction of flow of current. Thus the material opposes the flow of current.

The resistance is denoted by the symbol 'R' and is measured in **ohm** symbolically represented as Ω . We can define unit ohm as below.

> **Key Point** : *1 Ohm : The resistance of a circuit, in which a current of 1 Ampere generates the heat at the rate of one joules per second is said to be 1 ohm.*

Now $\quad$ 4.186 joules = 1 calorie

hence $\quad$ 1 joule = 0.24 calorie

Thus unit 1 ohm can be defined as that resistance of the circuit if it develops 0.24 calories of heat, when one ampere current flows through the circuit for one second.

Earlier we have seen that some materials possess large number of free electrons and hence offer less opposition to the flow of current. Such elements are classified as the **'Conductors'** of electricity. While in some materials the number of free electrons are very less and hence offering a large resistance to the flow of current. Such elements are classified as the **'Insulators'** of electricity.

Examples of good conductors are silver, copper, aluminium while examples of insulators are generally non metals like glass, rubber, wood, paper etc.

Let us see the factors affecting the resistance.

1.8.1 Factors Affecting the Resistance

1. Length of the material : The resistance of a material is directly proportional to the length. The resistance of longer wire is more. Length is denoted by 'l' .

2. Cross-sectional area : The resistance of a material is inversely proportional to the cross-sectional area of the material. More cross-sectional area allowed the passage of more number of free electrons, offering less resistance. The cross sectional area is denoted by 'a'.

3. The type and nature of the material : As discussed earlier whether it consists more number of free electrons or not, affects the value of the resistance. So material which is conductor has less resistance while an insulator has very high resistance.

4. Temperature : The temperature of the material affects the value of the resistance. Generally the resistance of the material increases as its temperature increases. Generally effect of small changes in temperature on the resistance is not considered as it is negligibly small.

So for a certain material at a certain temperature we can write a mathematical expression as,

$$R \propto \frac{l}{a}$$

and effect of nature of material is considered through the constant of proportionality denoted by ρ (rho) called **resistivity or specific resistance** of the material. So finally,

$$\boxed{R = \frac{\rho l}{a}}$$

Where l = length in metres

a = cross-sectional area in square metres

ρ = resistivity in ohms-metres

R = resistance in ohms

1.9 Resistivity and Conductivity

The resistivity or specific resistance of a material depends on nature of material and denoted by ρ(rho). From the expression of resistance it can be expressed as,

$$\boxed{\rho = \frac{Ra}{l} \quad \text{i.e.} \quad \frac{\Omega - m^2}{m} \quad \text{i.e.} \quad \Omega - m}$$

It is measured in Ω - m.

Definition : *The resistance of a material having unit length and unit cross-sectional area is known as its specific resistance or resistivity.*

The Table 1.3 gives the value of resistivity of few common materials.

Name of material	ρ in Ω - m
International Standard Copper	1.72×10^{-8}
Aluminium Cast	2.6×10^{-8}
Bronze	3.6×10^{-8}
Iron-Wrought	10.7×10^{-8}
Carbon Graphite	4.6×10^{-8}
Gold	2.36×10^{-8}
Silver Annealed	1.58×10^{-8}
Lead	22×10^{-8}

Table 1.3

> **Key Point** : *A material with highest value of resistivity is the best insulator while with poorest value of resistivity is the best conductor.*

1.9.1 Conductance (G)

The conductance of any material is reciprocal of its resistance and is denoted as G. It is the indication of ease with which current can flow through the material. It is measured in **siemens**.

So
$$\boxed{G = \frac{1}{R} = \frac{a}{\rho l} = \frac{1}{\rho}\left(\frac{a}{l}\right) = \sigma\left(\frac{a}{l}\right)}$$

1.9.2 Conductivity

The quantity $(1/\rho)$ is called **conductivity**, denoted as σ (sigma). **Thus the conductivity is the reciprocal of resistivity. It is measured in siemens/ m.**

> **Key Point** : *The material having highest value of conductivity is the best conductor while having poorest conductivity is the best insulator.*

➡ **Example 1.1 :** *The resistance of copper wire 25 m long is found to be 50 Ω. If its diameter is 1mm, calculate the resistivity of copper.*

Solution :

$$l = 25 \text{ m}, \qquad d = 1 \text{ mm}, \qquad R = 50\ \Omega$$

$$a = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(1^2) = 0.7853 \text{ mm}^2$$

Now $$\rho = \frac{Ra}{l} = \frac{50\times 0.7853\times 10^{-6}}{25} \qquad \ldots 1 \text{ mm} = 10^{-3} \text{ m}$$

$$= 1.57\times 10^{-6}\ \Omega\text{ - m} = \mathbf{1.57\ \mu\Omega\text{ - m}}$$

Example 1.2 : *Calculate the resistance of a 100 m length of wire having a uniform cross-sectional area of 0.02 mm² and having resistivity of 40 μΩ - cm.*

If the wire is drawn out to four time its original length, calculate its new resistance.

Solution : $l = 100$ m, a = 0.02 mm² and $\rho = 40\ \mu\Omega$ - cm

Now $$R = \frac{\rho l}{a} \qquad \text{express a in m}^2 \text{ and } \rho \text{ in } \Omega\text{ - m}$$

$$= \frac{40\times 10^{-6}\times 10^{-2}\times 100}{0.02\times 10^{-6}} = \mathbf{2000\ \Omega}$$

The wire is drawn out such that $l' = 4l$

But the volume of the wire must remain same before and after drawing the wire, which is the product of length and area.

$$\therefore \quad \text{Volume} = a\times l = a'\times l'$$

$$\therefore \quad a' = \frac{a\times l}{l'} = \frac{a\times l}{4l} = \frac{a}{4}$$

$$\therefore \quad R' = \text{new resistance} = \frac{\rho l'}{a'} = \frac{\rho(4l)}{\left(\frac{a}{4}\right)}$$

$$= 16\left(\frac{\rho l}{a}\right) = 16\text{ R} = \mathbf{32000\ \Omega}$$

Example 1.3 : *A silver wire has a resistance of 2.5 Ω. What will be the resistance of a manganin wire having a diameter, half of the silver wire and length one third ? The specific resistance of manganin is 30 times that of silver.*

Solution : R_s = silver resistance = 2.5 Ω, d_m = manganin diameter = $\frac{d_s}{2}$

l_m = manganin length = $\frac{l_s}{3}$, ρ_m = manganin specific resistance = 30 ρ_s

Now $$a_s = \frac{\pi}{4}(d_s)^2 = \text{area of cross-section for silver}$$

$$\therefore \quad R_s = \frac{\rho_s l_s}{a_s} = \frac{\rho_s l_s}{\frac{\pi}{4}(d_s)^2} = 2.5\ \Omega$$

and
$$R_m = \frac{\rho_m l_m}{a_m} = \frac{30\rho_s \times \left(\frac{l_s}{3}\right)}{\frac{\pi}{4}(d_m)^2} = \frac{10\rho_s l_s}{\frac{\pi}{4}\left(\frac{d_s}{2}\right)^2}$$

$$= 40\frac{\rho_s l_s}{\frac{\pi}{4}(d_s)^2} = 40\ R_s = 100\ \Omega$$ ... Resistance of manganin

Example 1.4 : *Prove that the length 'l' and diameter 'd' of a cylinder of copper are*

$$l = \left(\frac{rx}{\rho}\right)^{\frac{1}{2}} \text{ and } d = \left(\frac{16x\rho}{\pi^2 r}\right)^{\frac{1}{4}}$$

where x-volume, ρ-resistivity and r-resistance between opposite circular faces.

(May - 2001)

Solution : The resistance is given by,

$$r = \frac{\rho l}{a}$$

Now x = volume = $a \times l$

Multiplying numerator and denominator by l,

$$r = \frac{\rho l \times l}{a \times l} = \frac{\rho l^2}{x}$$

$$\therefore \quad l = \left(\frac{rx}{\rho}\right)^{1/2}$$ ... proved

Now
$$a = \frac{\pi}{4}d^2$$

$$r = \frac{\rho l \times a}{a \times a} = \frac{\rho x}{a^2}$$ ... multiplying and dividing by a

$$\therefore \quad r = \frac{\rho x}{\left(\frac{\pi}{4}d^2\right)^2}$$

$$\therefore \quad d^4 = \frac{16\rho x}{\pi^2 r}$$

$$\therefore \quad \boxed{d = \left(\frac{16\rho x}{\pi^2 r}\right)^{1/4}}$$ ... proved

1.10 Effect of Temperature on Resistance

The resistance of the material increases as temperature of a metal increases. Let us see the physical phenomenon involved in this process.

Atomic structure theory says that under normal temperature when the metal is subjected to potential difference, ions i.e. unmovable charged particles get formed inside the metal. The electrons which are moving randomly, get aligned in a particular direction as shown in the Fig. 1.6.

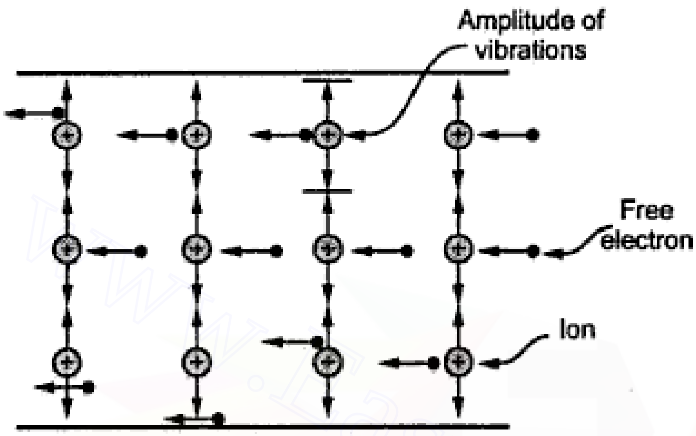


Fig. 1.6 Vibrating ions in a conductor

At low temperatures, the ions are almost stationary. But as temperature increases, the ions gain energy and start oscillating about their mean position. Higher the temperature, greater is the amplitude. Such vibrating ions cause obstruction to the flowing electrons. Similarly due to high amplitude of oscillating ions, chances of collision of electrons are more. Due to collision and obstruction due to higher amplitude of oscillations of ions, **the resistance of material increases as temperature increases.**

But this is not true for all materials. In some cases, the resistance decreases as temperature increases.

Key Point: *So effect of temperature on the resistance depends on nature of material.*

Let us see the effect of temperature on resistance of various category of materials.

1.10.1 Effect of Temperature on Metals

The resistance of all the pure metals like copper, iron, tungsten etc. increases linearly with temperature. For a copper, its resistance is 100 Ω at 0° then it increases linearly

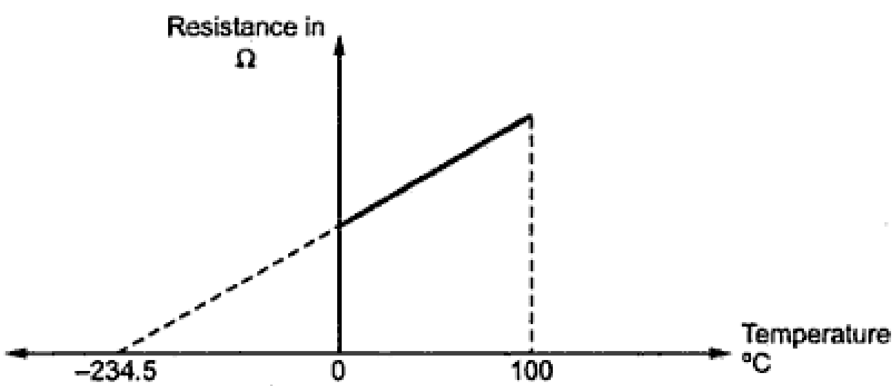


Fig. 1.7 Effect of temperature on metals

upto 100 °C. At a temperature of – 234.5 °C it is almost zero. Such variation is applicable to all the pure metals in the range of 0 °C to 100 °C. This is shown in the Fig. 1.7.

1.10.2 Effect of Temperature on Carbon and Insulators

The effect of temperature on carbon and insulators is exactly opposite to that of pure metals. **Resistance of carbon and insulators decreases as the temperature increases.** This can be explained with the help of atomic theory as below :

Insulators do not have enough number of free electrons and hence they are bad conductor of electricity. Now what happens in conductor is due to increase in temperature vibrations of ions increase but it does not increase number of free electrons. While in carbon and insulators due to increase in temperature, no doubt vibrations of ions increases but due to high temperature few electrons from atoms gain extra energy and made available as free electrons. So as number of free electrons increase though vibrations of ions increases overall difficulty to the flow of electrons reduces. This causes decrease in resistance.

> **Key Point**: *So in case of carbon and insulating materials like rubber, paper and all electrolytes, the resistance decreases as the temperature increases.*

1.10.3 Effect of Temperature on Alloys

The resistance of alloys increases as the temperature increases but rate of increase is not significant. In fact the alloys like Manganin (alloy of copper, managanese and nickel), Eureka (alloy of copper and nickel) etc. show almost no change in resistance for considerable change in the temperature. Due to this property alloys are used to manufacture the resistance boxes.

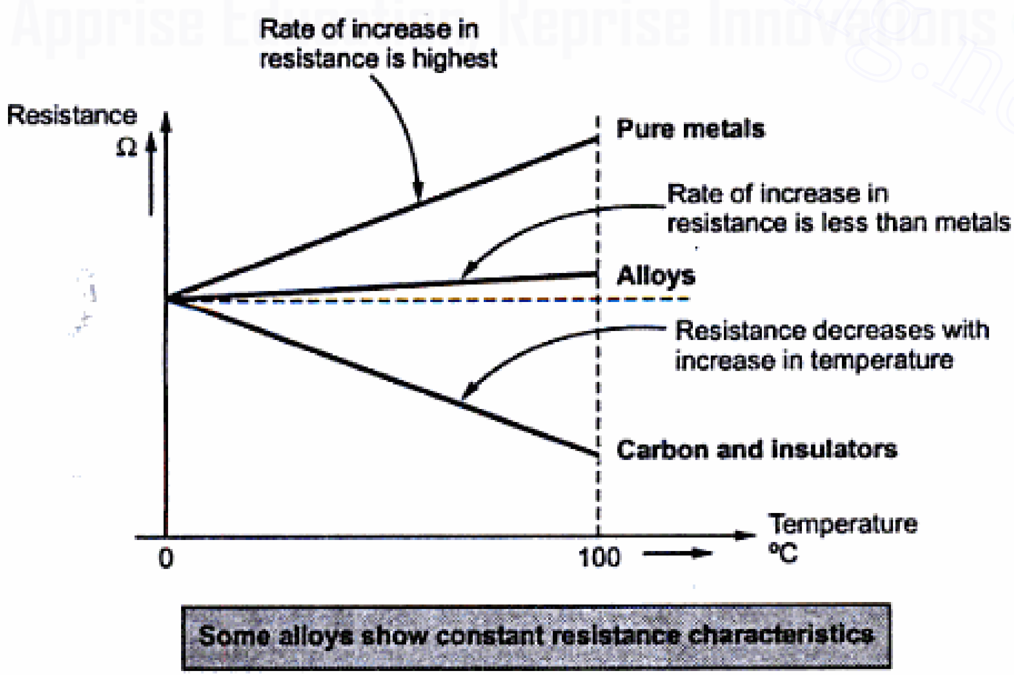


Fig. 1.8 (a) Effect of temperature on resistance

The Fig. 1.8 (a) shows the effect of temperature on metals, insulating materials and alloys.

The study of this, is very useful in finding out the temperature rise of cables, different windings in machines etc. Such study is possible by introducing the factor called resistance temperature coefficient of the material.

1.10.4 Effect of Temperature on Semiconductors

The materials having conductivity between that of metals and insulators are called semiconductors. The examples are silicon, germanium etc.

Key Point: *Semiconductors have negative temperature coefficient of resistivity hence as temperature increases, their resistance decreases.*

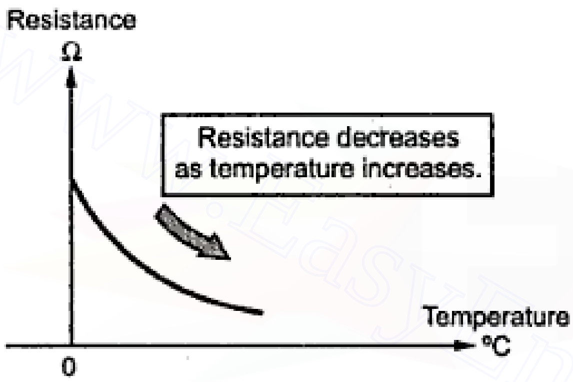


Fig. 1.8 (b) Effect of temperature on semiconductors

At normal temperature, the resistance of semiconductors is high. But as temperature increases, their resistance decreases with fast rate as shown in the Fig. 1.8(b). At absolute zero temperature, the semiconductors behave as perfect insulators. At higher temperature, more valence electrons acquire the energy and become free electrons. Due to increased number of free electrons, resistance of semiconductors decreases as temperature increases.

1.11 Resistance Temperature Coefficient (R.T.C.)

From the discussion uptill now we can conclude that the change in resistance is,

1) Directly proportional to the initial resistance.

2) Directly proportional to the change in temperature.

3) Depends on the nature of the material whether it is a conductor, alloy or insulator.

Let us consider a conductor, the resistance of which increases with temperature linearly

Let R_0 = Initial resistance at 0 °C

R_1 = Resistance at t_1 °C

R_2 = Resistance at t_2 °C

As shown in the Fig. 1.9, $R_2 > R_1 > R_0$.

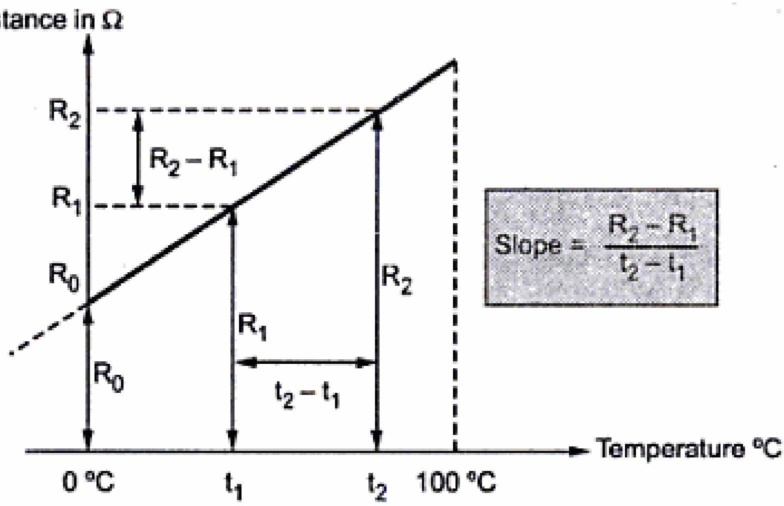


Fig. 1.9 Graph of resistance against temperature

> **Key Point** : *The change in resistance with temperature is according to the factor called resistance temperature coefficient (R.T.C.) denoted by α.*

Definition of R.T.C. : The resistance temperature coefficient at t °C is the ratio of change in resistance per degree celcius to the resistance at t °C.

$$\boxed{\text{R.T.C. at } t^\circ\text{C} = \frac{\Delta R \text{ per}^\circ\text{C}}{R_t} = \alpha_t}$$

From the Fig. 1.9, change in resistance = $R_2 - R_1$

change in temperature = $t_2 - t_1$

$\therefore$ change in resistance per °C = $\frac{\Delta R}{\Delta t} = \frac{R_2 - R_1}{t_2 - t_1}$

Hence according to the definition of R.T.C. we can write α_1 i.e. R.T.C. at t_1 °C as,

$$\boxed{\alpha_1 = \frac{\text{change in resistance per }^\circ\text{C}}{\text{resistance at } t_1{}^\circ\text{C}} = \frac{(R_2 - R_1 / t_2 - t_1)}{R_1}}$$

Similarly R.T.C. at 0 °C i.e. α_0 can be written as,

$$\alpha_0 = \frac{(R_1 - R_0 / t_1 - 0)}{R_0}$$

But $\frac{R_2 - R_1}{t_2 - t_1} = \frac{R_1 - R_0}{t_1 - 0}$ = slope of the graph

Hence R.T.C. at any temperature t °C can be expressed as,

$$\boxed{\alpha_t = \frac{\text{slope of the graph}}{R_t}}$$

1.11.1 Unit of R.T.C.

We know, $\alpha_t = \dfrac{\text{change in resistance per °C}}{\text{resistance at t °C}} \Rightarrow \dfrac{\Omega / °C}{\Omega} \Rightarrow /°C$

> Thus unit of R.T.C. is **per degree celcius** i.e. /°C

1.11.2 Use of R.T.C. in Calculating Resistance at t °C

Let α_0 = R.T.C. at 0 °C

R_0 = Resistance at 0 °C

R_1 = Resistance at t_1 °C

Then $$\alpha_0 = \frac{(R_1 - R_0 / t_1 - 0)}{R_0} = \frac{R_1 - R_0}{t_1 R_0}$$

$$\therefore \quad R_1 - R_0 = \alpha_0 t_1 R_0$$

$$\therefore \quad R_1 = R_0 + \alpha_0 t_1 R_0 = R_0 (1 + \alpha_0 \ t_1)$$

Thus resistance at any temperature can be expressed as,

$$\boxed{R_t = R_0 (1 + \alpha_0 t)}$$

So knowing R_0 and α_0 at 0 °C, the resistance at any t °C can be obtained.

Alternatively this result can be expressed as below,

Let R_1 = Resistance at t_1 °C

R_t = Resistance at t °C

$$\alpha_1 = \frac{(R_t - R_1 / t - t_1)}{R_1} \quad \text{... from definition}$$

$$\therefore \quad \alpha_1 R_1 (t - t_1) = R_t - R_1$$

$$\therefore \quad R_t = R_1 [1 + \alpha_1 (t - t_1)]$$

Where $t - t_1$ = change in temperature = Δt

In general above result can be expressed as,

$$R_{final} = R_{initial} [1 + \alpha_{initial} (\Delta t)]$$

So if initial temperature is t_1 and final is t_2, we can write,

$$\boxed{R_2 = R_1 [1 + \alpha_1 \Delta t]}$$

> **Key Point:** *Thus knowing resistance and R.T.C. of the material at any one temperature, the resistance of material at any other temperature can be obtained.*

1.11.3 Effect of Temperature on R.T.C.

From the above discussion, it is clear that the value of R.T.C. also changes with the temperature. As the temperature increases, its value decreases. For any metal its value is maximum at 0 °C .

From the result of section 1.11.2 we can write,

$$R_2 = R_1 \; [1 + \alpha_1 (t_2 - t_1)] \qquad \ldots (1)$$

where R_1 and α_1 are resistance and R.T.C. at t_1 °C and R_2 is resistance at t_2 °C.

If the same resistance is cooled from t_2 to t_1 °C and if α_2 is R.T.C. at t_2 °C then,

$$R_1 = R_2 \; [1 + \alpha_2 (t_1 - t_2)] \qquad \ldots (2)$$

Dividing equation (1) by R_2,

$$\therefore \quad 1 = \frac{R_1}{R_2} [1 + \alpha_1 (t_2 - t_1)]$$

$$\therefore \quad \frac{R_2}{R_1} = 1 + \alpha_1 (t_2 - t_1) \qquad \ldots (3)$$

Dividing equation (2) by R_1,

$$1 = \frac{R_2}{R_1} [1 + \alpha_2 (t_1 - t_2)]$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1}{1+\alpha_2 (t_1 - t_2)} \qquad \ldots (4)$$

Equating (3) and (4) we can write,

$$1 + \alpha_1 (t_2 - t_1) = \frac{1}{1+\alpha_2 (t_1 - t_2)}$$

$$\therefore \quad \alpha_1 (t_2 - t_1) = \frac{1}{1+\alpha_2 (t_1 - t_2)} - 1 = \frac{1-1-\alpha_2 (t_1 - t_2)}{1+\alpha_2 (t_1 - t_2)}$$

$$\therefore \quad \alpha_1 (t_2 - t_1) = \frac{-\alpha_2 (t_1 - t_2)}{1+\alpha_2 (t_1 - t_2)} = \frac{\alpha_2 (t_2 - t_1)}{1+\alpha_2 (t_1 - t_2)}$$

$$\therefore \quad \boxed{\alpha_1 = \frac{\alpha_2}{1+\alpha_2 (t_1 - t_2)} = \frac{1}{\frac{1}{\alpha_2} + (t_1 - t_2)}}$$

or

$$\boxed{\alpha_2 = \frac{\alpha_1}{1+\alpha_1 (t_2 - t_1)} = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}}$$

Using any of the above expression if α at any one temperature t_1 °C is known then α at any other temperature t_2 can be obtained.

If starting temperature is $t_1 = 0$ °C and α at t °C i.e. α_t is required then we can write,

$$\boxed{\alpha_t = \frac{\alpha_0}{1+\alpha_0(t-0)} = \frac{\alpha_0}{1+\alpha_0 t}}$$

This is very useful expression to obtain R.T.C. at any temperature t °C from α_0.

1.11.4 Effect of Temperature on Resistivity

Similar to the resistance, the specific resistance or resistivity also is a function of temperature. For pure metals it increases as temperature increases.

So similar to resistance temperature coefficient we can define temperature coefficient of resistivity as fractional change in resistivity per degree centigrade change in temperature from the given reference temperature.

i.e. if ρ_1 = resistivity at t_1 °C

ρ_2 = resistivity at t_2 °C

then temperature coefficient of resistivity at t_1 °C can be defined as,

$$\alpha_{t1} = \frac{(\rho_2-\rho_1)/(t_2-t_1)}{\rho_1}$$

Similarly we can write the expression for resistivity at time t °C as,

$$\boxed{\begin{aligned} \rho_t &= \rho_0(1+\alpha_0 t) \\ \rho_{t2} &= \rho_{t1}[1+\alpha_{t1}(t_2-t_1)] \end{aligned}}$$

Example 1.5 : *A certain winding made up of copper has a resistance of 100 Ω at room temperature. If resistance temperature coefficient of copper at 0 °C is 0.00428 /°C, calculate the winding resistance if temperature is increased to 50 °C. Assume room temperature as 25 °C.*

Solution : $t_1 = 25$ °C, $R_1 = 100\ \Omega$, $t_2 = 50$ °C, $\alpha_0 = 0.00428$ /°C

Now $\alpha_t = \dfrac{\alpha_0}{1+\alpha_0 t}$

$\therefore$ $\alpha_1 = \dfrac{\alpha_0}{1+\alpha_0 t_1} = \dfrac{0.00428}{1+0.00428\times 25} = 0.003866$ /°C

Use $R_2 = R_1[1+\alpha_1(t_2-t_1)] = 100[1+0.003866(50-25)]$

$= \mathbf{109.6657\ \Omega}$ ... Resistance at 50 °C

Example 1.6 : *The resistance of a wire increases from 40–ohm at 20 °C to 50 – ohm at 70 °C. Find the temperature co-efficient of resistance at 0 °C.* **(Dec. - 99, Dec. - 2000)**

Solution : $R_1 = 40\ \Omega$, $t_1 = 20\ °C$, $R_2 = 50\ \Omega$, $t_2 = 70\ °C$

Now, $$R_2 = R_1 [1 + \alpha_1 \Delta t]$$

$\therefore$ $$50 = 40 [1 + \alpha_1 (70 - 20)] \quad \text{i.e.} \quad \frac{5}{4} = 1 + \alpha_1 (50)$$

$\therefore$ $$50\ \alpha_1 = 0.25$$

$\therefore$ $$\alpha_1 = 5 \times 10^{-3}/°C \quad \text{i.e.} \quad \text{at } t_1 = 20\ °C$$

Now, $$\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t} \quad \text{i.e.} \quad \alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

$\therefore$ $$5 \times 10^{-3} = \frac{\alpha_0}{1 + \alpha_0 \times 20} \quad \text{i.e.} \quad 1 + 20\ \alpha_0 = 200\ \alpha_0$$

$\therefore$ $$180\ \alpha_0 = 1$$

$\therefore$ $$\alpha_0 = 5.55 \times 10^{-3}/°C$$ ... Temperature coefficient at 0 °C.

Example 1.7 : *A specimen of copper has a resistivity (ρ) and a temperature coefficient of 1.6 × 10^6 ohm - cm at 0 °C and 1/254.5 at 20 °C respectively. Find both of them at 60 °C.* **(May - 2001)**

Solution : $$\rho_0 = 1.6 \times 10^{-6}\ \Omega\text{-cm}, \quad \alpha_1 = \frac{1}{254.5}\ /°C \ \text{ at } 20\ °C$$

Now $$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

$\therefore$ $$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 \times 20}$$

$\therefore$ $$\frac{1}{254.5} = \frac{\alpha_0}{1+20\,\alpha_0}$$

$\therefore$ $$1 + 20\ \alpha_0 = 254.5\ \alpha_0$$

$\therefore$ $$\alpha_0 = \frac{1}{234.5}\ /°C \quad \text{at } 0\ °C$$

$\therefore$ $$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0 \times 60} = \frac{1/234.5}{1+\frac{60}{234.5}} = \frac{1}{294.5}\ /°C$$ ... at 60 °C

$$\rho_t = \rho_0 (1 + \alpha_0 t)$$

$\therefore$ $$\rho_{60} = 1.6 \times 10^{-6} \left(1+\frac{1}{234.5} \times 60\right) = 2 \times 10^{-6}\ \Omega\text{-cm}$$

Example 1.8 : *A resistance element having cross sectional area of 10 mm^2 and a length of 10 mtrs. takes a current of 4 A from a 220 V supply at ambient temperature of 20 °C. Find out, i) the resistivity of the material and ii) current it will take when the temperature rises to 60 °C. Assume α_{20} = 0.0003 /°C.* **(May - 2000)**

Solution : $a = 10\ mm^2 = 10\times10^{-6}\ m^2,\ V = 220\ V,\ l = 10\ m,\ I = 4\ A,\ t_1 = 20\ °C$

and $\alpha_{20} = 0.0003/°C,\ R_1 = \frac{V}{I} = \frac{220}{4} = 55\ \Omega$

Now, $R_1 = \frac{\rho_1 l}{a}$ i.e. $55 = \frac{\rho_1\times10}{10\times10^{-6}}$

$\therefore$ $\rho_1 = 0.000055\ \Omega\text{ - m} = 55\ \mu\Omega\text{ - m}$ ... at 20 °C

i) ρ_2 at t_2 = 60 °C, $\rho_2 = \rho_1[1+\alpha_1(t_2-t_1)]$

$= 0.000055\ [1 + 0.0003\ (60 - 20)]$

$= \mathbf{55.66\ \mu\Omega\text{ - m}}$

ii) $R_2 = \frac{\rho_2 l}{a}$

$\therefore$ $R_2 = \frac{55.66\times10^{-6}\times10}{10\times10^{-6}} = 55.66\ \Omega$

$\therefore$ $I = \frac{V}{R_2} = \frac{220}{55.66} = \mathbf{3.9525\ A}$ ... at 60 °C

Example 1.9 : *A coil has a resistance of 18 ohm at 20 °C and 22 ohm at 50 °C. Find the rise in the temperature when resistance becomes 24 ohm. The room temperature is 18 °C.* **(May-99, Dec.-2007)**

Solution : $R_1 = 18\ \Omega,\ t_1 = 20\ °C,\ R_2 = 22\ \Omega$ and $t_2 = 50\ °C$

Now, $R_2 = R_1[1+\alpha_1(t_2-t_1)]$ i.e. $22 = 18\ [1+\alpha_1(50-20)]$

Solving, $\alpha_1 = 0.007407\ /°C$

Now $R_3 = 24\ \Omega$ and $R_3 = R_1\ [1+\alpha_1\ (t_3 - t_1)]$

$\therefore$ $24 = 18\ [1 + 0.007407\ (t_3 - 20)]$

$\therefore$ $0.3333 = 0.007407\ (t_3 - 20)$

$\therefore$ $t_3 - 20 = 45$

$\therefore$ $t_3 = \mathbf{65\ °C}$

So room temperature is 18 °C given

$\therefore$ Temperature rise $= 65 - 18 = \mathbf{47\ °C}$

1.11.5 R.T.C. of Composite Conductor

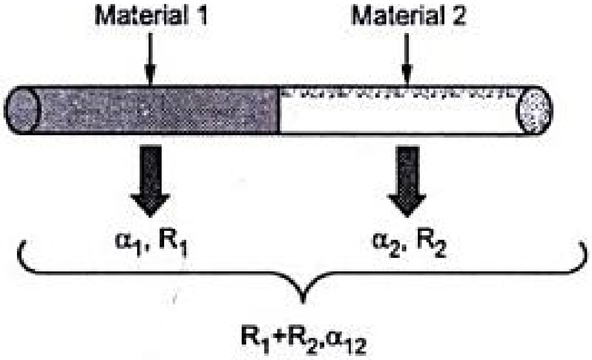


Fig. 1.10 Composite conductors

In many practical cases, it is necessary to manufacture conductors using two different types of materials, to achieve special requirements. Such a composite conductor is shown in the Fig. 1.10.

The material 1 has R.T.C. α_1 and its contribution in composite conductor is R_1.

The material 2 has R.T.C. α_2 and its contribution in composite conductor is R_2.

The combined composite conductor has resistance $R_1 + R_2$ while its RTC is neither α_1 nor α_2 but it is α_{12}, different than α_1 and α_2.

Analysis of Composite conductor : The analysis includes the calculation of α_{12} from α_1 and α_2.

Let R_1 = Resistance of material 1 at t_1 °C

R_2 = Resistance of material 2 at t_1 °C

t_2 = New temperature attained by composite conductor

R_{1t} = Resistance of material 1 at t_2 °C

R_{2t} = Resistance of material 2 at t_2 °C

R_{12} = Resistance of composite material at t_1 °C

R_{12t} = Resistance composite material at t_2 °C

It is known that,

$$R_{1t} = R_1 [1 + \alpha_1 (t_2 - t_1)] \quad \text{...(5)}$$

$$R_{2t} = R_2 [1 + \alpha_2 (t_2 - t_1)] \quad \text{...(6)}$$

$$R_{12t} = R_{12} [1 + \alpha_{12} (t_2 - t_1)] \quad \text{...(7)}$$

Where α_{12} = R.T.C. of composite conductor at t_1 °C

> **Key Point :** *The overall circuit is series connection of resistances at all temperatures, from electrical point of view.*

$$R_{12} = R_1 + R_2 \quad \text{... at } t_1 \text{ °C} \quad \text{...(8)}$$

And

$$R_{12t} = R_{1t} + R_{2t} \quad \text{... at } t_2 \text{ °C} \quad \text{...(9)}$$

Using (8) and (9) in (7),

$$[R_{1t} + R_{2t}] = [R_1 + R_2][1 + \alpha_{12}(t_2 - t_1)] \quad \text{...(10)}$$

Using (5) and (6) in (10),

$$R_1[1 + \alpha_1(t_2 - t_1)] + R_2[1 + \alpha_2(t_2 - t_1)] = [R_1 + R_2][1 + \alpha_{12}(t_2 - t_1)]$$

$$\therefore R_1 + R_1\alpha_1(t_2 - t_1) + R_2 + R_2\alpha_2(t_2 - t_1) = R_1 + R_1\alpha_{12}(t_2 - t_1) + R_2 + R_2\alpha_{12}(t_2 - t_1)$$

Cancelling R_1 and R_2 from both sides,

$$R_1\alpha_1(t_2 - t_1) + R_2\alpha_2(t_2 - t_1) = R_1\alpha_{12}(t_2 - t_1) + R_2\alpha_{12}(t_2 - t_1)$$

Cancelling $(t_2 - t_1)$ from both sides,

$$\alpha_{12}(R_1 + R_2) = R_1\alpha_1 + R_2\alpha_2 = \alpha_{12}(R_1 + R_2) \quad \text{... (11)}$$

$$\therefore \quad \boxed{\alpha_{12} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}} \quad \text{...(12)}$$

Thus α_{12} which is R.T.C. of composite conductor can be obtained at t_1 °C. Once this is known, α_{12} at any other temperature can be obtained as,

$$\boxed{\alpha_{12t} = \frac{\alpha_{12}}{1 + \alpha_{12}\Delta t} \quad \text{where } \Delta t \text{ is temperature rise}}$$

Prove that : $\dfrac{R_2}{R_1} = \dfrac{\alpha_1 - \alpha_{12}}{\alpha_{12} - \alpha_2}$

Divide the equation (11) by R_1,

$$\alpha_1 + \frac{R_2}{R_1}\alpha_2 = \alpha_{12} + \alpha_{12}\frac{R_2}{R_1}$$

$$\therefore \quad \alpha_1 - \alpha_{12} = \left(\alpha_{12}\frac{R_2}{R_1}\right) - \left(\frac{R_2}{R_1}\alpha_2\right)$$

$$\therefore \quad \alpha_1 - \alpha_{12} = \frac{R_2}{R_1}[\alpha_{12} - \alpha_2]$$

$$\therefore \quad \boxed{\frac{R_2}{R_1} = \frac{\alpha_1 - \alpha_{12}}{\alpha_{12} - \alpha_2}} \quad \text{... Proved}$$

Example 1.10 : *At a particular temperature the two resistances are 60 Ω and 90 Ω having temperature coefficients of 0.0037 /°C and 0.005 /°C respectively. Calculate the temperature coefficient of composite conductor at the same temperature, obtained by combining above two resistances in series.*

Solution : $R_1 = 60\ \Omega$, $R_2 = 90\ \Omega$, $\alpha_1 = 0.0037$ /°C, $\alpha_2 = 0.005$ /°C

Using the result derived,

$$\alpha_{12} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2} = \frac{(60 \times 0.0037) + (90 \times 0.005)}{(60 + 90)} = 0.00448\ /°\text{C}$$

Example 1.11 : *Two coils A and B have resistances 60 Ω and 30 Ω respectively at 20 °C. The resistance temperature coefficients for the two coils at 20 °C are 0.001 /°C and 0.004 /°C. Find the resistance of their series combination at 50 °C.*

Solution : The given values are,

For coil A, $R_{A1} = 60\ \Omega,\quad t_1 = 20\ °C,\quad \alpha_{A1} = 0.001\ /°C$

For coil B, $R_{B1} = 30\ \Omega,\quad t_1 = 20\ °C,\quad \alpha_{B1} = 0.004\ /°C$

Now $R_{A2} = R_{A1}\,[1 + \alpha_{A1}(t_2 - t_1)]$

$\therefore$ $R_{A2} = 60\,[1 + 0.001\,(50 - 20)] = 61.8\ \Omega$

This is resistance of coil A at 50 °C.

And $R_{B2} = R_{B1}\,[1 + \alpha_{B1}\,(t_2 - t_1)]$

$= 30\,[1 + 0.004 \times (50 - 20)] = 33.6\ \Omega$

This is resistance of coil B at 50 °C.

$\therefore$ Resistance of their series combination at 50 °C $= R_{A2} + R_{B2} = 61.8 + 33.6$

$= 95.4\ \Omega$

Example 1.12 : *Two coils A and B have resistances 100 Ω and 150 Ω respectively at 0 °C are connected in series. Coil A has resistance temperature coefficient of 0.0038 /°C while B has 0.0018 /°C. Find the resistance temperature coefficient of the series combination at 0 °C.*

Solution : At 0 °C, the series combination is $= R_A + R_B = 100 + 150 = 250\ \Omega$

Now $R_t = R_0\,(1 + \alpha_0\, t)$ i.e. $(R_{AB})_t = (R_{AB})_0\,[1 + \alpha_{AB0}\, t]$

where R_{AB} is a resistance of series combination.

α_{AB} is resistance temperature coefficient of series combination.

Now $(R_A)_t = (R_A)_0\,[1 + \alpha_{A0}\, t]$ and $(R_B)_t = (R_B)_0\,[1 + \alpha_{B0}\, t]$

$\therefore$ $(R_{AB})_t = (R_A)_t + (R_B)_t = (R_A)_0\,[1 + \alpha_{A0}\, t] + (R_B)_0\,[1 + \alpha_{B0}\, t]$

Substituting in above,

$(R_A)_0\,[1 + \alpha_{A0}\, t] + (R_B)_0\,[1 + \alpha_{B0}\, t] = (R_{AB})_0\,[1 + \alpha_{AB0}\, t]$

$(R_A)_0 = 100\ \Omega,\ \alpha_{A0} = 0.0038$

$(R_B)_0 = 150\ \Omega,\ \alpha_{B0} = 0.0018$

$(R_{AB})_0 = 250\ \Omega$

$\therefore\ 100\,[1 + 0.0038\, t] + 150\,[1 + 0.0018\, t] = 250\,[1 + (\alpha_{AB})_0\, t]$

$\therefore$ $100 + 0.38t + 150 + 0.27t = 250 + 250\,\alpha_{AB0}\, t$

$$\therefore \quad 0.65\ t = 250\ \alpha_{AB0}\ t$$

$$\therefore \quad \alpha_{AB0} = \mathbf{0.0026\ /^\circ C.}$$

This is the resistance temperature coefficient of the series combination at 0 °C.

Note : The example may be solved using the result derived as,

$$\alpha_{AB} = \frac{R_A \alpha_A + R_B \alpha_B}{R_A + R_B} = 0.0026\ /^\circ C$$

But in examination, such examples must be solved using basic procedure as used above and not by using direct expression derived.

Example 1.13 : *At any given temperature, two material A and B have resistance temperature coefficients of 0.004 and 0.0004 respectively. In what proportion resistances made up of A and B joined in series to give a combination having resistance temperature coefficient of 0.001 per °C ?*

Solution : Let R be resistance of material A then (x R) be resistance of material B.

The resistance of the series combination is,

$$R_{AB} = R_A + R_B$$

$$R_{AB} = R + xR = (1 + x)\ R\ \Omega$$

Let (α_{AB}) = resistance temperature coefficient of the series combination.

Let there be t °C change in temperature so,

$$R'_{AB} = (R_{AB})\ (\alpha_{AB})\ (t) = (1 + x)\ R\ (0.001)\ (t) \quad \ldots (1)$$

The resistance R'_{AB} is also $R'_A + R'_B$, where

R'_A is value of R_A due to change in temperature.

R'_B is value of R_B due to change in temperature.

$$R'_A = R_A \cdot (\alpha_A)\ (t) = R \cdot (0.004)\ (t)$$

and

$$R'_B = R_B \cdot (\alpha_B)\ (t) = xR \cdot (0.0004)\ (t)$$

$$R'_{AB} = R\ (0.004)\ (t) + x\ R\ (0.0004)\ (t)$$

$$R'_{AB} = R\ t(0.004 + 0.0004\ x) \quad \ldots (2)$$

Equating 1 and 2,

$$\therefore R\ t\ (1 + x)\ (0.001) = R\ t\ (0.004 + 0.0004x)$$

$$0.001 + 0.001\ x = 0.004 + 0.0004\ x \quad \text{i.e.} \quad 6 \times 10^{-4}\ x = 0.003$$

$\therefore \qquad x = 5 \quad$ i.e. $\quad R_B = 5\,R_A$

i.e. resistance R_A and R_B must be joined in the proportion 1 : 5.

Example 1.14 : *A resistor of 80 Ω resistance having a temperature coefficient of 0.0021 /°C at 0 °C is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material A the resistance is 80 Ω per 100 m and temperature coefficient is 0.003 /°C at 0 °C. For material B the corresponding figures are 60 Ω per 100 m and 0.0015 /°C at 0 °C. Calculate suitable lengths of the wires of materials A and B to be connected in series to get required resistor.*

Solution : R_{A0} = Resistance of A at 0 °C, R_{B0} = Resistance of B at 0°C

α_{A0} = R.T.C. of A at 0 °C = 0.003 /°C , α_{B0} = R.T.C. of B at 0 °C = 0.0015 /°C

R_{AB0} = Resistance of series combination of A and B at 0 °C = 80 Ω

α_{AB0} = R.T.C. of series combination at 0 °C = 0.0021 /°C

We know, $R_t = R_0\,(1+\alpha_0\,t)$ i.e. $R_{At} = R_{A0}(1+\alpha_{A0}\,t)$

$R_{Bt} = R_{B0}\,(1+\alpha_{B0}\,t)$ and $R_{ABt} = R_{AB0}\,(1+\alpha_{AB0}\,t)$

But $R_{ABt} = R_{At} + R_{Bt}$ ... series combination at t °C

Similarly $R_{AB0} = R_{A0} + R_{B0} = 80\ \Omega$ ... series combination at 0 °C

$$\therefore\ R_{AB0}\,(1+\alpha_{AB0}\,t) = R_{A0}\,(1+\alpha_{A0}\,t) + R_{B0}\,(1+\alpha_{B0}\,t)$$

$$\therefore\ 80\,(1+0.0021\,t) = R_{A0}\,(1+0.003\,t) + R_{B0}\,(1+0.0015\,t)$$

$$\therefore\ 80 + 0.168\,t = R_{A0} + 0.003\,R_{A0}\,t + R_{B0} + 0.0015\,R_{B0}\,t$$

$$\therefore\ 80 + 0.168\,t = (R_{A0} + R_{B0}) + 0.003\,R_{A0}\,t + 0.0015\,R_{B0}\,t$$

Now $R_{A0} + R_{B0} = 80$ and $R_{B0} = 80 - R_{A0}$

$$\therefore\ 80 + 0.168\,t = 80 + 0.003\,R_{A0}\,t + 0.0015\,(80 - R_{A0})\,t$$

$$\therefore\ 0.168\,t = [0.003\,R_{A0} + 0.0015\,(80 - R_{A0})]\,t.$$

$$\therefore\ 0.168 = 0.003\,R_{A0} + 0.12 - 0.0015\,R_{A0}$$

$$\therefore\ R_{A0} = 32\ \Omega$$

$$\therefore\ R_{B0} = 80 - 32 = 48\ \Omega$$

Now material A resistance is 80 Ω per 100 m so for 32 Ω the length required is,

$$\frac{32}{80}\times 100 = 40\ \text{m}$$

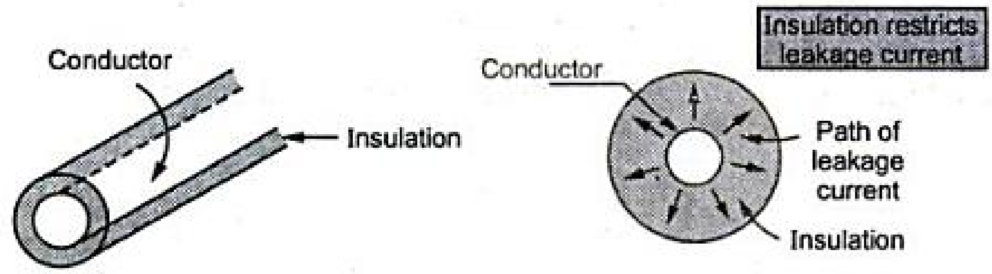
The material B has resistance of 60 Ω per 100 m so for 48 Ω the length required is ,

$$\frac{48}{60}\times 100 = 80\ \text{m}$$

1.12 Insulation Resistance

The **insulator** is a material which offers a **very high resistance** to the flow of current. Because of this property such insulating materials are used to insulate current carrying conductors so that current do not leak from them and come in contact with other bodies.

The conductor with insulation around it is shown in the Fig. 1.11 (a) while leakage current path is shown in the Fig. 1.11 (b).



(a) Insulation around conductor **(b) Direction of leakage current**

Fig. 1.11 Insulation and its function

Mainly such insulators are used to cover entire conductors which are required to be maintained at high potential with respect to earth. Such insulators try to avoid any leakage of current from conductors to earth. The resistance which offers opposition to the flow of leakage current is an ideal perfect insulation resistance. It stops the leakage current.

$$R_i = \frac{V}{I_l}$$

R_i = Insulation resistance, V = Voltage between conductor and earth, I_l = Leakage current.

> **Key Point**: *The value of insulation resistance is always very very high, of the order of megaohms.*

The commonly used insulating materials are rubber, paper, varnish, mica, percelain, glass etc. In practice the cables which are used to carry heavy currents are insulated with the help of number of layers of insulating materials. Let us derive the expression for the insulation resistance of a cable.

1.12.1 Insulation Resistance of a Cable

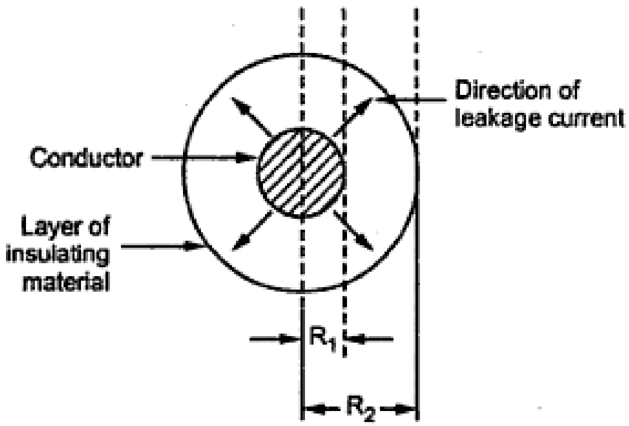


Fig. 1.12 Cable and path of leakage current

The Fig. 1.12 shows such a cable which is insulated with the help of layer of insulating material.

In such cables, leakage current flows radially from centre towards surface as shown in Fig. 1.12. Hence the cross-section of the path of such current is not constant but continuously changes with its length.

So to calculate its resistance, it is necessary to consider the section of cylindrical cable of radius r with small thickness as shown in the Fig. 1.13.

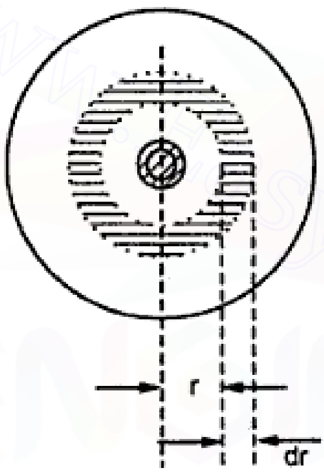


Fig. 1.13 Section of length dr

Let us find resistance of this ring. The radius of the ring is r and thickness dr. As leakage current flows redially the length in the direction of current, of this ring is dr only. While the total length of cable is say 'l' meters then its corss-sectional area perpendicular to the flow of current is surface area i.e. $2\pi r l$.

Hence the resistance of this ring can be written as

$$d R_i = \rho \frac{dr}{2\pi r l} \qquad \text{... as } R = \frac{\rho l}{a}$$

l = dr, $a = 2\pi r l$, ρ = resistivity

The total insulation resistance can be obtained by integrating this from inner radius up to outer radius i.e. R_1 to R_2.

$$R_i = \int_{R_1}^{R_2} d R_i = \int_{R_1}^{R_2} \frac{\rho\, dr}{2\pi r l}$$

$$= \frac{\rho}{2\pi l} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\rho}{2\pi l}[\log_e r]_{R_1}^{R_2}$$

$$= \frac{\rho}{2\pi l}[\log_e R_2 - \log_e R_1]$$

$$\therefore \qquad \boxed{R_i = \frac{\rho}{2\pi l} \log_e\left(\frac{R_2}{R_1}\right) \Omega.}$$

Thus insulation resistance is inversely proportional to its length.

> **Key Point** : *If two cables are joined in series, their conductor resistances come, in series but their insulation resistances are in parallel while if two cables are connected in parallel then conductor resistances are in parallel but insulation resistances are in series.*

Example 1.15 : *A single core cable has a conductor of diameter 1.2 cm and its insulation thickness of 1.6 cm. The specific resistance of the insulating material is 7.5×10^8 M Ω cm. Calculate the insulation resistance per kilometer of a cable. If now this resistance is to be increased by 20%. Calculate the thickness of the additional layer of insulation required.*

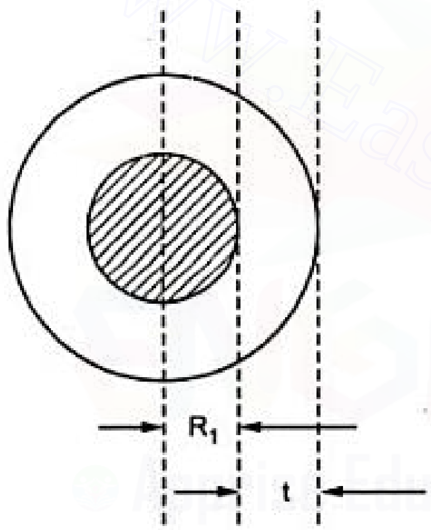


Fig. 1.14

Solution : Length l = 1 km = 1000 m

$$\rho = 7.5\times10^8 \text{ M}\Omega\text{-cm}$$

$$= 7.5\times10^8 \times 10^6 \ \Omega\text{-cm}$$

$$= 7.5\times10^8\times10^6\times10^{-2} \Omega\text{-m} = 7.5\times10^{12} \ \Omega\text{-m}$$

$$R_1 = \frac{D_1}{2} = \frac{1.2}{2} = 0.6 \text{ cm}$$

Insulation thickness = 1.6 cm

$$\therefore \qquad R_2 = R_1 + t = 2.2 \text{ cm}$$

$$R_i = \frac{\rho}{2\pi l}\log_e\left(\frac{R_2}{R_1}\right) = \frac{7.5\times10^{12}}{2\pi\times1000}\log_e\left(\frac{2.2}{0.6}\right)$$

$$= \mathbf{1550.9 \ M\Omega}$$

New R_i required is,

$$= 1550.9 + 0.2 \times 1550.9 \text{ (increased by 20 \%)}$$

$$= 1861.08 \text{ M}\Omega$$

$$\therefore \qquad 1861.08\times10^6 = \frac{\rho}{2\pi l}\log_e\left(\frac{R_2'}{R_1}\right)$$

$$\therefore \qquad 1861.08\times10^6 = \frac{7.5\times10^{12}}{2\pi\times1000}\log_e\left(\frac{R_2'}{0.6}\right)$$

$$\therefore \qquad R_2' = 2.8528 \text{ cm} = \text{New outermost radius.}$$

$\therefore$ Additional thickness required $= 2.8528 - R_1 - \text{(original thickness)}$

$$= 2.8528 - 0.6 - 1.6 = \mathbf{0.6528 \ cm}$$

Example 1.16 : *Two underground cables A and B, each has a conductor resistance of 0.6 Ω and 0.8 Ω respectively. Each has a insulation resistance of 600 MΩ and 400 MΩ respectively. If the cables are connected in,*

i) Series and ii) Parallel,

Calculate conductor resistance, and insulation resistance of the combination.

Solution : i) Cables are in series

Conductor resistances are also in series.

∴ Conductor resistance of combination = 0.6 + 0.8 = **1.4 Ω**

While insulation resistance are in parallel under series connection.

$$\therefore \quad R_i \text{ of combination} = \frac{R_A \times R_B}{R_A + R_B} = \frac{600 \times 10^6 \times 400 \times 10^6}{600 \times 10^6 + 400 \times 10^6}$$

$$= \mathbf{240\ M\Omega}$$

ii) Cables are in parallel

Conductor resistances are also in parallel.

$$\therefore \text{ Combinational conductor resistance} = \frac{0.6 \times 0.8}{0.6 + 0.8} = \mathbf{0.3428\ \Omega}$$

While insulation resistances are in series when cables are in parallel.

$$\therefore \quad R_i \text{ of combination} = R_A + R_B = 400 + 600 = \mathbf{1000\ M\Omega}$$

1.12.2 Effect of Temperature on Insulation Resistance

The insulation resistance depends on the temperature. When temperature increases, the valence electrons which are loosely bound to the nucleus, acquire thermal energy. Due to the additional energy acquired, these valence electrons become completely free from the force of attraction by the nucleus. These electrons are available as the free electrons.

> **Key Point:** *More the number of free electrons, better is the conductivity and less is the resistivity.*

Hence as temperature increases, the insulation resistance decreases while conductivity increases.

The effect can be mathematically expressed interms of an exponential relationship as,

$$\boxed{R_{it} = R_{i0}\, e^{-\alpha_0 t}}$$

Where R_{it} = Insulation resistance at t °C

R_{i0} = Insulation resistance at 0 °C

α_0 = R.T.C. at 0 °C

1.12.3 Effect of Moisture on Insulation Resistance

Practically insulation covering the conductor may absorb some moisture. Now water is very good conductor of electricity hence possibility of conduction increases through the insulation when it absorbs water. Thus moisture provides path for the leakage current.

Key Point : *As moisture content in insulation increases, the insulation resistance decreases as the water absorbed allows the leakage current to flow through it easily.*

1.13 Fundamental Quantities and Units

Scientists and engineers know that the terms they use, the quantities they measure must all be defined precisely. Such precise and standard measurements can be specified only if there is common system of indication of such measurements. This **common system of units is called 'SI' system i.e. International System of Units.**

The S.I. system is divided into seven base units and two supplementary units. The seven fundamental or base units are length, mass, time, electric current, temperature, amount of substance and luminous intensity. The two supplementary units are plane angle and solid angle. All other units are derived which are obtained from the above two classes of units. The derived units are classified into three main groups,

1. Mechanical units 2. Electrical units 3. Heat units

Let us discuss these groups in detail.

1.13.1 Mechanical Units

The various mechanical units are,

1. Mass : It is the matter possessed by the body. It is measured in kg and denoted as m.

2. Velocity : It is the distance travelled per unit time, measured in m/s.

3. Acceleration : It is the rate of change of velocity, measured in m/s^2.

4. Force : It is the push or pull which changes or tends to change the state of rest or uniform motion of body, measured in Newton.

One newton is the force required to give an acceleration of 1 m/s^2 to a mass of 1 kg.

$$\boxed{F = m \times a \quad N}$$

5. Weight : The gravitational force exerted by the earth on a body is called its weight, measured in Newtons.

$$\boxed{\text{Weight} = m \times g \quad \text{where } g = \text{gravitational acceleration} = 9.81 \ m/s^2}$$

6. Torque : It is the product of a force and a perpendicular distance from the line of action of force to the axis of rotation. It is measured in Nm.

$$\boxed{T = F \times r}$$ Where r = radial distance of rotation

7. Work : The work is said to be done when force acting on a body causes it to move. If body moves through distance d under the force F then,

$$\boxed{W = F \times d}$$

The work is measured in Joules.

8. Energy : It is the capacity to do the work. The work is done always at the cost of energy. The unit of energy is also Joules. The two forms of an energy are,

i) **Kinetic energy** which is the energy possessed by a body due to its motion. If body of mass m is moving with velocity v then the kinetic energy is,

$$\boxed{K.E. = \frac{1}{2} mv^2 \quad J}$$

ii) **Potential energy** which is the energy possessed by a body due to its position. When a body of mass m is lifted vertically through height of h then the potential energy is,

$$\boxed{P.E. = mgh = Wh \quad J \qquad \text{where } W = \text{weight}}$$

9. Power : The rate of doing work is power measured in J/sec i.e. watts

$$P = \frac{\text{work done}}{\text{time}} \quad J/sec \text{ i.e. } W$$

and $$\boxed{1\ W = 1\ J/sec}$$

Energy expended = Power × time = work done

1.13.1.1 Relation between Torque and Power

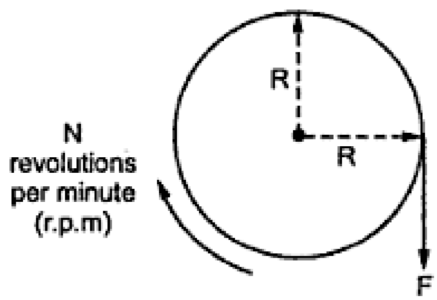


Fig. 1.15

Consider a pulley of radius R and F is the force applied as shown in the Fig. 1.15.

$$T = F \times R \quad Nm$$

Let speed of pulley is N revolutions per minute. Now work done in one revolution is force into distance travelled in one revolution.

$$d = \text{distance travelled in 1 revolution}$$
$$= 2\pi R$$

$$\therefore \quad W = \text{work done in 1 revolution} = F \times d = 2\pi R F \quad J$$

The time required for a revolution can be obtained from speed N r.p.m.

$$t = \text{time for a revolution} = \frac{60}{N} \text{ sec}$$

$$\therefore \quad P = \text{power} = \frac{W}{t} = \frac{2\pi R F}{\frac{60}{N}} = \left(\frac{2\pi N}{60}\right) \times (F \times R)$$

$$\therefore \quad \boxed{P = T \times \omega}$$

Where $T = F \times R$ = torque in Nm

$$\boxed{\omega = \frac{2\pi N}{60} = \text{angular velocity in rad/sec}}$$

The relation $P = T\,\omega$ is very important in analysing various mechanical systems.

1.13.2 Electrical Units

The various electrical units are,

1. Electrical work : In an electric circuit, movement of electrons i.e. transfer of charge is an electric current. The electrical work is done when there is a transfer of charge. The unit of such work is Joule.

> **Key Point:** *One joule of electrical work done is that work done in moving a charge of 1 coulomb through a potential difference of 1 volt.*

So if V is potential difference in volts and Q is charge in coulombs then we can write,

$$\text{Electrical work } W = V \times Q \quad J \qquad \text{But } I = \frac{Q}{t}$$

$$\therefore \quad \boxed{W = VIt \quad J} \qquad \text{Where } t = \text{Time in seconds}$$

2. Electrical power : The rate at which electrical work is done in an electric circuit is called an electrical power.

$$\text{Electrical power} \quad P = \frac{\text{electrical work}}{\text{time}} = \frac{W}{t} = \frac{V\,I\,t}{t}$$

$$\therefore \quad \boxed{P = VI \quad \text{J/sec i.e. watts}}$$

Thus power consumed in an electric circuit is 1 watt if the potential difference of 1 volt applied across the circuit causes 1 ampere current to flow through it.

Remember, 1 watt = 1 joule/sec

As unit of power watt is small, many a times power is expressed as kW (1000 watts) or MW (1×10^6 watts).

According to Ohm's law, $V = IR$ or $I = V/R$

Using this, power can be expressed as,

$$P = VI = I^2R = \frac{V^2}{R} \qquad \text{Where } R = \text{Resistance in } \Omega$$

3. Electrical energy : An electrical energy is the total amount of electrical work done in an electric circuit.

$$\therefore \qquad \boxed{\text{Electrical energy } E = \text{Power} \times \text{Time} = VIt \quad \text{joules}}$$

The unit of energy is joules or watt-sec.

The energy consumed by an electric circuit is said to be 1 joule or watt-sec when it utilises power of 1 watt for 1 second.

As watt-sec unit is very small, the electrical energy is measured in bigger units as watt-hour (Wh) and kilo watt-hour (kWh).

$$1 \text{ Wh} = 1 \text{ watt} \times 1 \text{ hour} = 1 \text{ watt} \times 3600 \text{ sec} = 3600 \text{ watt-sec i.e. J}$$

and $$1 \text{ kWh} = 1000 \text{ Wh} = 1 \times 10^3 \times 3600 \text{ J} = 3.6 \times 10^6 \text{ J}$$

When a power of 1 kW is utilised for 1 hour the energy consumed is said to be 1 kWh. This unit is called **Board of Trade Unit**.

Key point : *The electricity bills we are getting are charged based on this commercial unit of energy i.e. kWh or unit.*

1.13.3 Thermal Units

1. Heat energy : The flow of current through a material produces a heat. According to the principle of conservation of energy, the electrical energy spent must be equal to the heat energy produced. This is called **Joule's law**.

$$\boxed{\text{Heat energy } H = VIt = I^2 R t = \frac{V^2}{R} t \quad \text{joules}}$$

2. Specific heat capacity : The quantity of heat required to change the temperature of 1 kilogram of substance through 1 degree kelvin is called specific heat of that substance.

Key Point : *Its unit is Joules/kg - °K. This is denoted by 'C'.*

$$\boxed{C = \frac{Q}{m\Delta T} = \frac{Q}{m(T_2 - T_1)}}$$

Following table gives the values of specific heat capacity of various substances.

Substance	Specific heat capacity in J/kg - °K
Water	4187
Copper	390
Aluminium	950
Iron	500

Table 1.4

3. Sensible heat : The quantity of heat gained or lost when change in temperature occurs is called sensible heat. This can be calculated as,

$$\boxed{\text{Sensible heat} = m\,C\,\Delta t \quad \text{Joules.}}$$

m = Mass of substance in kg

C = Specific heat in J/kg -°K

Δt = $t_2 - t_1$ = change in temperature

4. Latent heat : The quantity of heat required to change the state of the substance i.e. solid to liquid to gas without change in its temperature is called latent heat.

It can be calculated as,

$$\boxed{\text{Latent Heat} = m \times L \quad \text{Joules.}}$$

Where m = Mass of substance in kg

L = Specific latent heat or specific enthalpy

The unit of L is J/kg while unit of latent heat is joules.

$$\boxed{\text{Total heat} = \text{Sensible heat} + \text{Latent heat}}$$

The various relations between electrical and thermal units are,

$$\boxed{\begin{aligned} 1 \text{ calorie} &= 4.186 \text{ joules} \\ 1 \text{ joule} &= \frac{1}{4.186} \text{ calorie} \\ &= 0.2389 \text{ calorie} \\ 1 \text{ kWh} &= 3.6 \times 10^6 \text{ J} = 860 \text{ k cal} \end{aligned}}$$

5. Specific enthalpy : It is the heat required to change the state of one kilogram mass of a substance without change in temperature.

Its unit is J/kg.

6. Calorific value : Heat energy can be produced by burning the fuels. The calorific value of a fuel is defined as the amount of heat produced by completely burning unit mass of that fuel.

It is measured in kJ/gram, kJ/kg etc.

$$\boxed{\text{Heat produced in joules} = \text{Mass in kg} \times \text{calorific value in J/kg}}$$

7. Water equivalent of container : When water is heated in a container then the container is also heated. So heat is wasted to heat the container. To take into account this heat, water equivalent of container must be known.

Key Point : *The water equivalent of container means whatever heat is required to heat body of container can be considered to be equivalent to heat energy required to heat equivalent mass of water.*

This is shown in the Fig. 1.16.

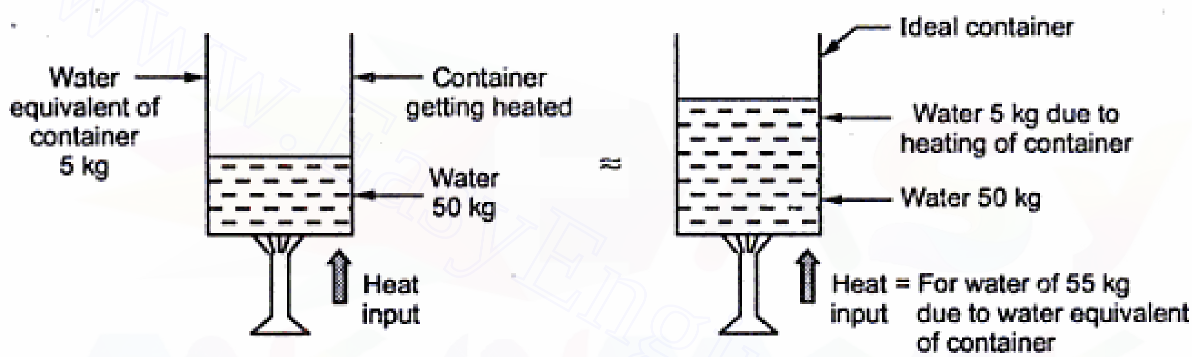


Fig. 1.16 Concept of water equivalent

1.13.4 Efficiency

The efficiency can be defined as the ratio of energy output to energy input. It can also expressed as ratio of power output to power input.

Its value is always less than 1. Higher its value, more efficient is the system of equipment. Generally expressed in percentage. Its symbol is η.

$$\boxed{\begin{aligned} \%\,\eta &= \frac{\text{Energy output}}{\text{Energy input}} \times 100 \\ &= \frac{\text{Power output}}{\text{Power input}} \times 100 \end{aligned}}$$

1.14 Cells and Batteries

A device which is used as a source of e.m.f. and which works on the principle of conversion of chemical energy into an electrical energy, is called a cell. But practically the voltage of a single cell is not sufficient to use in any practical application. Hence various cells are connected in series or parallel to obtain the required voltage level. The combination of various cells, to obtain the desired voltage level is called a **battery**.

The conductors of electricity can be classified in two categories as,

1. **Non electrolytes :** Conductors which are not affected by the flow of current through them are nonelectrolytes. The examples are metals, alloys, carbon and some other materials.
2. **Electrolytes :** Conductors which undergo decomposition due to the flow of current through them are electrolytes. The examples are various acids, bases, salt solutions and molten salts.

In any cell, two different conducting materials are immersed in an electrolyte. The chemical reaction results which separates the charges forming a new solution. The charges get accumulated on the conductors. Such charged conductors are called **electrodes**. The positively charged conductor is called **anode** while the negatively charged conductor is called **cathode**. Thus the charge accumulated on the electrodes creates a potential difference between the two conductors. The conductor ends are brought out as the terminals of the cell, for connecting the cell to an external circuit. The terminals are marked as positive and negative. Thus the chemical energy gets converted to an electrical energy. Hence the cell is an electrochemical device.

Key Point : *The chemical action in the cell continuously separates the charges to maintain the required terminal voltage.*

1.15 Types of Cells

The two types of cells are,

1. **Primary Cells :** The chemical action in such cells is not reversible and hence the entire cell is required to be replaced by a new one if the cell is down. The primary cells can produce only a limited amount of energy. Mostly the nonelectrolytes are used for the primary cells. The various examples of primary cells are zinc-carbon dry cell, zinc chloride cell, alkaline cells, mercury cell etc.
2. **Secondary Cells :** The chemical action in such cells is reversible. Thus if cell is down, it can be charged to regain its original state, by using one of the charging methods. In charging, the electrical energy is injected to the cell by passing a current in the opposite direction through it. In such cells, the electrical energy is stored in the form of chemical energy and the secondary cells are also called **storage cells, accumulators or rechargeable cells.** These are used to produce large amount of energy. The various types of secondary cells are Lead-acid cell, Nickel-cadmium alkaline cell etc. The most commonly used secondary cell is a Lead-acid cell or Lead-acid battery.

1.16 Cell Terminology

The various terminologies related to a cell are,

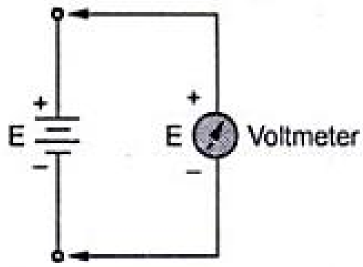


Fig. 1.17 E.M.F of a cell

1. E.M.F. of a cell : The voltage of a cell in an open circuit condition, measured by a very high resistance voltmeter is called e.m.f. of a cell. This is denoted as E, measured in volts. This is shown in the Fig. 1.17.

2. Internal resistance of a cell : The cell completes path of current from its positive terminal to negative terminal through **external circuit**. But to complete the closed path, the current flows from negative to positive terminal of cell, **internally**. The opposition by a cell to a current, when it flows internal to the cell is called the **internal resistance** of the cell. It is denoted as r and measured in ohms.

In an equivalent circuit of a cell, its internal resistance is shown in series with that cell. The Fig. 1.18 shows a cell and its internal resistance.

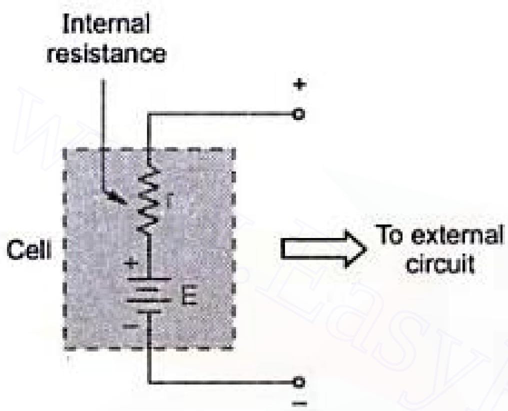


Fig. 1.18 Symbol of a cell

3. Terminal voltage : When an external resistance is connected across the terminals of the cell, the current I flows through the circuit. There is voltage drop 'Ir' across the internal resistance of the cell. The cell e.m.f. E has to supply this drop. Hence practically the voltage available at the terminals of the cell is less than E by the amount equal to 'Ir'. This voltage is called the **terminal voltage** V. This is shown in the Fig. 1.19.

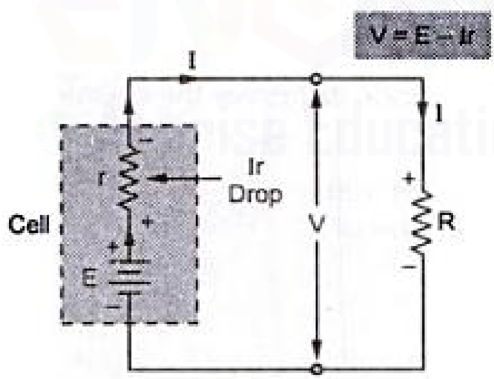


Fig. 1.19 Terminal voltage

Mathematically, the terminal voltage is given by,

$$\boxed{V = E - Ir}$$

From external resistance side we can write,

$$\boxed{V = IR}$$

Key Point : *Practically internal resistance of the cell must be as small as possible.*

It can also be observed that on no load i.e. external resistance not connected, the open circuit terminal voltage is same as e.m.f. of the cell, as current I = 0.

$\therefore$ $\boxed{V = E \quad \text{... on no load i.e. open circuit}}$

1.17 Primary Cells

It is seen that the primary cell is that which is required to replace by new one when it is run down.

The oldest types of primary cell are simple voltaic cell, Daniell cell, Leclanche cell etc. Some commonly used primary cells are,

1. Dry cell [Zinc-Carbon]
2. Mercury cell
3. Zinc-Chloride cell
4. Lithium cell
5. Alkaline Zinc-mercury oxide cell

Let us discuss primary cell in detail.

1.17.1 Dry Zinc-Carbon Cell

This is most common type of dry cell. It is the type of Leclanche cell.

Negative electrode → Zinc cup lined with paper	
Positive electrode → Centrally located carbon rod	

The space between the paper and carbon rod is filled with a paste of sal ammoniac, zinc chloride, manganese dioxide and carbon dust.

Key Point: *The paste is not dry and if it becomes dry, the cell becomes useless.*

The sal ammoniac acts as an electrolyte, the zinc chloride improves the chemical action and manganese dioxide acts as depolariser.

The Fig. 1.20 shows the cross-section of zinc-carbon dry cell.

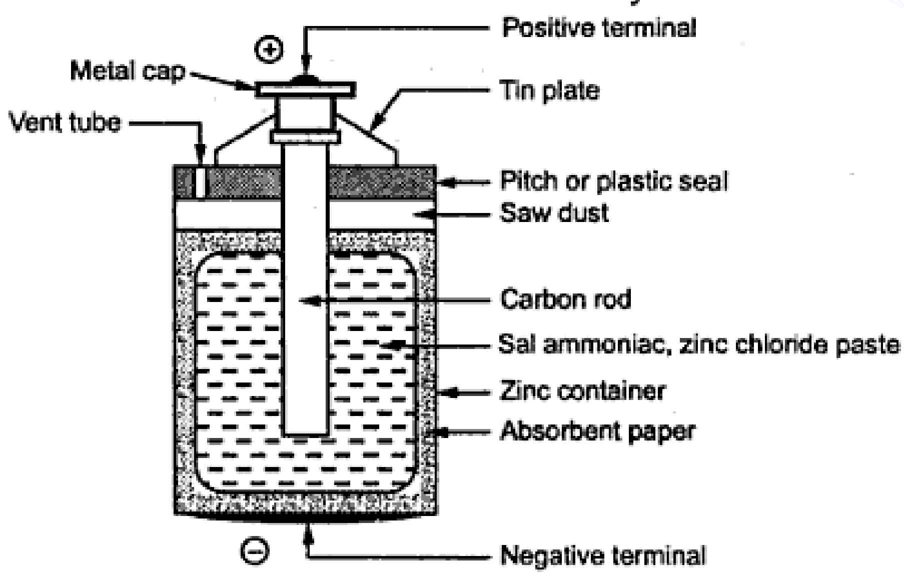


Fig. 1.20 Zinc-carbon dry cell

The zinc container is lined with paper to avoid direct reaction of zinc with carbon. The container is sealed with an insulator called pitch which is plastic seal of the cell. The tin plates are used at top and bottom which are positive and negative terminals of the cell respectively. The carbon dust added to the paste used to improve the conduction of the electrolyte. Externally the zinc container is covered with cardboard jacket to avoid any type of leakage.

1.17.1.1 Cell Reaction

When zinc atoms react with the electrolyte, the electrons are removed from the carbon rod. These electrons accumulate on the zinc electrode. As the electrons are negatively charged, the zinc electrode acts as **negative** terminal. The carbon rod from where negative charge is lost in the form of electrons, acts as **positive** terminal.

The sal ammoniac i.e. ammonium chloride paste reacts with zinc to liberate hydrogen.

$$Zn + 2\,NH_4\,Cl \longrightarrow Zn\,Cl_2 + 2\,NH_3 + H_2$$

The hydrogen reacts with manganese dioxide as,

$$H_2 + 2\,MnO_2 \longrightarrow Mn_2\,O_3 + H_2O$$

Key Point : *Hydrogen is liberated at faster rate while reaction of MnO_2 is slow. Due to this, hydrogen accumulates in the form of thin layer on carbon rod. This is called* ***polarisation****. Due to this, cell e.m.f. goes down if cell is operated for longer time.*

1.17.1.2 Features of Cell

1. The e.m.f. of new dry cell is about 1.5 V.
2. The internal resistance is about 0.1 to 0.4 Ω.
3. The capacity of 32 Ah when discharged through 20 Ω resistor till voltage drops to 0.5 V.
4. When not in use for long time, zinc gets attacked by the paste slowly and cell becomes useless though not in use.
5. In working condition, voltage drops due to polarisation hence used for intermittent service. When disconnected the depolarisation occurs to resotre the cell e.m.f.
6. Least expensive.
7. Portable and convenient to use.

1.17.1.3 Applications

Mostly used to get intermittent service to avoid polarisation. The various applications are,

1. Torch lights
2. Telephone and telegraph systems
3. Electronic apparatus and toyes
4. Wall clocks

5. Electric bells
6. Radio receivers and many other consumer applications

1.17.2 Mercury Cell

This is another type of a primary cell

Negative electrode →	Zinc cylinder
Positive electrode →	Mercury compound in contact with nickel plated steel or stainless steel

The concentrated solution of **potassium hydroxide** (KOH) and **zinc oxide** (ZnO) is used as an **electrolyte**.

It is available in cylindrical shape or miniature button shape.

The Fig. 1.21 shows the construction of a cylindrical mercury cell.

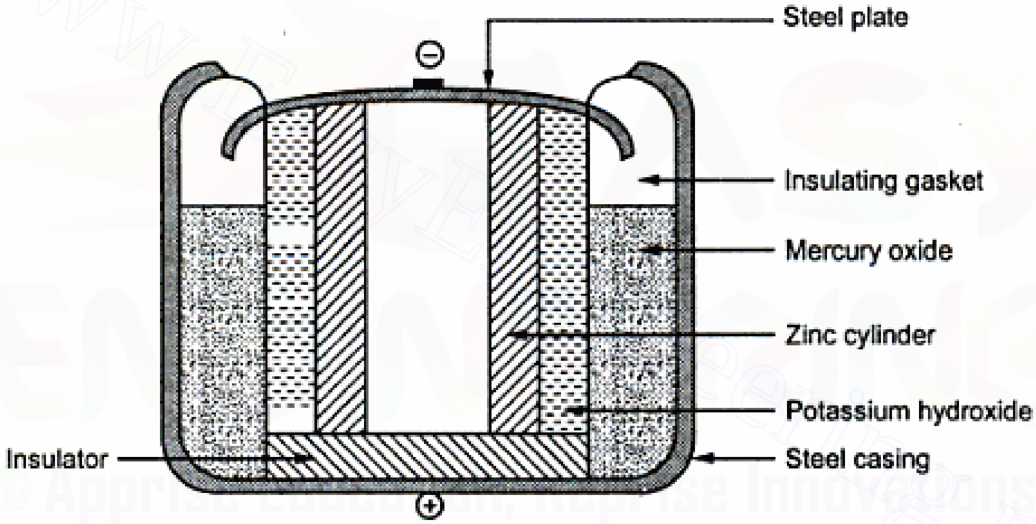


Fig. 1.21 Mercury Cell

The zinc electrode acting as negative terminal is made in the form of a hollow cylinder. The steel casing acts as a positive terminal. The layer of mercury oxide covers the electrolyte which is solution of KOH and ZnO. The cell is sealed with the help of insulating gasket.

1.17.2.1 Chemical reaction

The net chemical reaction involved in the cell is,

$$Zn + HgO \longrightarrow ZnO + Hg$$

1.17.2.2 Features of the cell

1. The chemical reaction does not evolve any gas hence no polarisation.
2. The cell maintains its e.m.f. for longer time in working condition.
3. The terminal voltage is about 1.2 to 1.3 V.
4. It has long life.
5. It has high ratio of output energy to weight of about 90 – 100 Wh/kg.
6. Costlier than dry cell.
7. It has high energy to volume ratio of about 500 – 600 Wh/L.
8. It has high efficiency.
9. Good resistance to shocks and vibrations.
10. Disposal is difficulty due to presence of poisonous materials inside.

1.17.2.3 Applications

The cells are preferred for providing power to small devices as available in miniature button shapes. The various applications are,

1. Hearing aids.
2. Electronic calculators.
3. Electronic clocks.
4. Guided missiles.
5. Medical electronic appliances such as pace makers.
6. Audio devices and cameras.

1.18 Secondary Cells

It is seen that the secondary cells are rechargeable cells as the chemical reactions in it are reversible. The two types of secondary cells are,

1. Lead acid cell
2. Alkaline cell

Let us discuss these cells in detail.

1.19 Lead Acid Battery

The various parts of lead acid battery are,

1. **Positive plate or Anode :** It is **lead peroxide** (PbO_2) plate of chocolate, dark brown colour.
2. **Negative plate or Cathode :** It is made up of **pure lead** (Pb) which is grey in colour.

3. **Electrolyte :** For the necessary chemical action aqueous solution of **sulphuric acid** (H_2SO_4) is used as an electrolyte.
4. **Seperators :** The positive and negative plates are arranged in groups and are placed alternately. The seperators are used to prevent them from coming in contact with each other short circuiting the cell.
5. **Container :** The entire assembly of plates along with the solution is placed in the plastic or ceramic container.
6. **Bottom blocks :** To prevent the short circuiting of cell due to the active material fallen from the plates, the space known as bottom blocks is provided at the bottom of the container.
7. **Plate connector :** The number of negative and positive plates are assembled alternately. To connect the positive plates together seperate connectors are used which are called plate connectors. The upward connections of the plate connectors are nothing but the terminals of the cell.
8. **Vent-plug :** These are made up of rubber and screwed to the cover of the cell. Its function is to allow the escape of gases and prevent escape of electroyte.

The Fig. 1.22 shows the construction of lead acid battery.

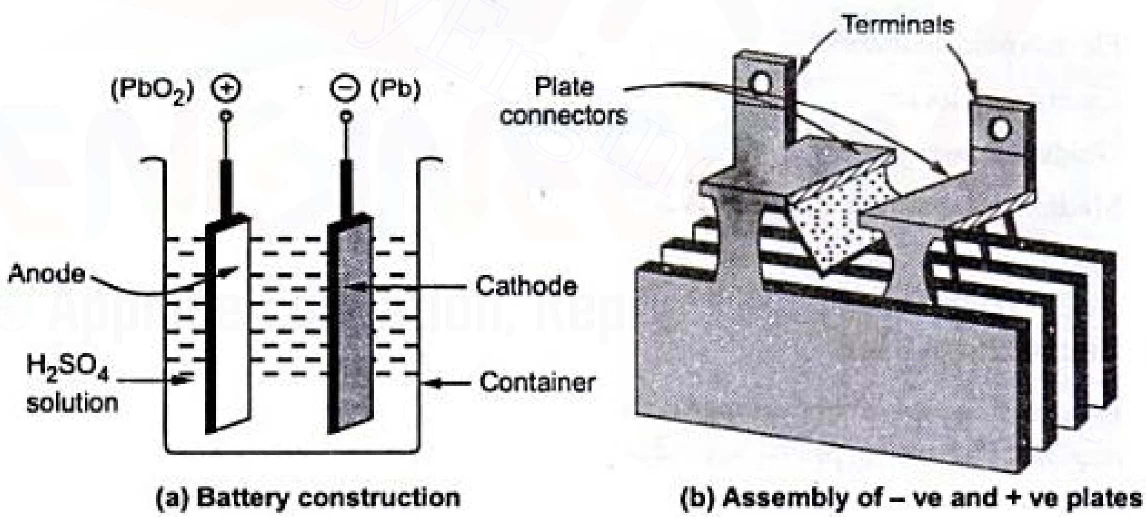


Fig. 1.22 Construction of lead acid battery

The various plates are welded to the plate connectors. The plates are immersed in H_2SO_4 solution. Each plate is a grid or frame work. Except some special assemblies, wide space between the plates is provided. In an alternate assembly of plates, the **negative plate is one more in number than positive.** So all the positive plates can work on both the sides.

1.19.1 Functions of Separators

The **separators** used have the following functions in the construction of lead acid battery :

1. Acting as mechanical spacer preventing the plates to come in contact with each other.
2. Prevent the growth of lead trees which may be formed on the negative plates and due to heavy accumulation may reach to positive plate to short circuit the cell.
3. Help in preventing the plates from shedding of the active material. The separators must be mechanically strong and must be porous to allow diffusion of the electrolyte through them.

1.19.2 Chemical Action in Lead Acid Battery

The chemical action in the lead acid battery can be divided into three processes :

1. First charging   2. Discharging   3. Recharging

Let us discuss these processes in detail.

1. First charging : When the current is passed for the first time through electrolyte, the H_2O in the electrolyte is electrolysed as,

$$H_2O \longrightarrow 2\,H^+ + O^-$$

The hydrogen ions as positively charged get attracted towards one of the electrodes which acts as cathode (negative). The hydrogen does not combine with lead and hence cathode retains its original state and colour.

The oxygen ion as negatively charged gets attracted towards the other lead plate which acts as anode (positive). But this oxygen chemically combines with the lead (Pb) to form lead peroxide (PbO_2). Due to the formation of lead peroxide the anode becomes dark brown in colour.

Thus anode is dark brown due to the layer of lead peroxide deposited on it while the cathode is spongy lead electrode.

So there exists a potential difference between the positive anode and the negative cathode which can be used to drive the external circuit. The electrical energy obtained from chemical reaction is drawn out of the battery to the external circuit, which is called discharging.

2. Discharging : When the external supply is disconnected and a resistance is connected across the anode and cathode then current flows through the resistance, drawing an electrical energy from the battery. This is **discharging.** The direction of current is opposite to the direction of current at the time of first charging. The discharging is shown in the Fig. 1.23.

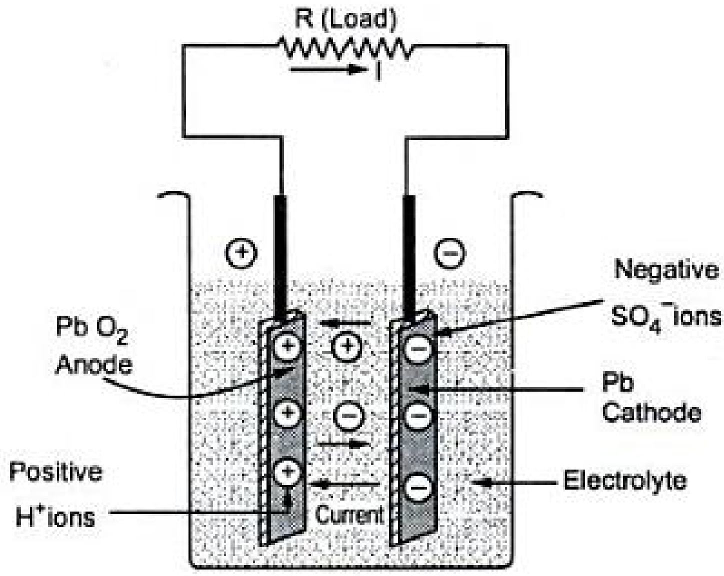


Fig. 1.23 Discharging

During the discharging, the directions of the ions are reversed. The H^+ ions now move towards anode and the SO_4^{--} ions move towards cathode.

This is because H_2SO_4 decomposes as,

$$H_2SO_4 \rightarrow 2H^+ + SO_4^{--}$$

At the anode, the hydrogen ions become free atoms and react with lead peroxide alongwith the H_2SO_4 and ultimately lead sulphate Pb SO_4 results as,

$$PbO_2 + H_2SO_4 + 2H \rightarrow PbSO_4 + 2H_2O \qquad \text{... At Anode}$$

At the cathode, each SO_4^{--} ion become free SO_4 which reacts with the metallic lead to get lead sulphate.

$$Pb + SO_4 \rightarrow PbSO_4 \qquad \text{... At Cathode}$$

Thus discharging results into formation of whitish lead sulphate on both the electrodes.

3. Recharging : The cell provides the discharge current for limited time and it is necessary to recharge it after regular time interval. Again an e.m.f. is injected through the cell terminals with the help of an external supply.

The charging is shown in the Fig. 1.24.

Due to this recharging current flows and following reactions take place,

At the cathode, $PbSO_4 + 2H \rightarrow Pb + H_2SO_4$

At the anode, $PbSO_4 + SO_4 + 2H_2O \rightarrow PbO_2 + 2H_2SO_4$

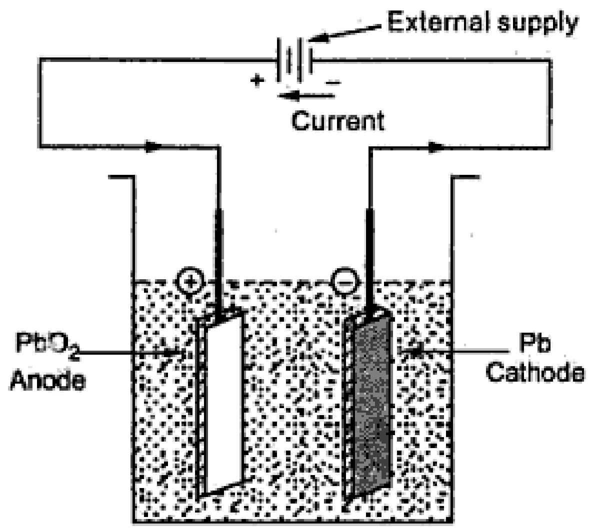


Fig. 1.24 Recharging the lead acid battery

Thus the PbO_2 gets formed at anode while lead sulphate layer on the cathode is reduced and gets converted to grey metallic lead. So the strength of the cell is regained. It can be seen from the reaction that water is used and H_2SO_4 is created. Hence the specific gravity of H_2SO_4 which is the charging indicator of battery, increases.

Key Point: *More the specific gravity of H_2SO_4, better is the charging.*

The specific gravity is 1.25 to 1.28 for fully charged battery while it is about 1.17 to 1.15 for fully discharged battery. The voltage also can be used as a charging indicator. For fully charged battery it is 2.2 to 2.5 volts.

The chemical reaction during charging and discharging can be represented using single equation as,

$$\underbrace{PbO_2}_{\text{Positive Anode}} + \underbrace{2H_2SO_4}_{\text{Electrolyte}} + \underbrace{Pb}_{\text{Negative Cathode}} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underbrace{2PbSO_4}_{\text{Both anode and cathode}} + \underbrace{2H_2O}_{\text{Water}}$$

Fig. 1.25

1.19.3 Features of Lead Acid Battery

The various features of lead acid battery are,

1. The capacity is about 100 to 300 ampere-hours.
2. The voltage is 2.2 V for fully charged condition.
3. The cost is low.

4. The internal resistance is very low.
5. The current ratings are high.
6. The ampere-hour efficiency is about 90 to 95% with 10 hour rate.

1.19.4 Conditions of a Fully Charged Battery

For identifying whether the battery is fully charged or not, following conditions must be observed,

1. The specific gravity of H_2SO_4 must be 1.25 to 1.28.
2. The voltage stops to rise and its value is about 2.2 to 2.5 V.
3. Violent gasing starts as battery is fully charged.
4. The colour of positive plate becomes dark brown while the colour of negative plate becomes slate grey.

1.19.5 Maintenances and Precautions to be taken for Lead Acid Battery

The following steps must be taken in the maintenance of the lead acid battery,

1. The battery must be recharged immediately when it discharges.
2. The level of the electrolyte must be kept above the top of plates so the plates remain completely immersed.
3. The rate of charge and discharge shold not be exceeded as specified by the manufacturers.
4. Maintain the specific gravity of the electrolyte between 1.28 to 1.18.
5. The loss of water due to evaporation and gasing must be made up using only distilled water.
6. The connecting plugs should be kept clean and properly tightened.
7. It should not be discharged till its voltages falls below 1.8 V.
8. When not in use, it should be fully charged and stored in a cool and dry place.
9. It should not be kept long in discharged condition. Otherwise $PbSO_4$ gets converted to hard substance which is difficult to remove by charging. This is called **sulphating**. Thus sulphating should be avoided.
10. The battery must be given periodic overcharge at half the normal rate to remove white sulphate.
11. The temperature of the battery should not exceed 45 °C otherwise plates deteriorate rapidly.
12. The battery terminals should not be shorted to check whether battery is charged or not.
13. Always keep the surface of the container dry.

14. No sulphuric acid should be added till it is sure that low specific gravity is due to under charge and not due to white sulphate formed on plates.
15. The acid used must be pure without any impurity and colourless.
16. The sparks and flames must be kept away from the battery.

1.19.6 Testing Procedure for Lead Acid Battery

1. Using hydrometer : The testing basically involves the checking of specific gravity of the sulphuric acid. It can be checked by the use of **hydrometer**. The hydrometer consists of a glass float with a calibrated stem placed in a syringe. The readings on hydrometer are shown in the Fig. 1.26.

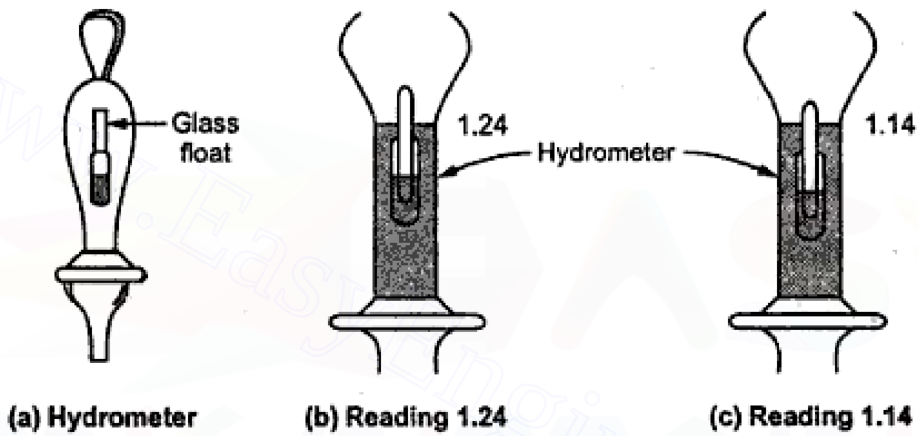


Fig. 1.26 Testing lead acid battery

2. Using cell tester : Another method of testing the lead acid battery is called **high discharge test** or **short circuit test**. The cell tester is used for this test. It consists of 0 - 3 V voltmeter shunted by a low resistance. The low resistance shunt is connected between two prongs as shown in the Fig. 1.27. The prongs are pressed against the cell terminals. A high discharge current flows through the low resistance and cell voltage is indicated by the voltmeter.

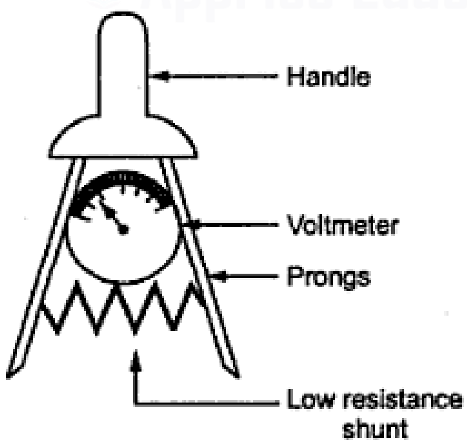


Fig. 1.27 Cell tester

The battery with full charge, without sulphatation occurred in it, shows proper reading on voltmeter. But battery which is sulphated, showing other indications of full charge will show low voltage reading. Thus this is reliable method of testing the lead acid battery.

Thus if the reading on voltmeter is less than 1.8 V, the battery needs charging while if it is more than 2.5 V it is overcharged. According to specific gravity reading the distilled water should be added to bring specific gravity back to its normal value.

1.19.7 Applications

The various applications of lead acid battery are,

1. In emergency lighting systems.
2. In automobiles for starting.
3. Uninterrupted power supply systems.
4. Railway signalling.
5. Electrical substations and the power stations.
6. For compensating feeder drops in case of heavy loads.
7. For energizing the trip coils of the relays and the switch gears.
8. As a source of supply in mines and telephone exchanges.

Apart from these applications, the lead acid batteries are used in various other areas also like telephone systems, aeroplanes, marine applications etc.

1.20 Battery Capacity

The battery capacity is specified in ampere-hours (Ah).

> **Key Point** : *It indicates the amount of electricity which a battery can supply at the specified discharge rate, till its voltage falls to a specified value.*

For a lead acid battery, the discharge rate is specified as 10 hours or 8 hours while the value of voltage to which it should fall is specified as 1.8 V.

Mathematically the product of discharge current in amperes and the time for discharge in hours till voltage falls to a specified value is the capacity of a battery.

$$\boxed{\text{Battery capacity} = I_D \times T_D \text{ (Ah)}}$$

Where I_D = Discharge current in amperes

T_D = Time of discharge in hours till voltage falls to a specified value.

Sometimes it is specified as **watt-hours (Wh)**. It is the product of the average voltage during discharge and the ampere hour capacity of a battery.

The battery capacity depends on the following factors,

1. **Discharge rate :** As the rate of discharge increases, the battery capacity decreases.
2. **Specific gravity of electrolyte :** More the specific gravity of electrolyte, more is the battery capacity as it decides internal resistance of the battery.
3. **Temperature :** As temperature increases, the battery capacity increases. This is shown in the Fig. 1.28.

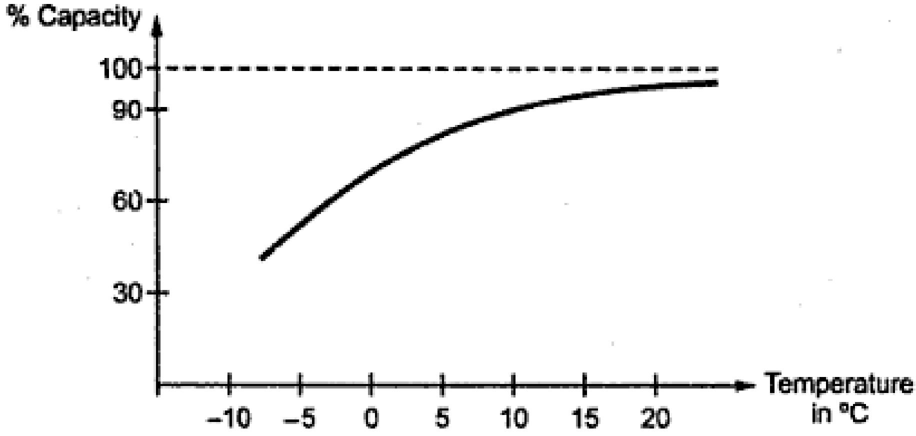


Fig. 1.28 Effect of temperature on battery capacity

4. **Size of the plates :** This is related to the amount of active material present in the battery.

1.21 Battery Efficiency

Mainly the battery efficiency is defined as the ratio of output during discharging to the input required during charging, to regain the original state of the battery.

It is defined in many ways as,

1. Ampere-hour efficiency or quantity efficiency
2. Watt-hour efficiency or energy efficiency

1.21.1 Ampere-hour Efficiency

It is defined as ratio of output in ampere-hours during discharging to the input in ampere-hours during charging. It is denoted as η_{Ah}.

$$\therefore \quad \eta_{Ah} = \frac{\text{Ampere-hours on discharge}}{\text{Ampere-hours on charge}}$$

$$\therefore \quad \boxed{\% \, \eta_{Ah} = \left[\frac{\text{Current} \times \text{Time on discharge}}{\text{Current} \times \text{Time on charge}}\right] \times 100}$$

For lead acid battery, it ranges between 80 % to 90 %.

1.21.2 Watt-hour Efficiency

It is defined as the ratio of output in watt-hours during discharging to the input in watt-hours during charging. It is denoted as η_{Wh}.

$$\therefore \quad \eta_{Wh} = \frac{\text{Watt-hours on discharge}}{\text{Watt-hours on charge}}$$

$$\therefore \% \eta_{Wh} = \left\{ \frac{[\text{Voltage during discharge (average)}] \times [\text{Current} \times \text{time at discharge}]}{[\text{Voltage during charge (average)}] \times [\text{Current} \times \text{time at charge}]} \right\} \times 100$$

$$= \eta_{Ah} \times \frac{\text{Average voltage during discharge}}{\text{Average voltage during charge}}$$

Key Point: *It can be seen that as average voltage during discharge is less than the average voltage during charge, the watt-hour efficiency is always less than the ampere-hour efficiency.*

For lead acid battery, watt-hour efficiency ranges between 70 % to 80 %.

1.22 Charge and Discharge Curves

The behaviour of battery voltage with respect to the time in hours of charging or discharging at normal rate is indicated by the curves called charge and discharge curves.

During discharge of the lead acid cell, the voltage decreases from about 2.1 V to 1.8 V, when cell is said to be completely discharged. The discharge rate is always specified as 8 hours, 10 hours etc.

During charging of the lead acid cell, the voltage increases from 1.8 V to about 2.5 V to 2.7 V, when cell is said to be completely charged. If the discharge rate is high, the curve is more drooping as voltage decreases faster. Such typical charge and discharge curves for lead-acid cell are shown in the Fig. 1.29. While discharging the voltage decreases to 2 V very fast, then remains constant for long period and at the end of discharge period falls to 1.8 V. While charging, initially it rises quickly to 2.1 to 2.2 V and then remains constant for long time. At the end of charging period it increases to 2.5 to 2.7 V.

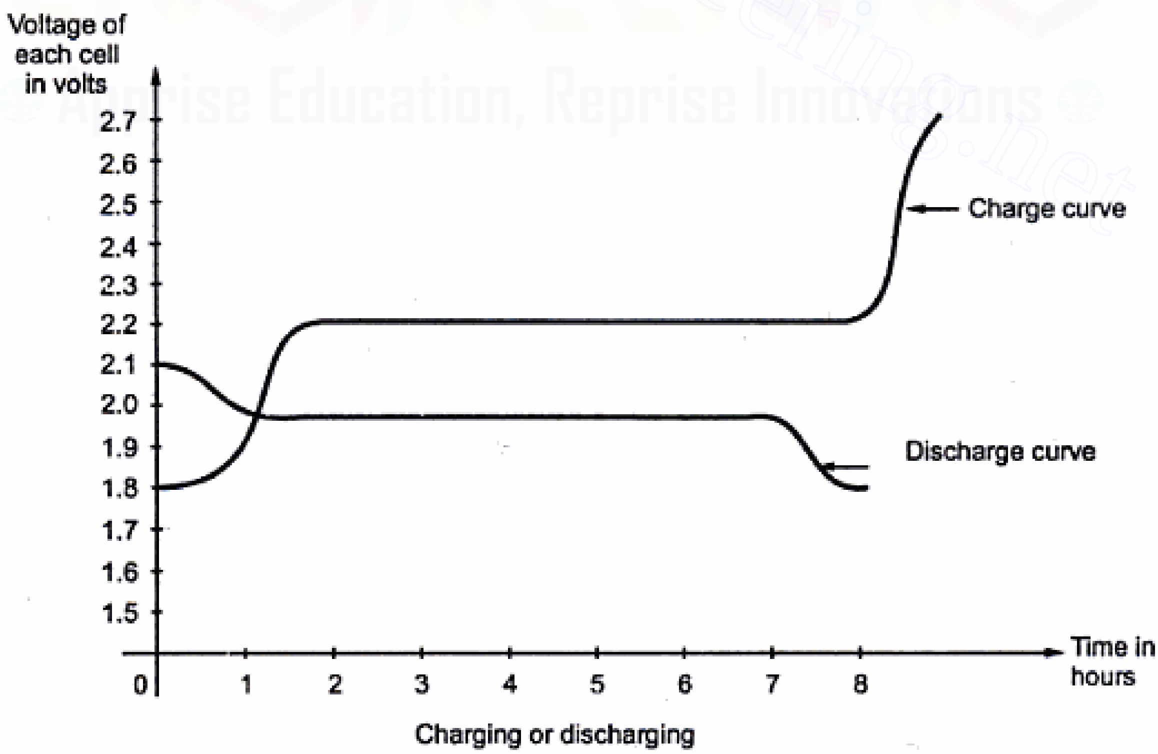


Fig. 1.29

1.23 Battery Charging

During charging, the chemical action takes place which is exactly opposite to that of discharging. Thus current in opposite direction to that at the time of discharge, is passed through the battery. For this the voltage applied is in excess of the voltage of the battery or cell. The battery voltage acts in opposite direction to that of the applied voltage and hence called back e.m.f. The charging current can be obtained as,

$$\boxed{\text{Charging current} = \frac{E_a - E_b}{R + r}}$$

Where E_a = Applied voltage

E_b = Back e.m.f. i.e. battery voltage

R = External resistance in the circuit

r = Internal resistance of the battery

Simple battery charging circuit used to charge the battery from d.c. supply is shown in the Fig. 1.30.

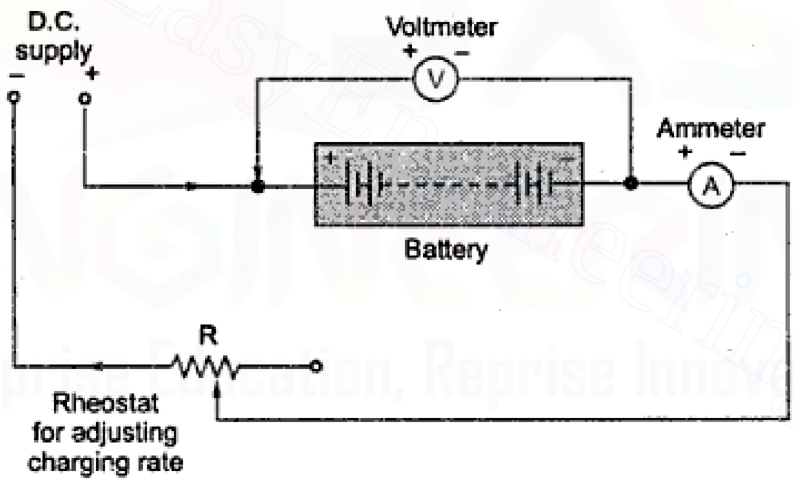


Fig. 1.30 Battery charging

The ammeter measures the charging current which is called **charging rate**, which can be adjusted using the external resistance R. The voltmeter measures the battery voltage. It is necessary that the positive terminal of the battery must be connected to the positive of the D.C. supply.

The charging current must be adjusted such that the temperature of the electrolyte will not increase beyond 100° to 110 °F.

1.23.1 Indications of Fully Charged Battery

The various indications of the fully charged cells are,

1. **Specific gravity :** The specific gravity of the fully charged cell increases upto 1.28 from about 1.18.
2. **Gassing :** When the cell is fully charged, it starts liberating the gas freely. In lead acid battery the hydrogen is liberated at cathode while oxygen at the anode. Gassing is a good indication of fully charged battery. Some acid particles may go out with the gases hence the charging room must be kept well ventilated.
3. **Voltage :** The voltage of the fully charged cell is about 2.7 V.
4. **Colour :** The colour of the plates changes for fully charged cell. Colour of the positive plate changes to dark chocolate brown while that of negative plate changes to grey colour. But as plates are immersed in the electrolyte, this indication is not clearly visible.

1.24 Charging Methods

The main methods of battery charging are,

1. Constant current method
2. Constant voltage method
3. Rectifier method

1.24.1 Constant Current Method

When the supply is high voltage but battery to be charged is of low voltage, then this method is used. The number of batteries which can be charged are connected in series across the available d.c. voltage. The constant current is maintained through the batteries with the help of variable resistor connected in series. The circuit is shown in the Fig. 1.31.

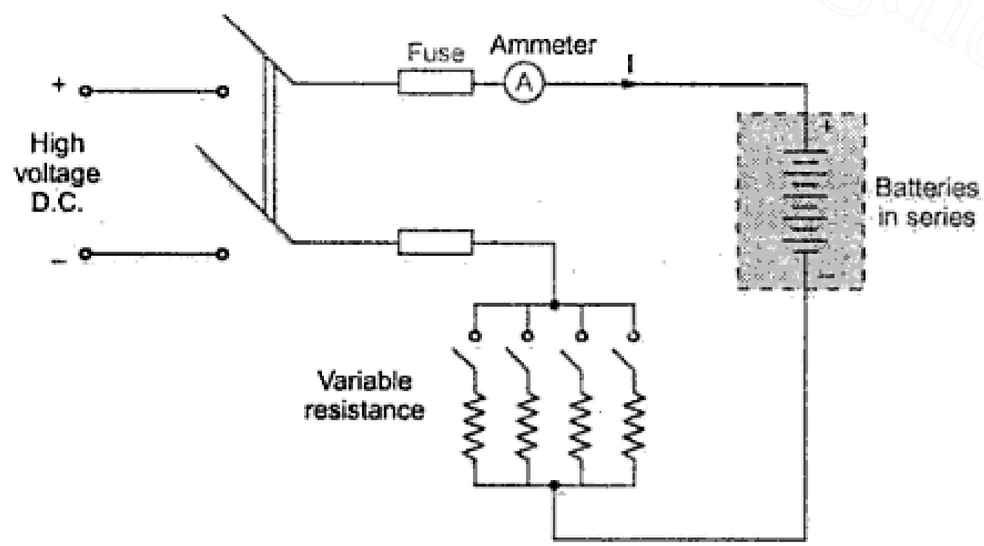


Fig. 1.31 Constant current method

The charging time required in this method is comparatively large. Hence in modern charger the number of charging circuits are used to give a variation of charging rates. Initially higher charging rate is used and later on lower charging rate is preferred.

1.24.2 Constant Voltage Method

In this method, the constant voltage is applied across the cells, connecting them in parallel. The charging current varies according to the state of the charge of each battery. The batteries to be charged are connected in 6 or 12 volt units across the positive and negative busbars i.e. mains supply. When the battery is first connected, a high charging current flows but as the terminal voltage of the battery increases, the charging current reduces automatically. At the end of the full charge, the voltage of the battery is equal to the voltage of the busbars and no current flows. The charging time required is much less in this method.

Another practically used method is called **trickle charge**. In this method, the charging current is maintained slightly more than the load current, through the battery. The load is constantly connected to the battery. So battery remains always in fully charged condition.

The Fig. 1.32 shows the cascade resistances used for the charging of batteries on d.c. mains supply.

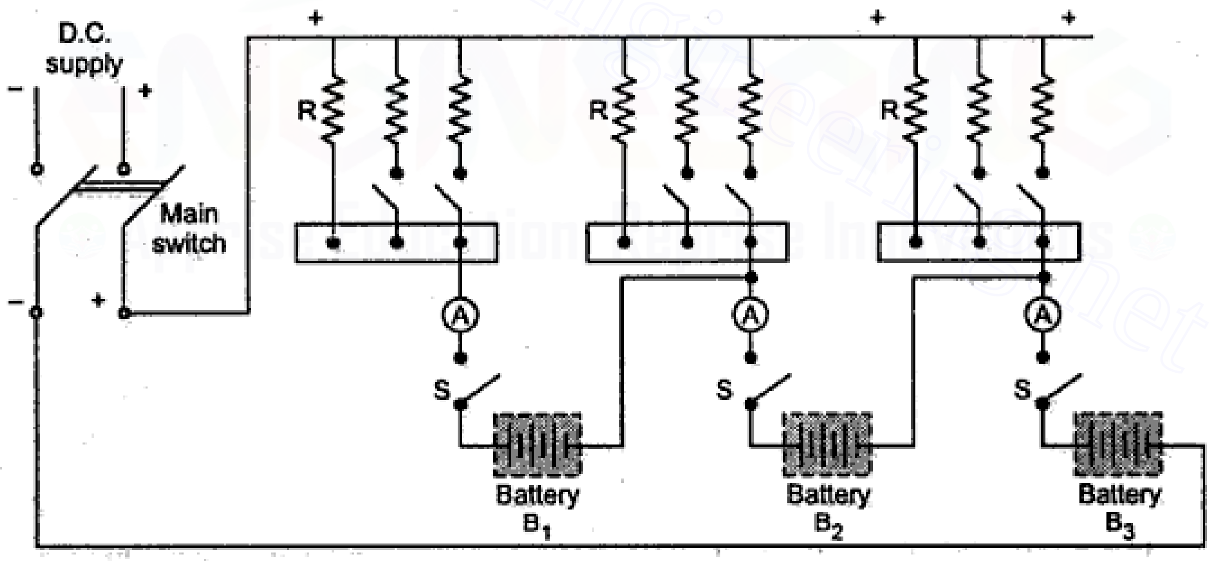


Fig. 1.32

The Fig. 1.33 shows the parallel charging circuit in which 2 separate groups each of 4 cells in series are connected in parallel across the mains.

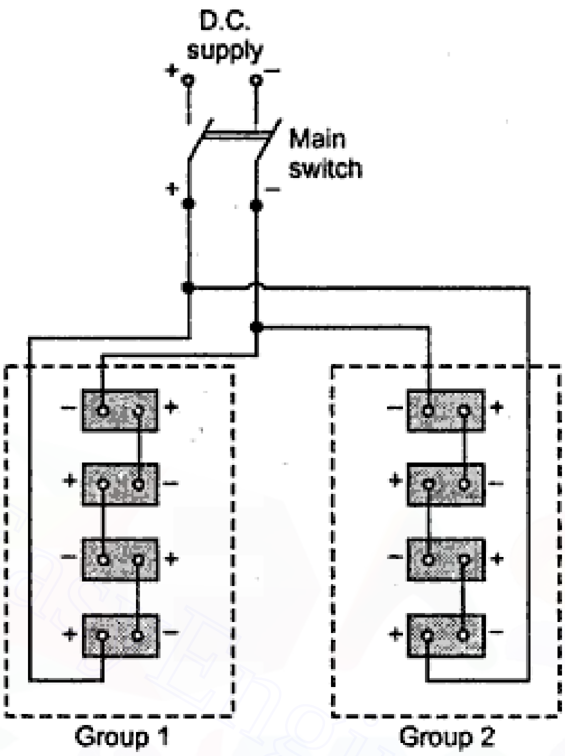


Fig. 1.33 Parallel charging circuit

1.24.3 Rectifier Method

When battery is required to be charged from a.c. supply, the rectifier method is used. The rectifier converts a.c. supply to d.c. Generally bridge rectifier is used for this purpose. The Fig. 1.34 shows the circuit used for rectifier method.

The step down transformer lowers the a.c. supply voltage as per the requirement. The bridge rectifier converts this low a.c. voltage to d.c. this is used to charge the battery.

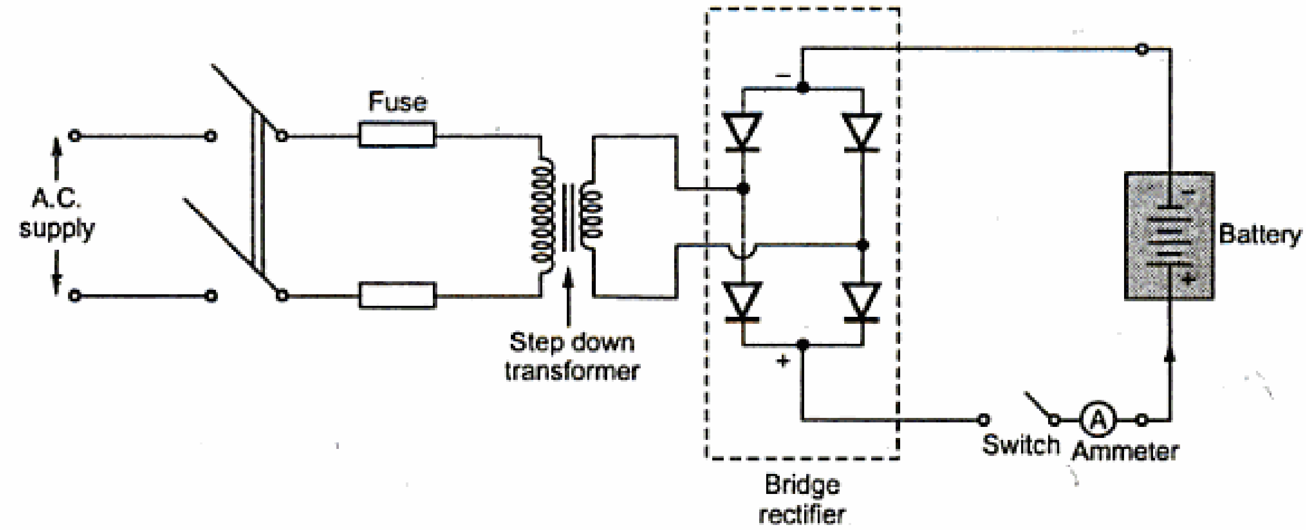


Fig. 1.34 Rectifier method

1.25 Grouping of Cells

The single cell is not sufficient to provide necessary voltage in many cases. Practically number of cells are grouped to obtain the **battery** which provides necessary voltage or current. The cells are grouped in three ways,

1. Series grouping
2. Parallel grouping
3. Series-parallel grouping

1.25.1 Series Grouping

The Fig. 1.35 shows the series grouping of cells so as to obtain the battery. There are n cells connected in series.

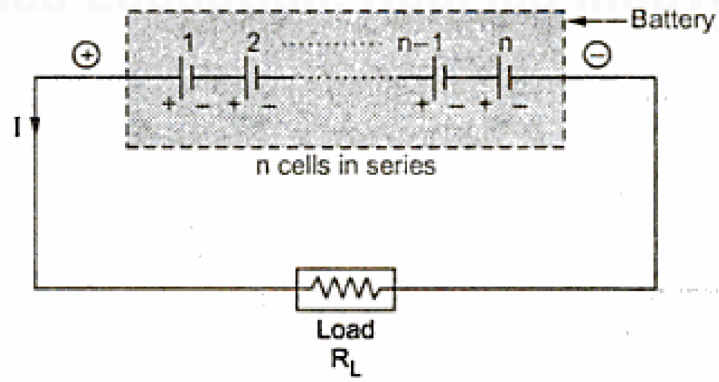


Fig. 1.35 Series grouping of cells

Let E = E.M.F. of each cell

r = Internal resistance of each cell

∴ V = Total voltage available = n × E volts

$$R_T = \text{Total circuit resistance} = \text{load} + \text{cells}$$
$$= R_L + n \times r$$

$$\therefore \quad \boxed{I = \frac{\text{Total voltage}}{\text{Total resistance}} = \frac{V}{R_T} = \frac{nE}{R_L + nr} \text{ A}}$$

Key Point : *In series circuit, current remains same. So this method does not improve current capacity. The current capacity is same as that of each cell connected in series. But voltage can be increased by increasing number of cells n.*

1.25.2 Parallel Grouping

In this method, positive terminals of cells are connected together and negative terminals are connected together as shown in the Fig. 1.36.

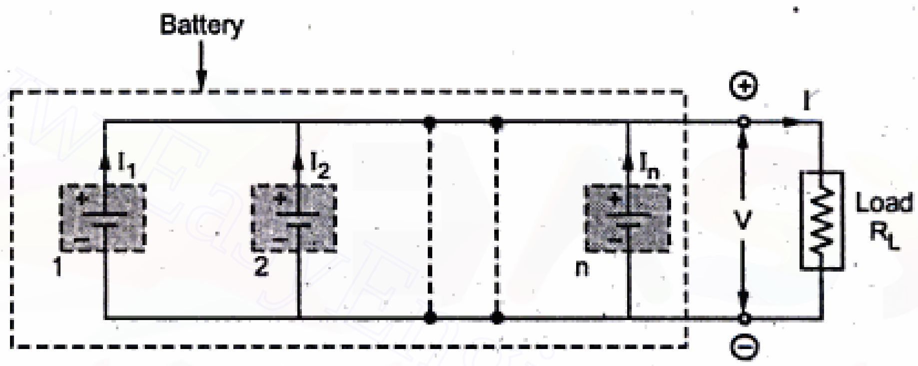


Fig. 1.36 Parallel grouping of cells

The terminal e.m.f. of each battery must be same as E.

$$\therefore \quad V = \text{Battery voltage} = E = \text{e.m.f. of each cell}$$
$$r = \text{Internal resistance of each cell}$$
$$I_n = \text{Current through nth branch}$$
$$\therefore \quad I = \text{Total current}$$
$$\therefore \quad \boxed{I = I_1 + I_2 + I_3 + \ldots + I_n}$$

Key Point : *It can be seen that in parallel grouping, the voltage remains same but by increasing number of cells the current capacity can be increased.*

In series grouping current rating of each cell must be same while in parallel grouping voltage rating of each cell must be same.

1.25.3 Series-Parallel Grouping

Practically the various groups can be connected in parallel where each group is a series combination of cells as shown in the Fig. 1.37.

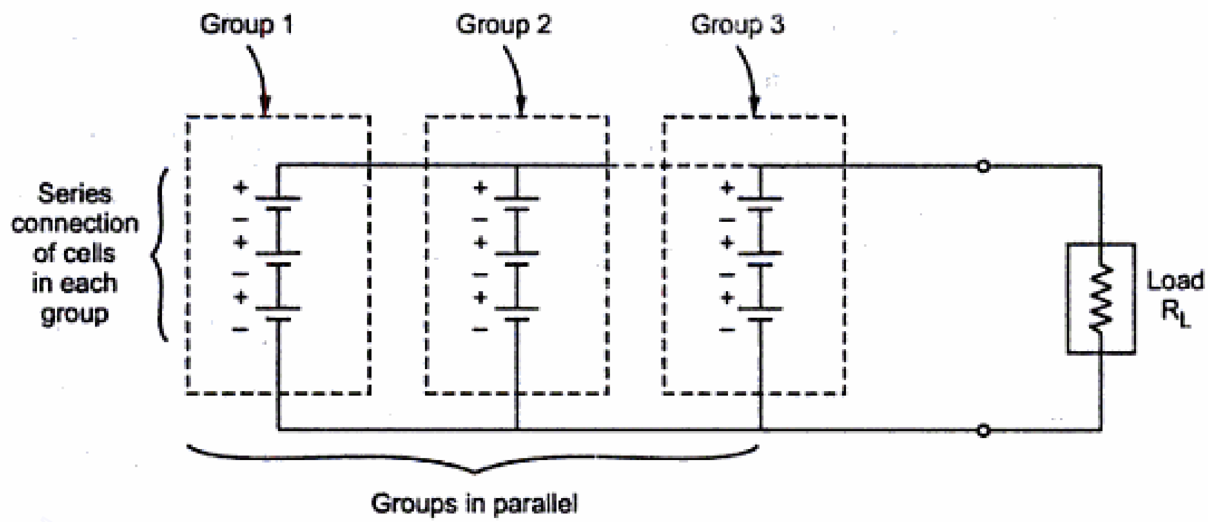


Fig. 1.37 Series-parallel grouping

This is used to satisfy both voltage and current requirement of the load.

1.26 Alkaline Cells

The secondary cells can be alkaline cells. These are of two types.

1. Nickel - iron cell or Edison cell
2. Nickel - cadmium or Nife Cell or Junger cell

1.27 Nickel - Iron Cell

In this cell,

Positive Plate → Nickel hydroxide [$(Ni(OH)_3$]
Negative Plate → Spongy iron (Fe)

The electrolyte is an alkali of 21 % solution of **potassium hydroxide solution** (KOH).

The insulated rods are used to seperate the positive and negative plates.

The Fig. 1.38 shows the construction of Nickel-iron cell.

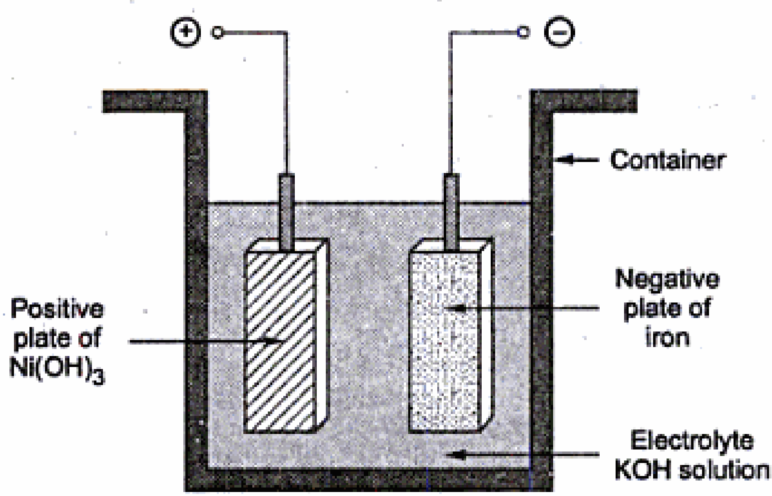


Fig. 1.38 Construction of Nickel-iron cell

1.27.1 Chemical Reaction

In a charged condition, the material of positive plate is $Ni(OH)_3$ and that of negative plate is iron. When it is connected to load and starts discharging, the nickel hydroxide gets converted to lower nickel hydroxide as $Ni(OH)_2$ while the iron on negative plate gets converted to ferrous hydroxide $Fe(OH)_2$. When charged again, reversible reaction takes place, regaining the material on each plate.

$$\underset{\text{Positive plate}}{2\,Ni(OH)_3} + \underset{\text{Electrolyte}}{2\,KOH} + \underset{\text{Negative plate}}{Fe} \underset{\text{Change}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{Positive plate}}{2\,Ni(OH)_2} + \underset{\text{Electrolyte}}{2\,KOH} + \underset{\text{Negative plate}}{Fe(OH)_2}$$

Fig. 1.39 Total reaction

> **Key Point** : *An electrolyte does not undergo any chemical change. Hence its specific gravity remains constant at about 1.2.*

By connecting various Nickel-iron cells properly, the Nickel-iron battery is obtained.

1.27.2 Electrical Characteristics

The electric characteristics indicates the variations in the terminal voltage of cell against the charging or discharging hours. The Fig. 1.40 shows the electrical characteristics of Nickel-iron cell during charging and discharging.

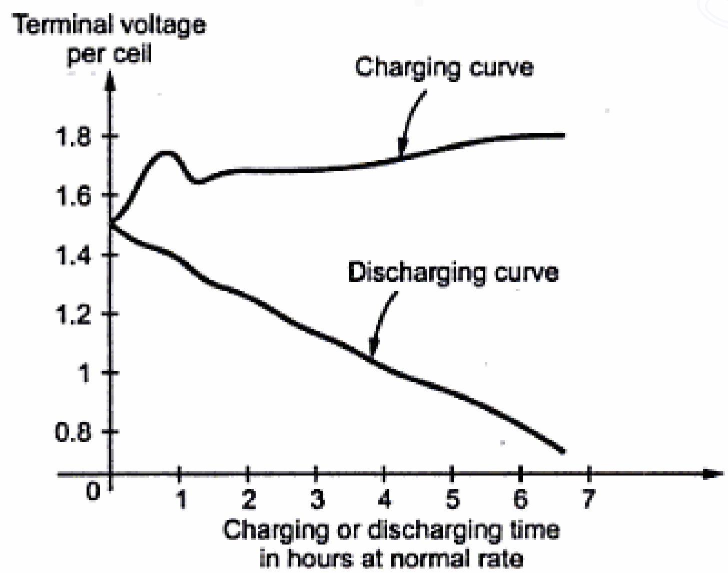


Fig. 1.40 Electrical characteristics of Nickel-iron cell

When fully charged its voltage is about 1.4 V and during discharging it reduces to about 1.1 to 1 V. During charging, the average charging voltage is 1.7 to 1.75 V.

> **Key Point**: *For Nickel-iron cell there is no specific minimum voltage below which the cell must not be discharged to avoid damage to the cell.*

1.27.3 Capacity

It is mentioned that electrolyte does not undergo any chemical change for this cell. Thus specific gravity of the electrolyte remains constant for long periods. Hence rate of discharge does not affect ampere-hour capacity of this cell significantly. **Thus Ah capacity of Nickel-iron cell remains almost constant.** But it does get affected by the temperature. The Fig. 1.41 shows the Ah capacity against discharging time curve for Nickel-iron cell.

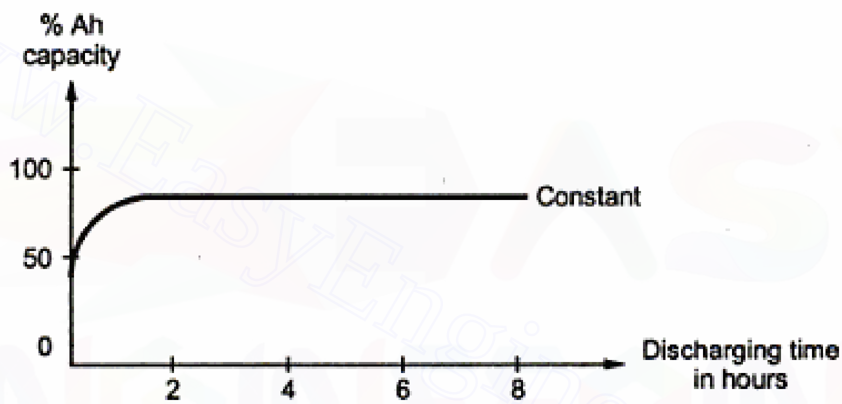


Fig. 1.41 Ah capacity against discharge time curve

1.27.4 Efficiency

The internal resistance of Nickel-iron cell is higher than lead acid cell hence both the efficiencies ampere-hour as well as watt-hour are less than that of lead acid cell. The ampere-hour efficiency is about 80 % while the watt-hour efficiency is about 60 %.

1.27.5 Advantages

The various advantages of Nickel-iron cell are,

1. Light in weight compared to lead acid cell.
2. Compact construction.
3. Mechanically strong and can sustain considerable vibrations.
4. Free from sulphatation and corrosion.
5. Less maintenance is required
6. Do not evolve dangerous attacking fumes.
7. Gives longer service life.

1.27.6 Disadvantages

The various disadvantages of Nickel-iron cell are,

1. High initial cost.
2. Low voltage per cell of about 1.2 V.
3. High internal resistance.
4. Lower operating efficiency.

1.27.7 Application

The Nickel-iron batteries are used in,

1. Mine locomotives and mine safety lamps
2. Space ship
3. Repeater wireless station
4. To supply power to tractors, submarines, aeroplanes etc.
5. In the railways for lighting and airconditioning purposes.

1.28 Nickel - Cadmium Cell

The construction of this cell is similar to the nickel-iron cell except the active material used for the negative plate.

Positive plate →	Nickel hydroxide $[Ni(OH)_3]$
Negative plate →	Cadmium (Cd)

The electrolyte used is again 21 % solution of potassium hydroxide (KOH) in distilled water. The specific gravity of the electrolyte is about 1.2.

Little iron is added to cadmium to get negative plate. The iron prevents the caking of active material and losing its porosity.

The Fig. 1.42 shows the construction of Nickel-cadmium cell.

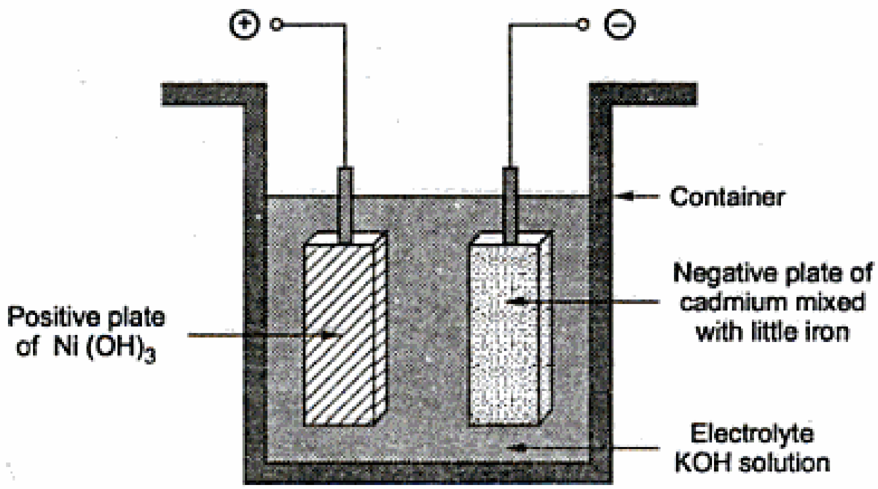


Fig. 1.42 Construction of Nickel-Cadmium cell

1.28.1 Chemical Reaction

In this cell also, in working condition $Ni(OH)_3$ gets converted to lower nickel hydroxide as $Ni(OH)_2$ while cadimum hydroxide $Cd(OH)_2$ gets formed at the negative plate. During charging reverse reaction takes place. The electrolyte does not undergo any chemical change.

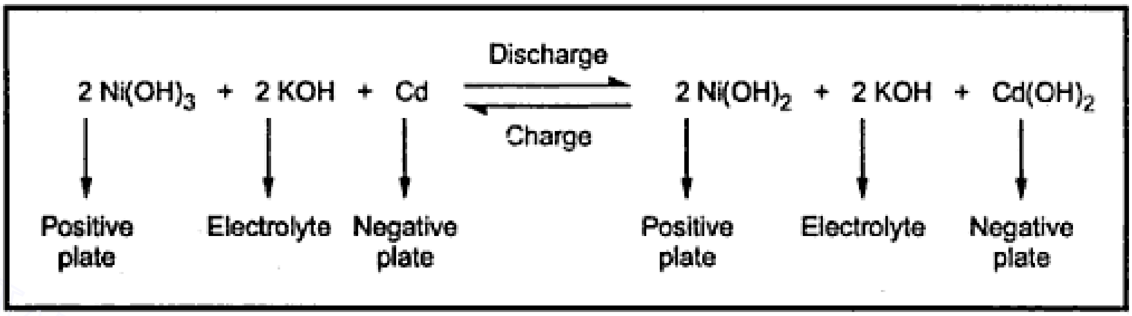


Fig. 1.43 Total reaction

1.28.2 Features

1. The electrical characeristics are similar to the Nickel-iron cell.
2. Due to use of cadmium, internal resistance is low.
3. The efficiencies are little bit higher than Nickel-iron cell.
4. Advantages and disadvantages are same as that of Nickel-iron cell.
5. The various charging methods such as constant current, constant voltage, trickel charging can be used.

1.28.3 Applications

The various applications of Nickel-cadmium battery are,

1. In railways for lighting and air conditioning systems.
2. In millitary aeroplanes, helicopters and commerical airlines for starting engines and provide emergency power supply.
3. In photographic equipments such as movie cameras and photoflash.
4. In electric shavers.
5. Due to small size in variety of cordless electronic devices.

1.29 Comparison of Various Batteries

Sr. No.	Particular	Lead acid cell	Nickel-Iron cell	Nickel-Cadmium Cell
1.	Positive plate	Lead peroxide (PbO_2)	Nickel hydroxide $Ni(OH)_3$	Nickel hydroxide $Ni(OH)_3$
2.	Negative plate	Lead (Pb)	Iron (Fe)	Cadmium (Cd)
3.	Electrolyte	Sulphuric acid H_2SO_4	Potassium hydroxide KOH	Potassium hydroxide KOH
4.	Average e.m.f.	2.0 V/cell	1.2 V/cell	1.2 V/cell
5.	Internal resistance	Low	High	Low
6.	Ah efficiency	90 to 95 %	70-80 %	70-80 %
7.	Wh efficiency	72 to 80%	55-60%	55-60%
8.	Ah capacity	Depends on discharge rate and temperature	Depends only on temperature	Depends only on temperature
9.	Cost	Less expensive	Almost twice the lead acid cell	Almost twice the lead acid cell
10.	Life	1250 charges and discharges	About 8 to 10 years	Very long life
11.	Weight	Moderate	Light	More heavy
12.	Mechanical strength	Poor	Good	Good

1.30 Comparison of Primary and Secondary Cells

Sr. No.	Primary Cells	Secondary Cells
1.	Electrical energy is directly obtained from chemical energy.	Electrical energy is present in the cell in the form of chemical energy and then converted to electrical energy.
2.	The chemical actions are irreversible.	The chemical actions are reversible.
3.	Cell is completely replaced when it goes down.	The cell is recharged back when it goes down.
4.	Polarisation is present.	Polarisation is absent.
5.	Low efficiency.	Efficiency is high

6.	Capacity is low.	Higher capacity.
7.	Less cost.	High initial cost.
8.	No maintenance required.	Frequent charging and other maintenance is required.
9.	Examples are dry cell, mercury cell, zinc-chloride cell	Examples are Nickel-iron, lead acid and Nickel-cadmium

1.31 NiMH Battery

Now a days, number of battery powered portable electronic devices are developed. The consumer demands higher energy rechargeable batteries which are capable of delivering longer service between recharges. The sealed nickel - metal hydride (NiMH) rechargeable battery satisfies the consumer demand and provides far improved performance compared to conventional rechargeable batteries. The NiMH battery has higher energy capacity and hence capable of providing longer service life.

1.31.1 Construction

This type of battery uses Nickel oxyhydroxide (NiOOH) as the active material in the positive electrode. While the negative active material is the hydrogen in the charged state, in the form of metal hydride. This metal hydride is an alloy which can undergo a reversible hydrogen absorbing and deabsorbing reaction when the battery is charged and discharged.

Typically there are two types of classes of the alloy which is used as the electrode material in the NiMH battery.

1. AB_5 alloy of which $LaNi_5$ is the example.
2. AB_2 alloy of which $TiMn_2$ or $ZrMn_2$ are examples.

Practically AB_5 alloys are more preferred as they offer better corosion resistance characteristics which provides longer cycle life and better rechargeability. These alloys have large hydrogen storing capability. The low hydrogen pressure alloy and highly pure materials can minimize the self discharge.

Both the electrodes use highly porous structure and large surface areas. This provides low internal resistance for the cell. The positive electrode is highly porous nickel-felt substrate into which the nickel compounds are pasted. While a perforated nickel-plated steel foil is used for the negative electrode onto which the plastic bonded active hydrogen storage alloy is coated.

The electrolyte used is an aqueous solution of pottasium hydroxide (KOH). In NiMH battery, the minimum amount of electrolyte is used with most of the liquid being absorbed by the seperator and the electrodes. This design helps the diffusion of oxygen to the negative electrode at the end of charge for the "oxygen recombination" reaction.

The NiMH battery is designed with a discharge and charge reserve in the negative electrode. This is shown in the Fig. 1.44

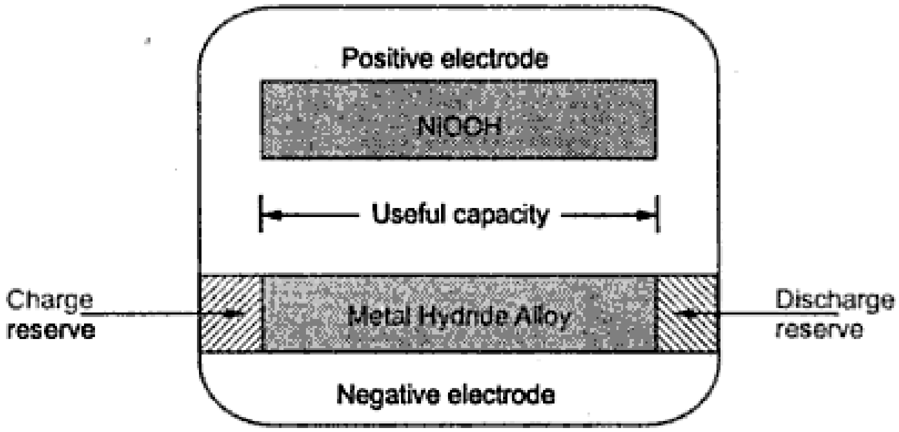


Fig. 1.44 Schematic representation of NiMH battery

In case of overdischarge, the gassing and the degradation of the cell is minimized by the discharge reserve.

In case of overcharge, the charge reserve make sure that the battery maintains low internal pressure.

The standard size NiMH batteries are constructed with cylindrical and prismatic type of nickel-metal hydride cells.

1.31.2 Cell Reactions

The cell reactions are divided into,

1. Cell reactions during charge.
2. Cell reactions during discharge.

1. During Charge : The positive electrode reaches to the full charge before negative electrode and causes the oxygen to evolve.

$$\boxed{2OH^- \longrightarrow H_2O + \frac{1}{2} O_2 + 2e^-}$$

Due to minimum amount of electrolyte, the oxygen gas diffuses through the seperator to the negative electrode. This design is called 'starved electrolyte' design of the cell.

The oxygen reacts with metal hydride at the negative electrode, to produce water.

$$\boxed{2\ MH + \frac{1}{2} O_2 \longrightarrow 2M + H_2O}$$

2. During Discharge : The nickel oxyhydroxide is converted to nickel hydroxide.

$$\boxed{NiOOH + H_2O + e^- \longrightarrow Ni\ (OH)_2 + OH^-}$$

While the metal hydriode is oxidized to the metal alloy (M).

$$\boxed{MH + OH^- \rightarrow M + H_2O + e^-}$$

Thus the overall reaction on discharge is,

$$MH + NiOOH \rightarrow M + Ni\,(OH)_2$$

1.31.3 Features

The various features of NiMH batteries are,

1. Higher capacity which is about 40 % longer life than ordinary NiCd battery of same size.
2. The charging is very fast in about one hour.
3. The cycle life is very long upto 500 charge / discharge cycles.
4. The internal resistance is very low due to 'starved electrolyte' type of design.
5. No pollution or effect on an environment as it does not contain any cadmium.
6. It is capable of performing well at extremes temperatures, on discharge from $-20\,°C$ to $+50\,°C$ and on charge from $0\,°C$ to $45\,°C$.
7. Due to higher energy density, battery volume and weight is minimum.
8. It has wide voltage range.
9. It is manufactured with special high impact and flame retardant polymers hence durable.
10. The various charging methods like quick charge, fast charge, trickle charge can be used for charging.

1.31.4 General Characteristics

The discharge characteristics of NiMH battery is similar to that of NiCd battery. On charging, the open circuit voltage ranges from 1.25 to 1.35 volts/cell. On discharge, the nominal voltage is 1.2 volts/cell and the typical end voltage is 1 volt/cell.

The Fig. 1.45 shows the discharge charactersitics of NiCd and NiMH batteries of same size. (See Fig. 1.45 on next page)

Both the batteries show the flat characteristics almost throughout the discharge. The midpoit voltage is between 1.25 to 1.1 V, which depends on the discharge load.

1.31.5 Self Discharge Characteristics

During storage, the NiMH battery discharges on its own. This is due to the reaction of residual hydrogen in the battery with the positive electrode. This causes slow and reversible decomposition of positive electrode. This is called self discharge of the cell. The rate of such a self discharge depends on the time for which the cell is stored and the temperature at which the cell is stored. At higher temperature, the rate of self discharge is

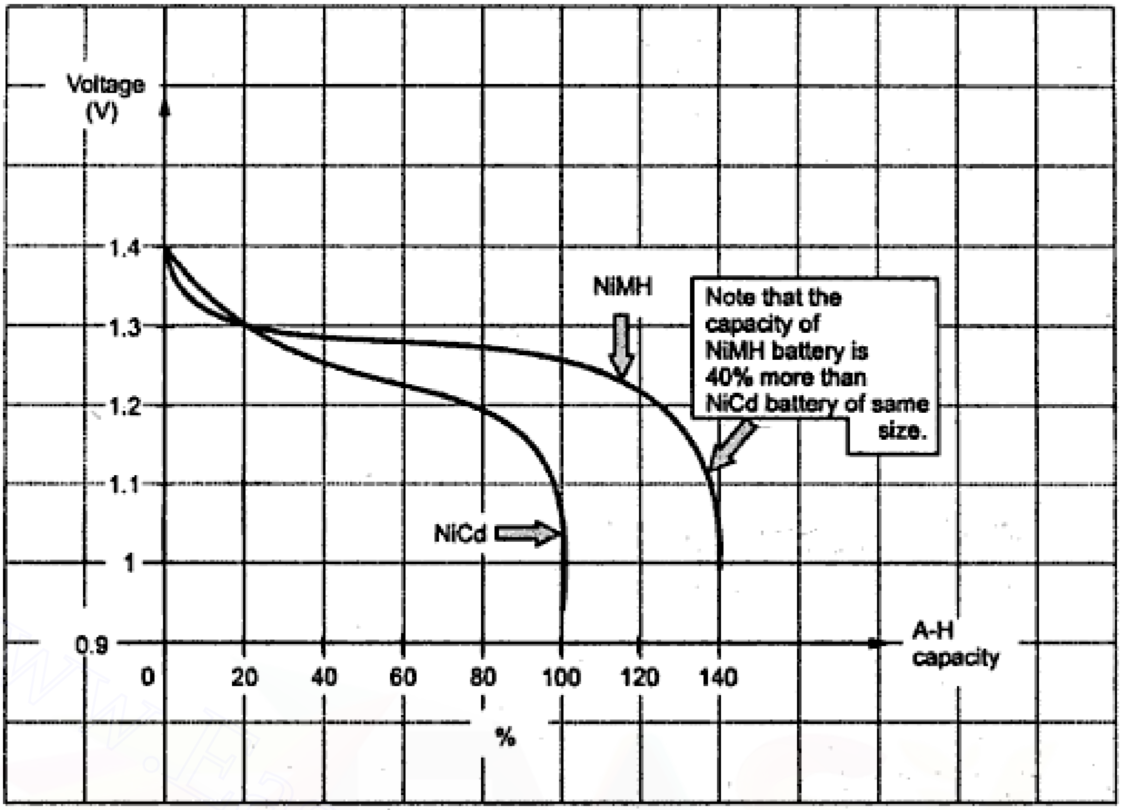


Fig. 1.45 Discharge characteristics

also high. The Fig. 1.46 shows the self discharge characteristics of NiMH cells at various temperatures.

The long term storage of NiMH battery in charged or discharged state has no permanent effect on the battery capacity.

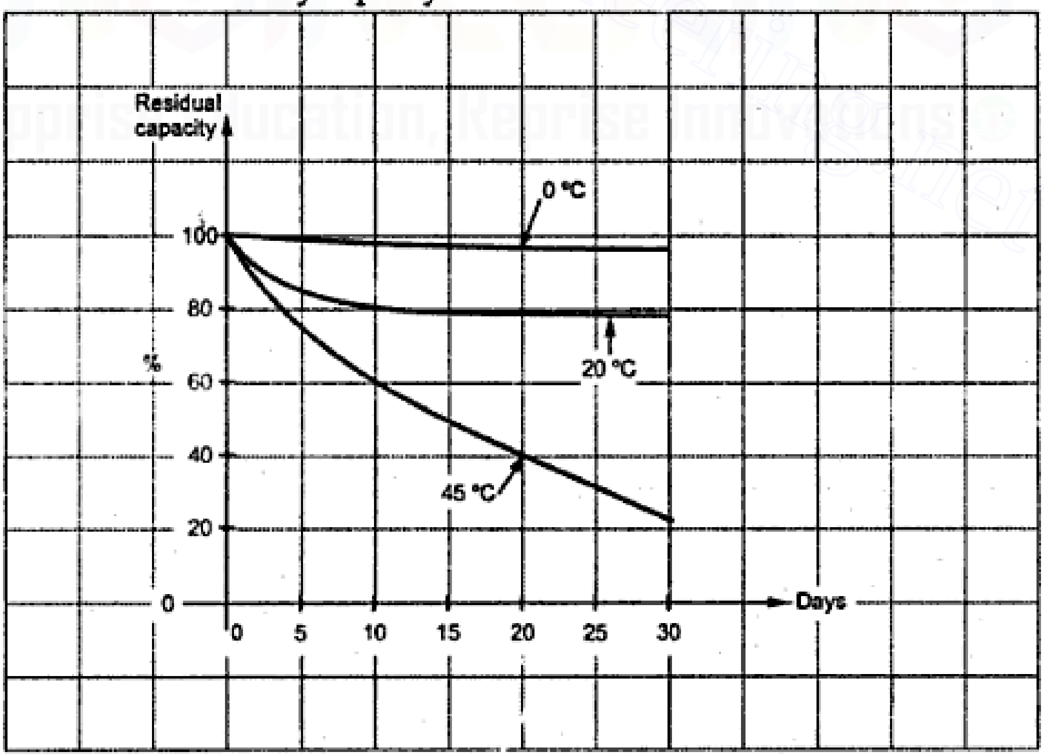


Fig. 1.46 Self discharge curves for NiMH cells

Key Point: *The loss in capacity due to self discharge is reversible and can be recovered to full capacity by repeated charge / discharge cycles.*

Even the capacity loss due to storage upto one year can be recovered. But long term storage at high temperatures can damage seals and separators. The proper temperature range for the storage of NiMH batteries is 10 °C to 30 °C.

1.31.6 Recharging Characteristics

The recharging characteristics of NiMH and NiCd batteries are almost same. For NiMH battery a proper charge control is necessary as it is more sensitive to overcharging.

The most common method used for charging the NiMH battery is constant current charge method with current controlled to avoid excessive temperature rise.

The Fig. 1.47 shows the typical charge-voltage characteristics of NiCd and NiMH batteries. The curves are almost flat when charged at constant current rate. Initially there is sharp increase in the voltage and similarly at about 80 % of charge, there is sharp rise in the voltages. But in the overcharging the NiCd batteries show the prominent voltage drop compared to NiMH batteries.

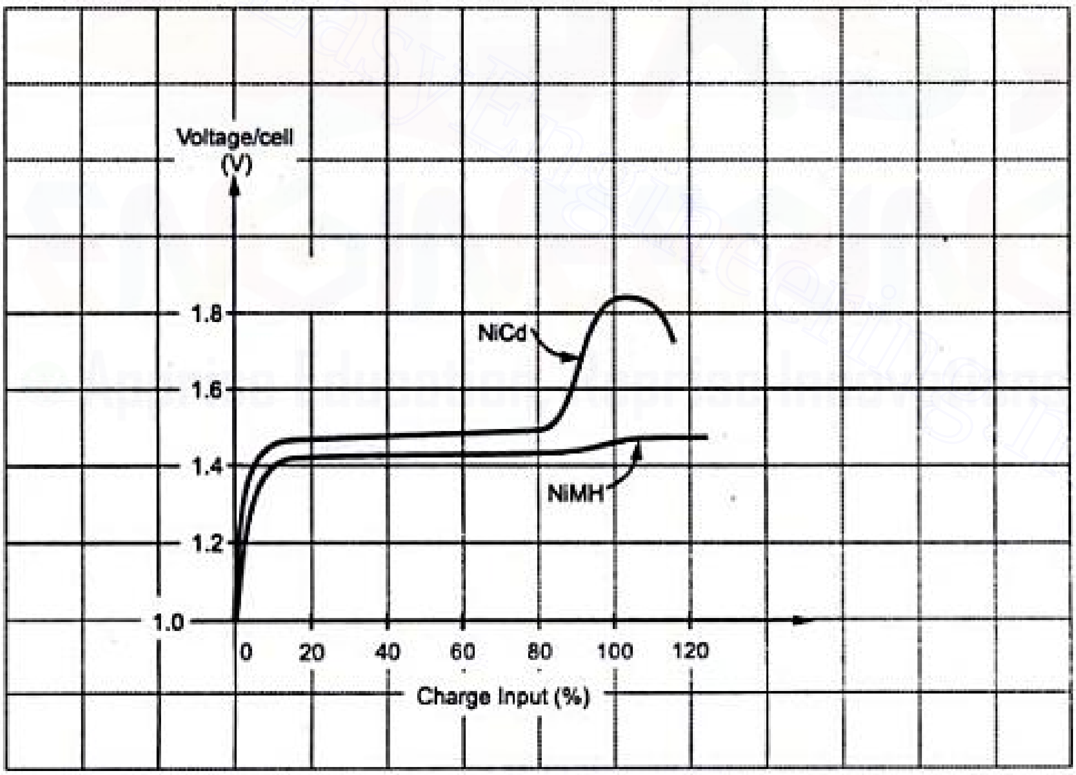


Fig. 1.47 Typical charge-voltage characteristics

1.31.7 Safety Precautions

The following care must be taken about the NiMH batteries for the safety purposes,

1. The battery should not be disassembled or opened. This can cause very high short circuit currents and may result into fire.

2. Keep and use the batteries away from the heat sources. Do not subject the battery to heat or do not dispose the battery in a fire. Store the battery at the recommended temperature.
3. Do not overcharge the battery. Due to this the battery may vent and hydrogen gas may be released. This gas can form explosive mixtures with the air.
4. Do not expose battery to a source of ignition or do not use in the air tight device compartments.
5. Do not use the battery for long time exceeding its specified ranges though the short term use may be possible.
6. Do not drop or do not subject to the strong mechanical shocks.

1.31.8 Applications

The various applications of NiMH battery are cellular phones, portable computers and many consumer electronic products. The batteries are used in digital cameras, cordless electronic devices, electronic toys and providing emergency supply to various electronic instruments. As the NiMH batteries are expensive, main application areas are cellular phones and laptop computers.

1.31.9 Comparison

No.	Parameter	Nickel-Cadmium (NiCd)	Nickel metal hydride (NiMH)
1.	Positive electrode	Nickel hydroxide $Ni(OH)_3$	Nickel oxyhydroxide NiOOH
2.	Negative electrode	Cadmium (Cd)	Metal hydride alloy (MH)
3.	Electrolyte	Potassium hydroxide (KOH)	Potassium hydroxide (KOH)
4.	Internal resistance	Low	Very low
5.	Life	Very long	Very very long
6.	Weight	Heavy	Light
7.	Mechanical strength	Good	Very good
8.	Gravimetric energy density	50 Wh / kg	55 Wh / kg
9.	Volumetric energy density	140 Wh / L	180 Wh / L
10.	Self discharge at 20 °C	15 - 20 % per month	20 - 30 % per month
11.	Cost	Best cost per performance value	Very high
12.	Protection	Internal short circuit protection is not provided.	Not supposed to produce internal shorts.

Note : The **gravimetric energy density** is a measure of how much energy a battery contains in comparison to its weight and typically expressed in watthour per kilogram (Wh/kg). The **volumetric energy density** is a measure of how much energy a battery contains in comparison to its volume and is typically expressed in wattthour per litre (Wh/L).

Examples with Solutions

Example 1.17 : *A copper coil when connected to a 30 volts supply initially takes current of 3 A and has a mean temperature of 20 °C. After sometime the current flowing in the coil falls to 2.85 A, supply voltage remaining same. The mean temperature of the coil is then 33.4 °C. Determine the temperature coefficient of resistance at 0 °C, i.e.* α_0.

Also state if it is true that α *can have*

i) zero value ii) negative value and if yes for which materials. **(Dec. - 2001)**

Solution : $V = 30$ volts, $I_1 = 3$ Amp, $t_1 = 20$ °C, $I_2 = 2.85$ Amp, $t_2 = 33.4$ °C

Now we have,
$$R_1 = \frac{V}{I_1} = \frac{30}{3} = 10\ \Omega \text{ at } t_1 = 20\,°C$$

$$R_2 = \frac{V}{I_2} = \frac{30}{2.85} = 10.52\ \Omega \text{ at } t_2 = 33.4\,°C$$

We can write,
$$R_t = R_0\,[1+\alpha_0\, t]$$

$$\therefore \quad R_1 = R_0\,[1+\alpha_0\, t_1] \quad \text{... (1)}$$

$$R_2 = R_0\,[1+\alpha_0\, t_2] \quad \text{... (2)}$$

Dividing equation (1) by (2),
$$\frac{R_1}{R_2} = \frac{1+\alpha_0\, t_1}{1+\alpha_0\, t_2}$$

$$\frac{10}{10.52} = \frac{1+\alpha_0\,(20)}{1+\alpha_0\,(33.4)}$$

$$\therefore \quad 0.9505\,(1+33.4\,\alpha_0) = 1+20\,\alpha_0$$

$$\therefore \quad 0.9505-1 = 20\,\alpha_0 - (0.9505)\,(33.4)\,\alpha_0$$

Solving,
$$\alpha_0 = \mathbf{4.2139\times10^{-3}\,/°C}$$

It is true that α can have zero value in case of alloys as with increase in temperature, alloys show almost no change in their resistance. The materials are Manganin and Eureka. It is also true that α can have negative value in case of insulating materials as with increase in temperature, resistance of insulating materials decreases. The materials are rubber, paper, mica, wood etc.

Example 1.18 : *It is required to maintain a loading of 5 kW in a heating unit. At an initial temperature of 15 °C, a voltage of 200 V is necessary for this purpose. When the unit is settled down to a steady temperature, a voltage of 220 V is required to maintain the same loading. Estimate the final temperature of the heating element, if the resistance temperature co-efficient of the heating element is 0.0006 per °C at 0 °C.* **(May - 2002)**

Solution : Power output = 5 kW = 5000 W, α_0 = 0.0006 /°C

$$\text{At } t_1 = 15\ °C,\ V_1 = 200 \text{ volt and } t_2,\ V_2 = 220 \text{ volt}$$

At 15 °C,

$$P = V_1 I_1 = V_1 \left(\frac{V_1}{R_1}\right) = \frac{V_1^2}{R_1}$$

$\therefore$

$$R_1 = \frac{V_1^2}{P} = \frac{(200)^2}{5000} = 8\ \Omega \qquad \text{... at } t_1 = 15\ °C$$

At t_2 °C,

$$P = V_2 I_2 = V_2 \left(\frac{V_2}{R_2}\right) = \frac{V_2^2}{R_2} \qquad \text{... power remains same}$$

$\therefore$

$$R_2 = \frac{V_2^2}{P} = \frac{(220)^2}{5000} = 9.68\ \Omega \qquad \text{... at } t_2\ °C$$

Now we have,

$$R_1 = R_0 [1 + \alpha_0 t_1] \qquad \text{... (1)}$$

$$R_2 = R_0 [1 + \alpha_0 t_2] \qquad \text{... (2)}$$

Dividing equation (2) by equation (1),

$$\frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1}$$

$\therefore$

$$1 + \alpha_0 t_2 = \frac{R_2}{R_1}(1 + \alpha_0 t_1) = \frac{9.68}{8}[1 + (0.0006)(15)]$$

$\therefore$

$$\alpha_0 t_2 = 1.2208 - 1 = 0.2208$$

$\therefore$

$$t_2 = 368.15\ °C$$

Example 1.19 : *If α_1 is the resistance temperature coefficient of a material at t_1 °C and α_2 at t_2 °C. Then prove that,*

$$(t_2 - t_1) = \frac{\alpha_1 - \alpha_2}{\alpha_1 \alpha_2}$$ **(Dec.-2005, Dec.-2007, May-2008)**

Solution : The resistance temperature coefficient at any temperature t °C can be obtained as,

$$\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}$$

So $\quad \alpha_1 = \dfrac{\alpha_0}{1+\alpha_0 t_1}$ and $\alpha_2 = \dfrac{\alpha_0}{1+\alpha_0 t_2}$

where $\quad \alpha_0$ = resistance temperature coefficient at 0 °C

$$\therefore \quad \frac{1}{\alpha_2} - \frac{1}{\alpha_1} = \frac{1+\alpha_0 t_2}{\alpha_0} - \frac{1+\alpha_0 t_1}{\alpha_0} = \frac{1+\alpha_0 t_2 - 1 - \alpha_0 t_1}{\alpha_0}$$

$$= \frac{\alpha_0(t_2 - t_1)}{\alpha_0} = t_2 - t_1$$

$$\therefore \quad \boxed{\frac{\alpha_1 - \alpha_2}{\alpha_1 \alpha_2} = t_2 - t_1}$$

... Proved

Example 1.20 : *It takes 30 minutes for an electric kettle to raise the temperature of 10 kg of water from 24 °C to 100 °C. The water equivalent of the container is 1 kg and the heat lost due to radiation is 400 kJ. Assuming the specific heat of water as 4200 J/kg°K, calculate the current taken by the kettle from a supply of 200 V.* **(May - 1999)**

Solution : Water equivalent of kettle =1 kg

$\therefore$ Total mass m = mass of water + equivalent of container = 10 + 1 = 11 kg

$$\Delta t = t_2 - t_1 = 100 - 24 = 76 \text{ °C}$$

Heat energy required to heat the water is,

$$H = m\,C\,\Delta t = 11 \times 4200 \times 76 = 3511.2 \text{ kJ}$$

Net input required = Energy required + Energy lost

$$= 3511.2 + 400 = 3911.2 \text{ kJ}$$

Time = 30 min = 1800 sec

$$\therefore \quad \text{power input} = \frac{\text{input in J}}{\text{time in sec}} = \frac{3911.2 \times 10^3}{1800}$$

$$= 2172.88 \text{ W}$$

$$\therefore \quad I = \frac{P_{in}}{V} = \frac{2172.88}{200} = 10.86 \text{ A}$$

Example 1.21 : *Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75 °C. To what height could a load 5 tonnes be raised with the expenditure of the same amount of energy ? Assume efficiency of heating and lifting equipment to be 90 % and 70 % respectively. Assume the specific heat capacity of water to be 4186 J/kg °K and 1 litre of water to have a mass of 1 kg and 1 tonne is equal to 1000 kg.* **(May - 1998, May-2007)**

Solution : m = 45 litres = 45 kg, Δt = 75 °C, C = 4186 J/kg °K

$$\therefore \quad \text{Output energy} = m\,C\,\Delta t = 45 \times 4186 \times 75$$

$$= 1.4127 \times 10^7 \text{ J}$$

$$\therefore \quad \text{Energy expended} = \frac{\text{Output energy}}{\eta \text{ of heating equipment}} = \frac{1.4127 \times 10^7}{0.9}$$

$$= 1.5697 \times 10^7 \text{ J}$$

Now this much energy is to be utilised to lift a load of mass m = 5 tonnes = 5000 kg.

$$\therefore \quad \text{Input energy} = 1.5697 \times 10^7 \text{ J}$$

$$\eta \text{ of lifting equipment} = 70\,\% = 0.7$$

$$\therefore \quad \text{Output energy} = \eta \times \text{input} = 0.7 \times 1.5697 \times 10^7$$

$$= 1.098 \times 10^7 \text{ J}$$

$$\text{Output energy} = mgh$$

$$\therefore \quad 1.098 \times 10^7 = 5000 \times 9.81 \times h$$

$$\therefore \quad h = 224.021 \text{ m}$$

So load can be raised to a height of 224.021 m.

Example 1.22 : *In case of power supply failure 7.5 kW electrical lighting and fan load of a commercial establishment is supplied by a small diesel generator set. Efficiency of diesel engine and the electrical generator are 55 % and 80 % respectively. Calculate the per kWh unit cost of this electricity if the set runs average 90 hours per month. The cost of diesel is Rs. 10/- per litre and the calorific value of diesel is 52,000 kJ/ltr.* **(Dec. - 1998)**

Solution : Consider the system shown in the Fig. 1.48.

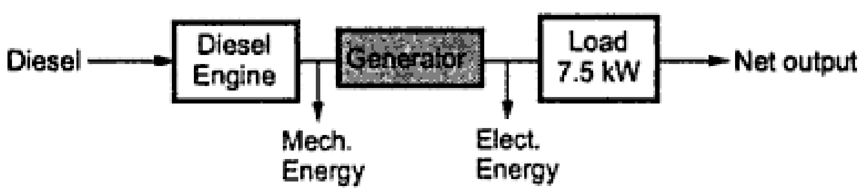


Fig. 1.48

$$\text{Output energy} = 7.5 \text{ kW}$$

$$\text{Monthly output} = 7.5 \times 90 = 675 \text{ kWh} = 675 \times 3600 \text{ kJ} = 2.43 \times 10^6 \text{ kJ}$$

$$\text{Engine} \quad \eta_1 = 55\,\% \text{ and Generator } \eta_2 = 80\,\%$$

$$\text{Monthly input} = \frac{\text{output}}{\eta_1 \times \eta_2} = \frac{2.43\times10^6}{0.55\times0.8} = 5.522\times10^6 \text{ kJ}$$

Hence for an entire month the diesel required is,

$$\text{Diesel in litres} = \frac{5.522\times10^6}{52000} = 106.206 \text{ litres}$$

Hence the total cost = 106.206 × 10 = Rs. 1062.06

This is required to supply 675 kWh hence,

$$\text{Cost per kWh} = \frac{1062.06}{675} = \textbf{Rs. 1.573}$$

Example 1.23 : *Determine the current flowing at the instant of switching a 60 watt lamp on a 230 V supply. The ambient temperature is 25 °C. The filament temperature is 2000 °C and the resistance temperature coefficient is 0.005 /°C at 0 °C.* **(Dec. - 1998)**

Solution : P = 60 W, V = 230 V, t_1 = 25 °C, t_2 = 2000 °C

R_2 = resistance of filament in ON condition

$$= \frac{V^2}{P} = \frac{(230)^2}{60} = 881.67\ \Omega$$

$$\alpha_1 = \text{R.T.C at } t_1 = \frac{\alpha_0}{1+\alpha_0 t_1} = \frac{0.005}{1+0.005\times25} = 4.44\times10^{-3}\ /°\text{C}$$

Now $R_2 = R_1(1+\alpha_1 \Delta t)$

$\therefore$ $881.67 = R_1[1 + 4.44\times10^{-3}\times(2000-25)]$

$\therefore$ $R_1 = 90.251\ \Omega$

$\therefore$ I = current at switching time

$$= \frac{V}{R_1} = \frac{230}{90.251} = \mathbf{2.548\ A}$$

Example 1.24 : *At the instant of switching a 40 W lamp on a 230 V supply, the current is observed to be 2.5 A. The R.T.C. of filament is 0.0048 /°C at 0°C. The ambient temperature is 27 °C. Find the working temperature of the filament and current taken during normal operation.* **(May-2007)**

Solution : P = 40 W, V = 230 V, I = 2.5 A, α_0 = 0.0048 /°C

At the time of switching, temperature is ambient i.e. t = 27 °C,

$$\therefore \quad R_{27} = \frac{V}{I} = \frac{230}{2.5} = 92\ \Omega$$

Under working condition, power consumption of lamp is 40 W.

$$\therefore \quad R_{t2} = \frac{V^2}{P} = \frac{(230)^2}{40} = 1322.5\ \Omega$$

Now $\quad R_{t2} = R_{27}[1 + \alpha_{27}(t_2 - 27)]$

$$\alpha_{27} = \frac{\alpha_0}{1+\alpha_0 t} = \frac{0.0048}{1+0.0048\times 27} = 4.2492\times 10^{-3}\ /^\circ C$$

$$\therefore \quad 1322.5 = 92\,[1 + 4.2492\times 10^{-3}(t_2 - 27)]$$

$$\therefore \quad 14.375 = 1 + 4.2492\times 10^{-3}(t_2 - 27)$$

$$\therefore \quad t_2 - 27 = 3147.6513$$

$$\therefore \quad t_2 = 3174.6513\ ^\circ C \quad \text{... working temperature}$$

$$\therefore \quad I(\text{working}) = \frac{V}{R_{t2}} = \frac{230}{1322.5} = 0.1739\ A \quad \text{... working current}$$

Example 1.25 : *An electric motor is driving a train weighing, 100 thousand kilogram up on an inclined track of 1 in 100 at a speed of 60 km/h. The frictional force of tracks is 10 kg. per 1000 kg. of its weight. If the motor operates on 11 kV, find the current taken by the motor assuming the overall efficiency of the system as 70 %.* **(Dec. - 1999, Dec. - 2000)**

Solution : The arrangement is shown in the Fig. 1.49.

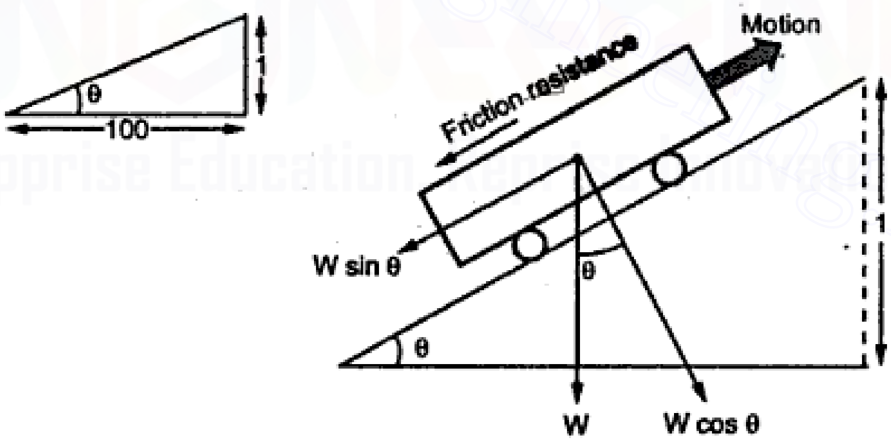


Fig. 1.49

Now, slope is 1 in 100.

$$\sin\theta = \tan\theta = \frac{1}{100} = 0.01 \quad \text{as } \theta \text{ is very small}$$

$$\therefore \quad W\sin\theta = 100\times 10^3 \times 0.01 = 1000\ kg = 1000\times 9.81 = 9810\ N$$

Track resistance = 10 kg per 1000 kg

$$= \frac{10}{1000} \times 100 \times 10^3 = 1000 \text{ kg} = 9810 \text{ N}$$

Now, W sin θ and track resistance, both are opposite to motion.

$\therefore$ Total resistance = 9810 + 9810 = 19620 N

Work done = Force × distance travelled in 1 sec = 19620 × d

Now, Speed = 60 km/h

$$d = \text{distance travelled in 1 sec.} = \frac{60 \times 10^3}{3600} = 16.67 \text{ m}$$

$\therefore$ Work done = 19620 × 16.67 = 327065.4 J

$$\therefore \text{ Power required by load} = \frac{\text{Work done}}{\text{time}} = \frac{327065.4}{1 \text{ sec}} = 327065.4 \text{ W}$$

P_{out} = Power required by load = 327065.4 W

Now, $\eta = \frac{P_{out}}{P_{in}}$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta} = \frac{327065.4}{0.7} = 467236.2857 \text{ W}$$

But, $P_{in} = V \times I$ i.e. $467236.2857 = 11 \times 10^3 \times I$

$\therefore$ I = **42.476 A**

➡ **Example 1.26 :** *An aluminium kettle weighs 2.8 kg and holds 2750 cm³ of water. When connected to 220 V supply, its heater element takes current of 10 A. The efficiency of the kettle is given as 75%. Find the time required to boil the water from the initial temperature of 27 °C. Assume specific heat capacity of water and aluminium as 4200 J/kg °K and 950 J/kg °K respectively.* **(Dec. - 97)**

Solution : Mass of kettle m_k = 2.8 kg and Mass of water m = 2750 cm^3

Now 1 m^3 water = 1000 kg

$\therefore$ $m = 2750 \times 10^{-6} \times 1000 = 2.75$ kg

Specific heat of kettle C_k = 950 J/kg °K

Specific heat of water C = 4200 J/kg °K

Heat required to raise temperature of water

$$= m\,C\,\Delta t = 2.75 \times 4200 \times (100 - 27)$$

$$= 843150 \text{ J}$$

Heat lost in heating the kettle = $m_k\,C_k\,\Delta t = 2.8 \times 950 \times (100 - 27) = 194180$ J

$\therefore$ Total heat required = 843150 + 194180 = 1037330 J

The efficiency η = 75 %

$$\therefore \quad \text{Heat input} = \frac{\text{Heat output}}{\eta} = \frac{1037330}{0.75} = 1383106.7 \text{ J}$$

$$\text{Power input} = V \times I = 220 \times 10 = 2200 \text{ W}$$

$$\text{Now} \quad P = \frac{\text{total heat input}}{\text{time required}}$$

$$\therefore \quad 2200 = \frac{1383106.7}{\text{time}}$$

$\therefore$ time = 628.68 sec = **10 min and 28.68 sec.**

Example 1.27 : *A fully charged car battery can give 200 Wh of energy for operating starting mechanism. At each start, the engine has to be cranked at 50 r.p.m. for 10 seconds, against a torque of 50 N-m. Assume overall efficiency of 40 % of the machine, estimate the number of starts before the battery is required to be recharged.*

Solution : For 1 start, N = 50 r.p.m.

$$\text{i.e.} \quad \omega = \frac{2\pi N}{60} = 5.2359 \text{ rad/sec.}$$

$$\text{Torque } T = 50 \text{ N-m}$$

$$\therefore \quad \text{Output of 1 start} = T \times \omega = 261.799 \text{ watt-sec i.e. J}$$

$$\text{Efficiency} = 40 \%$$

$$\therefore \quad \text{Input power required for 1 start} = \frac{\text{output}}{\eta} = \frac{261.799}{0.4}$$

$$= 654.498 \quad \text{watt-sec i.e. J}$$

$$\therefore \quad \text{Energy for 1 start} = \text{power} \times \text{time} = 654.498 \times 10$$

$$= 6544.98 \quad \text{joules i.e. watt-sec}$$

$$\text{Total energy battery can give} = 200 \text{ Wh.}$$

$$\therefore \quad \text{Energy required for 1 start} = 6544.98 \quad \text{watt-sec}$$

$$= \frac{6544.98}{3600} \quad \text{watt-hour i.e. Wh} = 1.81 \text{ Wh.}$$

$$\therefore \quad \text{Number of starts} = \frac{\text{Total energy}}{\text{Energy for 1 start}} = \frac{200 \text{ Wh}}{1.81 \text{ Wh}} = 110.497$$

$\therefore$ Number of starts are approximately **111.**

Example 1.28 : *An electric furnace is used in order to melt 50 kg of tin per hour. Melting temperature of tin is 235 °C and room temperature is 15 °C. Latent heat of fusion for tin is 13.31 kcal/kg. Specific heat of tin is 0.055 kcal/kg°K. If input to furnace is 5 kW, find the efficiency of the furnace.* **(Dec.-2005)**

Solution : $m = 50$ kg, $t_1 = 15$ °C, $t_2 = 235$ °C, $L = 13.31$ kcal/kg

$C = 0.055$ kcal/kg°K, $P_{in} = 5$ kW

The melting takes place in two steps :

1. Heat required to raise temperature from 15 °C to 235 °C = $m\,C\,\Delta t$
2. Latent heat required to convert solid state to liquid state = mL

$\therefore$ H = heat output required = $m\,C\,\Delta t + m\,L$

$= 50 \times 0.055 \times (235 - 15) + 50 \times 13.31 = 1270.5$ kcal

Now 1 cal = 4.2 J

$\therefore$ $H = 1270.5 \times 10^3 \times 4.2 \text{ J} = 5336.1$ kJ

time = 1 hour = 3600 sec

$$\therefore \quad P_{out} = \frac{H\text{ (output)}}{\text{time}} = \frac{5336.1 \times 10^3}{3600} = 1482.25 \text{ W}$$

While $P_{in} = 5$ kW

$$\therefore \quad \%\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{1482.25}{5 \times 10^3} \times 100 = 29.645\ \%$$

Example 1.29 : *A motor drives a load torque of 200 N-m at 750 r.p.m. drawing 18 kW from mains. Assuming temperature to remain constant and 1 joule equal 0.2392 cal, determine, i) efficiency of motor and ii) losses per minute in kcal*

Solution : $T = 200$ Nm, $N = 750$ r.p.m., $P_{in} = 18$ kW

$$P_{out} = T \times \omega = T \times \frac{2\pi N}{60} = \text{output of motor}$$

$$= \frac{200 \times 2\pi \times 750}{60} = 15.70796 \text{ kW}$$

$$\therefore \quad \%\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{15.70796}{18} \times 100 = 87.26\ \%$$

$$\text{Losses} = P_{in} - P_{out} = 18 \times 10^3 - 15.70796 \times 10^3 = 2292.04 \text{ W}$$

Losses in 1 minute = 2292.04 × (60 sec) = 137522.4 J

Now 1 J = 0.2392 cal

$\therefore$ Losses/min = 137522.4 × 0.2392 = **32.8953 kcal**

These losses appear in the form of heat.

Example 1.30 : *Given below are the different electric appliances used by a family, their ratings and number of hours they are used daily.*

Sr. No.	Appliance	Number	Rating	Hours (daily)
1	Tube lights	4	40 W each	5 hours, each
2	Bulbs	2	15 W each	6 hours, each
		1	40 W	2 hours.
3	Geyser	1	2 kW	1 hour.
4	Fans	3	60 W each	8 hours, each
5	Television	1	100 W	4 hours.

Find the electric bill per month if cost per unit (1 kWh) is 75 paise. (Assume 30 days month)

Solution : First calculate daily consumption of each appliance in watt hours.

Appliance	Energy consumption in watt-hour
Tube lights	4×40×5 = 800
Bulbs	2 × 15 × 6 = 180
	1×40×2 = 80
Geyser	1×2000×1 = 2000
Fans	3×60×8 = 1440
Television	1×100×4 = 400
	Total = 4900 Wh = 4.9 kWh

$\therefore$ Monthly energy consumption = 4.9×30

= 147 kWh = 147 units.

$\therefore$ Monthly bill = $147 \times (0.75)$ = **Rs. 110.25**

Example 1.31 : *In a hydroelectric generating station the difference in level (head) between the water surface and turbine driving the generators is 425 meters. If 1250 liters of water is required to generate 1 kWh of electric energy. Find the overall efficiency.*

Solution : 1 litre of water = 1 kg of water

$$\therefore \quad m = 1250 \text{ kg}, \quad h = 425 \text{ m}$$

$$\text{Energy produced by water} = mgh = 1250 \times 9.81 \times 425 = 5.21156 \times 10^6 \text{ J}$$

$$\therefore \text{ Energy produced in 1 hour by water} = \frac{5.21156 \times 10^6}{3600} = 1447.6563 \text{ Wh}$$

$$= 1.44765 \text{ kWh}$$

$$\text{While actual generation} = 1 \text{ kWh}$$

$$\therefore \quad \text{Efficiency} = \frac{1}{1.44765} \times 100 = 69.077 \ \%$$

Example 1.32 : *An electrically driven pump motor lifts 80 m^3 of water per minute through a height of 12 m. Efficiencies of motor and pump are 70 % and 80 % respectively.*

Calculate,

i) Current drawn by motor if it works on 400 volts supply.

ii) Energy consumption in kWh and cost of the energy at the rate of 75 paise/kWh, if pump operates 2 hours per day for 30 days.

Assume 1m^3 of water = 1000 kg **(Dec.-2006)**

Solution : The given values are,

$$m = 80 \text{ m}^3 = 80 \times 1000 \text{ kg}, \quad h = 12 \text{ m}, \quad \text{time} = 1 \text{ minute}$$

The arrangement is shown in the Fig. 1.50.

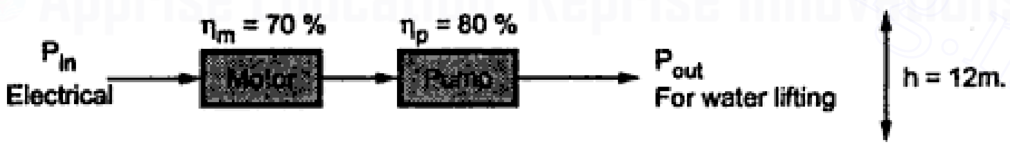


Fig. 1.50

$$P_{out} = mgh = 80 \times 1000 \times 9.81 \times 12 = 9.4176 \times 10^6 \text{ J}$$

$$\therefore \quad P_{out} \text{ in watts} = \frac{P_{out} \text{ in J}}{\text{Time}} = \frac{9.4176 \times 10^6}{60} \qquad \text{... time = 1 min = 60 sec}$$

$$= 156.96 \times 10^3 \text{ watts}$$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta_m \times \eta_p} = \frac{156.96 \times 10^3}{0.7 \times 0.8} = 280.2857 \times 10^3 \text{ watts}$$

$$\text{But} \quad P_{in} = VI \quad \text{i.e.} \quad 280.2857 \times 10^3 = 400 \times I$$

i) ∴ $I = 700.714$ A

ii) Daily energy consumption $= VI\,t$ where t = 2 hours per day

$= 400 \times 700.714 \times 2 = 560.5712 \times 10^3$ Wh

$= 560.5712$ kWh

∴ Monthly consumption $= 560.5712 \times 30 =$ **16817.136 kWh**

∴ cost of energy $= 16817.136 \times$ rate per kWh $= 16817.136 \times 0.75$

$=$ **Rs. 12612.85**

Example 1.33 : *An electric boiler has two heating elements each of 200 V, 4 kW rating. Boiler contains 20 litres of water at 20 °C. Assuming 8 % loss of heat from boiler, find the time required after switching on the boiler to heat the water up to 90 °C if*

a) two elements are in parallel and b) two elements are in series

Specific heat capacity of water is 4180 J/kg°K. Assume supply voltage as 200 V.

Solution : Mass of water = 20 litres = 20 kg

$$\Delta t = t_2 - t_1 = 90 - 20 = 70\ ^\circ C = 70\ ^\circ K$$

Heat energy required,

$$H = m\,C\,\Delta t = 20 \times 70 \times 4180 = 5.852 \times 10^6 \text{ J}$$

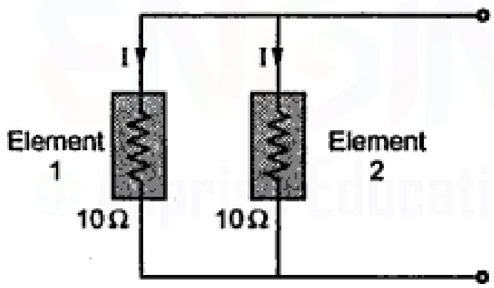


Fig. 1.51

a) Elements are in parallel :

Voltage across each element = 200 V

$$\text{Resistance of each} = \frac{V^2}{P} = \frac{(200)^2}{4000}$$

$$= 10\ \Omega$$

I = current by each element

$$= \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$

∴ Power by each element $= I^2 R$

$= (20)^2 \times 10 = 4$ kW

∴ Total power absorbed $= 2 \times 4 = 8$ kW

∴ Total input energy $=$ power × time $= 8000\,t$ J

Let 't' be time required to raise the temperature of water.

But there are 8 % losses i.e. $\eta = 92$ %

∴ Net heat energy required to raise the temperature

$$= \frac{H}{0.92} = 6.36086 \times 10^6 \quad J$$

$$\therefore \quad 6.36086 \times 10^6 = 8000\ t$$

$$\therefore \quad t = 795.1087 \text{ sec.} = 13.25 \text{ min.}$$

b) Elements in series :

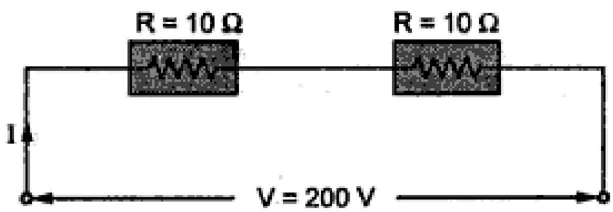


Fig. 1.52

Net heat energy required to raise the temperature of water due to losses remains same i.e. 6.36086×10^6 J.

But let us see how much energy can be absorbed when elements are in parallel.

$$\therefore \quad \text{Total resistance due to series} = R_1 + R_2 = 2\,R = 20\ \Omega$$

$$\therefore \quad \text{Total current drawn} = \frac{V}{R_{eq}} = \frac{200}{20} = 10\ A$$

$$\therefore \quad \text{Power by each element} = I^2 R = (10)^2 \times 10 = 1\ kW$$

$$\therefore \quad \text{Total power drawn} = 2 \times \text{power by each} = 2\ kW$$

$\therefore$ Total energy absorbed in time 't' sec

$$\text{power} \times \text{time} = 2000\ t \quad J$$

$$\therefore \quad 2000\ t = 6.36086 \times 10^6$$

$$\therefore \quad t = 3180.43 \text{ sec} = 53 \text{ min}$$

Example 1.34 : *The level difference (head) between water surface and turbine at hydroelectric generating station is 500 m. The capacity of station is of 250 MW and it supplies a full load for 8 hours a day. Overall efficiency of station is 90 %. Find how much volume of water is used, daily.*

Solution : It is connected to full load i.e.

$$P_{out} = 250 \quad MW = 250 \times 10^6 \quad \text{watt i.e. J/sec}$$

$$\therefore \quad \text{Energy output in 8 hours} = 250 \times 10^6 \times (8 \times 3600) = 7.2 \times 10^{12} \quad J$$

$$\text{Potential energy of water} = m\,g\,h = m \times 9.81 \times 500 \quad J$$

This must supply the energy input required for driving a load.

$$\therefore \quad \text{Input energy} = \frac{\text{output}}{\text{efficiency}} = \frac{7.2 \times 10^{12}}{0.9} = 8 \times 10^{12} \quad J$$

$$\therefore \quad 8\times10^{12} = m\times 9.81\times 500$$

$$\therefore \quad m = 1.6309\times10^{9}\ kg$$

Now $\quad 1\ m^3$ of water $= 1000$ kg

$$\therefore \quad 1.6309\times10^{9}\ kg = 1.6309\times10^{6}\ m^3$$

$\therefore$ Volume of water daily consumed is $1.6309\times10^{6}\ m^3$.

Example 1.35 : *A locomotive when driving a load 30 × 10^3 kg requires an output of 60 H.P. Load is moving up an incline of 2 in 100. The frictional resistance is 300 kg. The gearing efficiency of 80 % and motor efficiency is 90 %. Calculate the speed at which load is moving and current drawn by the motor if connected to 500 V mains.*

Assume 1 H.P. = 735.5 W.

Solution : mass of load = 30 × 10^3 kg, slope 2 in 100, friction = 300 kg

$$\eta_g = 80\ \%, \quad \eta_m = 90\ \%, \quad V = 500\ V$$

$$w = \text{Weight of load} = m\times g = 30\times10^3\times 9.81 = 294300\ N$$

The various forces acting on the locomotive are shown in the Fig. 1.53.

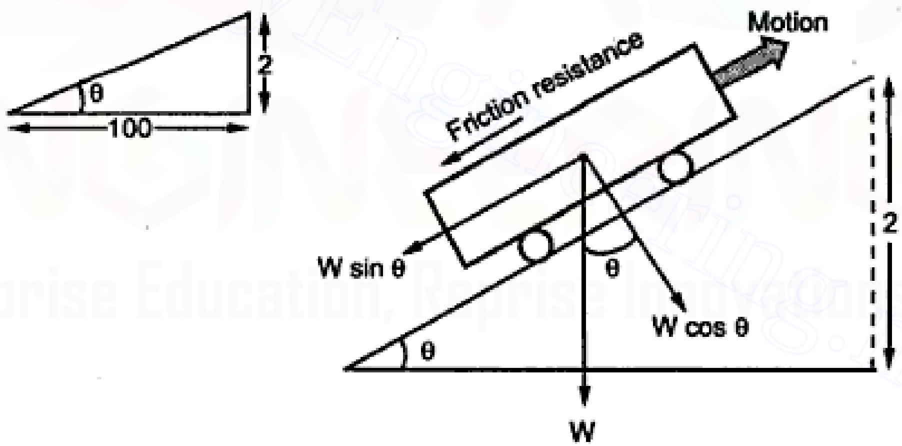


Fig. 1.53

The components opposite to motion are,

1. Friction resistance = 300 × 9.81 = 2943 N
2. Component of weight opposite to motion = W sin θ

$$= 294300\times\frac{2}{100} = 5886\ N$$

Remember that for small values of θ, $\sin\theta = \tan\theta = \frac{2}{100}$

$\therefore$ Total resistance = 2943 + 5886 = 8829 N

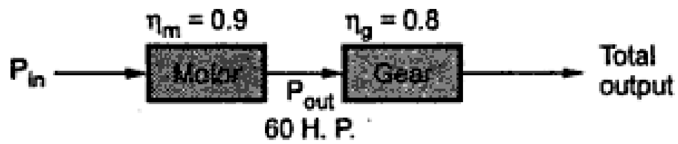


Fig. 1.54

$$P_{out} = \text{output of motor} = 60 \text{ H.P.} = 60 \times 735.5 = 44130 \text{ W}$$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta_m} = \frac{44130}{0.9} = 49033.33 \text{ W}$$

But $P_{in} = V I$

$$\therefore \quad 49033.33 = 500 \times I$$

$$\therefore \quad I = 98.067 \text{ A} \qquad \text{... current drawn}$$

$$\text{Total output} = P_{out} \times \eta_g = 44130 \times 0.8 = 35304 \text{ W}$$

$$\therefore \text{Total output energy} = \text{Total output} \times 1 \text{ sec} \quad \text{joules} \qquad \text{... as } 1 \text{ W} = 1 \text{ J/sec}$$

$$= 35304 \text{ J}$$

$$\text{Opposing force} = 8829 \text{ N}$$

$$\text{Work done} = \text{work done in overcoming opposition}$$

$$= \text{opposing force} \times \text{distance travelled in 1 sec}$$

$$= 8829 \times d \quad \text{J}$$

$$\therefore \quad 35304 = 8829 \times d$$

$$\therefore \quad d = 3.9986 \text{ m in 1 sec}$$

$$\therefore \quad \text{Speed of load} = 3.9986 \text{ m/sec} = \frac{3.9986 \times 3600}{1000} \text{ kmph} = \mathbf{14.39 \text{ kmph}}$$

Example 1.36 : *An electric lift makes 20 double journeys per hour. A load of 8 tonne is raised by it through a height of 60 m and it returns empty. The lift takes 80 sec to go up and 70 sec to return. The weight of cage is 1 tonne and that of counter weight 2500 kg. The efficiency of hoist is 75 % and of motor is 80 %. Find the energy consumption for 1 hour in kWh and power output rating of motor.*

Solution : The up and down journeys are shown in the Fig. 1.55 (a) and (b).

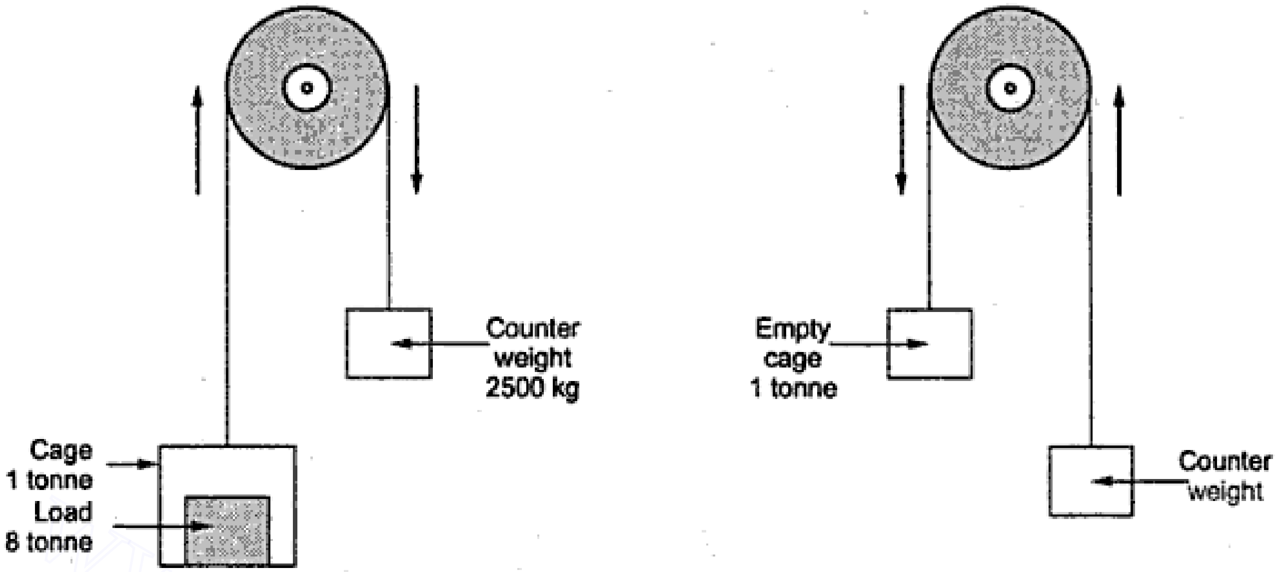


Fig. 1.55

i) During upward journey,

$$\text{Net mass} = \text{mass of load + mass of cage – counter weight}$$

$$= 8\times10^3 + 1\times10^3 - 2500 = 6500 \text{ kg}$$

$$\therefore \quad \text{Net weight} = 6500\times9.81 = 63765 \text{ N}$$

$$h = 60 \text{ m}$$

$$\therefore \quad \text{Work done} = mgh = 63765\times60 = 3.8259\times10^6 \text{ J}$$

ii) During downward journey,

$$\text{Net mass} = \text{counter weight – mass of cage}$$

$$= 2500 - 1\times10^3 = 1500 \text{ kg}$$

$$\therefore \quad \text{Work done} = mgh = 1500\times9.81\times60 = 882.9\times10^3 \text{ J}$$

Hence in a double journey i.e. upward and downward,

Total work done = $3.8259\times10^6 + 882.9\times10^3 = 4.7088\times10^6$ J

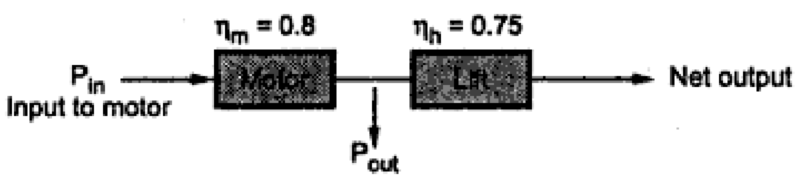


Fig. 1.56

In 1 hour lift completes 20 double journeys

$$\therefore \quad \text{Total work done in 1 hour} = 4.7088\times10^6\times20 = 94.176\times10^6 \text{ J}$$

$$P_{out} = \text{Output of motor} = \frac{\text{Net output in 1 hour}}{\eta_h}$$

$$= \frac{94.176\times10^6}{0.75} = 125.568\times10^6 \text{ J in 1 hour}$$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta_m} = \frac{125.568\times10^6}{0.8} = 156.96\times10^6 \text{ J in 1 hour}$$

$$\therefore \quad \text{Hourly consumption is kWh} = \frac{156.96\times10^6}{3600\times1000} = \mathbf{43.6\ kWh}$$

Key Point : *The power rating of motor depends on maximum power required out of two journeys i.e. upward or downward. The maximum is while upward journey, which decides power rating.*

$\therefore$ Work done in upward journey = 3.8259×10^6 J

$$\therefore \quad P_{out} = \frac{\text{Work done}}{\eta_h} = \frac{3.8259\times10^6}{0.75} = 5.1012\times10^6 \text{ J}$$

$$\text{Time required} = 80 \text{ sec for upward journey}$$

$$\therefore \quad \text{Output rating} = \frac{P_{out}}{\text{time}} = \frac{5.1012\times10^6}{80} = 63765 \text{ watts}$$

$$= \frac{63765}{735.5} = \mathbf{86.69\ H.P.} \qquad \dots 1 \text{ H.P.} = 735.5 \text{ W}$$

Example 1.37 : *A belt driven pulley of 0.1 m in radius rotates at a speed of 1500 revolutions per minute. The tension in the tight side is 35 kg. If it is producing output of 2 kW, calculate tension in the slack side of belt.*

Solution :

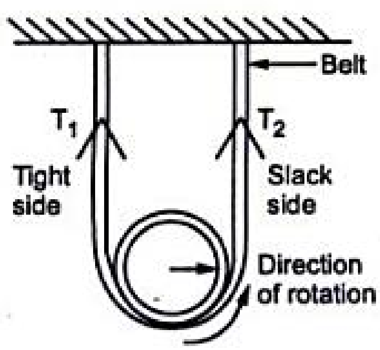


Fig. 1.57

Net tension acting on the pulley is difference between the two tensions,

i.e. $\quad F = (T_1 - T_2)\times 9.81$ in N

where T_1 and T_2 are in kg.

Torque produced = $F\times r$

Where r = radius of pulley

$= (T_1 - T_2)\times 9.81\times 0.1$

$$= (35 - T_2) \times 0.981 \text{ N-m}$$

Now power produced, $P = T \times \omega$

$$\therefore \quad 2 \times 10^3 = (35 - T_2) \times 0.981 \times \frac{2\pi N}{60}$$

as $\omega = \dfrac{2\pi N}{60}$ Where N is speed in r.p.m.

$$\therefore \quad 2 \times 10^3 = (35 - T_2) \times 0.981 \times \frac{2 \times \pi \times 1500}{60}$$

$$\therefore \quad T_2 = \mathbf{22.02 \text{ kg.}}$$

Example 1.38 : *Determine the power input necessary for an electric geyser to heat 8 litres of water from 25 °C to 75 °C in 10 min. Water equivalent of geyser is 150 grams. Heat lost in this period is 30 kJ. Find the efficiency of the geyser.*

Solution : Mass of water = 8 litre = 8 kg, water equivalent of geyser = 150 gm

$\Delta t = t_2 - t_1 = 75 - 25 = 50$, Time = 10 min = 600 sec, Heat lost = 30 kJ

$$\text{Total mass m} = 8 + 0.150 = 8.150 \text{ kg}$$

$$\therefore \quad \text{Heat required} = m\,C\Delta t = 8.150 \times 4190 \times 50 = 1.7074 \times 10^6 \text{ J}$$

Now

$$\text{Heat input} = \text{Heat required} + \text{Heat lost}$$

$$= 1.7074 \times 10^6 + 30 \times 10^3 = 1.737425 \times 10^6 \text{ J}$$

$$\text{Power input} = \frac{\text{Heat input}}{\text{Time}} = \frac{1.737425 \times 10^6}{600} = 2895.7083 \text{ W}$$

> **Key Point** : *While calculating efficiency of geyser we must consider energy output required to heat water alone and not along with the geyser.*

$$\therefore \quad \text{useful output} = 8 \times 4190 \times 50 = 1.676 \times 10^6 \text{ J}$$

$$\therefore \quad \text{Geyser } \eta = \frac{\text{useful output}}{\text{total input}} \times 100 = \frac{1.676 \times 10^6}{1.7374 \times 10^6} \times 100$$

$$= \mathbf{96.465 \ \%}$$

> **Key Point** : *Water equivalent of geyser means whatever heat is required to heat body of geyser can be considered to be equivalent to energy required to heat equivalent mass of water.*

Example 1.39 : *Determine the horsepower rating of an electrical motor which is required to be coupled to a centrifugal pump provided to lift water in to over head tank. The tank capacity is 10 kilo ltrs. The tank's water head is 10 mtrs. The efficiencies of pump and motor, at operating conditions, are 65 % and 85 % respectively. The tank is required to be filled up fully in 15 minutes. Water head loss due to pipe friction is equivalent to 1.15 mtrs. Further if the over head water tank is required to be fully filled up twice a day, what shall be the monthly (30 days) electricity bill if the electricity charges are Rs. 3.50 per unit kWh ?*

[Dec. - 2001]

Solution : m = 10 kilo litres = 10,000 lts = 10,000 kg, as 1 litre = 1 kg

$g = 9.81 \text{ m/s}^2$, t = 15 min = 900 sec, h = 10 m

Water head loss due to pipe friction = 1.15 m

$$\therefore \quad h_{eff} = 10+1.15 = 11.15 \text{ m}$$

$$\text{Net work done} = mg\,h_{eff}$$

$$P_{out} = \frac{\text{Net work done}}{t} = \frac{mg\,h_{eff}}{t} = \frac{10000\times 9.81\times 11.15}{900}$$

$$\text{Power output of pump} = 1215.35 \text{ W} = 1.2153 \text{ kW}$$

$$P_{in} \text{ to motor} = \frac{\text{Power output of pump}}{\eta_{pump}\times\eta_{motor}} = \frac{1215.35}{0.65\times 0.85}$$

$$= 2199.72 \text{ W} \approx 2.2 \text{ kW}$$

$$\text{Power output of motor} = \frac{\text{Power output of pump}}{\eta_{pump}}$$

$$= \frac{1.2153\times 10^3}{0.65} = 1869.69 \text{ W}$$

$$\text{Horsepower rating of motor} = (1869.69)/735.5 = 2.542 \text{ HP}$$

$$\text{Input energy required} = \text{Power input to motor} \times \text{twice a day}$$

$$= 2.2\times 10^3\times 2 = 4.4 \text{ kW}$$

$$\text{Total hrs required} = 2\times 15 \text{ min} = 30 \text{ min} = \frac{1}{2} \text{ hrs}$$

$$\therefore \quad \text{Total input} = 4.4 \text{ kW}\times\frac{1}{2} \text{ hrs} = 2.2 \text{ kWh daily}$$

Total electricity bill for month = Total input energy × Number of days × Rate of energy

$$= 2.2\times 30\times 3.50$$

$$\therefore \quad \text{Monthly bill} = \textbf{Rs. 231}$$

Example 1.40 : *A three blade wind mill is used to lift underground water and store it at ground level using a pump. For average wind speeds the value of torque developed is 20 N-m and the speed of this wind mill is 150 r.p.m. Actual head of water is 9 m and pipe friction is 1 m headloss. The wind mill mechanical efficiency and water pump efficiency are 40 % and 75 % respectively. Calculate the run of this wind mill to store water quantity of 20 kilo litres at ground level.* **(May - 2003)**

Solution : T = 20 N-m, N = 150 r.p.m. h_t = total head = 9 + 1 = 10 m

$$\eta_m = 40\ \%,\ \eta_{pump} = 75\ \%,\ m = 20 \text{ klitre} = 20\times10^3 \text{ litre}$$

$$P_{in} = T\times\omega \quad \text{Where} \quad \omega = \frac{2\pi N}{60} = \frac{20\times2\pi\times150}{60}$$

$$= 314.1592 \text{ W}$$

$$\therefore \quad P_{out} = P_{in}\times\eta_m\times\eta_{pump} = 314.1592\times0.4\times0.75$$

$$= 94.2477 \text{ W and W} = \text{J/sec}$$

So wind mill requires 1 sec to produce 94.2477 Joules. Now to lift 20×10^3 litres of water through 10 m it requires,

$$P_{total} = mgh_t \quad \text{Where } 1 \text{ litre} = 1 \text{ kg of water}$$

$$= (20\times10^3)\times9.81\times10 = 1962000 \text{ J}$$

$\therefore$ Time required by mill to produce P_{total} is,

$$t = \frac{P_{total}}{P_{out}} = \frac{1962000}{94.2477} = 20817.4841 \text{ sec} = \frac{20817.4841}{3600} \text{ hrs}$$

$$= 5.7826 \text{ hrs}$$

This is the required run of the mill.

Example 1.41 : *Three cells each having e.m.f. of 1.6 V and internal resistance 1.1 Ω are connected in (i) series and (ii) parallel, to a resistance of 2.5 Ω. Find in each case,*

a) Current b) p.d. across the external resistance c) power wasted in the external resistance.

Solution : Case i) Series connection

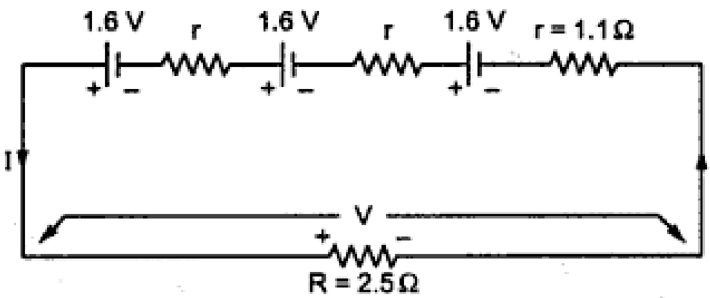


Fig. 1.58

Total e.m.f. = 3 × e.m.f. of each cell

$$E_T = 3\times1.6 = 4.8 \text{ V}$$

Total resistance of circuit

$$R_T = 3r + R$$

$$= 3 \times 1.1 + 2.5$$

$$= 5.8\ \Omega$$

$\therefore$ a) $I = \dfrac{E_T}{R_T} = \dfrac{4.8}{5.8} = \mathbf{0.8275\ A}$

b) $V =$ p.d. across $R = I \times R = 0.8275 \times 2.5 = \mathbf{2.0689\ V}$

c) $P = I^2R = (0.8275)^2 \times 2.5 = \mathbf{1.7118\ W}$

Case (ii) : Parallel connection

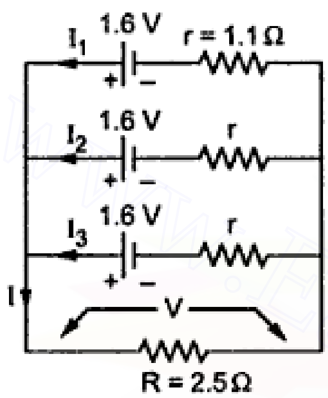


Fig. 1.59

In equivalent form, when batteries are in parallel, the total internal resistance is

$\boxed{r' = \dfrac{r}{n}}$ Where n = number of cells in parallel

$$= \frac{1.1}{3} = 0.3666\ \Omega$$

a) $\therefore$ $I = \dfrac{E}{R+r'} = \dfrac{1.6}{2.5+0.366} = \mathbf{0.55814\ A}$

b) $V = I \times R = 0.55814 \times 2.5 = \mathbf{1.3953\ V}$

c) $P = I^2R = (0.55814)^2 \times 2.5 = \mathbf{0.7788\ W}$

Example 1.42 : *When a resistance of 2 Ω is placed across the terminals of battery, the current is 2 A. When the resistance is increased to 5 Ω, the current falls to 1 A. Find e.m.f. of battery and its internal resistance.*

Solution : The two cases are shown in the Fig. 1.60.

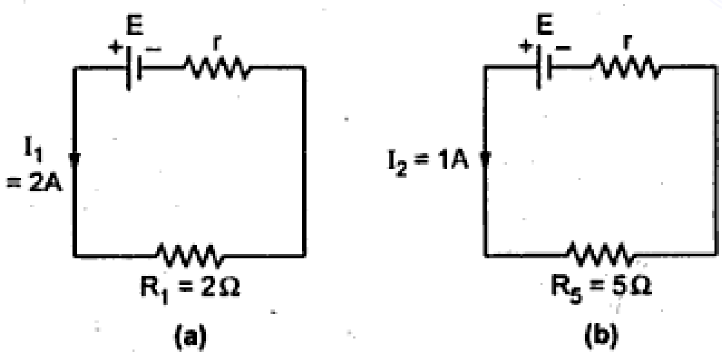


Fig. 1.60

Now $I_1 = \dfrac{E}{R_1 + r}$ i.e. $E = 2\,[2 + r] = 4 + 2r$ ...(1)

and $I_2 = \dfrac{E}{R_2 + r}$ i.e. $E = 1\,[5 + r] = 5 + r$ ...(2)

Subtracting (2) from (1),

$$0 = -1 + r \text{ i.e. } r = 1\,\Omega$$

and $$E = 6\text{ V}$$

Example 1.43 : *A bucket contains 15 - litres of water at 20 °C. A 2-kW immersion heater is used to raise the temperature of water to 95 °C. The overall efficiency of the process is 90 %, and the specific heat capacity of water is 4187 J/kg°K. Find the time required for the process.* **[Dec.-2003]**

Solution : m = 15 kg, 1 litre = 1 kg of water, C = 4187 J/kg °K

$$t_1 = 20\ ^\circ C,\ t_2 = 95\ ^\circ C,\ P_{in} = 2\text{ kW},\ \eta = 90\ \%$$

Energy required to heat the water is the output energy.

$$\therefore \quad \text{Output energy} = mC\,\Delta t = 15 \times 4187 \times (95-20) = 4.7103 \times 10^6\text{ J}$$

$$\text{Input energy} = \frac{\text{output}}{\eta} = \frac{4.7103 \times 10^6}{0.9} = 5.2337 \times 10^6\text{ J}$$

$$P_{in} = \frac{\text{input in J}}{\text{time in sec}}$$

$$\therefore \quad 2 \times 10^3 = \frac{5.2337 \times 10^6}{\text{time}}$$

$$\therefore \quad \text{time} = 2616.875\text{ sec} = \mathbf{43.614\ minutes}$$

Example 1.44 : *At 0 °C, a specimen of copper wire has its resistance equal to 4-milliohm and its temperature coefficient of resistance equal to (1 / 234.5) per °C. Find the values of its resistance and temperature coefficient of resistance at 70 °C.* **[Dec.-2003]**

Solution :

$$R_0 = 4\text{ m}\Omega, \quad \alpha_0 = \frac{1}{234.5}\ /^\circ C$$

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

$$\therefore \quad \alpha_{70} = \frac{(1/234.5)}{1+\frac{1}{234.5}\times 70} = 0.003284/^\circ C \quad \text{at } 70\ ^\circ C$$

Now $$R_t = R_0(1+\alpha_0 t) = 4\left[1+\frac{1}{234.5}\times 70\right] = 5.194\text{ m}\Omega \text{ at } 70\ ^\circ C$$

Example 1.45 : *An electric pump lifts 12 m^3 of water per minute to a height of 15-m. If its overall efficiency is 60%, find the input power. If the pump is used for 4-hours a day, find the daily cost of energy at Rs. 2.25 per unit.* **[Dec-2003]**

Solution :

Key Point : *1m³ of water = 1000 kg = 1 tonne*

$m = 12\ m^3 = 12 \times 1000$ kg as $1\ m^3$ of water = 1000 kg, h = 15 m, $\eta = 60\%$,

time = 1 minute = 60 sec

$$\text{Energy output} = mgh = 12 \times 1000 \times 9.81 \times 15 = 1.7658 \times 10^6 \text{ J}$$

$$\text{Energy input} = \frac{\text{output}}{\eta} = \frac{1.7658 \times 10^6}{0.6} = 2.943 \times 10^6 \text{ J i.e. watt-sec}$$

time for lifting = 1 minute = 60 sec

$$\therefore \quad P_{in} = \frac{\text{input}}{\text{time}} = \frac{2.943 \times 10^6}{60} = 49.05 \text{ kW}$$

For 4 hours, pump consumes $P_{in} \times 4$ = 196.2 kWh

Thus total units per day = 196.2

$\therefore$ Daily cost = 196.2×2.25 = Rs. **441.45**

Example 1.46 : *An electric pump lifts 60 m³ of water per hour to a height of 25 m. The pump efficiency is 82 % and the motor efficiency is 77 %. The pump is used for 3 hours daily. Find the energy consumed per week, if the mass of 1 - m³ of water is 1000 kg.*

[May-2004]

Solution : $1\ m^3 = 1000$ kg hence $m = 60\ m^3 = 60000$ kg

$h = 25$ m, $\eta_m = 77\ \%$, $\eta_p = 82\ \%$, time = 1 hour = 3600 sec

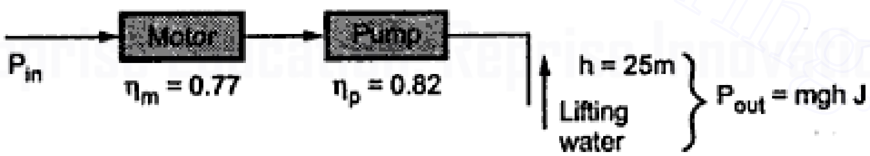


Fig. 1.61

$$P_{out} = mgh = 60000 \times 9.81 \times 25 = 14.715 \times 10^6 \text{ J}$$

$$P_{out \text{ in watts}} = \frac{P_{out} \text{ in J}}{\text{time}} = \frac{14.715 \times 10^6}{3600} = 4087.5 \text{ W}$$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta_m \times \eta_p} = \frac{4087.5}{0.77 \times 0.82} = 6473.7092 \text{ W}$$

Per day 3 hours running hence,

Daily consumption = 6473.7092×3 = 19.421 kWh

$\therefore$ Weekly power consumption = 7×19.421 = 135.947 kWh

$\therefore$ Weekly energy consumption = $135.947 \times 10^3 \times 3600 = 489.4124 \times 10^6$ J

Example 1.47 : *A single-core cable has its conductor diameter as 1.5 cm and outer diameter as 3.9 cm. The resistivities of conductor and insulator are 1.73×10^{-8} ohm-m and 8×10^{12} ohm-m respectively. Find, for a cable length of 100 m, its insulation resistance and the resistance of conductor.* **[May-2004]**

Solution : $D_1 = 1.5$ cm, outer diameter = 3.9 cm, $\rho_c = 1.73 \times 10^{-8}\ \Omega$ m,

$\rho_i = 8 \times 10^{12}\ \Omega$m, $l = 100$ m

For conductor, $R = \dfrac{\rho_c\, l}{a}$

and $$a = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times \left(1.5 \times 10^{-2}\right)^2 = 1.767 \times 10^{-4}\ m^2$$

$$\therefore \quad R = \frac{1.73 \times 10^{-8} \times 100}{1.767 \times 10^{-4}} = 9.7897 \times 10^{-3}\ \Omega$$

For insulation, $$R_1 = \frac{D_1}{2} = 0.75 \text{ cm}$$

$$R_2 = R_1 + t \quad \text{Where} \quad t = \frac{D_2 - D_1}{2} = 1.2 \text{ cm}$$

$$= 0.75 + 1.2 = 1.95 \text{ cm}$$

$$\therefore \quad R_i = \frac{\rho_i}{2\,\pi\, l}\, ln\left(\frac{R_2}{R_1}\right) = \frac{8 \times 10^{12}}{2\pi \times 100}\, ln\left(\frac{1.95}{0.75}\right)$$

$$= 1.2165 \times 10^{10}\ \Omega$$

Example 1.48 : *Two coils connected in series have resistances of 600 Ω and 400 Ω with temperature coefficient of 0.1 % and 0.4 % respectively at 20 °C. Find the effective temperature coefficient of series combination at 20 °C. When the combination is heated to 50 °C, find the resistance of the series combination.* **[Dec.-2004]**

Solution : $R_1 = 600\ \Omega$, $R_2 = 400\ \Omega$, $\alpha_1 = 0.1\ \% = \dfrac{0.1}{100}$ and $\alpha_2 = 0.4\ \% = \dfrac{0.4}{100}$

All values given at $t_1 = 20$ °C.

Let R'_1 and R'_2 are resistance values at t_2 °C

$$R'_1 = R_1\,[1 + \alpha_1\,(t_1 - t_1)] \quad \text{and} \quad R'_2 = R_2\,[1 + \alpha_2\,(t_2 - t_1)]$$

At t_1 °C, $R_{12} = R_1 + R_2$ and at t_2 °C, $R'_{12} = R'_1 + R'_2$

While $R'_{12} = R_{12}\,[1 + \alpha_{12}\,(t_2 - t_1)]$

Where α_{12} = R.T.C. of series combination at t_1 °C

$$\therefore \quad R'_1 + R'_2 = (R_1 + R_2)\,[1 + \alpha_{12}\,(t_2 - t_1)]$$

$\therefore \quad R_1 [1 + \alpha_1 (t_2 - t_1)] + R_2 [1 + \alpha_2 (t_2 - t_1)] = (R_1 + R_2) [1 + \alpha_{12} (t_2 - t_1)]$

Simplifying, $\alpha_{12} = \dfrac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}$ ... Refer section 1.11.5

$$= \frac{\left[600 \times \left(\frac{0.1}{100}\right)\right] + \left[400 \times \left(\frac{0.4}{100}\right)\right]}{600+400} = 2.2 \times 10^{-3} /°C$$

Now $\quad R_{12} = R_1 + R_2 = 600 + 400 = 1000\ \Omega$ at $t_1 = 20$ °C

$\therefore \quad R'_{12}$ = resistance of series combination at $t_2 = 50$ °C

$$= R_{12} [1 + \alpha_{12} (t_2 - t_1)] = 1000 [1 + 2.2 \times 10^{-3} (50 - 20)]$$

$$= 1066\ \Omega$$

Note : Alternatively find R'_1 and R'_2 at 50 °C and $R'_{12} = R'_1 + R'_2$

Example 1.49 : *The effective head of 100 MW power station is 220 m. Station supplies full load for 12 hours a day. The overall efficiency of power station is 86.4 %. Find the volume of water used.* **[Dec-2004]**

Solution : h = 220 m , Full load output P_{out} = 100 MW = 100×10^6 W

t = 12 hours, η = 86.4 %, $P_{out} = 100 \times 10^6$ W i.e. J/sec

$\therefore$ Energy output for full load = P_{out} × hours per day = $100 \times 10^6 \times 12 = 1200 \times 10^6$ Wh

$$= 1200 \times 10^6 \times 3600\ \text{J} = 4.32 \times 10^{12}\ \text{J}$$

$$\therefore \quad \text{Input energy} = \frac{\text{Energy output}}{\eta} = \frac{4.32 \times 10^{12}}{0.864} = 5 \times 10^{12}\ \text{J}$$

This is potential energy of water mgh.

$\therefore \quad$ P.E. of water = Input energy supplied

$\therefore \quad m \times g \times h = 5 \times 10^{12}$

$$\therefore \quad m = \frac{5 \times 10^{12}}{9.81 \times 220} = 2.3167 \times 10^9\ \text{kg}$$

But 1 m^3 of water = **1000 kg**

$$\therefore \quad \text{volume of water used} = \frac{2.3167 \times 10^9}{1000}\ m^3 = 2.3167 \times 10^6\ m^3$$

Example 1.50 : *An electric furnace is used to melt aluminium. Initial temperature of the solid aluminium is 32 °C and its melting point is 680 °C. Specific heat capacity of aluminium is 0.95 kJ/kg°K, and the heat required to melt 1 kg of aluminium at its melting point is 450 kJ. If the input power drawn by the furnace is 20 kW and its overall efficiency is 60%, find the mass of aluminium melted per hour.* **[May-2005]**

Solution : $t_1 = 32\ ^\circ C$, $t_2 = 680\ ^\circ C$, $C = 0.95$ kJ/kg°K, $P_{in} = 20$ kW, $\eta = 60\% = 0.6$

Heat required to melt 1 kg of Al at melting point is 450 kJ is the information related to Latent Heat.

$$\text{Total heat} = \text{sensible heat} + \text{Latent heat} = m\,C\,\Delta t + mL$$

Where $L = 450$ kJ/kg as given

$\therefore$ Total output energy $= m\,[C\,\Delta t + L] = m\,[0.95 \times (680 - 32) + 450]$

$= m \times 1.0656 \times 10^3$ kJ ... Note both C and L in kJ

$P_{in} = 20$ kW

and time = 1 hour as mass of Al per hour to be obtained

$\therefore$ Input energy $= P_{in} \times \text{time} = 20 \times 10^3 \times 3600 = 72 \times 10^6$ J

$\therefore$ Output energy $= \text{Input} \times \eta = 72 \times 10^6 \times 0.6 = 43.2 \times 10^6\ \text{J} = 43.2 \times 10^3$ kJ

Equating with total output energy required,

$m \times 1.0656 \times 10^3 = 43.2 \times 10^3$

$\therefore$ m = **40.5405 kg** aluminium per hour will be melted

Example 1.51 : *Show how four cells, each rated 1.5 V, 0.1 A, can be connected as batteries in three different ways to obtain different voltage and current ratings. State the voltage and current ratings of each type.* **[May-2005]**

Solution : The three ways in which cells can be connected are,

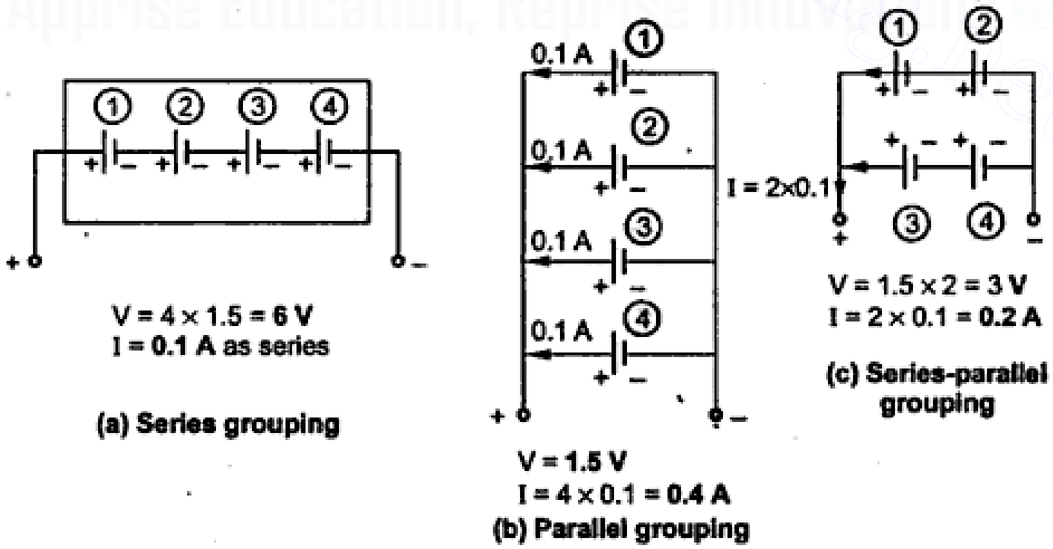


Fig. 1.62

Example 1.52 : *At 0 °C, the resistances and their temperature coefficients of resistance of two resistors 'A and 'B' are 80 ohm and 120 ohm, and 0.0038 per °C and 0.0018 per °C, respectively. Find the temperature-coefficient of resistance at 0 °C of their seires combination.*

[May-2005]

Solution : At 0° C, $R_1 = 80\ \Omega$, $R_2 = 120\ \Omega$, $\alpha_1 = 0.0038$ /°C, $\alpha_2 = 0.0018$ /°C

Refer section 1.11.5,

$$\alpha_{12} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2} \qquad \text{... R.T.C. of combination at 0 °C}$$

Key Point: *Students must derive the result and then use.*

$$\therefore \quad \alpha_{12} = \frac{80\times0.0038+120\times0.0018}{80+120} = 2.6\times10^{-3}\ /°C$$

Example 1.53 : *In a thermal generating station the heat energy obtained by burning 1 kg of coal is 16,000 kJ. Find the mass of coal required to get an output electrical energy of 1 kWh from the station, if its overall efficiency is 18 %.* **[May-2005]**

Solution : m = 1 kg, Heat energy = 16000 kJ, output = 1 kWh, η = 18 %

$$\text{Calorific value of coal} = \text{heat energy/kg} = 16000 \text{ kJ/kg}$$

$$\therefore \quad \text{Total input energy} = m \times 16000 \times 10^3 \text{ J} \qquad \text{... (1)}$$

$$m = \text{mass of coal burned}$$

$$\text{Output required} = 1 \text{ kWh} = 1 \times 10^3 \text{ Wh}$$

$$= 1 \times 10^3 \times 3600 \text{ W-sec i.e. J}$$

$$\therefore \quad \text{Input required} = \frac{\text{Output energy in J}}{\eta} = \frac{1\times10^3\times3600}{0.18}$$

$$= 20 \times 10^6 \text{ J} \qquad \text{... (2)}$$

Equating (1) and (2),

$$m \times 16000 \times 10^3 = 20 \times 10^6$$

$$\therefore \quad m = 1.25 \text{ kg} \qquad \text{... mass of coal required}$$

Example 1.54 : *Calculate the current required by a 1500 V D.C. locomotive when driving a total load of 100×10^3 kg at 25 km per hour up on incline of 1 in 100. Assume tractive resistance of 0.069 N/kg and efficiency of motor's gearing as 70 %.*

[May-2006]

Solution : The arrangement is shown in the Fig. 1.63.

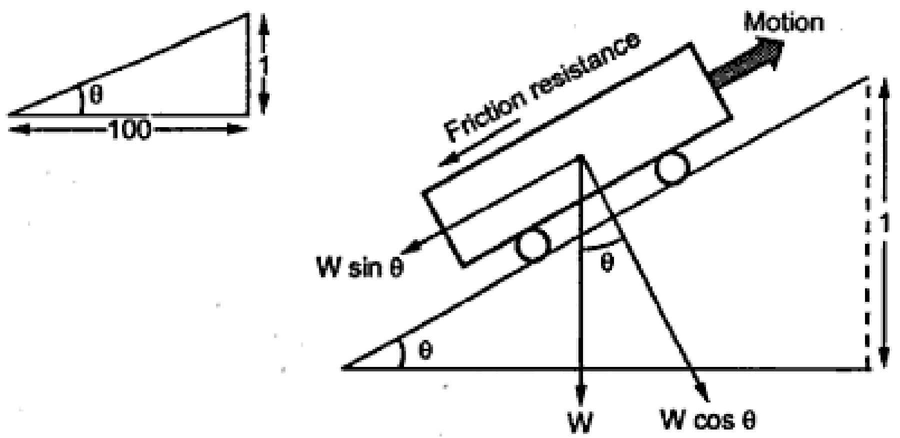


Fig. 1.63

$$\sin\theta = \tan\theta = \frac{1}{100} = 0.01 \qquad \text{... } \theta \text{ is very small}$$

$$W \sin\theta = 100\times10^3\times0.01 = 1000 \text{ kg} = 9810 \text{ N}$$

$$\text{Track resistance} = 0.069 \text{ N/kg} = 0.069\times100\times10^3 = 6900 \text{ N}$$

$$\text{Total resistance} = 9810 + 6900 = 16710 \text{ N}$$

$$\text{Work done} = \text{Force}\times\text{distance travelled in 1 sec} = 16710\times d$$

Now $\text{speed} = 25 \text{ km/hr}$

$$\therefore \quad d = \frac{25\times10^3}{3600} = 6.944 \text{ m in 1 sec}$$

$$\therefore \quad \text{work done} = 16710\times6.944 = 116040.924 \text{ J}$$

$$\therefore \quad P_{out} = \frac{W}{\text{time}} = \frac{116040.924}{1 \text{ sec}} = 116040.924 \text{ W}$$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta_{gear}} = \frac{116040.924}{0.7} = 165.77274\times10^3 \text{ W}$$

But $P_{in} = V\times I$

$$\therefore \quad I = \frac{P_{in}}{V} = \frac{165.77274\times10^3}{1500} = \mathbf{110.5151 \text{ A}} \qquad \text{... current required}$$

Example 1.55 : *A D.C. shunt motor, after running several hours on constant voltage of 400 V, takes field current of 1.6 A. If temperature rise is 40 °C, what value of extra resistance is required in field circuit to maintain field current equal to 1.6 A. Assume motor started from cold at 20 °C and* $\alpha_{20} = 0.0043/°C$. **[May-2006]**

Solution : $V = 400\ V, I_f = 1.6\ A, \Delta t = 40\,°C, T_1 = 20\,°C, \alpha_{20} = 0.0043\ /°C$

$$R_2 = \frac{V}{I_f} = \frac{400}{1.6} = 250\ \Omega$$

... Field circuit resistance after 4 hours at t_2 °C

$$R_2 = R_1[1+\alpha_1 \Delta t] \quad \text{where } \alpha_1 = \alpha_{20}$$

$$250 = R_1[1+0.0043\times 40]$$

$$\therefore \quad R_1 = 213.3105\ \Omega \quad \text{... Field circuit resistance at } t_1\ °C$$

But I_f is to be maintained constant.

$$\therefore \quad I_f = \frac{V}{R_1+R_x} = 1.6$$

$$\therefore \quad R_1+R_x = \frac{400}{1.6} = 250$$

$$\therefore \quad R_x = 250 - 213.3105 = \mathbf{36.6894\ \Omega} \quad \text{... Extra resistance required.}$$

Example 1.56 : *An immersion heater is used for heating 9 liters of water. Its resistance is 50 ohm and has efficiency of 83.6 %. How much time required to heat water from 20 °C to 70 °C, when connected to 250 V supply. Specific heat capacity of water is 4180 J/kg °K.*

[Dec.-2006, 6 Marks]

Solution : 1 liter = 1 kg, $V = 250\ V$, $t_2 = 70\ °C$, $t_1 = 20\ °C$.

$$\therefore \quad m = 9\ kg,\ R = 50\ \Omega, \quad \eta = 83.6\ \%, \quad C = 4187\ J/kg°K$$

Output energy = Energy required to heat the water

$$= m\,C\,\Delta t = 9\times 4187\times(70-20) = 1.88415\times 10^6\ J$$

$$\therefore \quad \text{Input energy} = \frac{\text{Output}}{\eta} = \frac{1.88415\times 10^6}{0.836} = 2.2537\times 10^6\ J$$

$$P_{in} = \frac{V^2}{R} = \frac{(250)^2}{50} = 1250\ W$$

$$\text{Also,} \quad P_{in} = \frac{\text{Input energy in J}}{\text{Time in sec}}$$

$$\therefore \quad 1250 = \frac{2.2537\times 10^6}{\text{Time}}$$

$$\therefore \quad \text{Time} = \mathbf{1802.96\ sec = 30.05\ min.}$$

Example 1.57 : *A single core Cu cable has conductor diameter of 3 cm and insulation thickness of 2 cm. The resistivity of Copper and insulation is 1.73×10^{-8} and 9×10^{12} ohm-meter respectively. Determine resistance of conductor and insulator of the cable for 150 meter length.* **[Dec.-2007]**

Solution : $\rho_c = 1.73 \times 10^{-8}$ Ω-m, $\rho_i = 9 \times 10^{12}$ Ω-m, $l = 150$ m,

For conductor, D_1 = diameter = 3 cm , $R_1 = \frac{D_1}{2} = 1.5$ cm

$$\therefore \quad a_c = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(3)^2 = 7.0685 \text{ cm}^2 = 7.0685 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad R = \frac{\rho_c l}{a_c} = \frac{1.73 \times 10^{-8} \times 150}{7.0685 \times 10^{-4}} = \mathbf{3.6711 \times 10^{-3}\ \Omega}$$

For insulation, $R_2 = R_1 + t = 1.5 + 2 = 3.5$ cm

$$\therefore \quad R_i = \frac{\rho_i}{2\pi l} \ln\left(\frac{R_2}{R_1}\right) = \frac{9 \times 10^{12}}{2\pi \times 150} \ln\left(\frac{3.5}{1.5}\right) = \mathbf{8.091 \times 10^{9}\ \Omega}$$

Example 1.58 : *How long it will take to raise the temperature of 880 gm of water from 16 °C to boiling point. The heater takes 2 Amp at 220 V supply and has efficiency of 90 %.* **[Dec.-2007]**

Solution : m = 880 gm, $\Delta t = 100\ °C - 16\ °C = 84$, $\eta = 90\ \%$, I = 2 A, V = 220 V

For the water, $C = 4190$ J/kg°K

Output Heat required = $m\, C\, \Delta t = (880 \times 10^{-3}) \times 4190 \times 84 = 309.7248$ kJ

$$\text{Heat input} = \frac{\text{Heat output}}{\eta} = \frac{309.7248}{0.9} = 344.1386 \text{ kJ}$$

$$\text{power input} = VI = 220 \times 2 = 440 \text{ W i.e. J/s}$$

$$\text{But} \quad \text{power input} = \frac{\text{Heat input}}{\text{Time required}}$$

$$\therefore \quad 440 = \frac{344.1386 \times 10^{3}}{\text{Time required}}$$

$$\therefore \quad \text{Time required} = \mathbf{782.1333\ sec = 13.035\ min}$$

Example 1.59 : *A resistance element having cross sectional area of 10 mm² and length 10 meter takes a current of 4 Amp from 220 V supply at temperature of 20 °C. Find (i) the resistivity of the material and (ii) current it will take when temperature rises 60 °C. Assume $\alpha_{20} = 0.0003$ /°C.* **[May-200**

Solution : $a = 10\ mm^2 = 10 \times 10^{-6}\ m^2$, $l = 10$ m, $I = 4$ A, $V = 220$ V, $t_1 = 20$ °C

i) $$R_1 = \frac{V}{I} = \frac{220}{4} = 55\ \Omega$$

But $$R_1 = \frac{\rho l}{a} \quad \text{i.e.} \quad \rho = \frac{R_1 a}{l} = \frac{55 \times 10 \times 10^{-6}}{10}$$

∴ $$\rho = 55 \times 10^{-6}\ \Omega\text{-m}$$ ...Resistivity

ii) $$t_2 = 60\ °C, \quad \alpha_1 = \alpha \text{ at } 20\ °C = 0.0003\ /°C$$

$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)] = 55 [1 + 0.0003 (60 - 20)] = 55.66\ \Omega$$

∴ $$\text{New current} = \frac{V}{R_2} = \frac{220}{55.66} = 3.9525\ A$$

Review Questions

1. *What is charge ? What is the unit of measurement of charge ?*
2. *Explain the relation between charge and current.*
3. *What is the difference between e.m.f. and potential difference ?*
4. *What is the resistance ? Which are the various factors affecting the resistance ?*
5. *Define the resistivity and conductivity of the material, stating their units.*
6. *Explain the effect of temperature on the resistance of, i) Metals ii) Insulators and iii) Alloys.*
7. *Define resistance temperature coefficient. Derive its units.*
8. *Explain the use of R.T.C. in calculating resistance at t °C.*
9. *Explain the effect of temperature on R.T.C.*
10. *Write the notes on,*
    i) Mechanical units ii) Electrical units iii) Thermal units
11. *An electric kettle is required to heat 10 litres of water from room temperature of 20 °C to 100 °C, in 2 minutes, the supply voltage being 200 V d.c. If the efficiency of kettle is 90 %, calculate the resistance of the heating element. Assume the specific heat capacity of water 4190 J/kg °K.*
    (Ans. : 1.2887 Ω)
12. *Find the rating of a tin-melting furnace in order to melt 50 kg of tin per hour. The melting temperature of tin is 230 °C while its initial temperature is 20 °C. Specific heat of tin is 0.055 kcal/kg while its latent heat is 13.31 kcal/kg. Assume furnace efficiency as 70 %.*
    (Ans. : 2.0645 kW)
13. *An electric kettle contains 1.2 kg of water at 20 °C. It takes 20 minutes to raise the temperature of the water to 100 °C. Assuming the heat lost due to radiation and heating the kettle to be 60 kJ, find the current taken by the kettle from the supply of 230 V. Assume specific heat capacity of water to be 4190 J/kg °K.* **(Ans. : 1.675 A)**

14. *In a hydroelectric generating station the difference in level (head) between the water surface and the turbine driving the generators is 425 meters. If 1250 litres of water are require to generate 2 kWh of electrical energy, find the over all efficiency. (1 litre of water has a mass of 1 kg).*
(Ans. : 69.079 %)

15. *An electric lift makes 120 double journeys in a day and a load of 6 tonne is raised to a height of 100 m in one and half minutes. In the return journey the cage of the lift is empty and it completes the journey in 70 secs. The weight of the cage is 600 kg and the counter weight is 3 tonne. Calculate, h.p rating of motor.*
Assume the efficiency of the lift as 80 % and that of motor is 88 %. **(Ans. : 66 H.P.)**

16. *One tonne of brass is to be melted in an electric furnace. If the charge is to be melted in 45 min, what is the power input to the furnace ? Assume furnace efficiency as 65 %.*
Specific heat of brass is 0.094 cal/gm °K, Latent heat of brass is 40 cal/gm, Melting point of brass is 927 °C. Initial temperature of brass is 25 °C. **(Ans. : 297.629 kW)**

17. *A pump which is gear driven by a d.c. electric motor delivers 1000 kg of water per minute to a tank, 22 m above the level of the pump. If the efficiency of pump is 80% and that of the gearing is 90 % while that of motor is 85 %, what current does the motor take from 400 V supply.*
(Ans. : 14.69 A)

18. *The field winding of a d.c. machine takes a current of 20 Amp. from 240 V d.c. supply at 25 °C. After a run of a 4 hours, the current drops to 15 Amp, supply voltage remaining constant. Determine its temperature rise.* **(Ans. : 86.5 °C)**

19. *A coil has resistance of 18 Ω at 20 °C and 20 Ω at 50 °C. Find its temperature rise when its resistance is 21 Ω and ambient temperature is 15 °C.* **(Ans. : 50 °C)**

20. *The current at the instant of switching a 40 W, 240 V lamp is 2 Amp. The resistance temperature co-efficient of the filament material is 0.0055 at the room temperature of 20°C. Find the working temperature of lamp.* **(Ans. : 2020 °C)**

21. *The resistance of a copper wire is 50 Ω at a temperature of 35 °C. If the wire is heated to a temperature of 80 °C. find its resistance at that temperature. Assume the temperature co-efficient of resistance of copper at 0 °C to be 0.00427 /°C. Also find the temperature co-efficient at 35 °C.*
(Ans. : 58.35 Ω, 3.71×10^{-3}/°C)

22. *The field winding of a d.c. motor is connected across a 440 V supply. When the room temperature is 17°C, winding current is 2.3 A. After the machine has been running for few hours, the current has fallen to 1.9 A, the voltage remaining unaltered. Calculate the average temperature throughout the winding, assuming α_0 of copper = 0.00426 /°C.* **(Ans. : 70 °C)**

23. *What is cell and battery ? State and explain the various types of cells.*
24. *Explain the construction of lead acid battery.*
25. *Explain first charging, discharging and recharging in case of lead acid battery.*
26. *State the features and application areas of lead acid battery.*
27. *State the maintenance procedure for lead acid batteries.*
28. *How battery capacity is defined ? On which factors it depends ?*

29. *What is battery efficiency? In how many ways battery efficiency is expressed ?*
30. *State and explain what is ampere-hour efficiency and watt-hour efficiency.*
31. *Write a note on charge and discharge curves for lead acid battery.*
32. *With a basic charging circuit, explain the battery charging.*
33. *Explain the two methods of battery charging.*
34. *State the indications for fully charged battery.*
35. *Explain the construction of NiMH battery.*
36. *State the cell reactions of NiMH cell.*
37. *State the features of NiMH cell.*
38. *Draw and explain the following charactersitics of NiMH batteries,*
    i) Discharge characteristics
    ii) Self discharge characteristics
    iii) Charge-voltage characteristics
39. *State the safety precautions while using the NiMH batteries.*
40. *What are the applications of NiMH batteries.*
41. *Compare Nickel-Cadmium and Nickel metal hydride batteries.*

❑❑❑

2 D.C. Circuits

2.1 Introduction

In practice, the electrical circuits may consist of one or more sources of energy and number of electrical parameters, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements alongwith various sources of energy gives rise to complicated electrical circuits, generally referred as **networks**. The terms **circuit** and **network** are used synonymously in the electrical literature. The d.c. circuits consist of only resistances and d.c. sources of energy. And the circuit analysis means to find a current through or voltage across any branch of the circuit. This chapter incıudes various techniques of analysing d.c. circuits.

The chapter explains the basic terminology used in the network analysis and classification of networks. It explains Ohm's law, Kirchhoff's laws and various network simplification techniques such as series-parallel combinations, star-delta transformation, source transformation etc. These techniques are very basic and useful, which can be further applied to understand various network theorems. The network theorems such as Superposition, Thevenin's, Norton's and Maximum power transfer as applied to d.c. circuits are also included in this chapter.

2.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

2.2.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network.** Such a network is shown in the Fig. 2.1.

2.2.2 Network Element

Any individual circuit element with two terminals which can be connected to other circuit element, is called a **network element.**

Network elements can be either active elements or passive elements. Active elements are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements. Passive elements are the elements which either store energy or dissipate energy in the form of heat. Resistor, inductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

2.2.3 Branch

A part of the network which connects the various points of the network with one another is called **a branch.** In the Fig. 2.1, AB, BC, CD, DA, DE, CF and EF are the various branches. A branch may consist more than one element.

2.2.4 Junction Point

A point where three or more branches meet is called **a junction point.** Point D and C are the junction points in the network shown in the Fig. 2.1.

2.2.5 Node

A point at which two or more elements are joined together is called **node.** The junction points are also the nodes of the network. In the network shown in the Fig. 2.1, A, B, C, D, E and F are the nodes of the network.

2.2.6 Mesh (or Loop)

Mesh (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any node twice. In the Fig. 2.1 paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc. are the loops of the network.

In this chapter, the analysis of d.c. circuits consisting of pure resistors and d.c. sources is included.

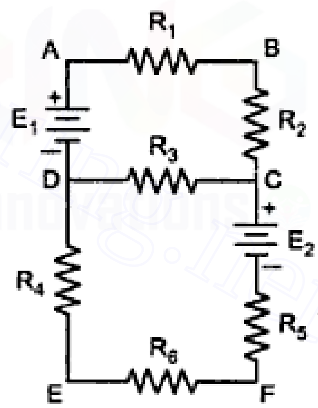


Fig. 2.1 An electrical network

2.3 Classification of Electrical Networks

The behaviour of the entire network depends on the behaviour and characteristics of its elements. Based on such characteristics electrical network can be classified as below :

i) Linear Network : A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network.** The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

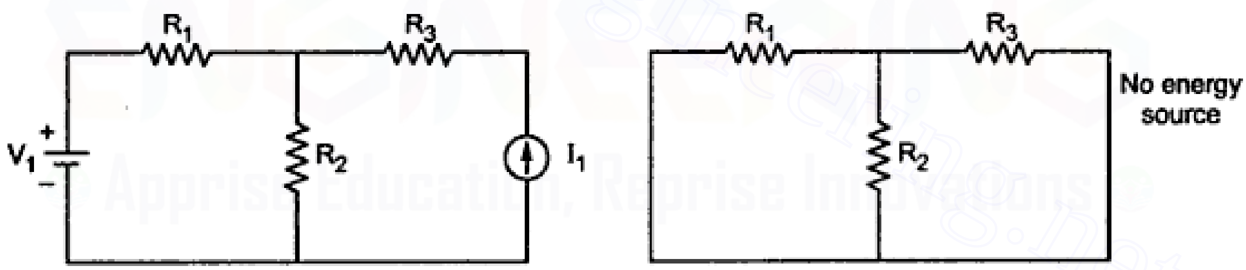
ii) Non linear Network : A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network** . The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

iii) Bilateral Network : A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network.** Network consisting only resistances is good example of bilateral network.

iv) Unilateral Network : A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network.** Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

v) Active Network : A circuit which contains at least one source of energy is called **active.** An energy source may be a voltage or current source.

vi) Passive Network : A circuit which contains no energy source is called **passive circuit.** This is shown in the Fig. 2.2.



(a) Active network (b) Passive network

Fig. 2.2

vii) Lumped Network : A network in which all the network elements are physically separable is known as **lumped network.** Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

viii) Distributed Network : A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network.** The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, any where in the circuit.

The classification of networks can be shown as,

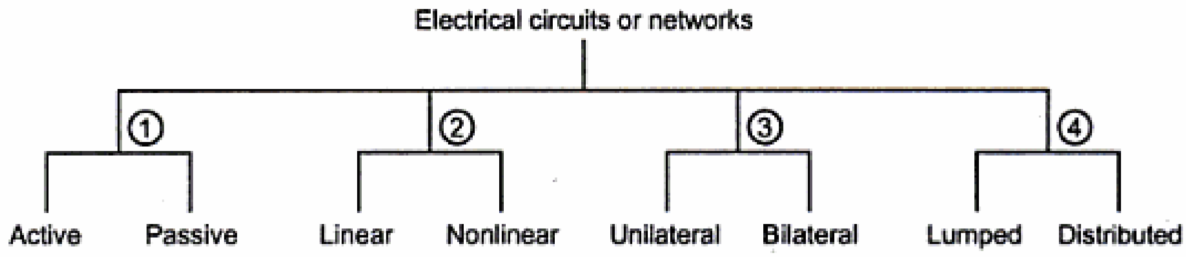


Fig. 2.3 Classification of networks

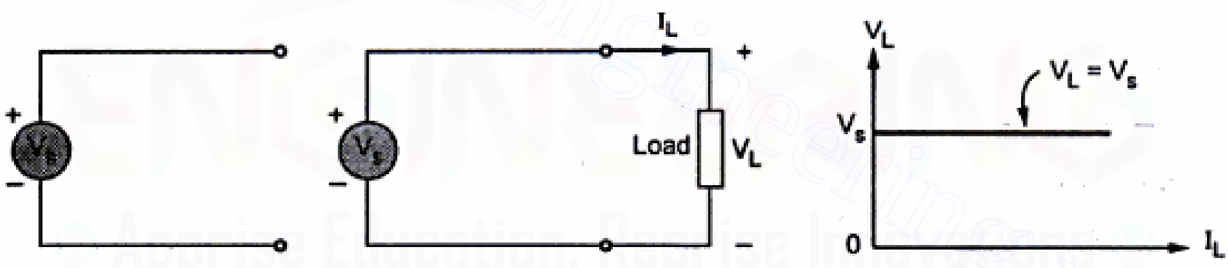
2.4 Energy Sources

There are basically two types of energy sources ; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

2.4.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 2.4 (a). This is connected to the load as shown in Fig. 2.4 (b). At any time the value of voltage at load terminals remains same. This is indicated by V- I characteristics shown in the Fig. 2.4 (c).

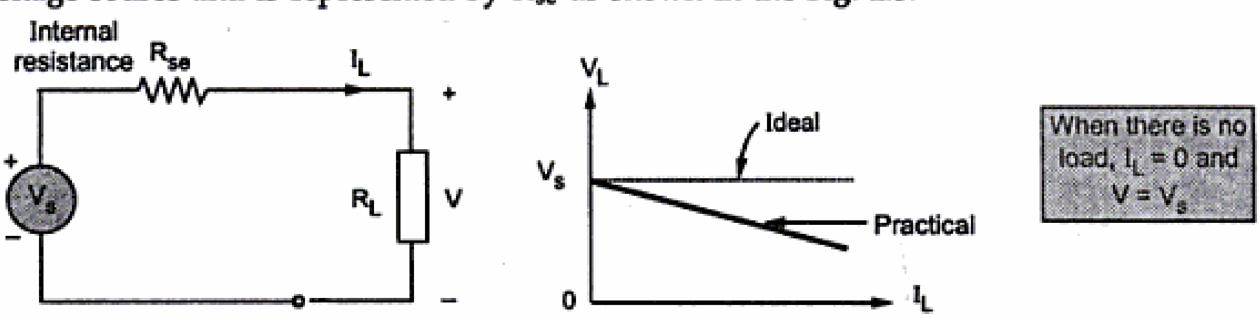


(a) Symbol (b) Circuit (c) Characteristics

Fig. 2.4 Ideal voltage source

Practical voltage source :

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 2.5.



(a) Circuit (b) Characteristics

Fig. 2.5 Practical voltage source

Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$\boxed{V_L = -(R_{se})\, I_L + V_S = V_S - I_L\, R_{se}}$$

Key Point : *For ideal voltage source, $R_{se} = 0$*

Voltage sources are further classified as follows,

i) Time Invariant Sources :

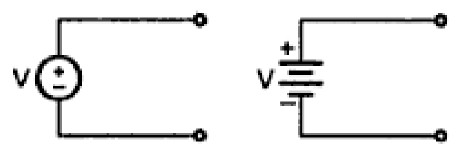


Fig. 2.6 (a) D. C. source

The sources in which voltage is not varying with time are known as **time invariant voltage sources** or **D.C. sources**. These are denoted by capital letters. Such a source is represented in the Fig. 2.6 (a).

ii) Time Variant Sources :

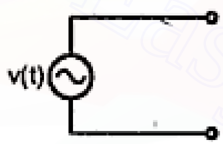


Fig. 2.6 (b) A. C. source

The sources in which voltage is varying with time are known as **time variant voltage sources or A.C. sources.** These are denoted by small letters. This is shown in the Fig. 2.6 (b).

2.4.2 Current Source

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 2.7 (a). This is connected to the load as shown in the Fig. 2.7 (b). At any time, the value of the current flowing through load I_L is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 2.7 (c).

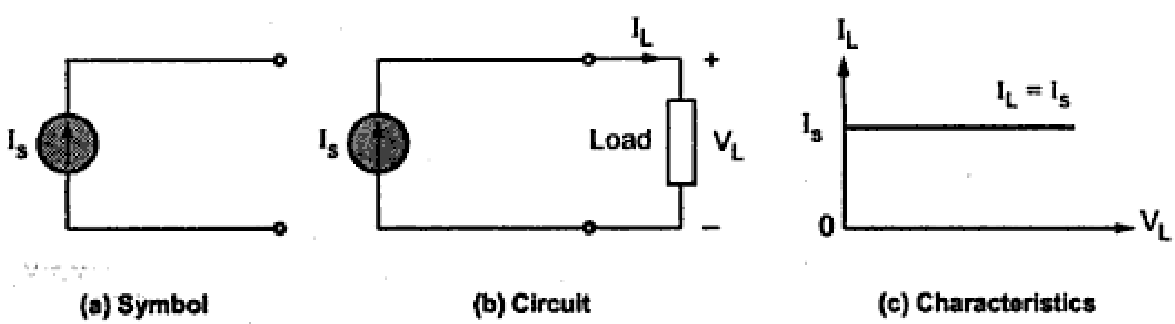


Fig. 2.7 Ideal current source

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 2.8.

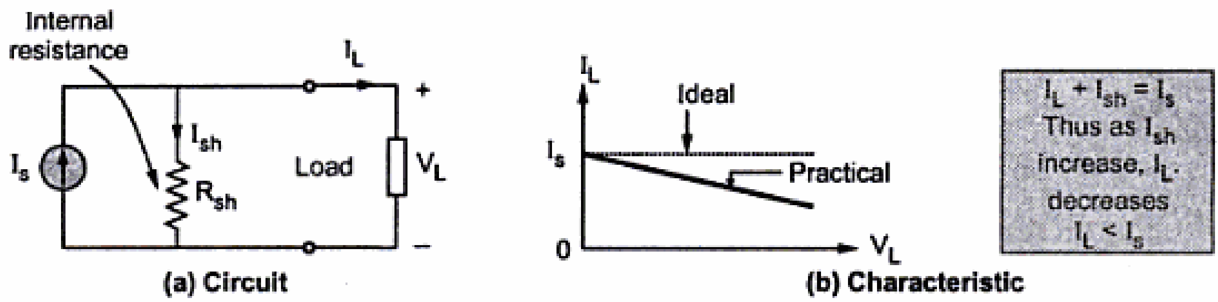


Fig. 2.8 Practical current source

Because of R_{sh}, current through its terminals decreases slightly with increase in voltage at its terminals.

Key Point: *For ideal current source, $R_{sh} = \infty$.*

Similar to voltage sources, current sources are classified as follows :

i) Time Invariant Sources :

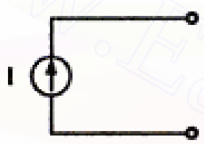


Fig. 2.9 (a) D. C. source

The sources in which current is not varying with time are known as **time invarient current sources** or **D.C. sources**. These are denoted by capital letters.

Such a current source is represented in the Fig. 2.9 (a).

ii) Time Variant Sources :

The sources in which current is varying with time are known as **time variant current sources** or **A.C. sources**. These are denoted by small letters.

Such a source is represented in the Fig. 2.9 (b).

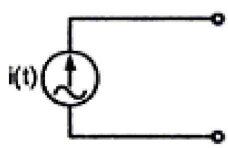


Fig. 2.9 (b) A. C. source

The sources which are discussed above are independent sources because these sources does not depend on other voltages or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside

2.4.3 Dependent Sources

Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 2.10 and further classified as,

i) Voltage Dependent Voltage Source : It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS**. It is shown in the Fig. 2.10 (a).

ii) Current Dependent Current Source : It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. 2.10 (b).

iii) Current Dependent Voltage Source : It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. 2.10 (c).

iv) Voltage Dependent Current Source : It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 2.10 (d).

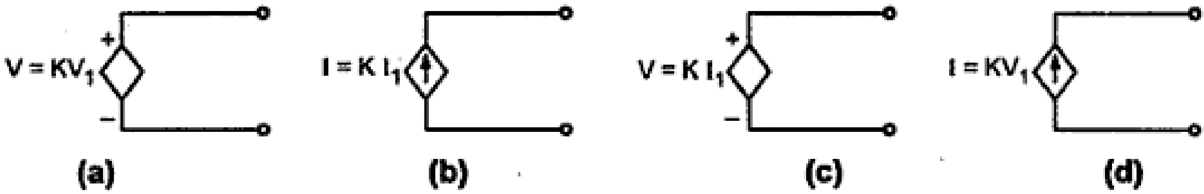


Fig. 2.10

K is constant and V_1 and I_1 are the voltage and current respectively, present elsewhere in the given circuit. The dependent sources are also known as controlled sources.

In this chapter, d.c. circuits consisting of independent d.c. voltage and current sources are analysed.

2.5 Ohm's Law

This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit. Dr. **Ohm** in 1827 discovered a law called **Ohm's Law.** It states,

Ohm's Law : *The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.*

Mathematically, $\boxed{I \propto \dfrac{V}{R}}$

Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 2.11.

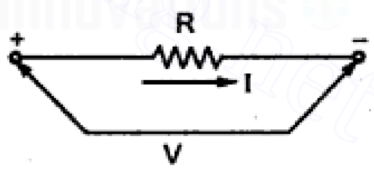


Fig. 2.11 Ohm's law

Now $$I = \frac{V}{R}$$

The unit of potential difference is defined in such a way that the constant of proportionality is unity.

Ohm's Law is,	$I = \dfrac{V}{R}$	amperes
	$V = IR$	volts
	$\dfrac{V}{I}$ = constant = R	ohms

The Ohm's law can be defined as,

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the conductor remains constant.

Key Point: *Ohm's Law can be applied either to the entire circuit or to the part of a circuit. If it is applied to entire circuit, the voltage across the entire circuit and resistance of the entire circuit should be taken into account. If the Ohm's Law is applied to the part of a circuit, then the resistance of that part and potential across that part should be used.*

2.5.1 Limitations of Ohm's Law

The limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.

2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = k\,I^m \quad \text{where k, m are constants.}$$

2.6 Series Circuit

A series circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade** connection. There is only one path for the flow of current.

Current same voltage division

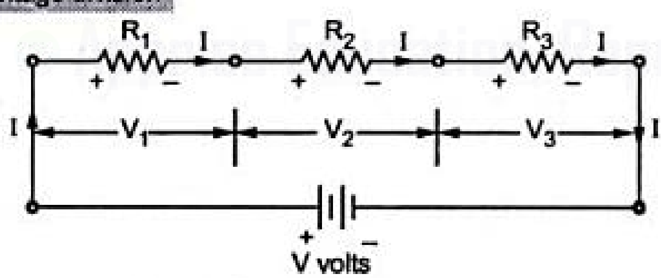


Fig. 2.12 A series circuit

Consider the resistances shown in the Fig. 2.12.

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. e.g. the chain of small lights, used for the decoration purposes is good example of series combination.

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively

Then, $$V = V_1 + V_2 + V_3$$

Now according to Ohm's law, $$V_1 = I R_1,\ V_2 = I R_2,\ V_3 = I R_3$$

Current through all of them is same i.e. I

$\therefore$ $$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I\,R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistences in series,	$R = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

2.6.1 Characteristics of Series Circuits

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots\ldots + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.
4) The equivalent resistance is the largest of all the individual resistances.

i.e $$R > R_1,\ R > R_2,\ \ldots\ldots R > R_n$$

2.7 Parallel Circuit

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the Fig. 2.13.

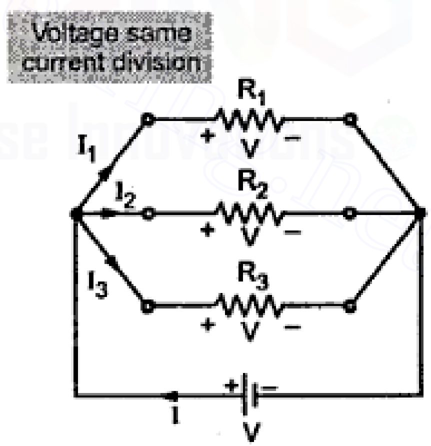


Fig. 2.13 A parallel circuit

In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V '.

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. While the voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1,\quad V = I_2 R_2,\quad V = I_3 R_3$$

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \text{... (1)}$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}} \quad \text{... (2)}$$

where R_{eq} = Total or equivalent resistance of the circuit

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}}$$

Conductance (G) :

It is known that, $\frac{1}{R} = G$ (conductance) hence,

$$\therefore \quad \boxed{G = G_1 + G_2 + G_3 + \ldots\ldots + G_n} \quad \text{... For parallel circuit}$$

Important result :

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad \boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

This formula is directly used hereafter, for two resistances in parallel.

2.7.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.

2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + ... + I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.

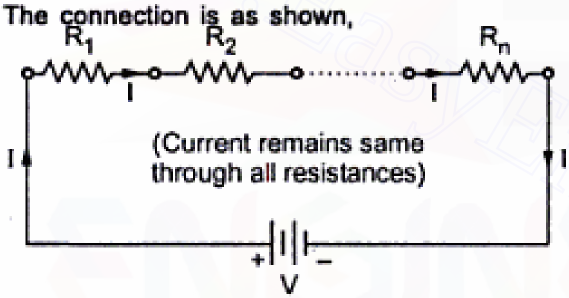
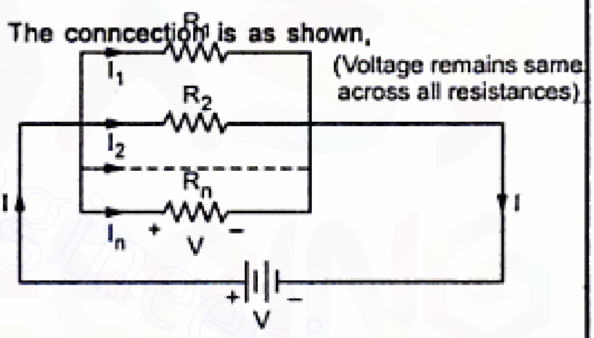
4) The equivalent resistance is the smallest of all the resistances.

$$R < R_1, \quad R < R_2, ..., R < R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

Key Point : *The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.*

2.8 Comparison of Series and Parallel Circuits

Sr. No.	Series Circuit	Parallel Circuit
1.	The connection is as shown, 	The connection is as shown, 
2.	The same current flows through each resistance.	The same voltage exists across all the resistances in parallel.
3.	The voltage across each resistance is different.	The current through each resistance is different.
4.	The sum of the voltages across all the resistances is the supply voltage. $V = V_1 + V_2 + V_3 + ... + V_n$	The sum of the currents through all the resistances is the supply current. $I = I_1 + I_2 + ... I_n$
5.	The equivalent resistance is, $R_{eq} = R_1 + R_2 + ... + R_n$	The equivalent resistance is, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$
6.	The equivalent resistance is the largest than each of the resistances in series. $R_{eq} > R_1, R_{eq} > R_2 ...... R_{eq} > R_n$	The equivalent resistance is the smaller than the smallest of all the resistances in parallel.

Example 2.1 : *Find the equivalent resistance between the two points A and B shown in the Fig. 2.14.*

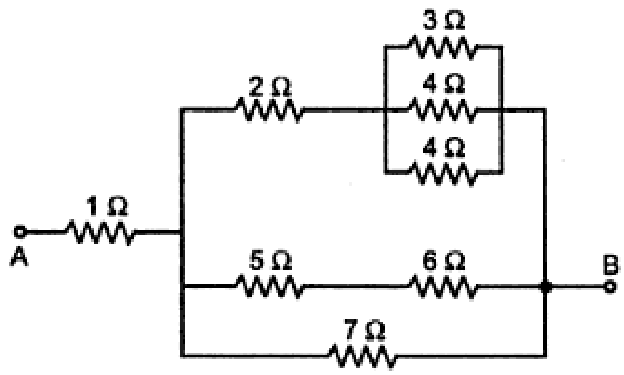


Fig. 2.14

Solution : Identify combinations of series and parallel resistances.

The resistances 5 Ω and 6 Ω are in series, as going to carry same current.

So equivalent resistance is 5 + 6 = 11 Ω

While the resistances 3 Ω , 4 Ω, and 4 Ω are in parallel, as voltage across them same but current divides.

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$

∴ $R = \frac{12}{10} = 1.2\ \Omega$

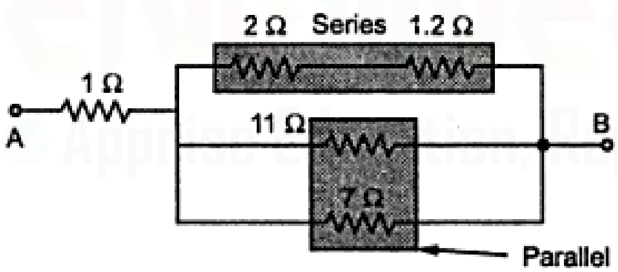


Fig. 2.14 (a)

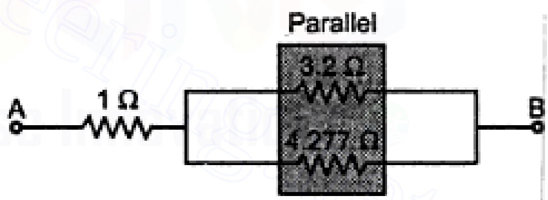


Fig. 2.14 (b)

Replacing these combinations redraw the figure as shown in the Fig. 2.14 (a).

Now again 1.2 Ω and 2 Ω are in series so equivalent resistance is 2 + 1.2 = 3.2 Ω while 11 Ω and 7 Ω are in parallel.

Using formula $\frac{R_1 R_2}{R_1+R_2}$ equivalent resistance is $\frac{11\times 7}{11+7} = \frac{77}{18} = 4.277\Omega$.

Replacing the respective combinations redraw the circuit as shown in the Fig. 2.14 (b).

Now 3.2 and 4.277 are in parallel.

∴ Replacing them by $\frac{3.2\times 4.277}{3.2+4.277} = 1.8304\ \Omega$

∴ $R_{AB} = 1 + 1.8304 = 2.8304\ \Omega$

2.9 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the nework plays an important role.

2.9.1 Short Circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire, the two points are said to be short circuited. **The resistance of such short circuit is zero.**

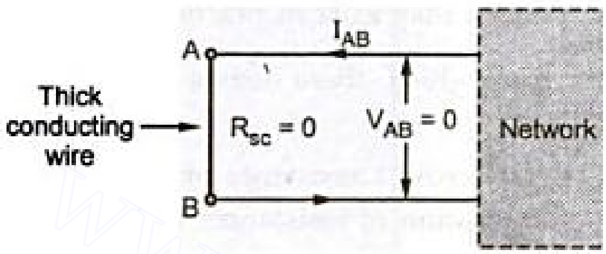


Fig. 2.15

The part of the network, which is short circuited is shown in the Fig. 2.15. The points A and B are short circuited. The resistance of the branch AB is $R_{sc} = 0\ \Omega$.

The curent I_{AB} is flowing through the short circuited path.

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0 \text{ V}$$

Key Point : *Thus, voltage across short circuit is always zero though current flows through the short circuited path.*

2.9.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

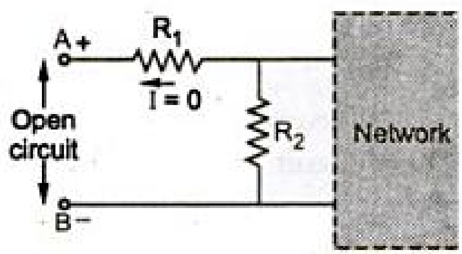


Fig. 2.16

As there is no direct connection in an open circuit, **the resistance of the open circuit is ∞.**

The part of the network which is open circuited is shown in the Fig. 2.16. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{oc} = \infty\ \Omega$.

There exists a voltage across the points AB called open circuit voltage, V_{AB} but $R_{oc} = \infty\ \Omega$.

According to Ohm's law,

$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0 \text{ A}$$

Key Point : *Thus, current through open circuit is always zero though there exists a voltage across open circuited terminals.*

2.9.3 Redundant Branches and Combinations

The redundant means excessive and unwanted.

> **Key Point** : *If in a circuit there are branches or combinations of elements which do not carry any current then such branches and combinations are called redundant from circuit point of view.*

The redundant branches and combinations can be removed and these branches do not affect the performance of the circuit.

The two important situations of **redundancy** which may exist in practical circuits are,

Situation 1 : Any branch or combination across which there exists a short circuit, becomes redundant as it does not carry any current.

If in a network, there exists a direct short circuit across a resistance or the combination of resistances then that resistance or the entire combination of resistances becomes inactive from the circuit point of view. Such a combination is redundant from circuit point of view.

To understand this, consider the combination of resistances and a short circuit as shown in the Fig. 2.17 (a) and (b).

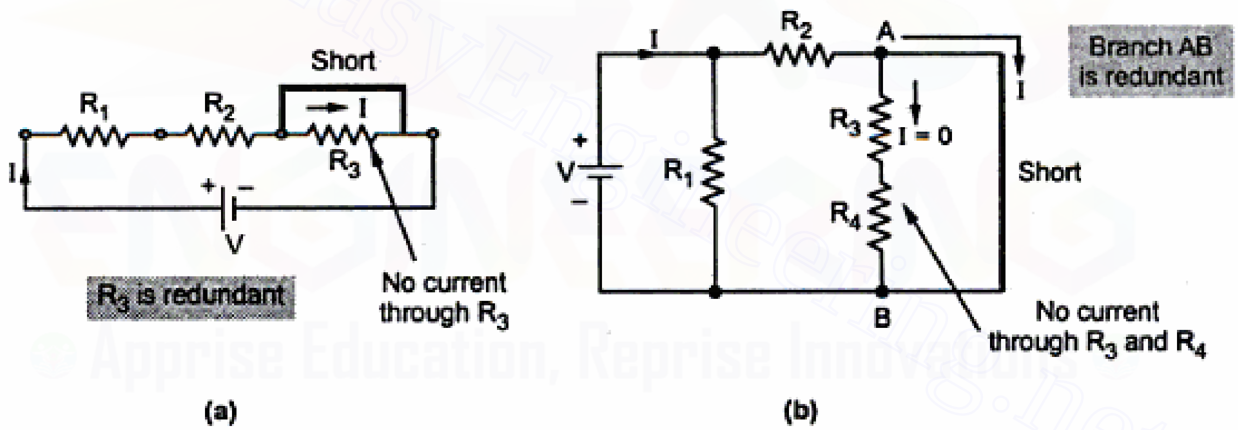


Fig. 2.17 Redundant branches

In Fig. 2.17 (a), there is short circuit across R_3. The current always prefers low resistance path hence entire current I passes through short circuit and hence resistance R_3 becomes redundant from the circuit point of view.

In Fig. 2.17 (b), there is short circuit across combination of R_3 and R_4. The entire current flows through short circuit across R_3 and R_4 and no current can flow through combination of R_3 and R_4. Thus that combination becomes meaningless from the circuit point of view. **Such combinations can be elliminated while analysing the circuit.**

Situation 2 : If there is open circuit in a branch or combination, it can not carry any current and becomes redundant.

In Fig. 2.18 as there exists open circuit in branch BC, the branch BC and CD can not carry any current and are become redundant from circuit point of view.

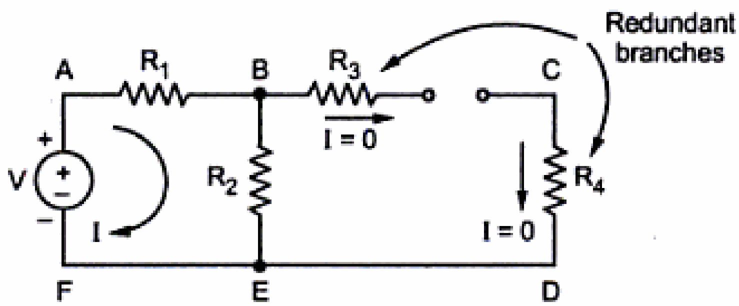


Fig. 2.18 Redundant branches due to open circuit

2.10 Voltage Division in Series Circuit of Resistors

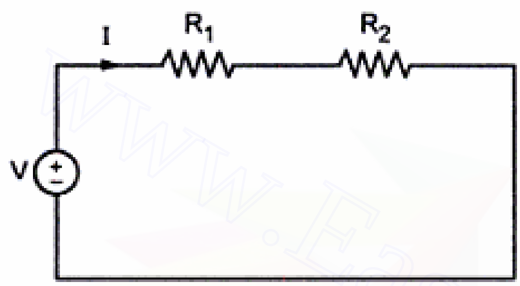


Fig. 2.19

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts.

As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I. Then applying KVL, we get,

$$V = I\,R_1 + I\,R_2$$

$$\therefore \quad I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of voltage drops V_{R1} and V_{R2} across R_1 and R_2 respectively.

$$\therefore \quad V_{R1} = I \,.\, R_1$$

$$\therefore \quad \boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

Similarly, $V_{R2} = I \,.\, R_2$

$$\therefore \quad \boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So this circuit is a **voltage divider circuit.**

Key Point : *So in general, voltage drop across any resistor, or combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage.*

Example 2.2 : *Find the voltage across the three resistances shown in the Fig. 2.20.*

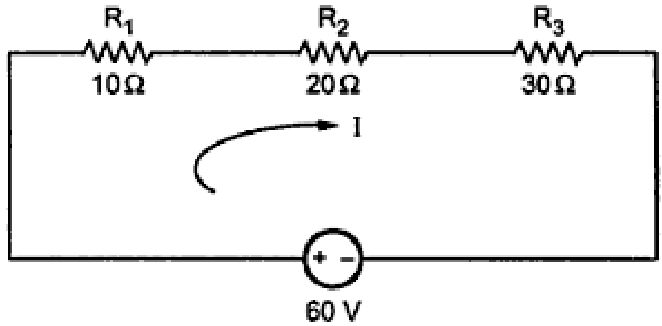


Fig. 2.20

Solution :

$$I = \frac{V}{R_1+R_2+R_3} \quad \text{... series circuit}$$

$$= \frac{60}{10+20+30} = 1\text{ A}$$

$$\therefore \quad V_{R1} = IR_1 = \frac{V \times R_1}{R_1+R_2+R_3} = 1\times 10 = \mathbf{10\ V}$$

$$\therefore \quad V_{R2} = IR_2 = \frac{V \times R_2}{R_1+R_2+R_3} = 1\times 20 = \mathbf{20\ V}$$

$$\text{and} \quad V_{R3} = IR_3 = \frac{V \times R_3}{R_1+R_2+R_3} = 1\times 30 = \mathbf{30\ V}$$

Key Point *: It can be seen that voltage across any resistance of series circuit is ratio of that resistance to the total resistance, multiplied by the source voltage.*

2.11 Current Division in Parallel Circuit of Resistors

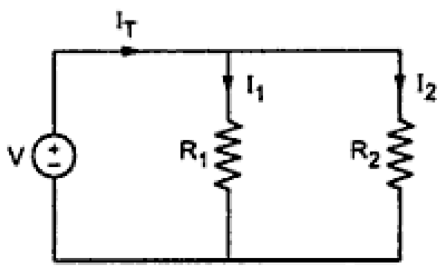


Fig. 2.21

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.

Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

$$\therefore \quad I_T = I_1 + I_2$$

$$\text{But} \quad I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$$

$$\text{i.e.} \quad V = I_1 R_1 = I_2 R_2$$

$$\therefore \quad I_1 = I_2\left(\frac{R_2}{R_1}\right)$$

Substituting value of I_1 in I_T,

$$\therefore \quad I_T = I_2\left(\frac{R_2}{R_1}\right) + I_2 = I_2\left[\frac{R_2}{R_1}+1\right] = I_2\left[\frac{R_1+R_2}{R_1}\right]$$

$$\therefore \quad \boxed{I_2 = \left[\frac{R_1}{R_1+R_2}\right] I_T}$$

Now

$$I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1+R_2}\right] I_T$$

$$\therefore \quad I_1 = \left[\frac{R_1+R_2-R_1}{R_1+R_2}\right] I_T$$

$$\therefore \quad \boxed{I_1 = \left[\frac{R_2}{R_1+R_2}\right] I_T}$$

Key Point : *In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.*

Example 2.3 : *Find the magnitudes of total current, current through R_1 and R_2 if,*

$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.

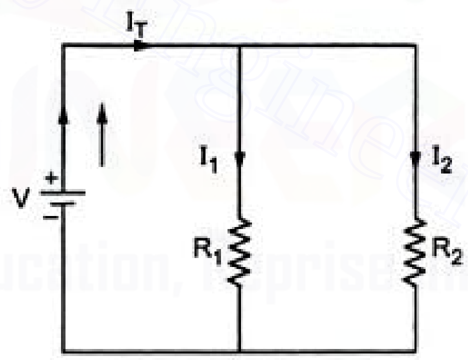


Fig. 2.22

Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 R_2}{R_1+R_2} = \frac{10\times 20}{10+20} = 6.67\ \Omega$$

$$\therefore \quad I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = 7.5 \text{ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T\left(\frac{R_2}{R_1+R_2}\right) = 7.5\times\left(\frac{20}{10+20}\right)$$

$$= 5 \text{ A}$$

and $$I_2 = I_T\left(\frac{R_1}{R_1+R_2}\right) = 7.5\times\left(\frac{10}{10+20}\right)$$

$$= 2.5 \text{ A}$$

It can be verified that $I_T = I_1 + I_2$

2.12 Source Transformation

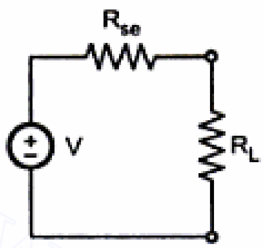


Fig. 2.23 (a) Voltage source

Consider a practical voltage source shown in the Fig. 2.23 (a) having internal resistance R_{se}, connected to the load having resistance R_L.

Now we can replace voltage source by equivalent current source.

> **Key Point :** *The two sources are said to be equivalent, if they supply equal load current to the load, with same load connected across its terminals*

The current delivered in above case by voltage source is,

$$I = \frac{V}{(R_{se}+R_L)}, \quad R_{se} \text{ and } R_L \text{ in series} \qquad ...(1)$$

If it is to be replaced by a current source then load current must be $\frac{V}{(R_{se}+R_L)}$

Consider an equivalent current source shown in the Fig. 2.23 (b).

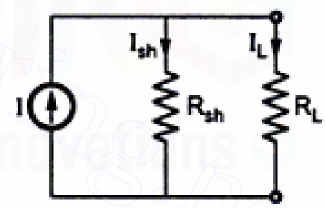


Fig. 2.23 (b) Current source

The total current is 'I'.

Both the resistances will take current proportional to their values.

From the current division in parallel circuit we can write,

$$I_L = I\times\frac{R_{sh}}{(R_{sh}+R_L)} \qquad ...(2)$$

Now this I_L and $\frac{V}{R_{se}+R_L}$ must be same, so equating (1) and (2),

$$\therefore \quad \frac{V}{R_{se}+R_L} = \frac{I\times R_{sh}}{R_{sh}+R_L}$$

Let internal resistance be, $R_{se} = R_{sh} = R$ say.

Then, $V = I \times R_{sh} = I \times R$

or $I = \dfrac{V}{R_{sh}}$

$\therefore$ $$\boxed{I = \frac{V}{R} = \frac{V}{R_{se}}}$$

Key Point: *If voltage source is converted to current source, then current source* $I = \dfrac{V}{R_{se}}$ *with parallel internal resistance equal to* R_{se}.

Key Point: *If current source is converted to voltage source, then voltage source* $V = I\,R_{sh}$ *with series internal resistance equal to* R_{sh}.

The direction of current of equivalent current source is always from –ve to + ve, internal to the source. While converting current source to voltage source, polarities of voltage is always as +ve terminal at top of arrow and –ve terminal at bottom of arrow, as direction of current is from –ve to +ve, internal to the source. This ensures that current flows from positive to negative terminal in the external circuit.

Note the directions of transformed sources, shown in the Fig. 2.23 (a), (b), (c) and (d).

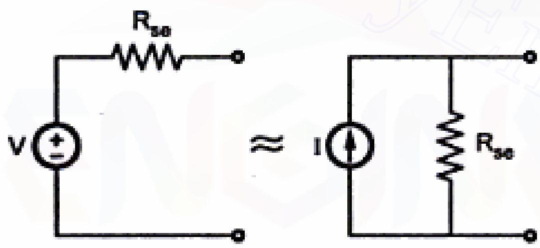


Fig. 2.24 (a) $I = \dfrac{V}{R_{se}}$

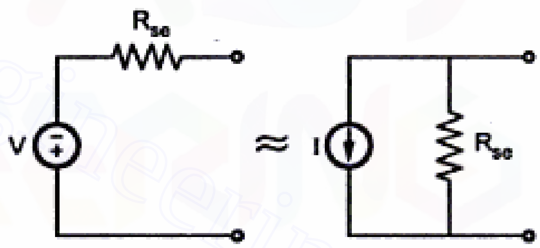


Fig 2.24 (b) $I = \dfrac{V}{R_{se}}$

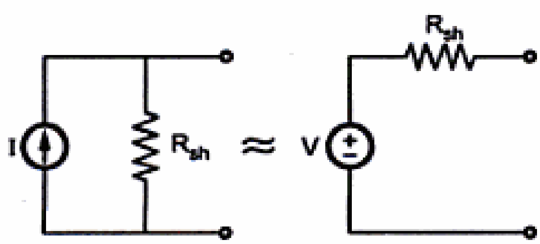


Fig. 2.24 (c) $V = I \times R_{sh}$

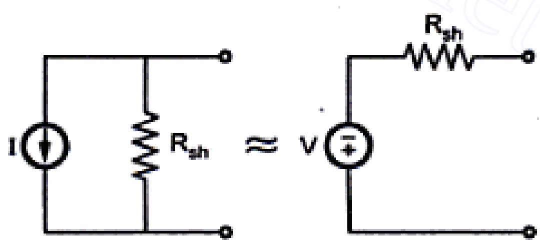


Fig. 2.24 (d) $V = I \times R_{sh}$

Example 2.4 : *Transform a voltage source of 20 volts with an internal resistance of 5 Ω to a current source.*

Solution: Refer to the Fig. 2.25 (a).

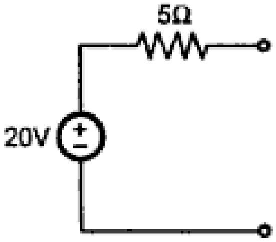


Fig. 2.25 (a)

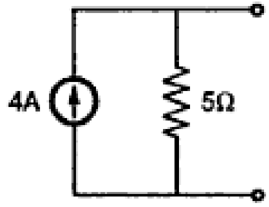


Fig. 2.25 (b)

Then current of current source is, $I = \frac{V}{R_{sc}} = \frac{20}{5} = 4$ A with internal parallel resistance same as R_{sc}.

$\therefore$ Equivalent current source is as shown in the Fig. 2.24 (b).

Example 2.5 : *Convert the given current source of 50 A with internal resistance of 10 Ω to the equivalent voltage source.*

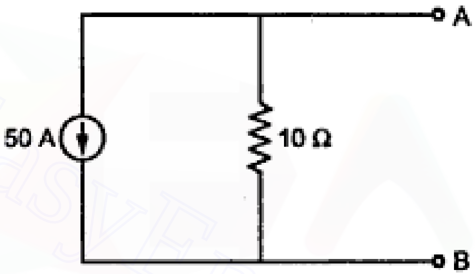


Fig. 2.26

Solution : The given values are, I = 50 A and R_{sh} = 10 Ω

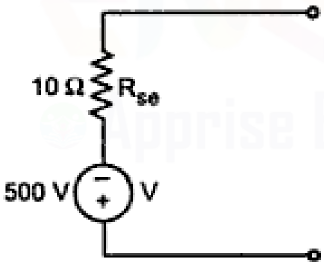


Fig. 2.26 (a)

For the equivalent voltage source,

$$V = I \times R_{sh} = 50 \times 10$$

$$= 500 \text{ V}$$

$$R_{se} = R_{sh} = 10\ \Omega \text{ in series.}$$

The equivalent voltage source is shown in the Fig. 2.26 (a).

Note the polarities of voltage source, which are such that + ve at top of arrow and – ve at bottom.

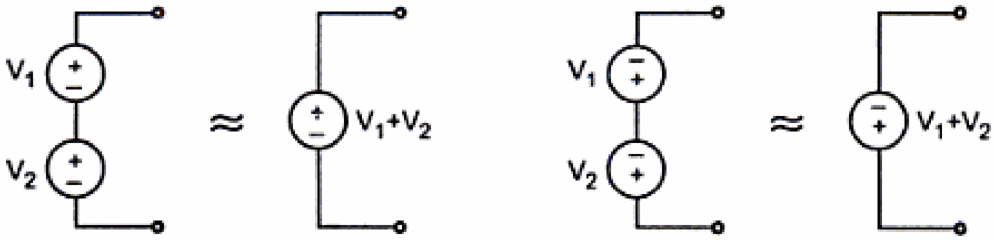
2.13 Combinations of Sources

In a network consisting of many sources, series and parallel combinations of sources exist. If such combinations are replaced by the equivalent source then the network simplification becomes much more easy. Let us consider such series and parallel combinations of energy sources.

2.13.1 Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources.

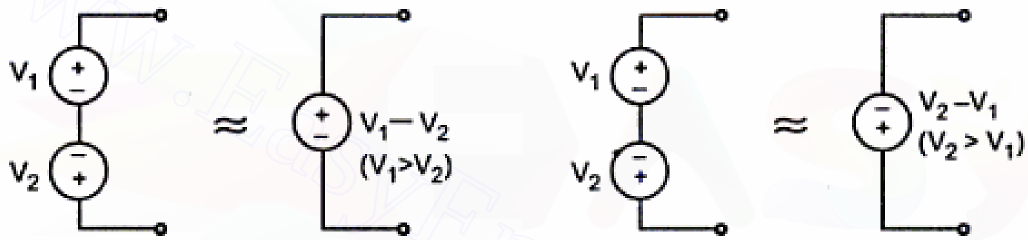
Consider the two sources as shown in the Fig. 2.27.



(a) **Fig. 2.27** (b)

Thus if the polarities of the two sources are same then the equivalent single source is the addition of the two sources with polarities same as that of the two sources.

Consider the two sources as shown in the Fig. 2.28.



(a) **Fig. 2.28** (b)

Thus if the polarities of the two sources are different then the equivalent single source is the difference between the two voltage sources. The polarities of such source is same as that of the greater of the two sources.

> **Key Point** : *The voltage sources to be connected in series must have same current ratings though their voltage ratings may be same or different.*

The technique can be used to reduce the series combination of more than two voltage sources connected in series.

2.13.2 Voltage Sources in Parallel

Consider the two voltage sources in parallel as shown in the Fig. 2.29.

The equivalent single source has a value same as V_1 and V_2.

It must be noted that at the terminals open circuit voltage provided by each source must be equal as the sources are in parallel.

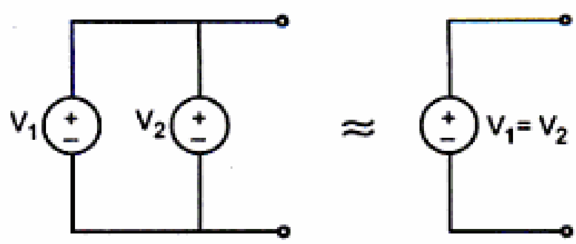


Fig. 2.29

> **Key Point** : *Hence the voltage sources to be connected in parallel must have same voltage ratings though their current rating may be same or different.*

2.13.3 Current Sources in Series

Consider the two current sources in series as shown in the Fig. 2.30.

The equivalent single source has a value same as I_1 and I_2.

> **Key Point :** *The current through series circuit is always same hence it must be noted that the current sources to be connected in series must have same current ratings though their voltage ratings may be same or different.*

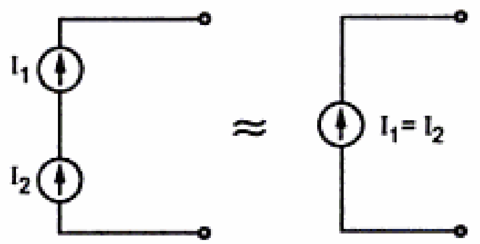
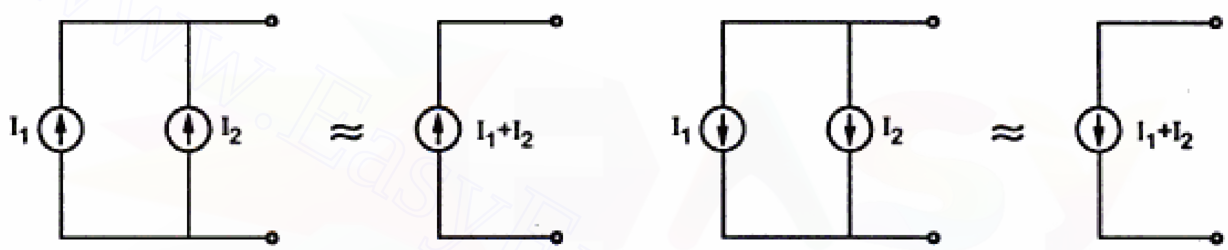


Fig. 2.30

2.13.4 Current Sources in Parallel

Consider the two current sources in parallel as shown in the Fig. 2.31.

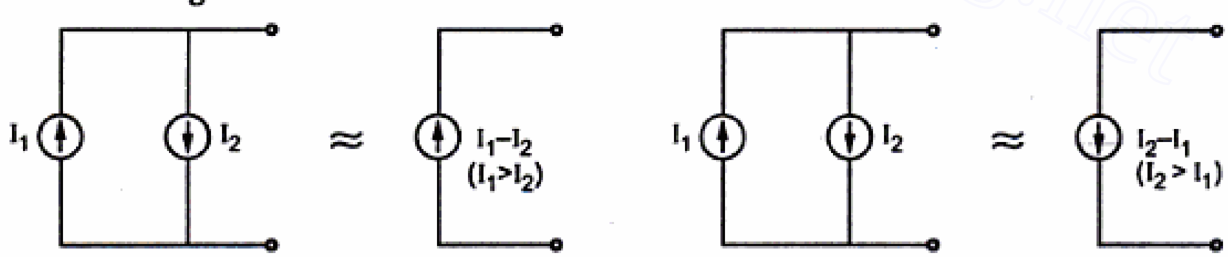


(a) (b)

Fig. 2.31

Thus if the directions of the curents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources.

Consider the two current sources with opposite directions connected in parallel as shown in the Fig. 2.32.



(a) (b)

Fig. 2.32

Thus if the directions of the two sources are different then the equivalent single source has a direction same as greater of the two sources with a value equal to the difference between the two sources.

> **Key Point :** *The current sources to be connected in parallel must have same voltage ratings though their current ratings may be same or different.*

2.14 Kirchhoff's Laws

In 1847, a German Physicist, Kirchhoff, formulated two fundamental laws of electricity. These laws are of tremendous importance from network simplification point of view.

2.14.1 Kirchhoff's Current Law (KCL)

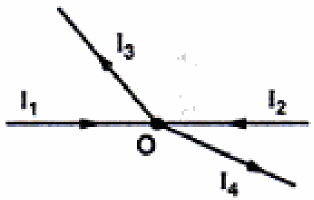


Fig. 2.33 Junction point

Consider a junction point in a complex network as shown in the Fig. 2.33.

At this junction point if I_1= 2A, I_2 = 4A and I_3 = 1A then to determine I_4 we write, total current entering is 2 + 4 = 6A while total current leaving is 1+ I_4 A

And hence, $I_4 = 5$ A.

This analysis of currents entering and leaving is nothing but the application of Kirchhoff's Current Law. The law can be stated as,

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

The word algebraic means considering the signs of various currents.

$$\boxed{\sum I \text{ at junction point } = 0}$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

e.g. Refer to Fig. 2.33, currents I_1 and I_2 are positive while I_3 and I_4 are negative.

Applying KCL, $\sum I$ at junction O = 0

$$I_1 + I_2 - I_3 - I_4 = 0 \text{ i.e. } I_1 + I_2 = I_3 + I_4$$

The law is very helpful in network simplification.

2.14.2 Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f s in the path"

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\boxed{\text{Around a closed path } \sum V = 0}$$

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

This law is very useful in loop analysis of the network.

2.14.3 Sign Conventions to be Followed while Applying KVL

When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.

(a) Fig. 2.34 (b)

In the Fig. 2.34 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.

In the Fig. 2.34 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Once all such polarities are marked in the given circuit, we can apply KVL to any closed path in the circuit.

Now while tracing a closed path, if we go from – ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise.**

For example, if the branch AB is traced from A to B then the drop across it must be considered as rise and must be taken as + IR while writing the equations.

While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop.**

For example, in the Fig. 2.34 (a) only, if the branch is traced from B to A then it should be taken as negative, as – IR while writing the equations.

Similarly in the Fig. 2.34 (b), if branch is traced from A to B then there is a voltage drop and term must be written negative as – IR while writing the equation. If the branch is traced from B to A, it becomes a rise in voltage and term must be written positive as + IR while writing the equation.

Key Point:

1) ***Potential rise*** *i.e. travelling from negative to positively marked terminal, must be considered as* ***Positive.***
2) ***Potential drop*** *i.e. travelling from positive to negatively marked terminal, must be considered as* ***Negative.***
3) *While tracing a closed path, select any one direction clockwise or anticlockwise. This selection is totally independent of the directions of currents and voltages of various branches of that closed path.*

2.14.4 Application of KVL to a Closed Path

Consider a closed path of a complex network with various branch currents assumed as shown in the Fig. 2.35 (a).

As the loop is assumed to be a part of complex network, the branch currents are assumed to be different from each other.

Due to these currents the various voltage drops taken place across various resistances are marked as shown in the Fig. 2.35 (b).

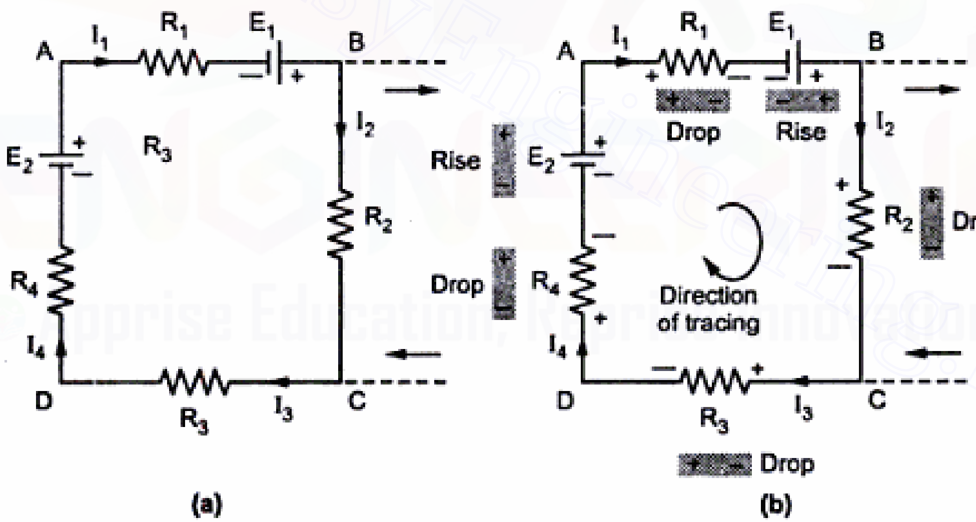


Fig. 2.35 (a), (b) Closed loop of a complex network

The polarity of voltage drop along the current direction is to be marked as positive (+) to negative (–).

Let us trace this closed path in clockwise direction i.e. **A-B-C-D-A.**

Across R_1 there is voltage drop $I_1 R_1$ and as getting traced from +ve to –ve, it is drop and must be taken as negative while applying KVL.

Battery E_1 is getting traced from negative to positive i.e. it is a rise hence must be considered as positive.

Across R_2 there is a voltage drop $I_2 R_2$ and as getting traced from +ve to −ve, it is drop and must be taken negative.

Across R_3 there is a drop $I_3 R_3$ and as getting traced from +ve to −ve, it is drop and must be taken as negative.

Across R_4 there is drop $I_4 R_4$ and as getting traced from +ve to −ve, it is drop must be taken as negative.

Battery E_2 is getting traced from −ve to +ve, it is rise and must be taken as positive.

∴ We can write an equation by using KVL around this closed path as,

$$-I_1 R_1 + E_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 + E_2 = 0 \qquad \text{... Required KVL equation}$$

i.e. $$E_1 + E_2 = I_1 R_1 + I_2 R_2 + I_3 R_3 + I_4 R_4$$

If we trace the closed loop in opposite direction i.e. along **A-D-C-B-A** and follow the same sign convention, the resulting equation will be same as what we have obtained above.

> **Key Point**: *So while applying KVL, direction in which loop is to be traced is not important but following the sign convention is most important.*

The same sign convention is followed in this book to solve the problems.

2.14.5 Steps to Apply Kirchhoff's Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1 : Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

Step 2 : Mark all the branch curretns with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

Assumed directions may be wrong, in such case answer of such current will be mathematically negative which indicates the correct direction of the current. A particular current leaving a particular source has some magnitude, then same magnitude of current should enter that source after travelling through various branches of the network.

Step 3 : Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

Step 4 : Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any previous equation.

> **Key Point** : *KVL must be applied to sufficient number of loops such that each element of the network is included at least once in any of the equations.*

Step 5 : Solve the simultaneous equations for the unknown currents. From these currents unknown voltages and power consumption in different resistances can be calculated.

What to do if current source exists ?

> **Key Point :** *If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, the loops should not be considered involving current source. The loop equations must be written to those loops which do not include any current source. This is because drop across current source is unknown.*

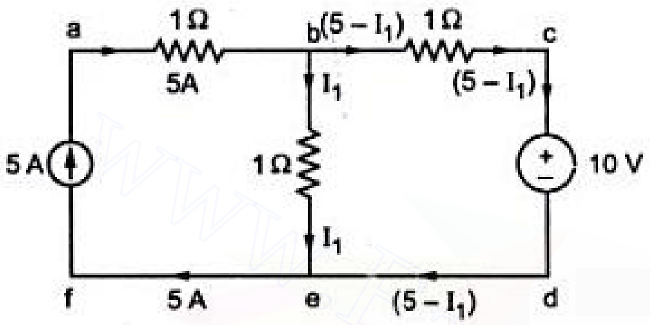


Fig. 2.36

For example, consider the circuit shown in the Fig. 2.36. The current distribution is completed interms of current source value. Then KVL must be applied to the loop bcdeb, which does not include current source. **The loop abefa should not be used for KVL application,** as it includes current source. Its effect is already considered at the time of current distribution.

2.15 Cramer's Rule

If the network is complex, the number of equations i.e. unknowns increases. In such case, the solution of simultaneous equations can be obtained by **Cramer's Rule** for determinants.

Let us assume that set of simultaneous equations obtained is, as follows :

$$\boxed{\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n &= C_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n &= C_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots + a_{nn}x_n &= C_n \end{aligned}}$$

where $C_1, C_2, \ldots\ldots\ldots\ldots C_n$ are constants.

Then Cramer's rule says that form a system determinant Δ or D as,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D$$

Then obtain the subdeterminants D_j by replacing j^{th} column of Δ by the column of constants existing on right hand side of equations i.e. $C_1, C_2, \ldots C_n$;

$$D_1 = \begin{vmatrix} C_1 & a_{12} & \dots & a_{1n} \\ C_2 & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ C_n & a_{n2} & \dots & a_{nn} \end{vmatrix}, \quad D_2 = \begin{vmatrix} a_{11} & C_1 & \dots & a_{1n} \\ a_{21} & C_2 & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & C_n & \dots & a_{nn} \end{vmatrix}$$

and

$$D_n = \begin{vmatrix} a_{11} & a_{12} & \dots & C_1 \\ a_{21} & a_{22} & \dots & C_2 \\ \vdots & & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & C_n \end{vmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$\boxed{X_1 = \frac{D_1}{D},\ X_2 = \frac{D_2}{D},\ \dots\dots\dots\dots,\ X_n = \frac{D_n}{D}}$$

where $D_1, D_2, \dots, D_n$ and D are values of the respective determinants.

Example 2.6 : *Apply Kirchhoff's current law and voltage law to the circuit shown in the Fig. 2.37.*

Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative ?

If yes, explain the significance of the negative sign.

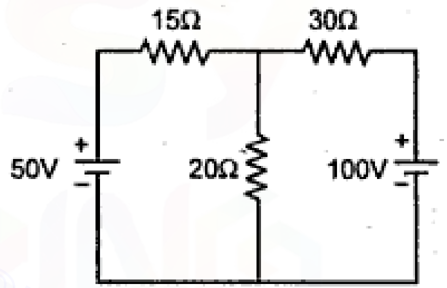


Fig. 2.37

Solution : Application of Kirchhoff's law :

Step 1 and 2 : Draw the circuit with all the values which are same as the given network. Mark all the branch currents starting from +ve of any of the source, say +ve of 50 V source.

Step 3 : Mark all the polarities for different voltages across the resistances. This is combined with step 2 shown in the network below in Fig. 2.37 (a).

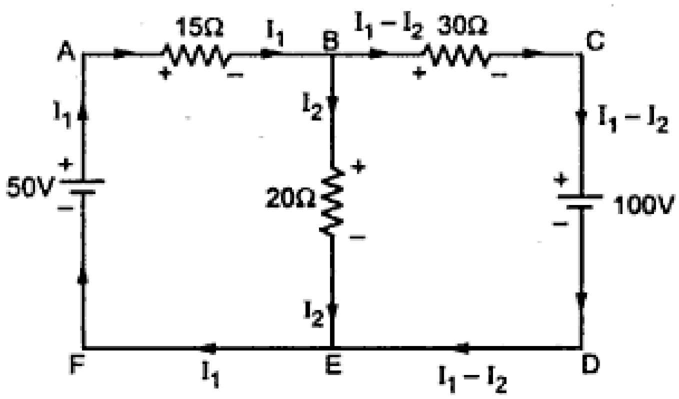


Fig. 2.37 (a)

Step 4 : Apply KVL to different loops.

Loop 1 : A-B-E-F-A., $-15\ I_1 - 20\ I_2 + 50 = 0$ ... (1)

Loop 2 : B-C-D-E-D, $-30\ (I_1 - I_2) - 100 + 20\ I_2 = 0$ ... (2)

Rewriting all the equations, taking constants on one side.

$15\ I_1 + 20\ I_2 = 50$ ...(1) and $-30\ I_1 + 50\ I_2 = 100$ ...(3)

Apply Cramer's rule, $D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$

Calculating D_1, $D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500$

$$I_1 = \frac{D_1}{D} = \frac{500}{1350} = \mathbf{0.37\ A}$$

Calculating D_2, $D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000$

$$I_2 = \frac{D_2}{D} = \frac{3000}{1350} = \mathbf{2.22\ A}$$

For I_1 and I_2, as answer is positive, assumed direction is correct.

$\therefore$ For I_1 answer is 0.37 A. For I_2 answer is 2.22 A

$$I_1 - I_2 = 0.37 - 2.22 = -1.85\ A$$

Negative sign indicates assumed direction is wrong.

i.e. $I_1 - I_2 = 1.85$ A flowing in opposite direction to that of the assumed direction.

2.16 Star and Delta Connection of Resistances

In the complicated networks involving large number of resistances, Kirchhoff's laws give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case application of Star-Delta or Delta-Star transformation, considerably reduces the complexity of the network and brings the network into a very simple form. This reduces the number of unknowns and hence network can be analysed very quickly for the required result. These transformations allow us to replace three star connected resistances of the network, by equivalent delta connected resistances, without affecting currents in other branches and vice-versa.

Let us see what is Star connection ?

If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in **Star**.

The Fig. 2.38 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 2.38 (a) and (b) are exactly identical. The Fig. 2.38 (b) can be redrawn as Fig. 2.38 (a) or vice-versa, in the circuit from simplification point of view.

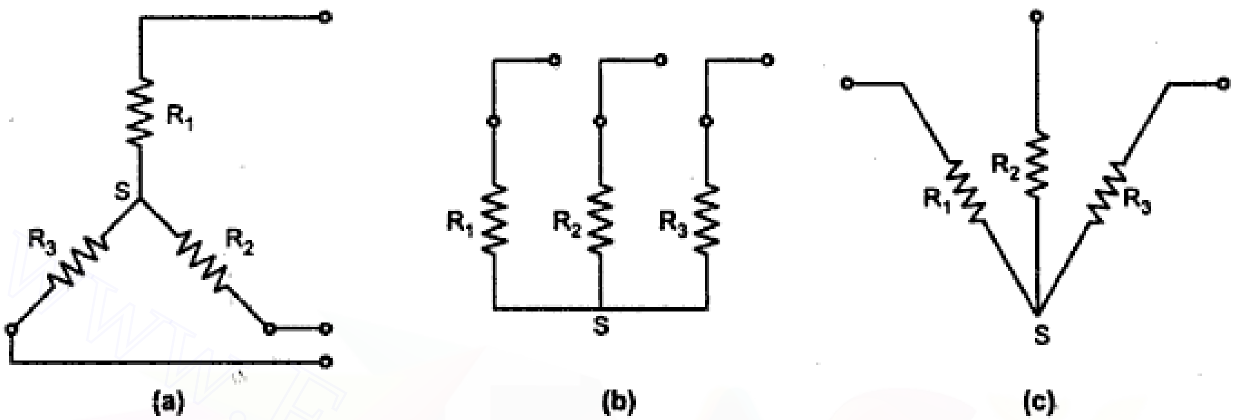


Fig. 2.38 Star connection of three resistances

Let us see what is delta connection ?

If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in **Delta**.

Key Point: *Delta connection always forms a loop, closed path.*

The Fig. 2.39 (a) and (b) show delta connection of three resistances. The Fig. 2.39 (a) and (b) are exactly identical.

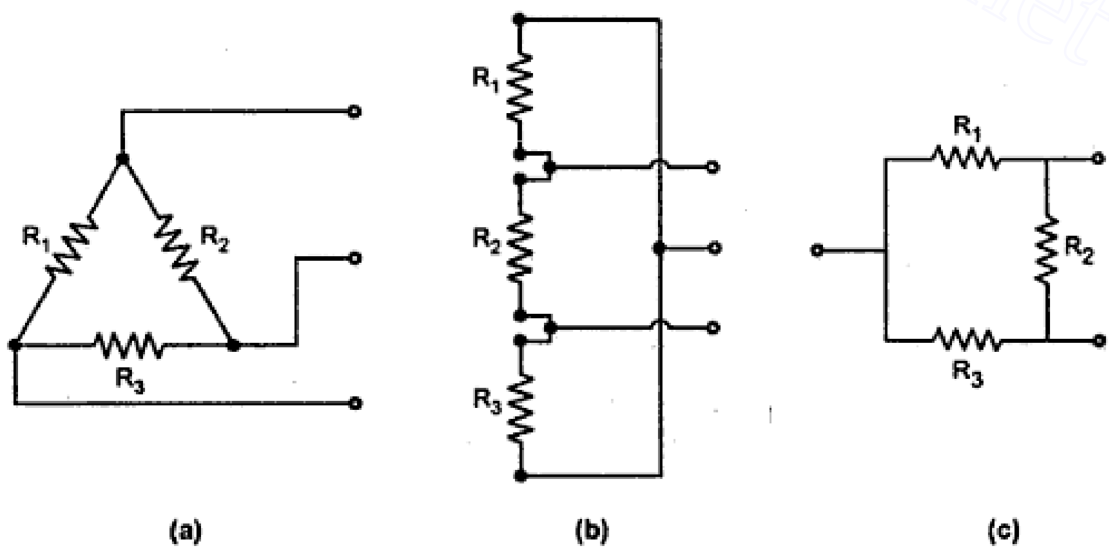
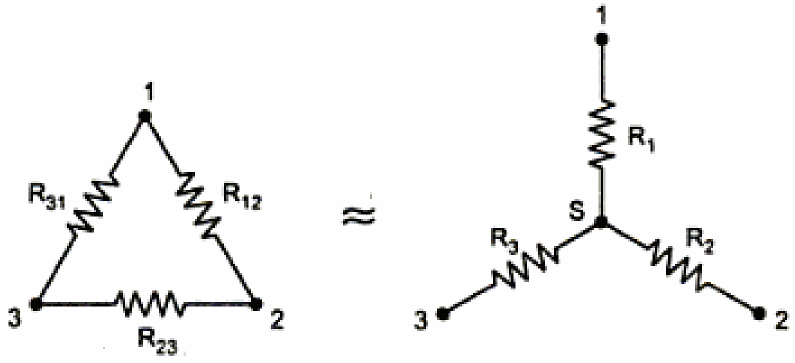


Fig. 2.39 Delta connection of three resistances

2.16.1 Delta-Star Transformation



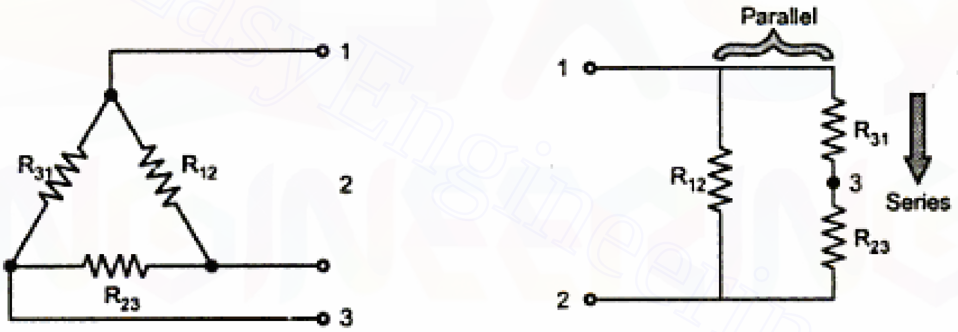
Given Delta Fig. 2.40 Equivalent Star

Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta as shown in the Fig. 2.40. The terminals between which these are connected in Delta are named as 1, 2 and 3.

Now it is always possible to replace these Delta connected resistances by three equivalent Star connected resistances R_1, R_2, R_3 between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 2.40 which is called **equivalent Star of Delta connected resistances.**

> **Key Point :** *Now to call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.*

Let us analyse Delta connection first, shown in the Fig. 2.40 (a).



(a) Given Delta Fig. 2.40 (b) Equivalent between 1 and 2

Now consider the terminals (1) and (2). Let us find equivalent resistance between (1) and (2). We can redraw the network as viewed from the terminals (1) and (2), without considering terminal (3). This is shown in the Fig. 2.40(b).

Now terminal '3' we are not considering, so between terminals (1) and (2) we get the combination as,

R_{12} parallel with $(R_{31} + R_{23})$ as R_{31} and R_{23} are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} \qquad \text{...(a)}$$

[using $\dfrac{R_1 R_2}{R_1 + R_2}$ for parallel combination]

Now consider the same two terminals of equivalent Star connection shown in the Fig. 2.41.

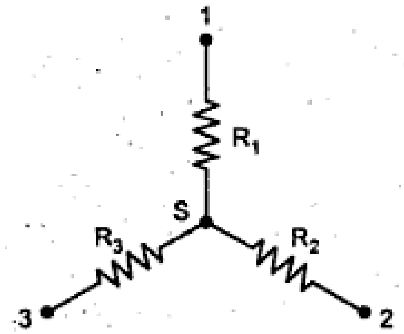


Fig. 2.41 Star connection

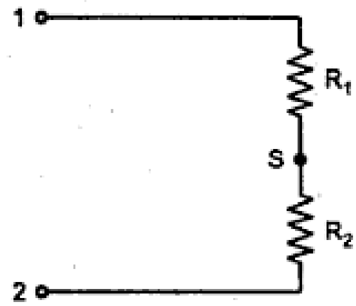


Fig. 2.42 Equivalent between 1 and 2

Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).

And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 2.42.

$\therefore$ Between (1) and (2) the resistance is = $R_1 + R_2$ ... (b)

This is because, two of them found to be in series across the terminals 1 and 2 while 3 found to be open.

Now to call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence equating equations (a) and (b),

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \qquad \text{...(c)}$$

Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \qquad \text{...(d)}$$

Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \qquad \text{...(e)}$$

Now we are interested in calculating what are the values of R_1, R_2, R_3 interms of known values R_{12}, R_{23}, and R_{31}.

Subtracting (d) from (c),

$$\frac{R_{12}(R_{31}+R_{23}) - R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore \qquad R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \qquad \text{...(f)}$$

Adding (f) and (e),

$$\frac{R_{12} R_{31} - R_{23} R_{31} + R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\therefore \frac{R_{12} R_{31} - R_{23} R_{31} + R_{31} R_{12} + R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = 2R_1$$

$$2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\boxed{R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (c), (d) and (e) we can get,

$$\boxed{R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}}$$

and

$$\boxed{R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

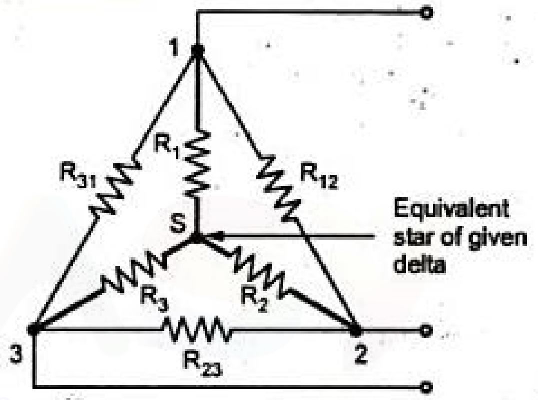


Fig. 2.43 Delta and equivalent Star

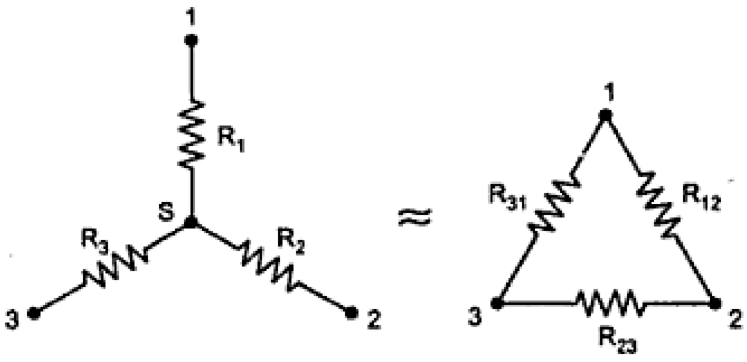
Easy way of remembering the result :

The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

So if we want equivalent resistance between terminal (2) and star point i.e. R_2 then it is the product of two resistances in delta which are connected to same terminal i.e. terminal (2) which are R_{12} and R_{23} divided by sum of all delta connected resistances i.e. R_{12}, R_{23} and R_{31}.

$$\therefore \qquad R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$

2.16.2 Star-Delta Transformation



Given Star Fig. 2.44 Equivalent Delta

Consider the three resistances R_1, R_2 and R_3 connected in Star as shown in Fig. 2.44.

Now by Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances R_{12}, R_{23} and R_{31}, between the same terminals. This is called **equivalent Delta of the given star.**

Now we are interested in finding out values of R_{12}, R_{23} and R_{31} interms of R_1, R_2 and R_3.

For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12}\,R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(g)}$$

$$R_2 = \frac{R_{12}\,R_{23}}{R_{12}+R_{23}+R_{31}} \quad \text{...(h)}$$

$$R_3 = \frac{R_{23}\,R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(i)}$$

Now multiply (g) and (h), (h) and (i), (i) and (g) to get following three equations.

$$R_1R_2 = \frac{R_{12}^2\,R_{31}\,R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(j)}$$

$$\therefore \quad R_2R_3 = \frac{R_{23}^2\,R_{12}\,R_{31}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(k)}$$

$$R_3R_1 = \frac{R_{31}^2\,R_{12}\,R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(}l\text{)}$$

Now add (j) ,(k) and (*l*)

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}^2R_{31}R_{23}+R_{23}^2R_{12}R_{31}+R_{31}^2R_{12}\,R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}R_{31}R_{23}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = \frac{R_{12}R_{31}\,R_{23}}{R_{12}+R_{23}+R_{31}}$$

But $$\frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} = R_1 \quad \text{From equation (g)}$$

∴ Substituting in above in R.H.S. we get,

$$\therefore \quad R_1R_2+R_2R_3+R_3R_1 = R_1 R_{23}$$

$$\therefore \quad \boxed{R_{23} = R_2+R_3+\frac{R_2 R_3}{R_1}}$$

Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1+R_2+\frac{R_1 R_2}{R_3}}$$

and $$\boxed{R_{31} = R_3+R_1+\frac{R_1 R_3}{R_2}}$$

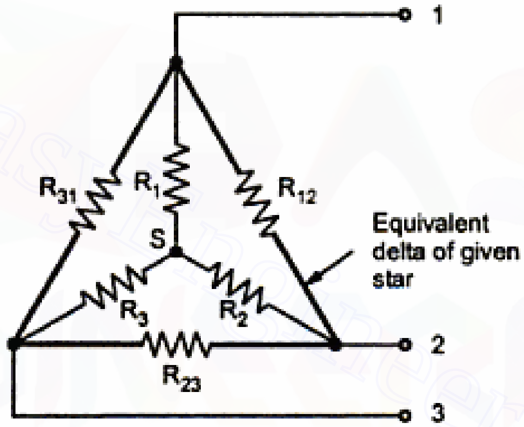


Fig. 2.45 Star and equivalent Delta

Easy way of remembering the result :

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

So if we want equivalent delta resistance between terminals (3) and (1), then take sum of the two resistances connected between same two terminals (3) and (1) and star point respectively i.e. terminal (3) to star point R_3 and terminal (1) to star point i.e. R_1. Then to this sum of R_1 and R_3, add the term which is the product of the same two resistances i.e. R_1 and R_3 divided by the third star resistance which is R_2.

∴ We can write, $R_{31} = R_1+R_3+\dfrac{R_1R_3}{R_2}$ which is same as derived above.

Result for equal resistances in star and delta :

If all resistances in a Delta connection have same magnitude say R, then its equivalent Star will contain,

$$R_1 = R_2 = R_3 = \frac{R \times R}{R+R+R} = \frac{R}{3}$$

i.e. equivalent Star contains three equal resistances, each of magnitude one third the magnitude of the resistances connected in Delta.

If all three resistances in a Star connection are of same magnitude say R, then its equivalent Delta contains all resistances of same magnitude of ,

$$R_{12} = R_{31} = R_{23} = R+R+\frac{R \times R}{R} = 3R$$

i.e. equivalent delta contains three resistances each of magnitude thrice the magnitude of resistances connected in Star.

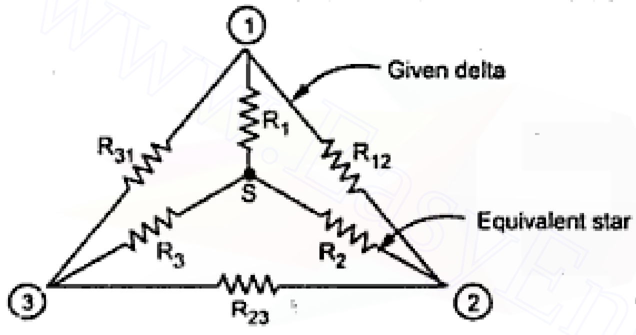
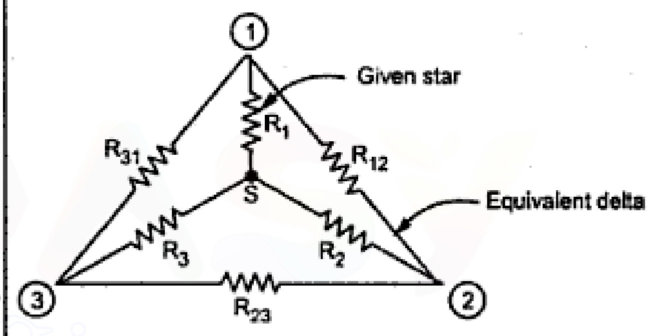
Delta-Star	Star-Delta
	
$R_1 = \dfrac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{12} = R_1 + R_2 + \dfrac{R_1R_2}{R_3}$
$R_2 = \dfrac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$	$R_{23} = R_2 + R_3 + \dfrac{R_2R_3}{R_1}$
$R_3 = \dfrac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$	$R_{31} = R_3 + R_1 + \dfrac{R_1R_3}{R_2}$

Table 2.1 Star-Delta and Delta-Star Transformations

Example 2.7 : *Convert the given Delta in the Fig. 2.46 into equivalent Star.*

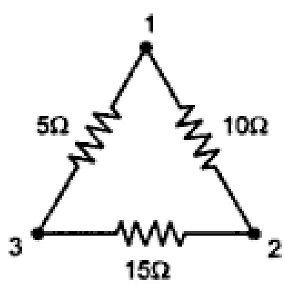


Fig. 2.46

Solution : Its equivalent star is as shown in the Fig. 2.47.

where

$$R_1 = \frac{10 \times 5}{5+10+15} = 1.67\ \Omega$$

$$R_2 = \frac{15 \times 10}{5+10+15} = 5\ \Omega$$

$$R_3 = \frac{5 \times 15}{5+10+15} = 2.5\ \Omega$$

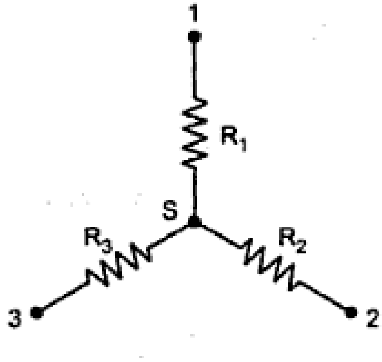


Fig. 2.47

Example 2.8 : *Convert the given star in the Fig. 2.48 into an equivalent delta.*

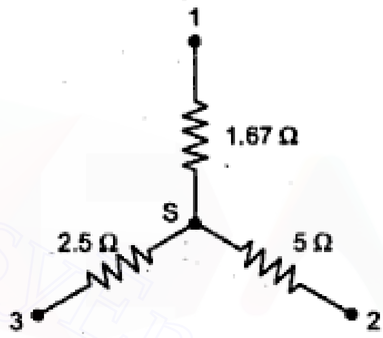


Fig. 2.48

Solution : Its equivalent delta is as shown in the Fig. 2.48 (a).

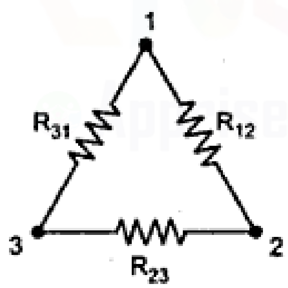


Fig. 2.48 (a)

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10\ \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15\ \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = 5\ \Omega$$

Example 2.9 : *Find equivalent resistance between points A-B.*

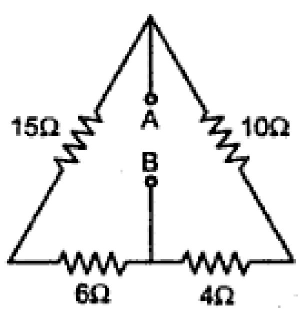


Fig. 2.49

Solution : Redrawing the circuit,

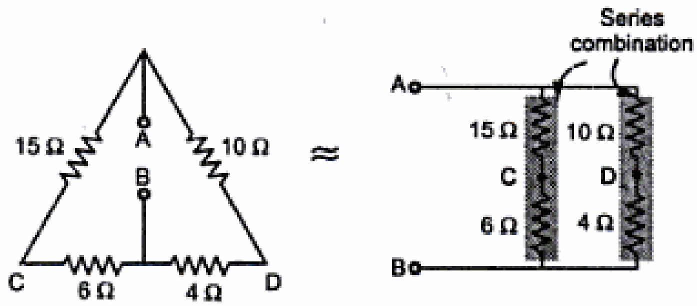
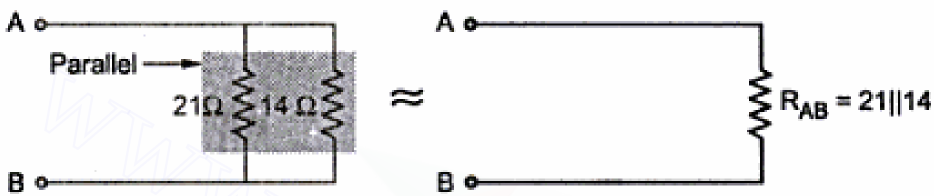


Fig. 2.49 (a)



$$R_{AB} = \frac{21 \times 14}{21 + 14} = 8.4\ \Omega$$

Fig. 2.49 (b)

Example 2.10 : *Find equivalent resistance between points A-B.*

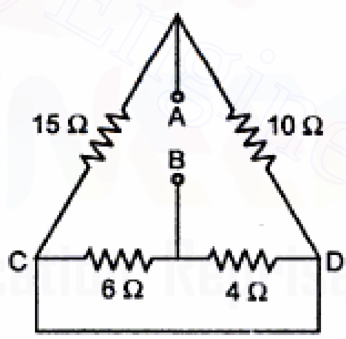


Fig. 2.50

Solution : Redraw the circuit,

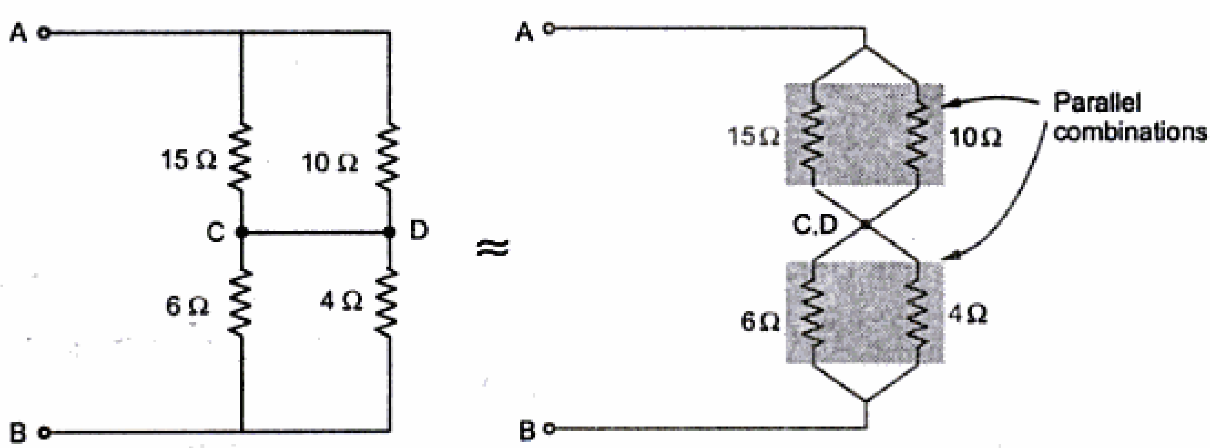


Fig. 2.50 (a)

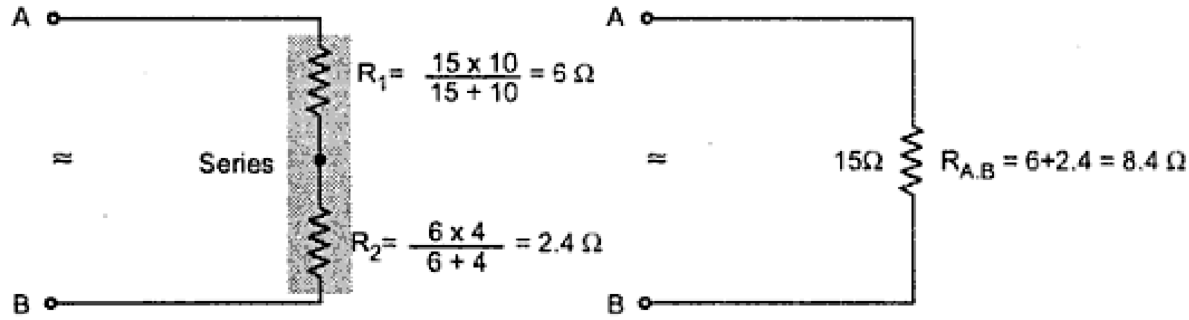


Fig. 2.50 (b)

$\therefore \quad R_{AB} = 8.4\ \Omega$

Example 2.11 : *Calculate the effective resistance between points A and B in the given circuit in Fig. 2.51.* **(Dec. - 97)**

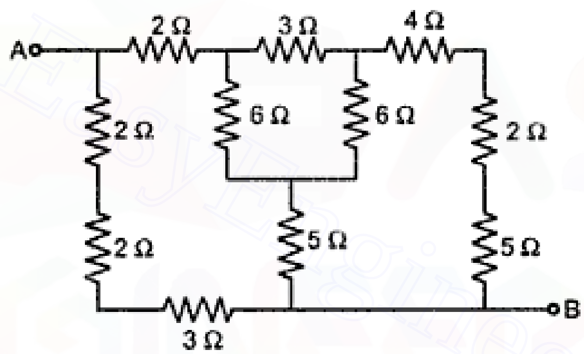


Fig. 2.51

Solution : The resistances 2, 2 and 3 are in series while the resistances 4, 2, and 5 are in series.

$\therefore \quad 2 + 2 + 3 = 7\ \Omega$

and $\quad 4 + 2 + 5 = 11\ \Omega$

The circuit becomes as shown in Fig. 2.51 (a).

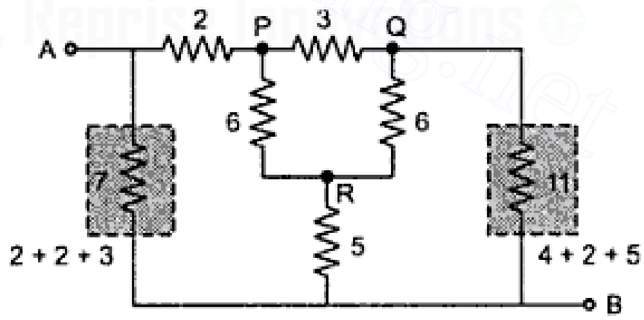


Fig. 2.51 (a)

Converting Δ PQR to equivalent star,

$$R_{PN} = \frac{6 \times 3}{6+3+6} = 1.2\ \Omega$$

$$R_{PN} = \frac{6 \times 6}{6+3+6} = 2.4\ \Omega$$

$$R_{QN} = \frac{6 \times 3}{6+3+6} = 1.2\ \Omega$$

Hence the circuit becomes as shown in the Fig. 2.51 (b).

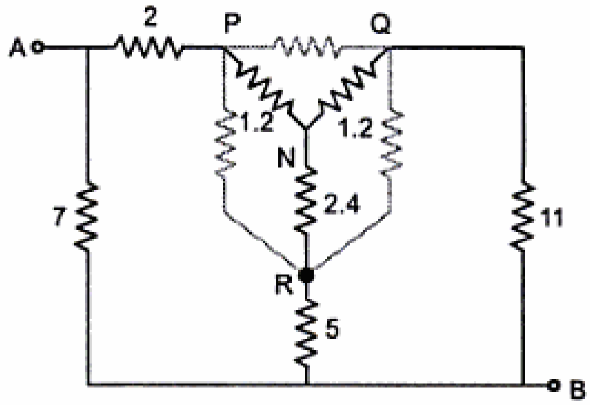


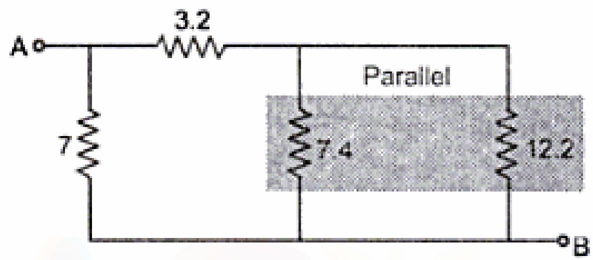
Fig. 2.51 (b)

The resistances 2 and 1.2 are in series.

1.2 and 11 are in series.

5 and 2.4 are in series.

∴ Circuit becomes after simplification as shown in the Fig. 2.51 (c).



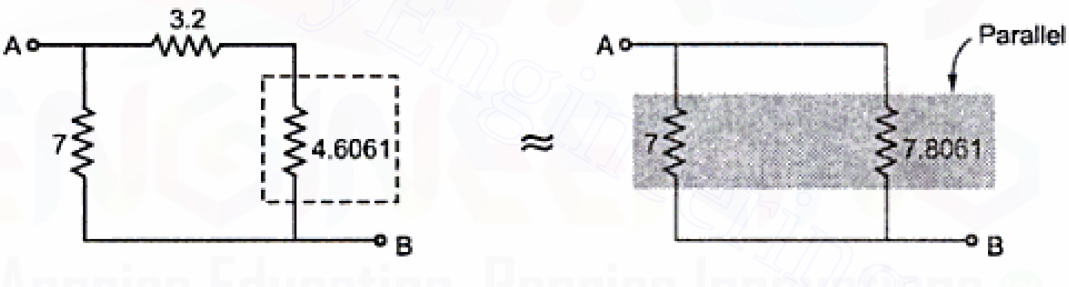
The resistances 7.4 and 12.2 are in parallel.

$$\therefore \quad 7.4 \parallel 12.2 = \frac{7.4 \times 12.2}{7.4 + 12.2} = 4.6061\ \Omega$$

So circuit becomes,

Now the two resistances are in parellel as shown in the Fig. 2.51(e).

$$\therefore \quad R_{AB} = \frac{7 \times 7.8061}{7 + 7.8061} = 3.69\ \Omega$$



(d) **Fig. 2.51** (e)

Example 2.12 : *Determine the current supplied by each battery in the circuit shown in the Fig. 2.52 by using Kirchhoff's laws.* **(May - 98)**

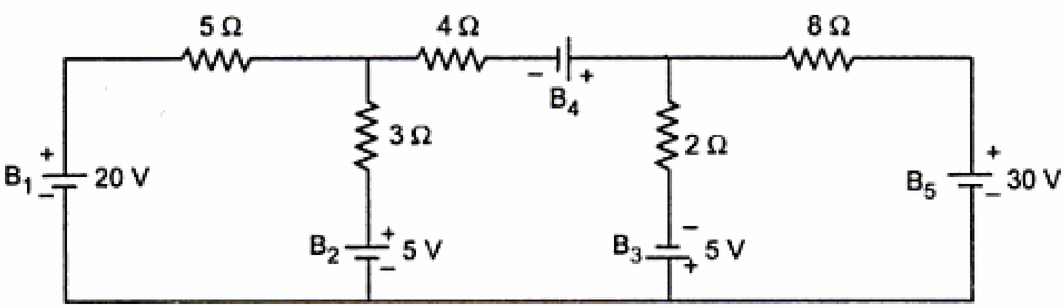


Fig. 2.52

Solution : Show all the branch currents and the polarities of voltage drops across the resistances due to the respective currents as shown in the Fig. 2.53.

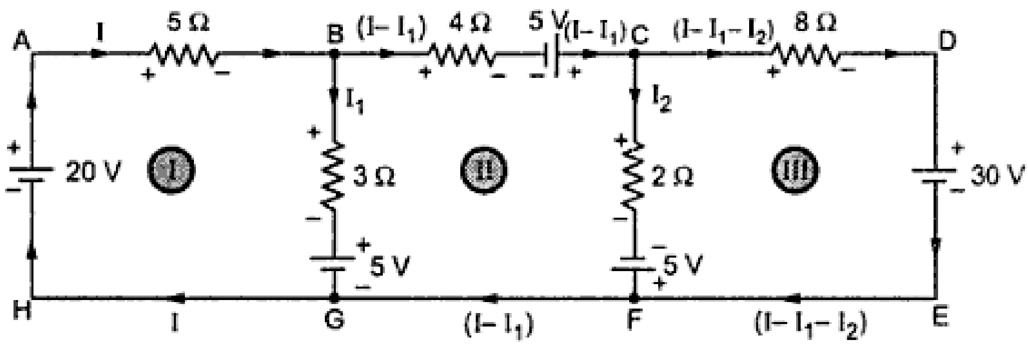


Fig. 2.53

Applying KVL to various loops :

For loop 1, ABGHA

$$-5\,I - 3\,I_1 - 5 + 20 = 0 \quad \text{i.e.} \quad +5\,I + 3\,I_1 = 15 \qquad \ldots(1)$$

For loop 2, BCFGB

$$-4\,(I - I_1) + 5 - 2\,I_2 + 5 + 5 + 3\,I_1 = 0 \quad \text{i.e.}\ 4I - 7I_1 + 2I_2 = 15 \qquad \ldots(2)$$

For loop 3, CDEFC

$$-8\,(I - I_1 - I_2) - 30 - 5 + 2I_2 = 0 \quad \text{i.e.} \ -8I + 8I_1 + 10I_2 = 35 \qquad \ldots(3)$$

Solving (1), (2) and (3)

$\therefore$ $I = 2.558$ A, $I_1 = 0.7357$ A, $I_2 = 4.9581$ A

Hence the current supplied by various batteries can be calculated as below :

Current supplied by $B_1 = I =$ **2.558 A**

Current supplied by $B_2 = I_1 =$ **0.7357 A**

Current supplied by $B_3 = I_2 =$ **4.9581 A**

Current supplied by $B_4 = (I - I_1) = (2.558 - 0.7357) =$ **1.8223 A**

Current supplied by $B_5 = (I - I_1 - I_2) = (2.558 - 0.7357 - 4.9581)$

$=$ **– 3.1358 A** ... – ve sign means opposite direction

Example 2.13 : *Find the equivalent resistance between terminals B and C of the circuit shown in the Fig. 2.54.* **(May - 99)**

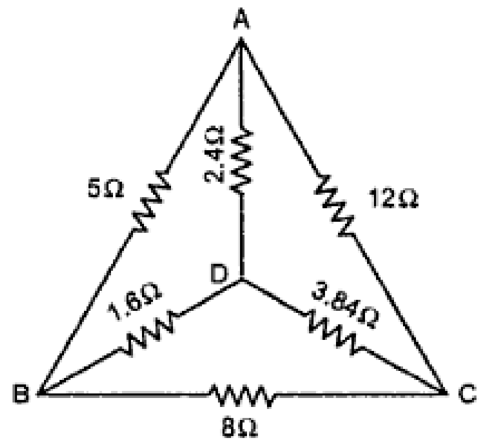
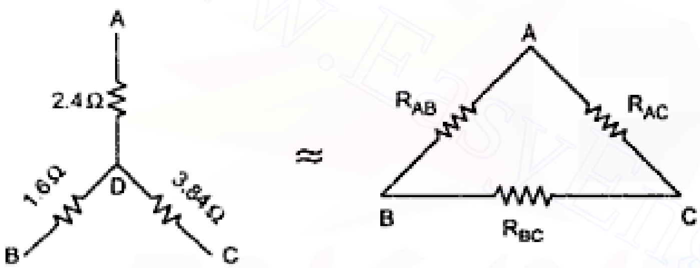


Fig. 2.54

Solution : Solution is also possible by converting Delta to Star which gives solution in less steps.

Converting star ADCB to delta ACB.



$$R_{AB} = 2.4 + 1.6 + \frac{2.4 \times 1.6}{3.84} = 5\ \Omega$$

$$R_{AC} = 2.4 + 3.84 + \frac{2.4 \times 3.84}{1.6} = 12\ \Omega$$

$$R_{BC} = 1.6 + 3.84 + \frac{1.6 \times 3.84}{2.4} = 8\ \Omega$$

Fig. 2.54 (a)

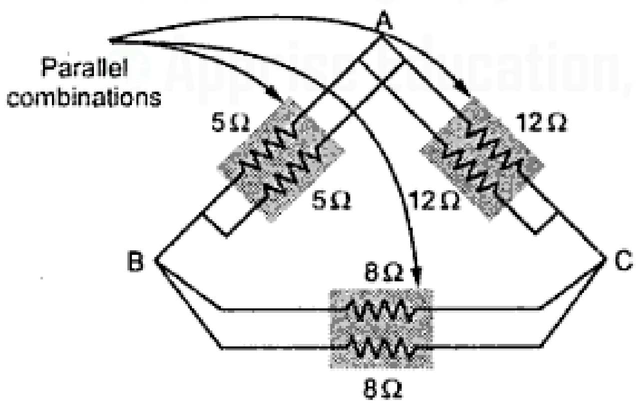


Fig. 2.54 (b)

$\approx$

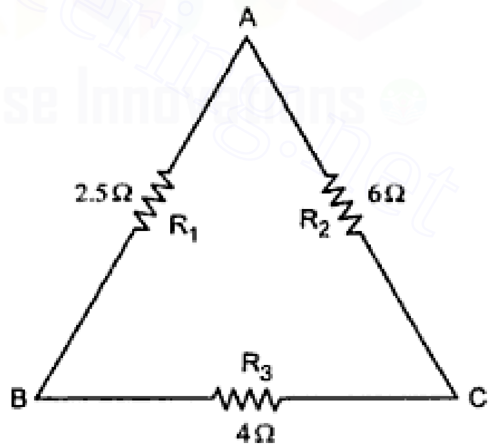


Fig. 2.54 (c)

$$R_1 = \frac{5 \times 5}{5+5} = 2.5\ \Omega, \quad R_2 = \frac{12 \times 12}{12+12} = 6\ \Omega, \quad R_3 = \frac{8 \times 8}{8+8} = 4\ \Omega$$

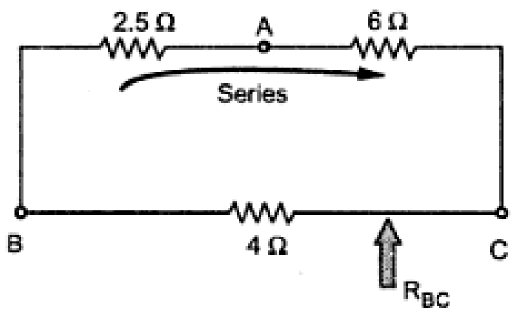


Fig. 2.54 (d)

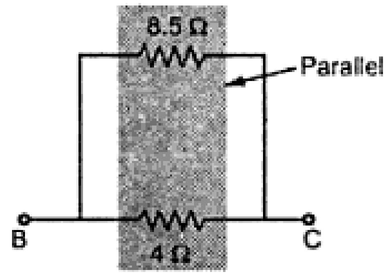


Fig. 2.54 (e)

$$R_{BC} = \frac{4 \times 8.5}{4+8.5} = 2.72\ \Omega$$

Example 2.14 : *Using Kirchhoff's laws, calculate the current delivered by the battery shown in Fig. 2.55.* **(May - 99)**

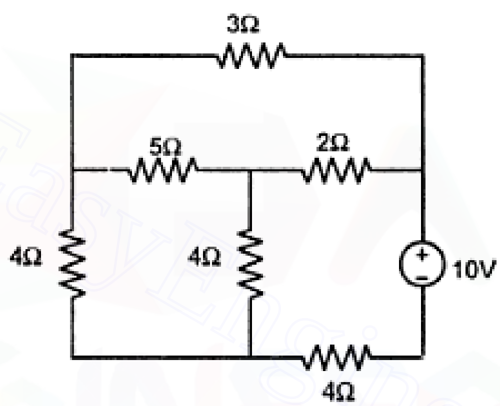


Fig. 2.55

Solution : The various branch currents are shown in the Fig. 2.55 (a).

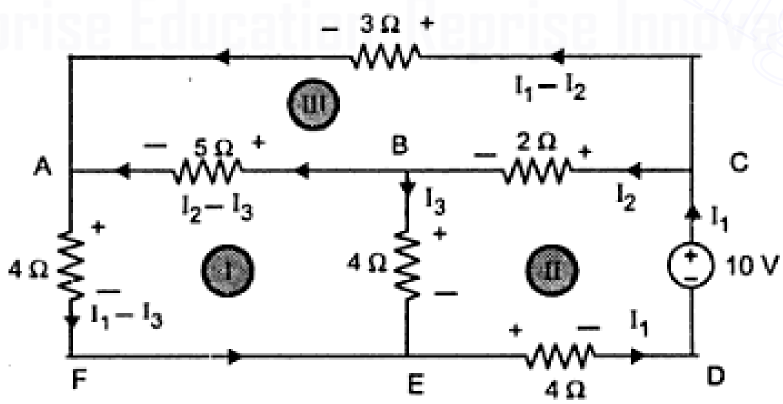


Fig. 2.55 (a)

Consider loop ABEFA,

$$+5(I_2 - I_3) - 4I_3 + 4(I_1 - I_3) = 0 \quad \text{i.e. } 4I_1 + 5I_2 - 13I_3 = 0 \qquad \dots (1)$$

Consider loop BCDEB,

$$+2I_2 - 10 + 4I_1 + 4I_3 = 0 \quad \text{i.e. } 4I_1 + 2I_2 + 4I_3 = 10 \qquad \dots (2)$$

Consider loop ABCA,

$$+5(I_2 - I_3) + 2I_2 - 3(I_1 - I_2) = 0 \quad \text{i.e. } -3I_1 + 10I_2 - 5I_3 = 0 \qquad \text{... (3)}$$

Using Cramer's rule, I_1 = **1.3852 A**

This is the current delivered by the battery.

Example 2.15 : *Find the resistance between (1) B & C and (2) A & C in the network shown in the Fig. 2.56.* **(Dec. - 99, Dec. - 2000)**

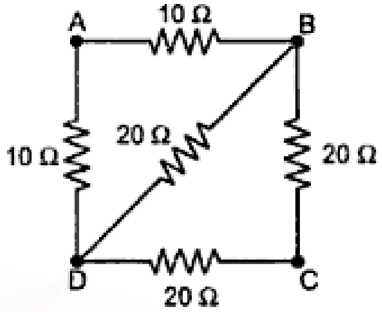


Fig. 2.56

Solution : (i) Between B and C

As looking through B and C, 10 Ω and 10 Ω are in series, as both carry same current.

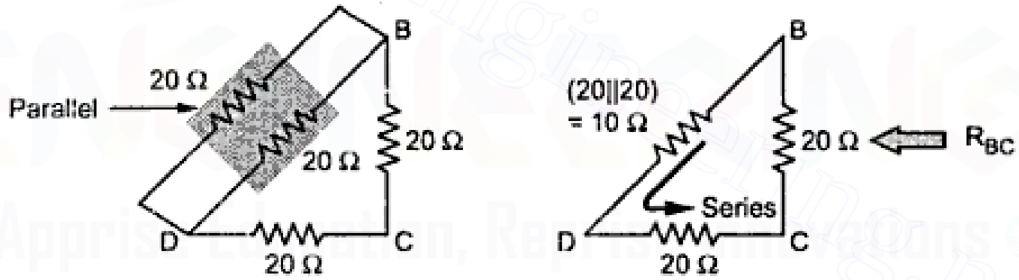


Fig. 2.56 (a) **Fig. 2.56 (b)**

Again, 10 Ω and 20 Ω are in series.

$$R_{BC} = 20 \| 30 = \frac{20 \times 30}{20 + 30}$$

$$= \mathbf{12\ \Omega}$$

(ii) Between A and C

Converting delta BCD to equivalent star,

$$R_{BS} = R_{CS} = R_{DS} = \frac{20 \times 20}{(20 + 20 + 20)}$$

$$= 6.67\ \Omega$$

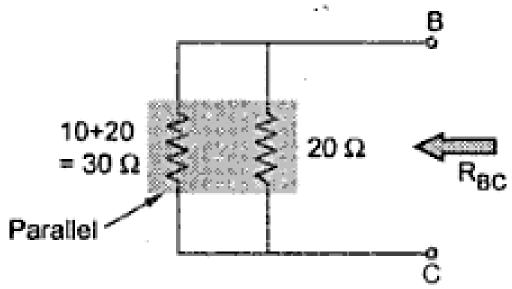


Fig. 2.56 (c)

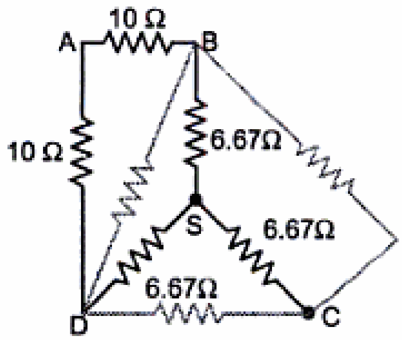


Fig. 2.56 (d)

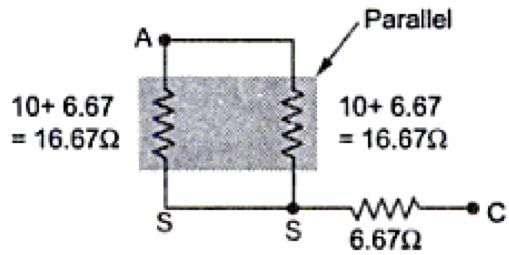


Fig. 2.56 (e)

$\therefore$ $$R_{AC} = 8.33 + 6.67$$

$$= 15\ \Omega$$

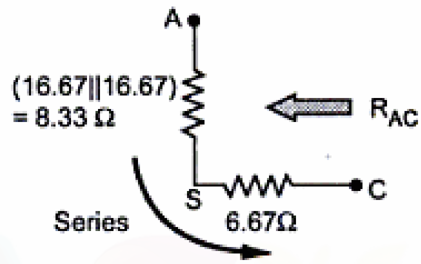


Fig. 2.56 (f)

Example 2.16 : *Find the V_{CE} and V_{AG} for the circuit shown in Fig. 2.57.* **(May-2006)**

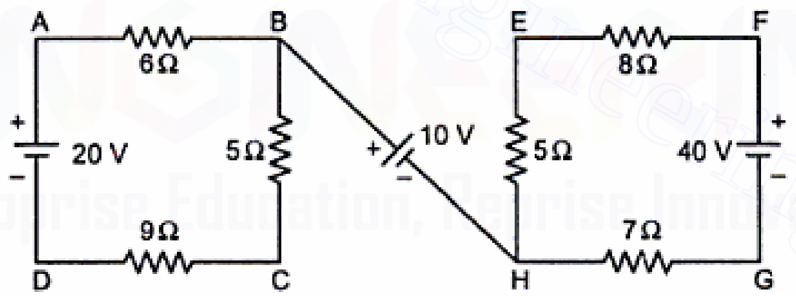


Fig. 2.57

Solution : Assume the two currents as shown in the Fig. 2.57 (a)

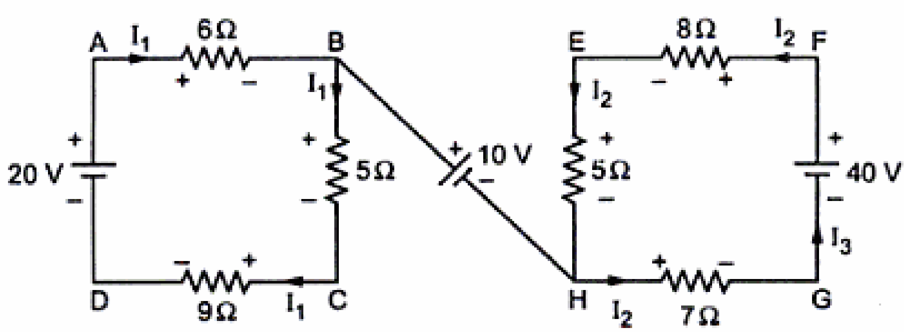


Fig. 2.57 (a)

Applying KVL to the two loops,

$$-6\,I_1 - 5\,I_1 - 9\,I_1 + 20 = 0 \quad \text{and} \quad -8\,I_2 - 5\,I_2 - 7\,I_2 + 40 = 0$$

$$\therefore \qquad I_1 = 1\text{ A and } I_2 = 2\text{ A}$$

i) Trace the path C-E,

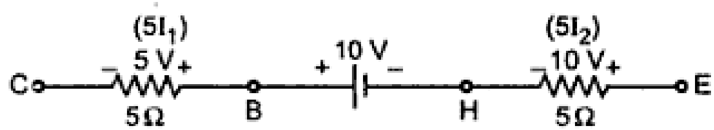


Fig. 2.57 (b)

$$\therefore \qquad V_{CE} = -5\text{ V}$$

$$= \textbf{5 V with C negative}$$

ii) Trace the path A-G,

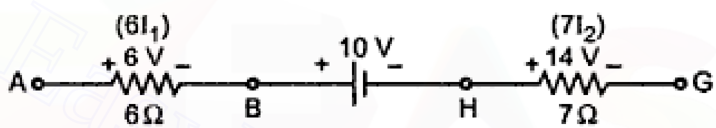


Fig. 2.57 (c)

$$\therefore \qquad V_{AC} = \textbf{30 V with A positive}$$

Example 2.17 : *Calculate the equivalent resistance between the terminals (X) and (Y) for the circuit shown in Fig. 2.58.* **(Dec. - 2007)**

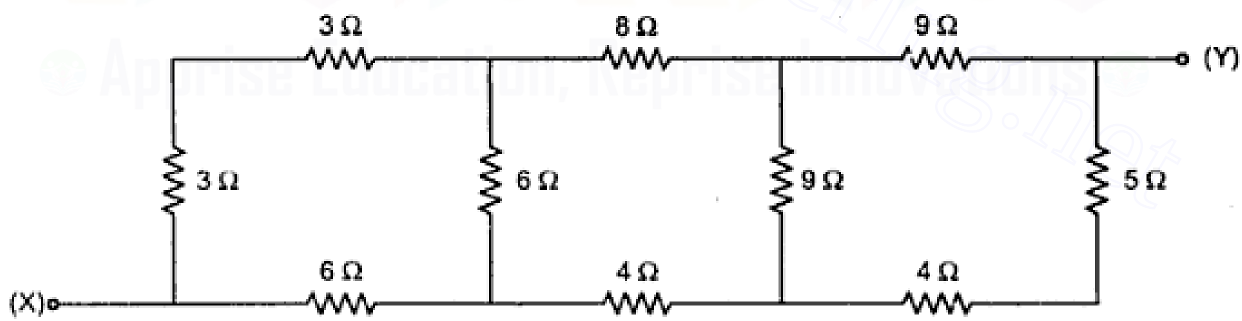


Fig. 2.58

Solution : On the left side 3 Ω and 3 Ω are in series while on the right side 5 Ω and 4 Ω are in series.

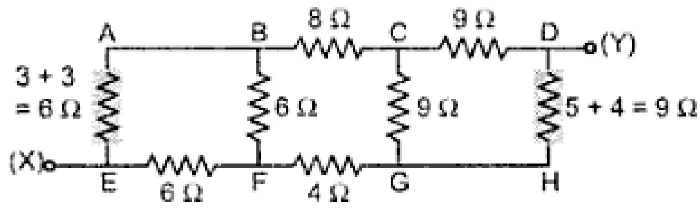


Fig. 2.58 (a)

Convert Δ ABFE to equivalent star.

Convert Δ CDHG to equivalent star.

As all the resistance of Δ ABFE are equal,

all the resistances of equivalent star are also equal, given by,

$$R = \frac{6 \times 6}{6+6+6} = 2\ \Omega$$

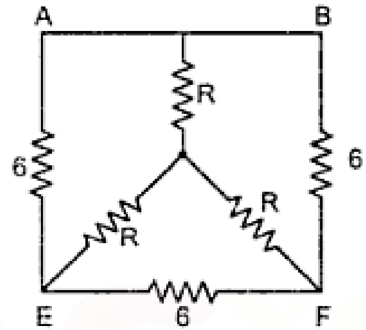


Fig. 2.58 (b)

Similarly in equivalent star of CDGH, each resistance is equal say R′ given by,

$$R' = \frac{9 \times 9}{9+9+9} = 3\ \Omega$$

The circuit reduces as shown in the Fig. 2.58 (d).

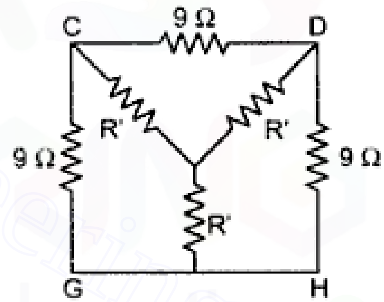


Fig. 2.58 (c)

The resistances 2 Ω, 8 Ω and 3 Ω are in series. The resistances 2 Ω, 4 Ω and 3 Ω are in series.

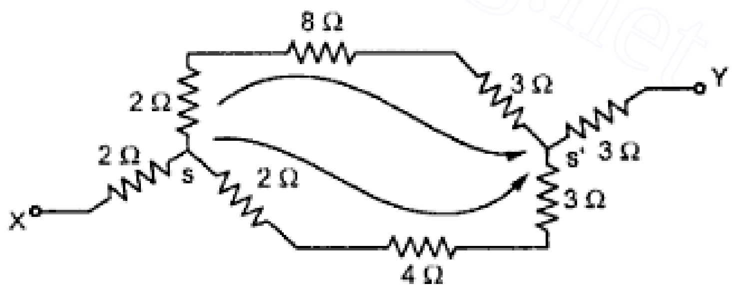


Fig. 2.58 (d)

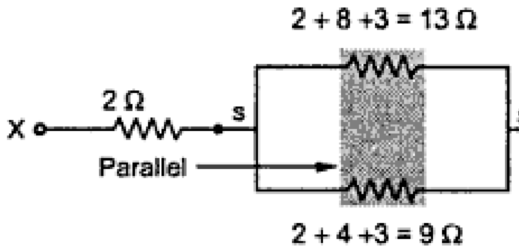


Fig. 2.58 (e)

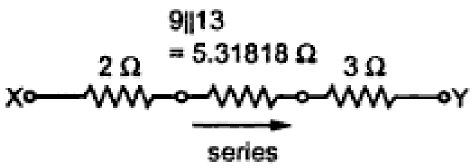


Fig. 2.58 (f)

$$\therefore \qquad R_{XY} = 2 + 5.31818 + 3 = \mathbf{10.31818\ \Omega}$$

Example 2.18 : *Find the current in the branch A - B in the d.c. circuit shown in the Fig. 2.59, using Kirchhoff's laws.* **(Dec. - 2000)**

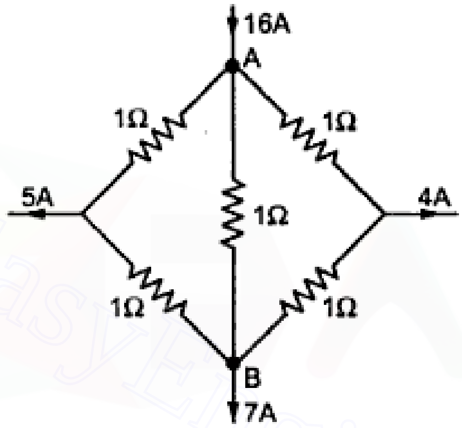


Fig. 2.59

Solution : The various branch currents are shown in the Fig. 2.59 (a).

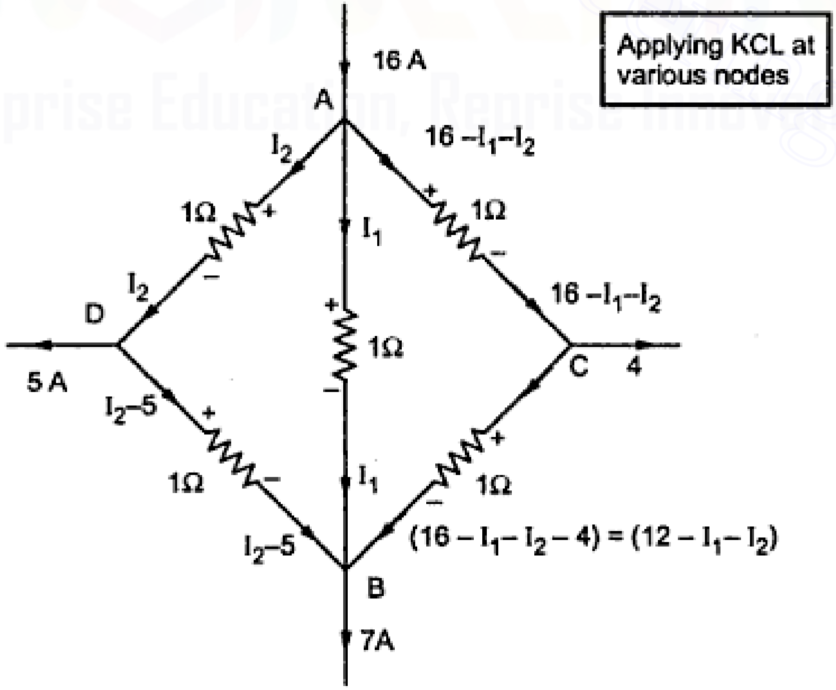


Fig. 2.59 (a)

Applying KVL to loop ADBA

$$-I_2 - (I_2 - 5) + I_1 = 0$$

$$\therefore \quad I_1 - 2I_2 = -5 \qquad \dots (1)$$

Applying KVL to the loop ACBA,

$$-(16 - I_1 - I_2) - (12 - I_1 - I_2) + I_1 = 0$$

$$\therefore \quad -16 + I_1 + I_2 - 12 + I_1 + I_2 + I_1 = 0$$

$$\therefore \quad 3I_1 + 2I_2 = 28 \qquad \dots (2)$$

Add (1) and (2), $4I_1 = 23$

$$\therefore \quad I_1 = 5.75 \text{ A}$$ ... This is the current through branch AB.

Example 2.19 : *A circuit is shown in the Fig. 2.60 (a). Using delta-star analysis, reduce it to the circuit as shown in the Fig. 2.60 (b).*

Find the values of R_a, R_b and R_c in the equivalent form of the circuit.

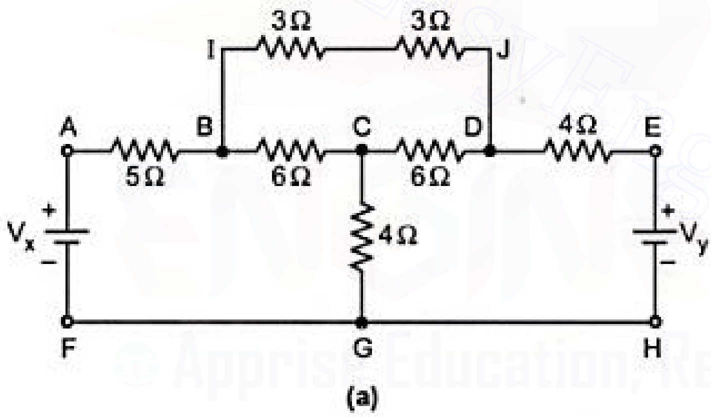


Fig. 2.60

Solution :

> **Key Point** : *The arrangement of resistances is to be analysed and rearranged in equivalent delta, formed of R_a, R_b and R_c.*

The 3 Ω resistances in branch IJ are in series giving 3 + 3 = 6 Ω

The delta is formed of CBIJDC which is to be converted to star.

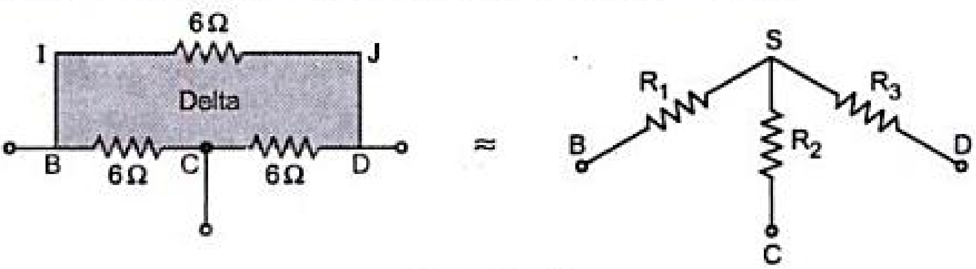


Fig. 2.60 (c)

As each resistance of Δ is 6Ω, in equivalent star,

$$R_1 = R_2 = R_3 = \frac{6\times6}{6+6+6} = 2\Omega$$

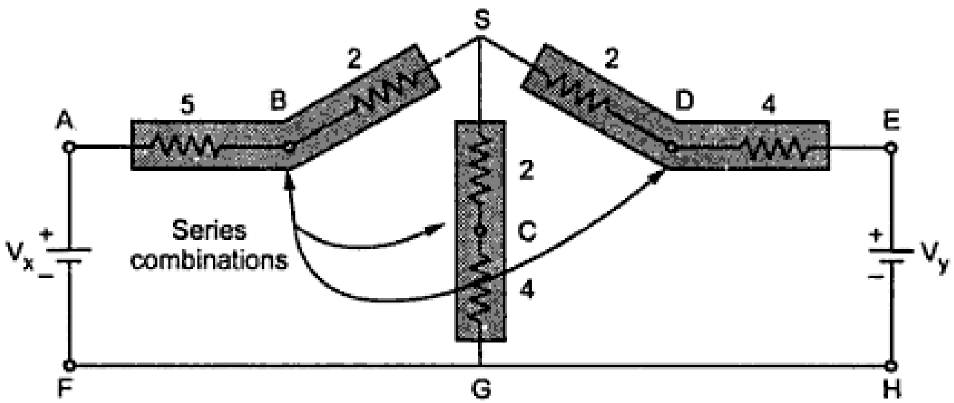


Fig. 2.60 (d)

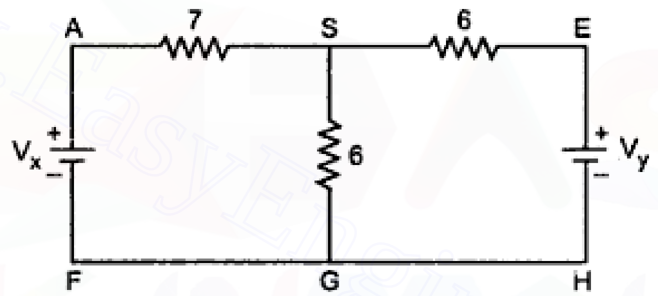


Fig. 2.60 (e)

The Fig. 2.60 (e) represent equivalent star to be converted to delta to get required form,

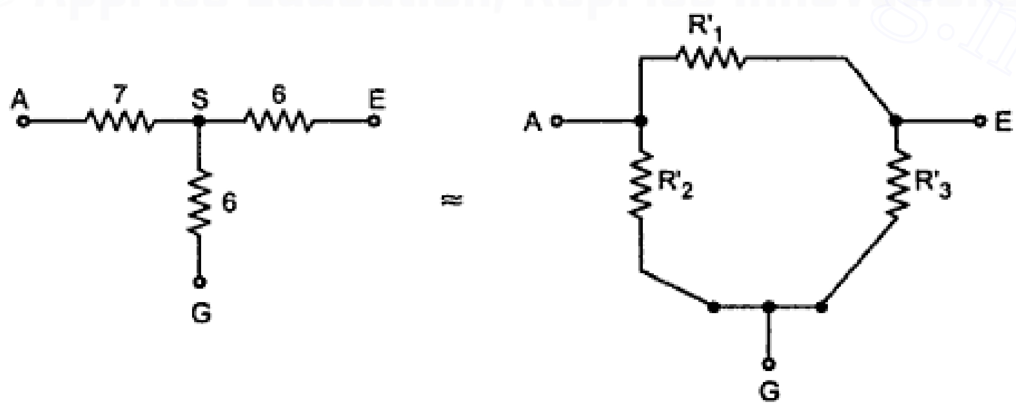


Fig. 2.60 (f)

$$R'_1 = 7+6+\frac{7\times6}{6} = 20\Omega$$

$$R'_2 = 7+6+\frac{7\times 6}{6} = 20\,\Omega$$

$$R'_3 = 6+6+\frac{6\times 6}{7} = 17.1428\,\Omega$$

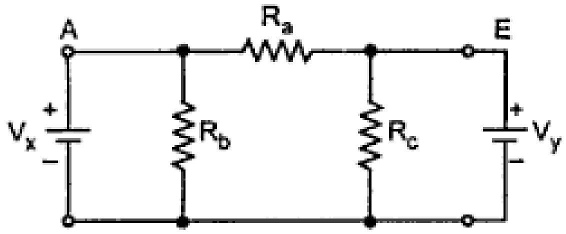


Fig. 2.60 (g)

Thus final form of the circuit which is expected is as shown in the Fig. 2.60 (g).

$$R_a = R'_1 = 20\ \Omega$$

$$R_b = R'_2 = 20\ \Omega$$

$$R_c = R'_3 = 17.1428\ \Omega$$

Example 2.20 : *The circuit is shown in the Fig. 2.61.*

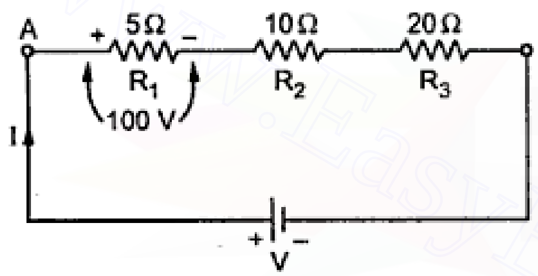


Fig. 2.61

i) Find the equivalent resistance across the supply.

ii) If voltage drop across 5Ω is 100 V, find the supply voltage.

iii) Find the power consumed by each resistance.

Solution : It is series combination of resistances.

i) $R_{eq} = R_1 + R_2 + R_3 = 5 + 10 + 20 = 35\Omega$

ii) The drop across R_1 is 100 V given. The current remains same through R_1, R_2 and R_3.

$\therefore \quad V_1 = \text{drop across } R_1 = I \times R_1 = 100 \text{ V}$

$\therefore \quad I = \frac{100}{R_1} = \frac{100}{5} = 20 \text{ A}$

$\therefore \quad V_2 = \text{drop across } R_2 = I \times R_2 = 20 \times 10 = 200 \text{ V}$

$\therefore \quad V_3 = \text{drop across } R_3 = I \times R_3 = 20 \times 20 = 400 \text{ V}$

$\therefore \quad V = V_1 + V_2 + V_3 = 100 + 200 + 400 = 700 \text{ V}$ ... supply voltage

iii) $P_1 = \text{power consumed by } R_1 = V_1 I \text{ or } I^2R_1 = 2000 \text{ W}$

$P_2 = \text{power consumed by } R_2 = V_2 I \text{ or } I^2R_2 = 4000 \text{ W}$

$P_3 = \text{power consumed by } R_3 = V_3 I \text{ or } I^2R_3 = 8000 \text{ W}$

Example 2.21 : *The four resistances 40 Ω, 32 Ω, 60 Ω and R_4 Ω are connected in parallel across d.c. supply. Current in 40Ω is 3 A while the total current from supply is 25.8 A. Find, i) Supply voltage ii) R_4 iii) Equivalent resistance across supply.*

Solution : The circuit diagram is shown in the Fig. 2.62.

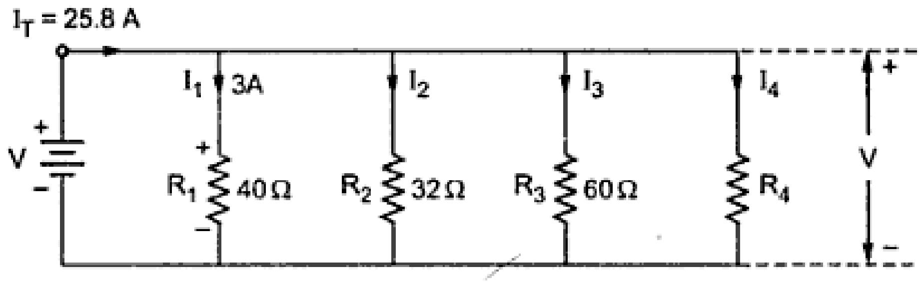


Fig. 2.62

In parallel circuit voltage across each resistance is same equal to supply voltage.

i) Supply voltage $V = I_1R_1 = I_2R_2 = I_3R_3 = I_4R_4$

$\therefore \quad V = I_1R_1 = 3 \times 40 = 120 \text{ V}$

ii) $\quad 120 = I_2 \times 32 = I_3 \times 60 = I_4 \times R_4$

$\therefore \quad I_2 = 3.75 \text{ A}, \quad I_3 = 2 \text{ A}$

But $\quad I_T = I_1 + I_2 + I_3 + I_4$

$\therefore \quad 25.8 = 3 + 3.75 + 2 + I_4$

$\therefore \quad I_4 = 17.05 \text{ A}$

And $\quad I_4 \times R_4 = V$ i.e. $17.05 R_4 = 120$

$\therefore \quad R_4 = 7.0381 \, \Omega$

iii) For parallel circuit, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} = \dfrac{1}{40} + \dfrac{1}{32} + \dfrac{1}{60} + \dfrac{1}{7.0381}$

$\therefore \quad R_{eq} = 4.6511 \, \Omega$

2.17 Superposition Theorem

This theorem is applicable for linear and bilateral networks. Let us see the statement of the theorem.

Statement : *In any multisource complex network consisting of linear bilateral elements, the voltage across or current through any given element of the network is equal to the algebraic sum of the individual voltages or currents, produced independently across or in that element by each source acting independently, when all the remaining sources are replaced by their respective internal resistances.*

Key Point : *If the internal resistances of the sources are unknown then the independent voltage sources must be replaced by short circuit while the independent current sources must be replaced by an open circuit.*

The theorem is also known as Superposition principle. In other words, it can be stated as, the response in any element of linear, bilateral network containing more than one sources is the sum of the responses produced by the sources, each acting independently. The response means the voltage across the element or the current in the element. The superposition theorem does not apply to the power as power is proportional to square of the current, which is not a linear function.

2.17.1 Explanation of Superposition Theorem

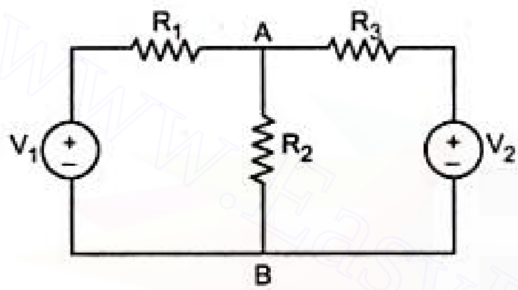


Fig. 2.63

Consider a network, shown in the Fig. 2.63, having two voltage sources V_1 and V_2.

Let us calculate, the current in branch A-B of the network, using superposition theorem.

Step 1) According to Superposition theorem, consider each source independently. Let source V_1 volts is acting independently. At this time, other sources must be replaced by internal impedances.

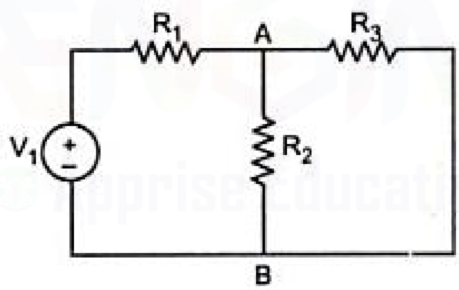


Fig. 2.63 (a)

But as internal impedance of V_2 is not given, the source V_2 must be replaced by short circuit. Hence circuit becomes, as shown in the Fig. 2.63 (a).

Using any of the network reduction techniques discussed earlier, obtain the current through branch A-B i.e. I_{AB} due to source V_1 alone.

Step 2) Now consider source V_2 volts alone, with V_1 replaced by a short circuit, to obtain the current through branch A-B. The corresponding circuit is shown in the Fig. 2.63 (b).

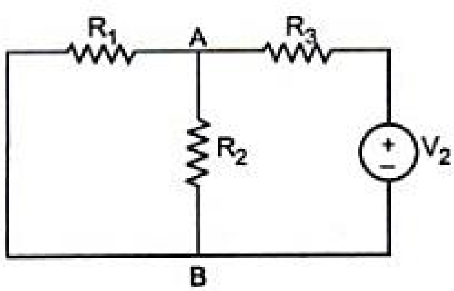


Fig. 2.63 (b)

Obtain I_{AB} due to V_2 alone, by using any of the network reduction techniques discussed earlier.

Step 3) According to the Superposition theorem, the total current through branch A-B is the sum of the currents through branch A-B produced by each source acting independently.

$$\therefore \quad \text{Total } I_{AB} = I_{AB} \text{ due to } V_1 + I_{AB} \text{ due to } V_2$$

2.17.2 Steps to Apply Superposition Theorem

Step 1 : Select a single source acting alone. Short the other voltage sources and open the current sources, if internal resistances are not known. If known, replace them by their internal resistances.

Step 2 : Find the current through or the voltage across the required element, due to the source under consideration, using a suitable network simplification technique.

Step 3 : Repeat the above two steps for all the sources

Step 4 : Add the individual effects produced by individual sources, to obtain the total current in or voltage across the element.

Example 2.22 : *Use the Superposition theorem to calculate the current in branch PQ of the circuit shown in Fig. 2.64.* **(Dec. - 97)**

Solution : In a superposition principle, each source is to be considered independently.

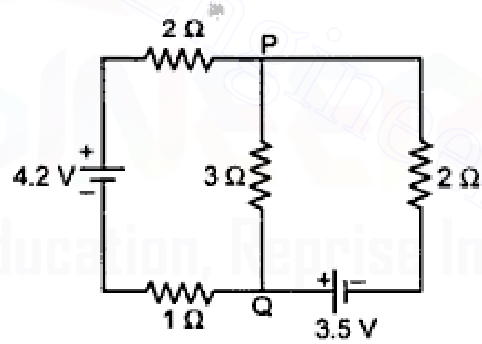


Fig. 2.64

Step 1 : Let us consider 4.2 V, replacing other by short circuit.

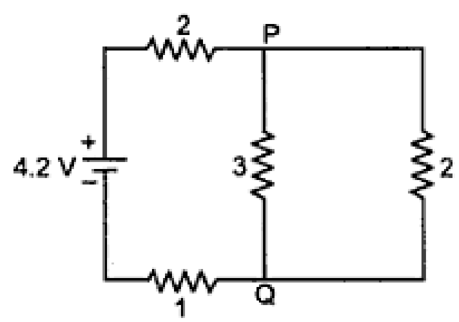


Fig. 2.64 (a)

The resistances 3 Ω and 2 Ω are in parallel

$$\therefore \quad 3 \parallel 2 = \frac{3\times 2}{3+2} = 1.2\ \Omega$$

$$\therefore \quad I = \frac{4.2}{(2+1.2+1)} = 1\ \text{A}$$

Now we want I_{PQ}, hence using current division formula,

$$I'_{PQ} = 1\ \text{A} \times \frac{2}{2+3} = 0.4\ \text{A} \downarrow$$

... due to 4.2 V alone

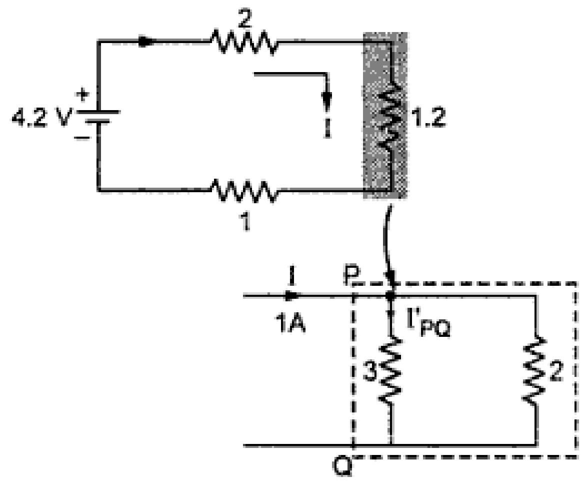


Fig. 2.64 (b)

Step 2 :

Now consider 3.5 V source, replacing other by a short circuit.

The resistances 2 and 1 are in series hence

$$2 \text{ series } 1 = 2 + 1 = 3\ \Omega$$

The resistances 3 and 3 are in parallel.

$$\therefore \quad 3 \parallel 3 = \frac{3\times 3}{3+3} = 1.5\ \Omega$$

$$I = \frac{3.5}{(2+1.5)} = 1\ \text{A}$$

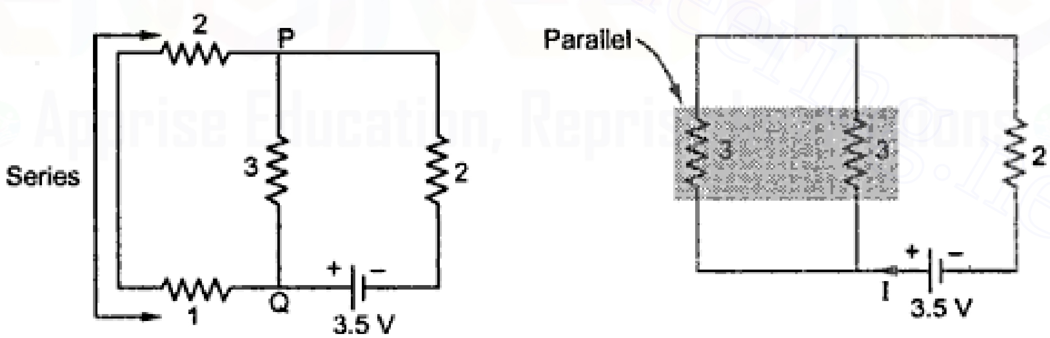


Fig. 2.64 (c)

Fig. 2.64 (d)

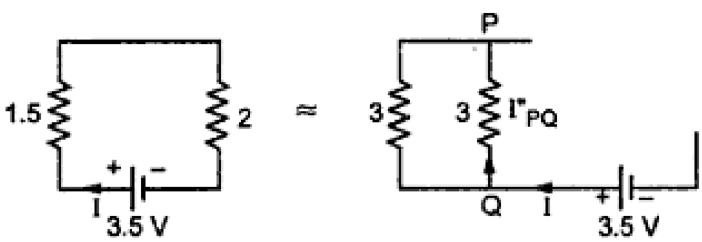


Fig. 2.64 (e)

But we want I_{PQ}, hence using current division formula we get,

$$I''_{PQ} = I \times \frac{3}{(3+3)}$$

$$= 1 \times \frac{3}{(3+3)}$$

$$= 0.5 \uparrow \text{A}$$

... due to 3.2 V alone

Hence total current through PQ branch

$= 0.4 \text{ A} \downarrow + 0.5 \text{ A} \uparrow = \mathbf{0.1 \text{ A}} \uparrow$

Example 2.23 : *Calculate current through the 15 Ω resistance using Kirchhoff's law and verify your answer using Superposition theorem as well. The circuit is shown in the Fig. 2.65.* **(Dec. - 98)**

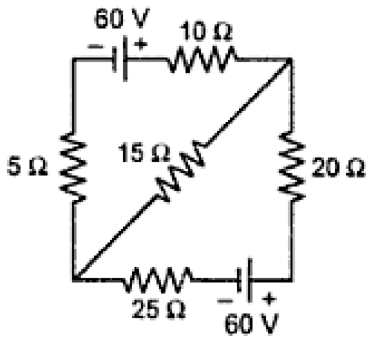


Fig. 2.65

Solution : To use **Kirchhoff's law**, let us indicate various currents as shown in the Fig. 2.65 (a).

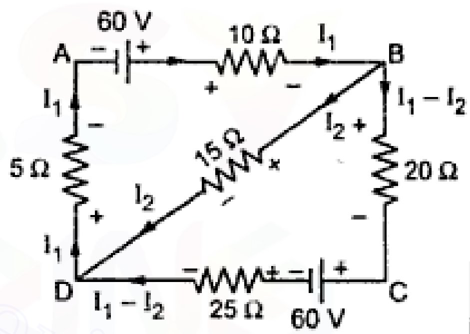


Fig. 2.65 (a)

Consider loop ABDA,

$$+ 60 - 10\, I_1 - 15\, I_2 - 5\, I_1 = 0$$

$$\therefore \quad 15\, I_1 + 15\, I_2 = 60$$

$$\therefore \quad I_1 + I_2 = 4 \qquad \dots (1)$$

Consider loop BDCB,

$$- 15\, I_2 + 25\,(I_1 - I_2) + 60 + 20\,(I_1 - I_2) = 0$$

$$- 45\, I_1 + 60\, I_2 = 60$$

$$\therefore \quad - 3\, I_1 + 4 I_2 = 4 \qquad \dots (2)$$

Add [3 × equation (1)] to (2),

$$- 3\, I_1 + 4 I_2 = 4$$

$$+ 3\, I_1 + 3 I_2 = 12$$

$$\therefore \quad 7\, I_2 = 16$$

$$\therefore \quad I_2 = 2.2857 \text{ A}$$

So current through 15 Ω resistance is **2.2857 A** from B to D.

Now let us use **Superposition Theorem,** consider upper 60 V battery alone the lower battery is replaced by short circuit as shown in the Fig. 2.65 (b).

The circuit further reduces to,

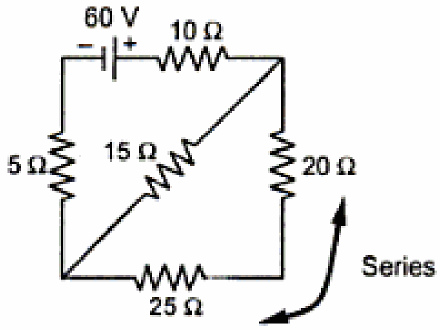


Fig. 2.65 (b)

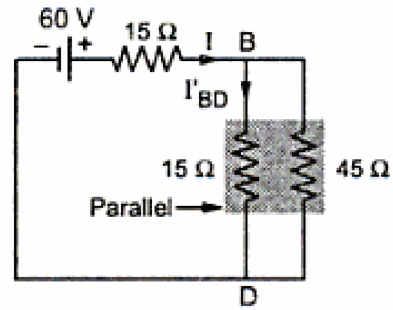


Fig. 2.65 (c)

Hence the total current is,

$$I = \frac{60}{15+(15||45)} = \frac{60}{15+11.25} = 2.2857 \text{ A} \qquad \text{... Current division rule}$$

$$\therefore \qquad I'_{BD} = I \times \frac{45}{(45+15)} = 2.2857 \times \frac{45}{60}$$

$$= \mathbf{1.7142\ A} \downarrow \text{ due to one 60 V battery.}$$

Consider other 60 V battery now, hence circuit reduces as,

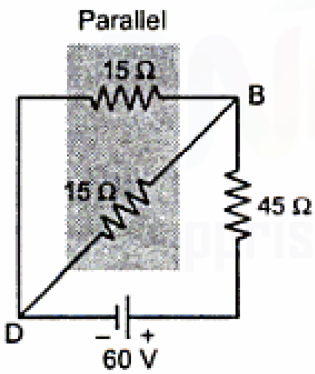


Fig. 2.65 (d)

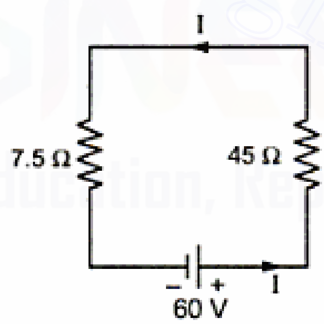


Fig. 2.65 (e)

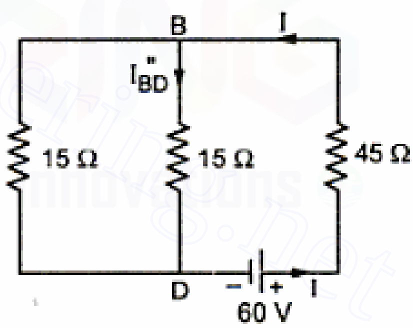


Fig. 2.65 (f)

$$\therefore \quad \text{Total current} \quad I = \frac{60}{45+(15||15)} = \frac{60}{45+7.5} = 1.1428 \text{ A}$$

$$\therefore \qquad I''_{BD} = I \times \frac{15}{15+15} = 1.1428 \times \frac{1}{2} = 0.5714 \text{ A} \downarrow$$

$$\therefore \qquad I''_{BD} = \mathbf{0.5714\ A} \downarrow \text{ due to other 60 V battery.}$$

Hence according to Superposition Theorem,

$$I_{15\Omega} = 1.7142 + 0.5714 = \mathbf{2.2857\ A} \downarrow$$

This is same as calculated by Kirchhoff's laws.

Example 2.24 : *Use Superposition theorem to find the current through the 20 ohm resistance shown in the Fig. 2.66.* **(May - 99)**

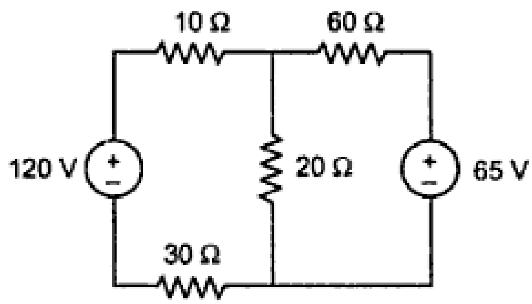


Fig. 2.66

Solution : Step 1 : Consider 120 V battery alone, shorting 65 V battery.

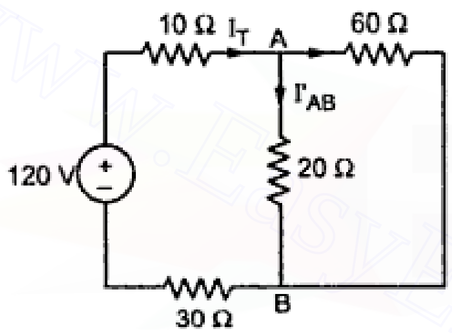


Fig. 2.66 (a)

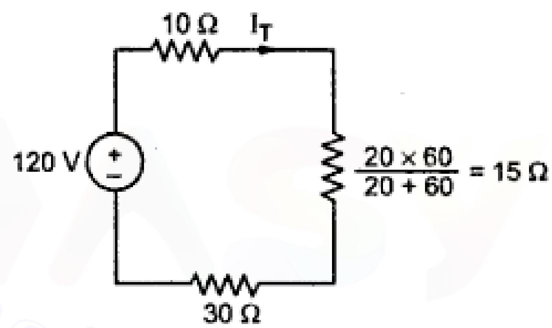


Fig. 2.66 (b)

$$\therefore \quad I_T = \frac{120}{10+15+30} = 2.1818 \text{ A}$$

$$\therefore \quad I'_{AB} = I_T \times \frac{60}{20+60} \quad \text{... Current division rule}$$

$$= \mathbf{1.6363\ A} \text{ due to 120 V battery} \downarrow$$

Step 2 : Consider 65 V battery alone, shorting 120 V battery.

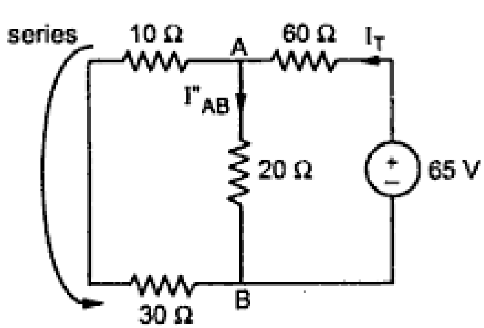


Fig. 2.66 (c)

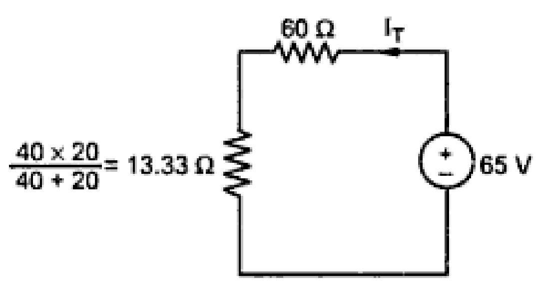


Fig. 2.66 (d)

$$I_T = \frac{65}{73.33} = 0.8863 \text{ A}$$

$$\therefore \quad I''_{AB} = I_T \times \frac{40}{20+40} = \textbf{0.5909 A due to 65 V battery} \downarrow$$

$\therefore$ Total current through 20 Ω resistance, according to Superposition theorem is,

$$I_{20\Omega} = 1.6363 + 0.5909 \text{ both in same direction}$$

$$= \textbf{2.2272 A} \downarrow$$

Example 2.25 : *In the circuit shown, find current through branch AB by Superposition theorem.* **(Dec. - 99)**

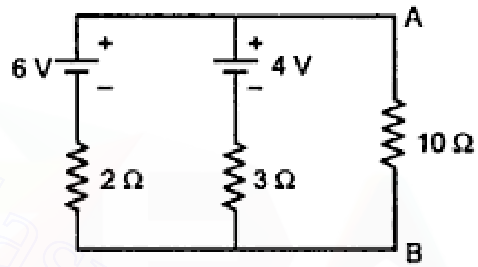


Fig. 2.67

Solution : Step 1 : Consider 6 V source alone

Now, resistances 10 Ω and 3 Ω are in parallel. Hence total current, I is

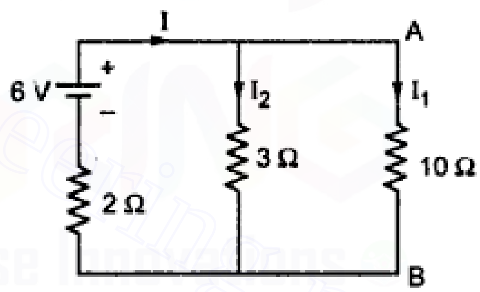


Fig. 2.67 (a)

$$I = \frac{6}{2+(3||10)} = \frac{6}{2+\left(\frac{3\times10}{3+10}\right)} = \frac{6}{2+2.307}$$

$$\therefore \quad I = 1.3928 \text{ A}$$

As per current distribution in parallel branches,

$$I_1 = I \times \frac{3}{3+10} = \frac{1.3928\times3}{13} = \textbf{0.3214 A} \downarrow \quad \text{... (6 V alone)}$$

This is I_{AB} due to 6 V battery alone.

Step 2 : Consider 4 V battery alone.

Now, the resistances 2 Ω and 10 Ω are in parallel. Hence, current I can be obtained as,

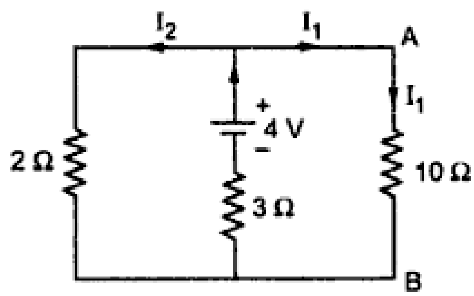


Fig. 2.67 (b)

$$I = \frac{4}{3+(21||0)} = \frac{4}{3+\left(\frac{2\times10}{2+10}\right)}$$

$$= \frac{4}{3+1.67} = 0.8571 \text{ A}$$

According to current distribution in parallel branches,

$$I_1 = I \times \frac{2}{(2+10)} = 0.8571 \times \frac{2}{12}$$

$$= \mathbf{0.1428\ A} \downarrow \quad \text{... (4 V alone)}$$

This is I_{AB} due to 4 V battery alone.

According to Superposition theorem,

$$\text{Total } I_{AB} = 0.3214\ \text{A} \downarrow + 0.1428\ \text{A} \downarrow = \mathbf{0.4642\ A} \downarrow \quad \text{...Total current}$$

2.18 Thevenin's Theorem

Let us see the statement of the theorem.

Statement : *Any combination of linear bilateral circuit elements and active sources, regardless of the connection or complexity, connected to a given load R_L, may be replaced by a simple two terminal network consisting of a single voltage source of V_{TH} volts and a single resistance R_{eq} in series with the voltage source, across the two terminals of the load R_L. The voltage V_{TH} is the open circuit voltage measured at the two terminals of interest, with load resistance R_L removed. This voltage is also called* ***Thevenin's equivalent voltage****. The R_{eq} is the* ***equivalent resistance*** *of the given network as viewed through the terminals where R_L is connected, but with R_L removed and all the active sources are replaced by their internal resistances.*

> **Key Point** : *If the internal resistances are not known then independent voltage sources are to be replaced by the short circuit while the independent current sources must be replaced by the open circuit.*

2.18.1 Explanation of Thevenin's Theorem

The concept of Thevenin's equivalent across the terminals of interest can be explained by considering the circuit shown in the Fig. 2.68 (a). The terminals A-B are the terminals of interest across which R_L is connected. Then Thevenin's equivalent across the load terminals A-B can be obtained as shown in the Fig. 2.68 (b).

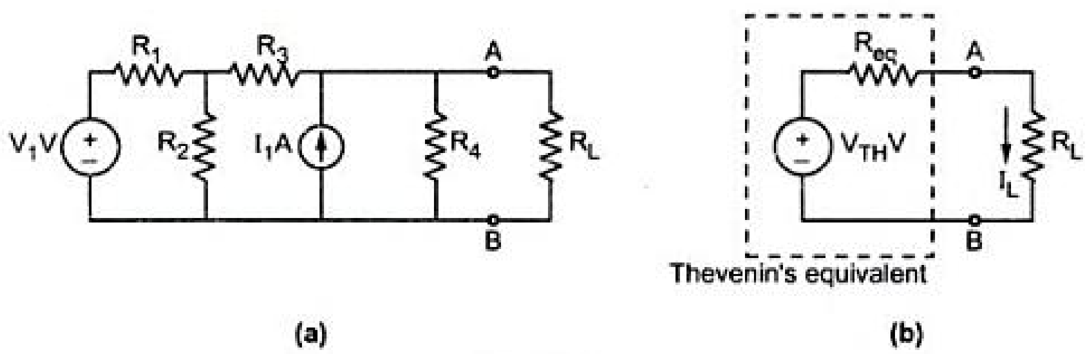
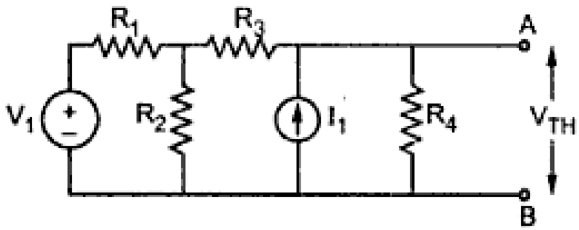
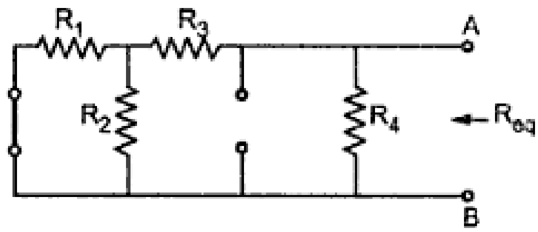


Fig. 2.68

The voltage V_{TH} is obtained across the terminals A-B with R_L removed. Hence V_{TH} is also called open circuit Thevenin's voltage. The circuit to be used to calculate V_{TH} is shown in the Fig. 2.69 (a), for the network considered above. While R_{eq} is the equivalent resistance obtained as viewed through the terminals A-B with R_L removed, voltage sources replaced by short circuit and current sources by open circuit. This is shown in the Fig. 2.69 (b).



(a) Calculation of V_{TH}



(b) Calculation of R_{eq}

Fig. 2.69

While obtaining V_{TH}, any of the network simplification techniques can be used.

When the circuit is replaced by Thevenin's equivalent across the load resistance, then the load current can be obtained as,

$$\boxed{I_L = \frac{V_{TH}}{R_L + R_{eq}}}$$

By using this theorem, current through any branch of the circuit can be obtained, treating that branch resistance as the load resistance and obtaining Thevenin's equivalent across the two terminals of that branch.

2.18.2 Steps to Apply Thevenin's Theorem

Step 1 : Remove the branch resistance through which current is to be calculated.

Step 2 : Calculate the voltage across these open circuited terminals, by using any of the network simplification techniques. This is V_{TH}.

Step 3 : Calculate R_{eq} as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all independent sources by their internal resitances. If the internal reistances are not known then replace independent voltage sources by short circuits and independent current sources by open circuits.

Step 4 : Draw the Thevenin's equivalent showing source V_{TH}, with the resistance R_{eq} in series with it, across the terminals of branch of interest.

Step 5 : Reconnect the branch resistance. Let it be R_L. The required current through the branch is given by,

$$I = \frac{V_{TH}}{R_{eq} + R_L}$$

2.18.3 Limitations of Thevenin's Theorem

The limitations of Thevenin's theorem are,

1. Not applicable to the circuits consisting of nonlinear elements.
2. Not applicable to unilateral networks.
3. There should not be magnetic coupling between the load and circuit to be replaced by Thevenin's theorem.
4. In the load side, there should not be controlled sources, controlled from some other part of the circuit.

Example 2.26 : *For the circuit shown in the Fig. 2.70 find the Thevenin's equivalent across 16 Ω resistance and hence find the current through it.* **(May - 84)**

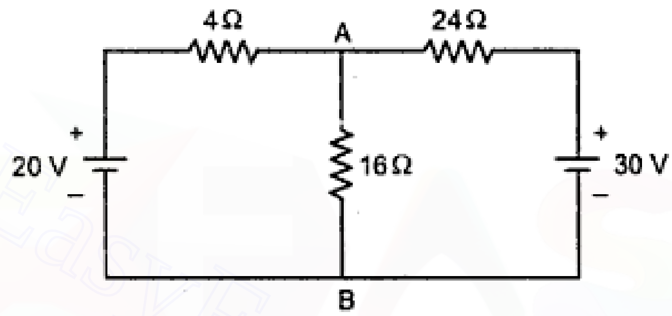


Fig. 2.70

Solution : **Step 1 :** Remove 16 Ω resistance.

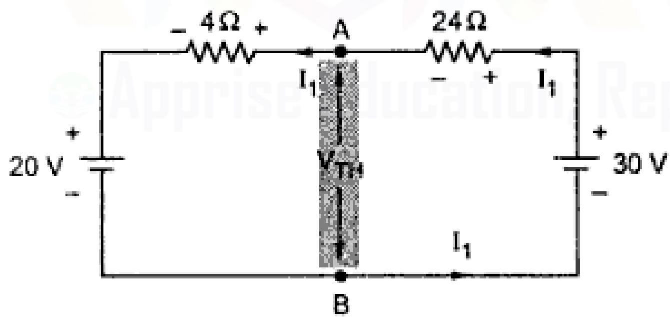


Fig. 2.70 (a)

Step 2 : Find open circuit voltage V_{TH}.

$\therefore - 24\ I_1 - 4I_1 - 20 + 30 = 0$

$\therefore\ 28\ I_1 = 10$

$\therefore\quad I_1 = \dfrac{10}{28}\ A$

$\therefore$ Drop across 4 Ω is $= \dfrac{10}{28} \times 4$

$= 1.4285\ V$

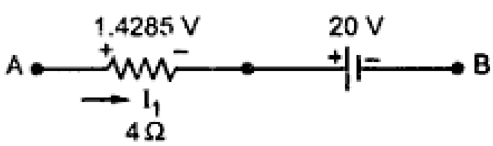


Fig. 2.70 (b)

Trace the path from A to B and arrange the voltage drops as shown in the Fig. 2.70 (b).

$\therefore\ V_{AB} = V_{TH} = 20 + 1.4285$

$=$ **21.4285 V with A positive**

Step 3 : Calculate R_{eq} shorting both the voltage sources.

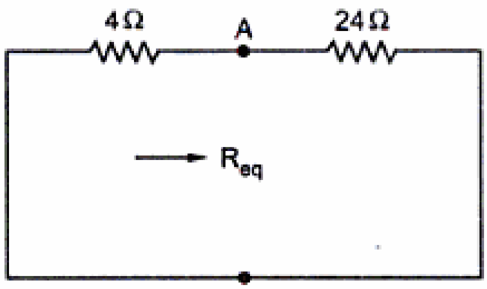


Fig. 2.70 (c)

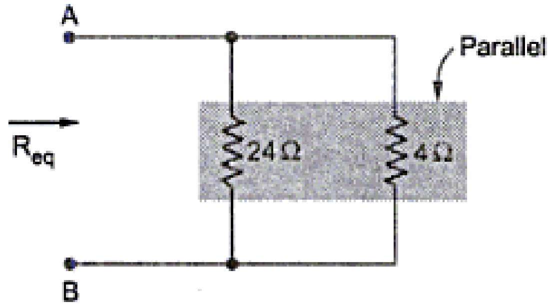


Fig. 2.70 (d)

$$\therefore \qquad R_{eq} = R_{AB} = 24 \parallel 4 = 3.4285\ \Omega$$

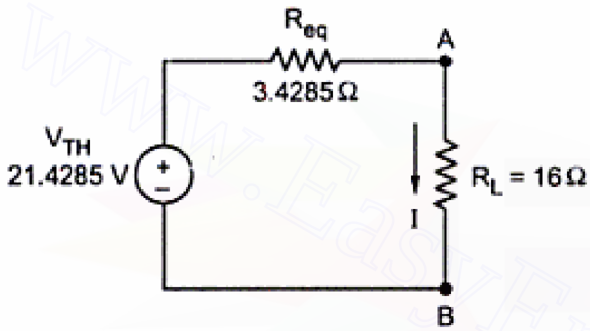


Fig. 2.70 (e)

Step 4 : Thevenin's equivalent is shown in the Fig. 2.70 (e).

Step 5 : Hence current I is,

$$I = \frac{V_{TH}}{R_{eq} + R_L} = \frac{21.4285}{3.4285 + 16}$$

$$= 1.1029\ \text{A} \downarrow$$

Example 2.27 : *Find the current I_2, in Fig. 2.71, by application of Thevenin's theorem.*

(Dec-2003)

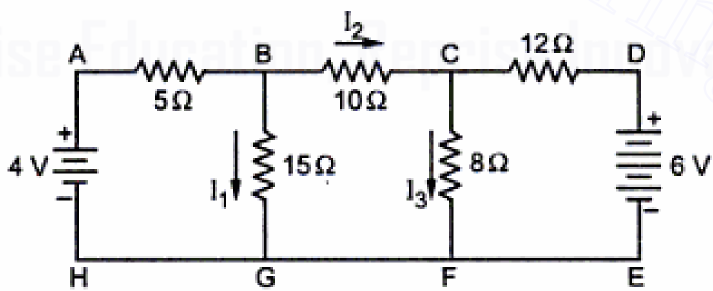


Fig. 2.71

Solution : Thevenin's theorem

Step 1 : Remove the branch of 10 Ω through which current is required.

Step 2 : Find open circuit voltage V_{TH}.

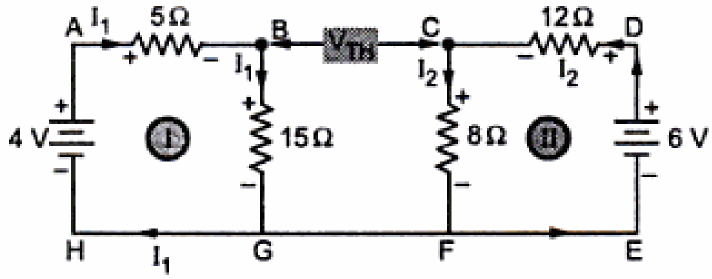


Fig. 2.71 (a)

Applying KVL to the two loops,

$$-5\,I_1 - 15\,I_1 + 4 = 0 \quad \text{i.e.} \quad I_1 = \frac{4}{20} = 0.2\text{ A}$$

$$\therefore \quad \text{Drop } V_{BG} = 15 \times 0.2 = 3\text{ V}$$

$$-12\,I_2 - 8\,I_2 + 6 = 0 \quad \text{i.e.} \quad I_2 = \frac{6}{20} = 0.3\text{ A}$$

$$\therefore \quad \text{Drop } V_{CF} = 8 \times 0.3 = 2.4\text{ V}$$

Trace path from B to C as shown in the Fig. 2.71 (b), showing drops with proper polarities.

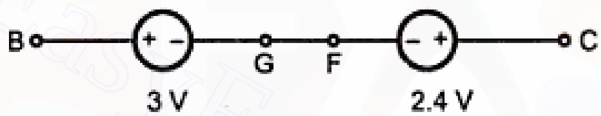


Fig. 2.71 (b)

Both drops are in opposite direction.

$$\therefore \quad V_{TH} = V_{BC} = 3 - 2.4 = 0.6\text{ V with B positive}$$

Step 3 : Find $R_{eq} = R_{BC}$ with voltage sources replaced by short circuits.

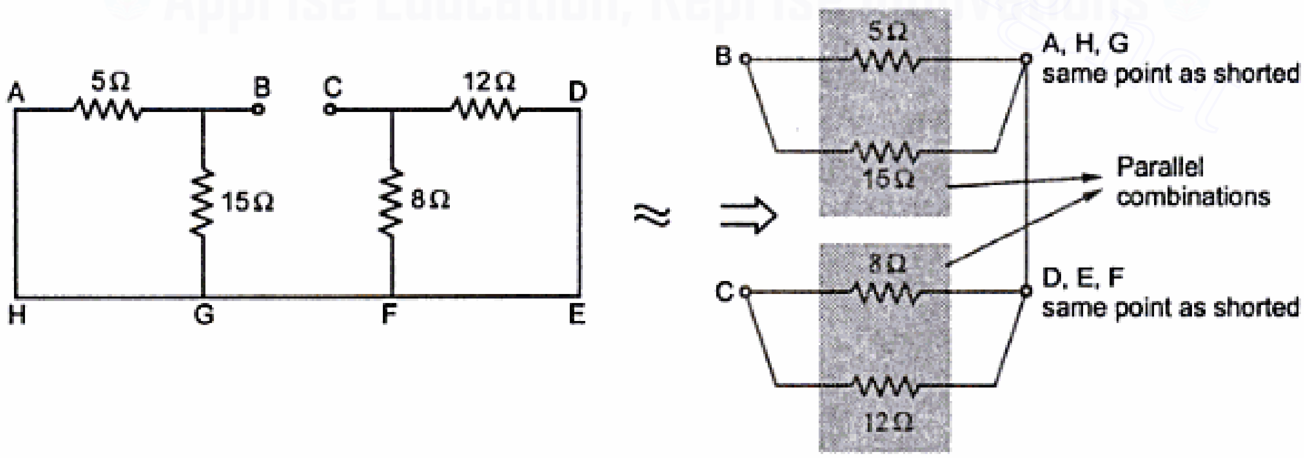


Fig. 2.71 (c)

$$\therefore \quad R_{BC} = (5 \,||\, 15) + (8 \,||\, 12) = 8.55\text{ W} = R_{eq}$$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.71 (d).

Step 5 : Hence current I_2 is,

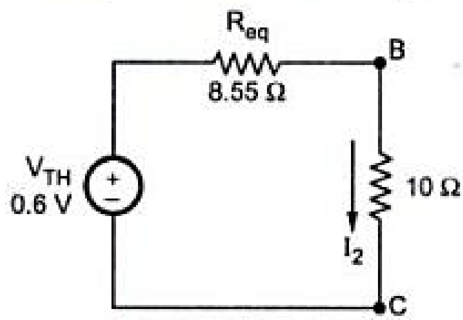


Fig. 2.71 (d)

$$I_2 = \frac{V_{TH}}{R_{eq}+10} = \frac{0.6}{8.55+10}$$

$$= 0.032345 \text{ A}$$

$$= 32.345 \text{ mA}$$

2.19 Norton's Theorem

The Norton's theorem can be stated as below,

Statement : *Any combination of linear bilateral circuit elements and active sources, regardless of the connection or complexity, connected to a given load R_L, can be replaced by a simple two terminal network, consisting of a single current source of I_N amperes and a single impedance R_{eq} in parallel with it, across the two terminals of the load R_L. The I_N is the short circuit current flowing through the short circuited path, replaced instead of R_L. It is also called* **Norton's current**. *The R_{eq} is the equivalent impedance of the given network as viewed through the load terminals, with R_L removed and all the active sources are replaced by their internal impedances. If the internal impedances are unknown then the independent voltage sources must be replaced by short circuit while the independent current sources must be replaced by open circuit, while calculating R_{eq}.*

> **Key Point :** *Infact the calculation of R_{eq} and its value remains same, whether the theorem applied to the network is Thevenin or Norton, as long as terminals of interest remain same.*

2.19.1 Explanation of Norton's Theorem

Consider the network shown in the Fig. 2.72 (a). The terminals A-B are the load terminals where R_L is connected. According to the Norton's theorem, the network can be replaced by a current source I_N with equivalent resistance R_{eq} parallel with it, across the load terminals, as shown in the Fig. 2.72 (b).

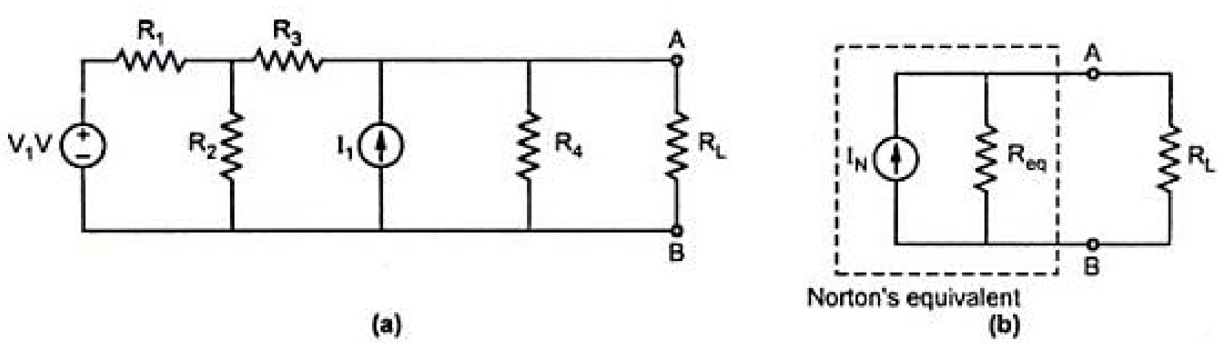


Fig. 2.72

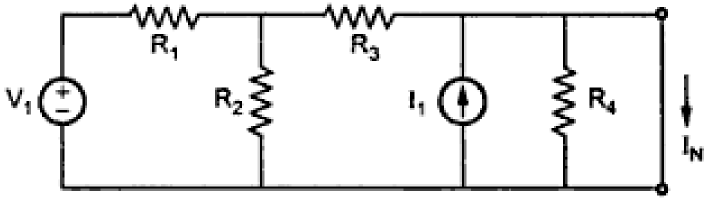


Fig. 2.73 (a)

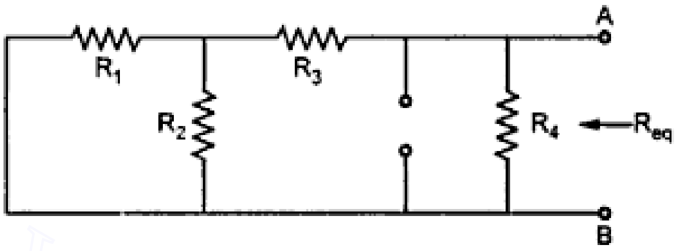


Fig. 2.73 (b)

For obtaining the current I_N, short the load terminals AB as shown in the Fig. 2.73 (a). Then find current I_N by using any of the network simplification techniques discussed earlier. This is Norton's current. While to calculate R_{eq} use same procedure as discussed earlier for Thevenin's theorem. For the convenience of reader circuit for calculation of R_{eq} is shown in the Fig. 2.73 (b).

This theorem is called **dual of the Thevenin's theorem.** This is because, if the Thevenin's equivalent voltage source is converted to equivalent current source using source transformation, we get the Norton's equivalent. This is shown in the Fig. 2.73 (c).

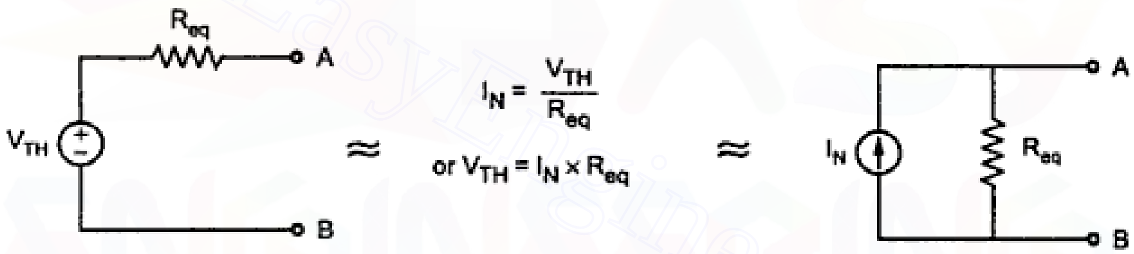


Fig. 2.73 (c)

2.19.2 Steps to Apply Norton's Theorem

Step 1 : Short the branch through which the current is to be calculated.

Step 2 : Obtain the current through this short circuited branch, using any of the network simplification techniques. This current is Norton's current I_N.

Step 3 : Calculate the equivalent resistance R_{eq}, as viewed through the terminals of interest, by removing the branch resistance and making all the independent sources inactive.

Step 4 : Draw the Norton's equivalent across the terminals of interest, showing a current source I_N with the resistance R_{eq} parallel with it.

Step 5 : Reconnect the branch resistance. Let it be R_L. Then using current division in parallel circuit of two resistances, current through the branch of interest can be obtained as,

$$I = I_N \times \frac{R_{eq}}{R_{eq}+R_L}$$

Example 2.28 : *Using Thevenin's theorem determine the current flowing through 2 Ω resistance in the network shown in Fig. 2.74.* **(Dec.-2005)**

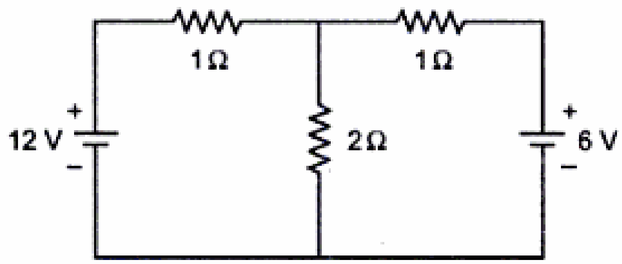


Fig. 2.74

Verify the answer using Norton's theorem.

Sol. : Thevenin's theorem :

Step 1 : Remove 2 Ω resistance.

Step 2 : Find open circuit voltage V_{TH}.

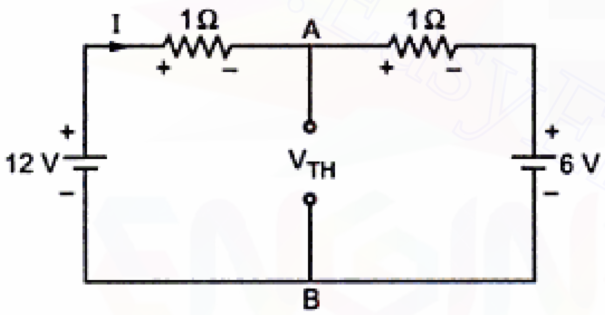


Fig. 2.74 (a)

Applying KVL to the loop,

$$-I - I - 6 + 12 = 0$$

$$\therefore \quad 2I = 6 \text{ i.e. } I = 3 \text{ A}$$

Drop across 1 Ω = 3×1 = 3 V

Tracing path from A to B through 12 V source as shown in the Fig. 2.75 (b).

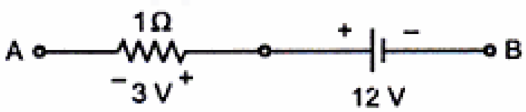


Fig. 2.74 (b)

$$\therefore \quad V_{AB} = V_{TH}$$

$$= 12 - 3 = 9 \text{ V} \quad \text{with A positive}$$

Step 3 : Calculate R_{eq}, shorting voltage sources.

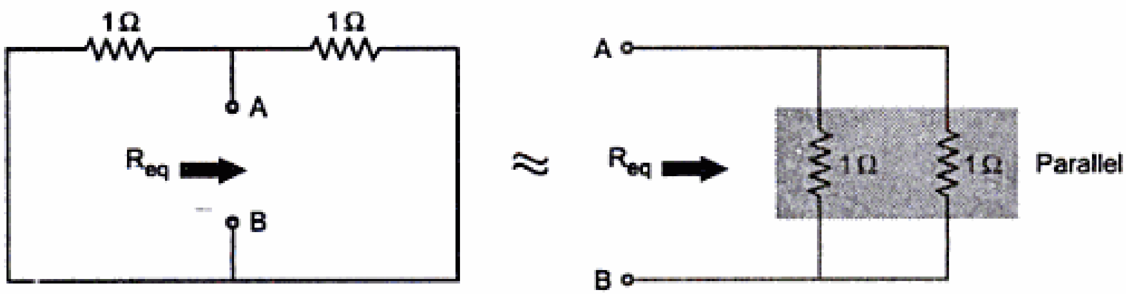


Fig. 2.74 (c)

$\therefore \quad R_{eq} = 1 \,||\, 1 = 0.5\ \Omega$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.74 (d).

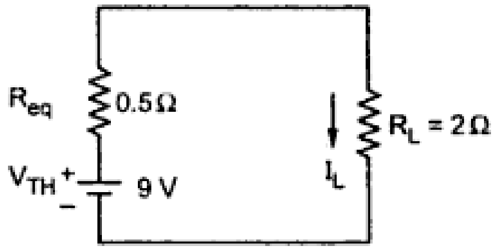


Fig. 2.74 (d)

Step 5 : Current through 2 Ω is,

$$I_L = \frac{V_{TH}}{R_L + R_{eq}} = 3.6\ \text{A} \downarrow$$

Norton's Theorem :

Step 1 : Short the branch of 2 Ω.

Step 2 : Calculate the short circuit current I_N.

Apply KVL to the two loops,

$$-I_1 + 12 = 0 \quad \text{i.e. } I_1 = 12\ \text{A} \quad \text{... Loop I}$$

$$-(I_1 - I_N) - 6 = 0 \quad \text{... Loop II}$$

$$\therefore \quad -I_1 + I_N - 6 = 0$$

$$\therefore \quad I_N = 6 + I_1 = 18\ \text{A}$$

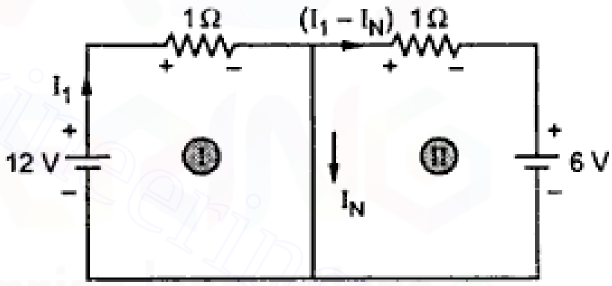


Fig. 2.75 (a)

Step 3 : Calculate R_{eq} shorting voltage sources. This is same as calculated above.

$$\therefore \quad R_{eq} = 0.5\ \Omega$$

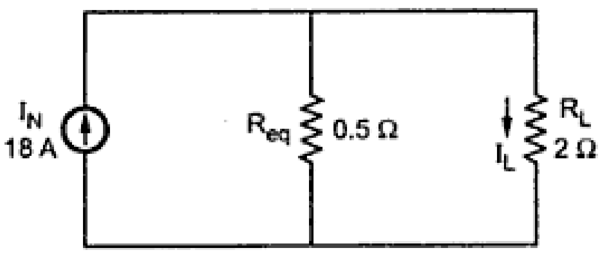


Fig. 2.75 (b)

Step 4 : Norton's equivalent is shown in the Fig. 2.75 (b).

Step 5 : Current through 2 Ω is,

$$I_L = I_N \times \frac{R_{eq}}{R_L + R_{eq}} = \frac{18 \times 0.5}{2.5} = 3.6\ \text{A} \downarrow$$

Example 2.29 : *Replace the given network by Norton's equivalent across the terminals A-B.*

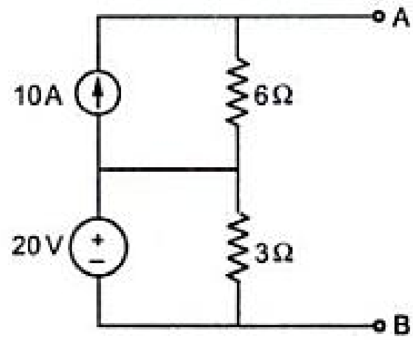


Fig. 2.76

Solution : Step 1 : Short the branch AB.

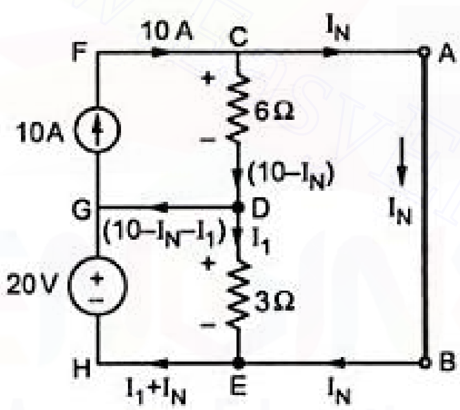


Fig. 2.76 (a)

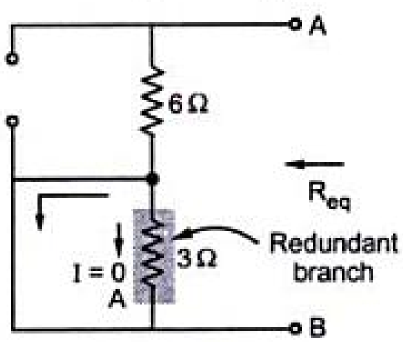


Fig. 2.76 (b)

Step 2 : Calculate the short circuit current using Kirchhoff's laws. As there is current source, **apply KVL to these loops only which do not include current source.** The current source value is considered, for current distribution using KCL.

Loop CABEC,

$$+3I_1 + 6(10 - I_N) = 0$$

$$\therefore 3I_1 - 6\, I_N = -60 \quad \text{... (1)}$$

Loop GDEHG,

$$-3I_1 + 20 = 0$$

$$\therefore \quad I_1 = \frac{20}{3} = 6.667 \text{ A} \quad \text{... (2)}$$

$$\therefore \quad \mathbf{I_N = 13.333\ A} \downarrow$$

Step 3 : To calculate R_{eq}, replace voltage source by short circuit and current source by open circuit.

> **Key Point :** *There is direct short circuit across 3Ω resistance hence it becomes redundant from the circuit point of view.*

$$\therefore \qquad R_{eq} = 6\ \Omega$$

Step 4 : Norton's equivalent across A-B is shown in the Fig. 2.76 (c).

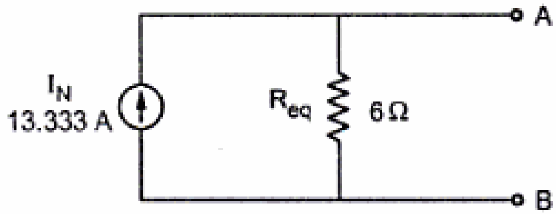


Fig. 2.76 (c)

> **Key Point :** *Do not apply KVL to the loop consisting current source. The effect of current source is taken care of while obtaining current distribution. Then apply KVL to those loops which do not include any current source.*

2.20 Maximum Power Transfer Theorem

Let us see the statement of the theorem.

Statement : *In an active resistive network, maximum power transfer to the load resistance takes place when the load resistance equals the equivalent resistance of the network as viewed from the terminals of the load.*

2.20.1 Proof of Maximum Power Transfer Theorem

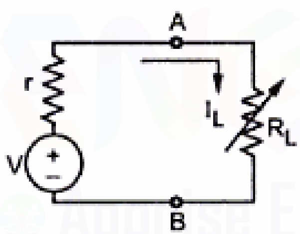


Fig. 2.77 (a)

Consider a d.c. source of voltage V volts and having internal resistance of r ohms connected to a variable load resistance R_L as shown in the Fig. 2.77 (a). The load current is I_L and is given by,

$$I_L = \frac{V}{r+R_L}$$

The power consumed by the load resistance R_L, is

$$P = I_L^2\ R_L = \left[\frac{V}{(r+R_L)}\right]^2 R_L$$

If R_L is changed, I_L is also going to change and at a particular value of R_L, power transferred to the load is maximum. Let us calculate value of R_L for which power transfer to load is maximum. To satisfy maximum power transfer we can write,

$$\frac{d\,P}{d\,R_L} = 0$$

$$\frac{d\,P}{d\,R_L}\left[\frac{V}{(r+R_L)}\right]^2 R_L = 0$$

$$\therefore \qquad V^2\frac{d}{d\,R_L}\left[\frac{R_L}{(r+R_L)^2}\right] = 0 \qquad \text{... as voltage is constant}$$

$$\therefore \quad (r+R_L)^2 \frac{d(R_L)}{dR_L} - R_L \frac{d}{dR_L}(r+R_L)^2 = 0$$

$$\therefore \quad (r+R_L)^2 (1) - R_L\, 2\,(r+R_L) = 0$$

$$\therefore \quad (r+R_L-2R_L) = 0$$

$$\boxed{R_L = r}$$

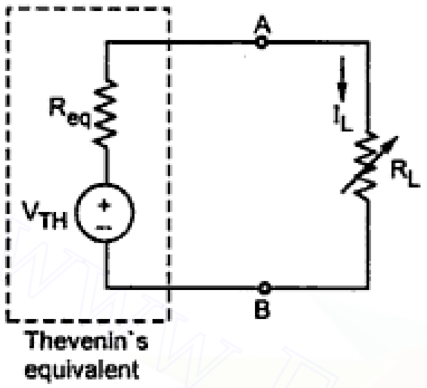


Fig. 2.77 (b)

Thus when load resistance is equal to the internal resistance of source the maximum power transfer takes place.

Now any complex network can be represented with a single voltage source of V_{TH} volts with equivalent resistance R_{eq} in series with it, using Thevenin's theorem across the load terminals. Thus the variable load resistance R_L, in such case must be equal to R_{eq} to have maximum power transfer to the load.

$$\therefore \quad \boxed{R_L = R_{eq}} \quad \text{... for maximum power transfer}$$

Let us calculate the magnitude of maximum power transfer. It can be obtained by substituting $R_L = R_{eq}$ in the expression of power.

$$\therefore \quad P_{max} = \left(\frac{V_{TH}}{R_{eq}+R_L}\right)^2 R_L \qquad \text{with } R_L = R_{eq}$$

$$\therefore \quad \boxed{P_{max} = \frac{V_{TH}^2}{(2R_{eq})^2} \times R_{eq} = \frac{V_{TH}^2}{4R_{eq}} \text{ watts}}$$

2.20.2 Steps to Apply Maximum Power Transfer Theorem

Step 1 : Calculate Thevenin's voltage V_{TH} or Norton's current I_N.

Step 2 : Calculate R_{eq} as viewed through the load terminals.

Step 3 : Draw Thevenin's equivalent or Norton's equivalent.

Step 4 : $R_L = R_{eq}$ gives the condition for maximum power transfer to load.

Step 5 : And maximum power is given by,

$$P_{max} = \frac{V_{TH}^2}{4R_{eq}}$$

Key Point : *If in the problem only the value R_L for maximum power transfer is asked then only R_{eq} is to be calculated. In such case there is no need to calculate V_{TH} or I_N. These values are required only if magnitude of P_{max} is required.*

Example 2.30 : *Find the value of R_L for maximum power transfer and the magnitude of maximum power dissipated in the resistor R_L in the circuit shown in the Fig. 2.78.*

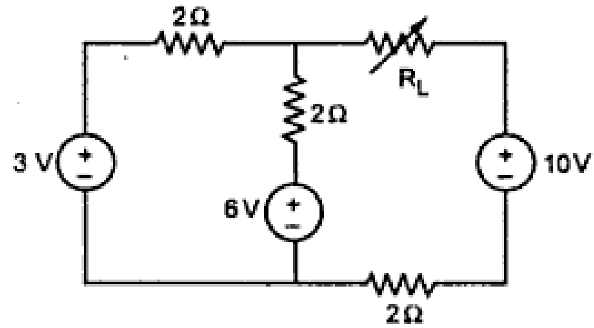


Fig. 2.78

Solution : Step 1 : Remove the load resistance R_L.

Step 2 : Find the Thevenin's voltage V_{TH} across the open terminals as magnitude of maximum power is asked.

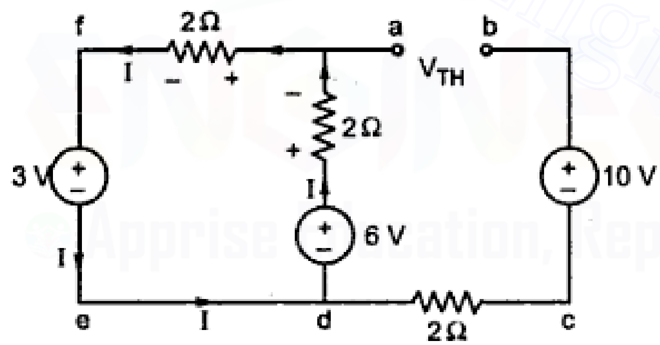


Fig. 2.78 (a)

Applying KVL to the loop,

$$-2I - 2I - 3 + 6 = 0$$

$$\therefore \quad 4I = 3$$

$$\therefore \quad I = 0.75 \text{ A}$$

$\therefore$ Drop across $2\ \Omega = 2 \times 0.75 = 1.5$ V

The 2 Ω resistance in branch cd is not carrying any current hence drop across it is zero volts. Trace the path from a to b and show the various voltage drops as shown in the Fig. 2.78 (b).

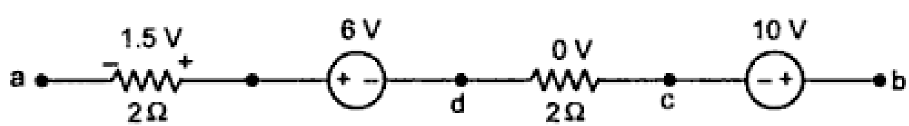


Fig. 2.78 (b)

$$\therefore \quad V_{TH} = 11.5 - 6 = 5.5 \text{ V with b positive with respect to a.}$$

Step 3 : Calculate R_{eq}, replacing all voltage sources by short circuits.

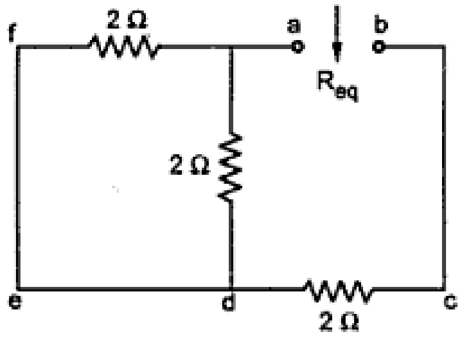


Fig. 2.78 (c)

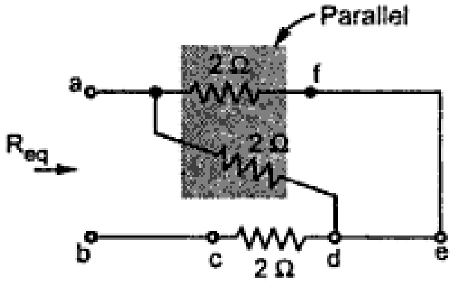


Fig. 2.78 (d)

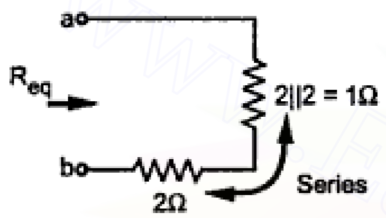


Fig. 2.78 (e)

As seen from the Fig. 2.78 (d) and 2.78 (e) we can write,

$$R_{eq} = [2 \parallel 2] + 2 = 1 + 2 = 3\ \Omega$$

Thus for maximum power transfer to load,

$$R_L = R_{eq} = 3\ \Omega$$

Step 4 : The maximum power transfer is given by,

$$P_{max} = \frac{V_{TH}^2}{4\,R_{eq}}$$

$$= \frac{(5.5)^2}{4 \times 3} = 2.5208\ \text{W}$$

Example 2.31 : *Find the value of R_{AD} for maximum power transfer, in the circuit shown in the Fig. 2.79.*

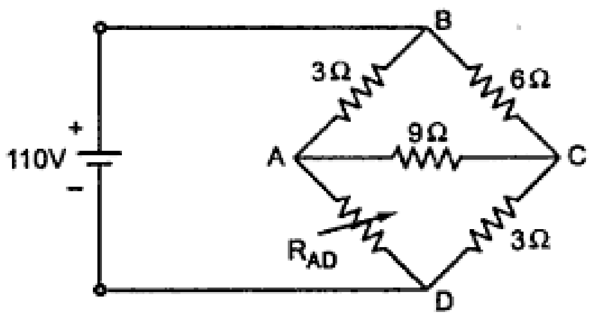


Fig. 2.79

Solution : As magnitude of P_{max} is not required, only R_{eq}, as seen through terminals AD is to be obtained with voltage source shorted.

As points B and D are directly connected, the circuit can be redrawn as shown in the Fig. 2.79 (b), showing B and D as a single point.

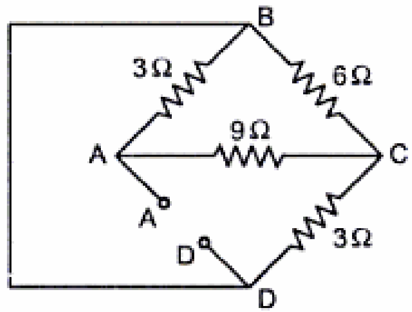


Fig. 2.79 (a)

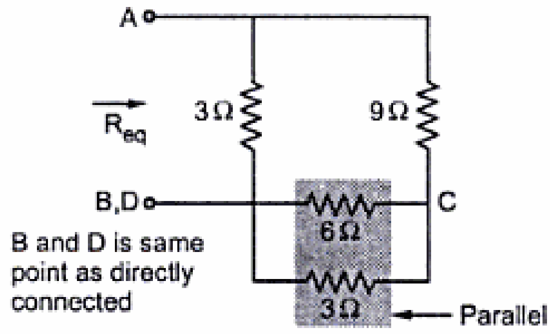


Fig. 2.79 (b)

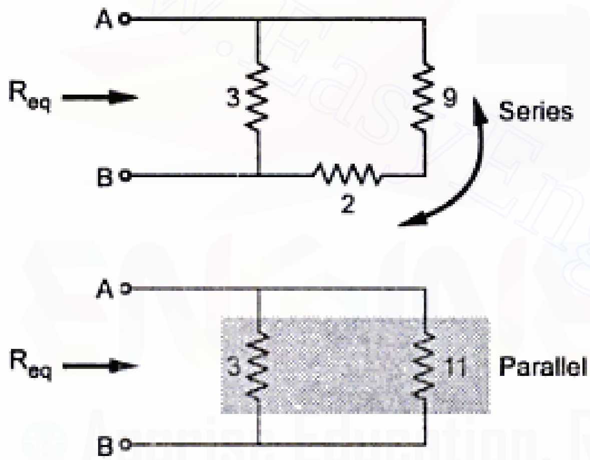


Fig. 2.79 (c)

$$\therefore \quad R_{eq} = (3) \parallel [9 + (6 \parallel 3)]$$

$$= (3) \parallel [9 + 2]$$

$$= (3) \parallel (11)$$

$$= 2.3571\ \Omega$$

For P_{max}, $R_{AD} = R_{eq} = 2.3571\ \Omega$

2.21 Concept of Loop Current

A loop current is that current which simultaneously links with all the branches, defining a particular loop.

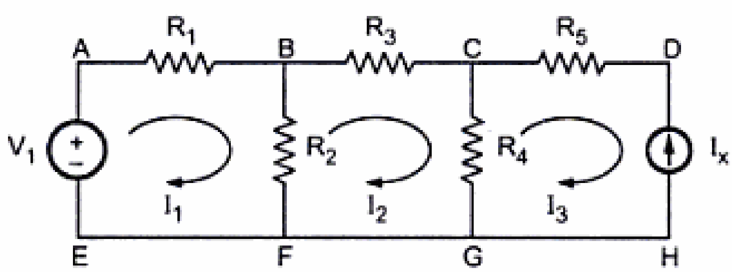


Fig. 2.80 Concept of loop current

The Fig. 2.80 shows a network. In this, I_1 is the loop current for the loop ABFEA and simultaneously links with the branches AB, BF, FE and EA. Similarly I_2 is the second loop current for the loop BCGFB and I_3 is the third loop current for the loop CDFGC.

Observe :

1. For the common branches of the various loops, multiple loops currents get associated. For example to the branch BF, both I_1 and I_2 are associated.
2. The branch current is always unique hence a branch current can be expressed interms of associated loop currents.

Key Point : *The total branch current is the algebraic sum of all the loop currents associated with that branches.*

$$I_{BF} = I_1 - I_2 \text{ from B to F}$$

$$I_{CG} = I_2 - I_3 \text{ from C to G}$$

3. The branches consisting current sources, directly decide the values of the loop currents flowing through them.

The branch DH consists current source of I_x amperes and only the loop current I_3 is associated with the branch DH in opposite direction. Hence $I_3 = -I_x$.

4. Assuming such loop currents and assigning the polarities for the drops across the various branches due to the assumed loop currents, the Kirchhoff's voltage law can be applied to the loops. Solving these equations, the various loop currents can be obtained. Once the loop currents are obtained, any branch current can be calculated.

Note : From the syllabus point of view, in this book, the branch current method is used to solve the problems. If loop currents are given in the problem, mark the branch currents interms of given loop currents and then use KVL, to solve the problem.

Examples with Solutions

Example 2.32 : *Calculate the resistance between terminals A-B.*

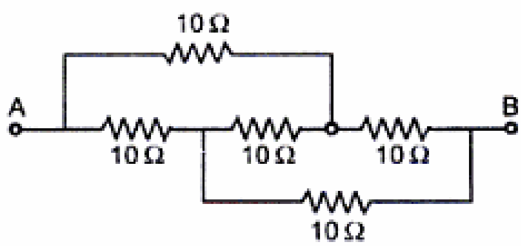


Fig. 2.81

Solution : Refer Fig. 2.81 (a),

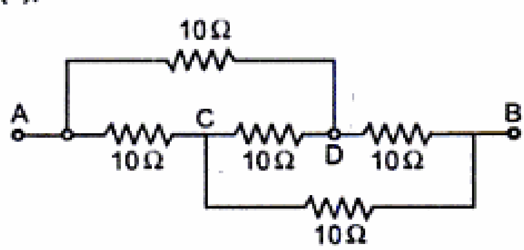
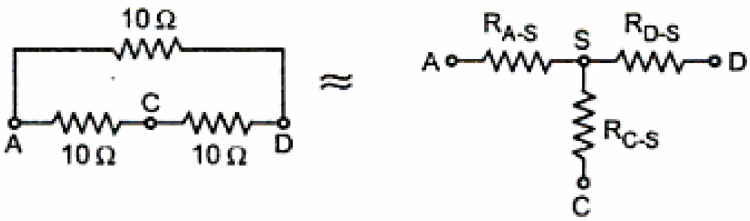


Fig. 2.81 (a)

Loop A-C-D forms Δ converting to Star,



$$R_{AS} = \frac{10 \times 10}{10+10+10}$$

$$= 3.33\ \Omega$$

$$= R_{CS} = R_{DS}$$

Fig. 2.81 (b)

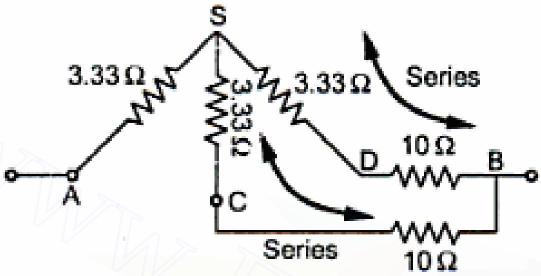


Fig. 2.81 (c)

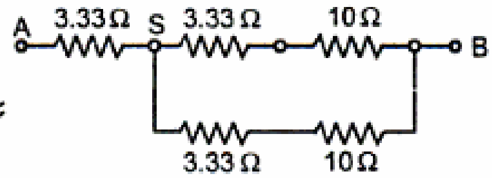


Fig. 2.81 (d)

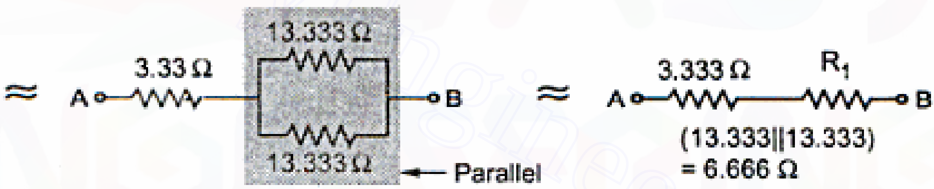
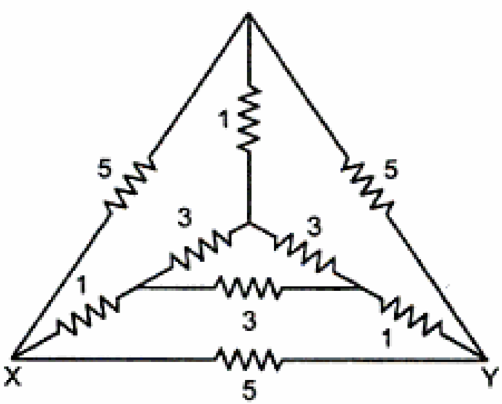


Fig. 2.81 (e)

$$R_{AB} = 3.333 + 6.666 = 10\ \Omega$$

Example 2.33 : *Determine the resistance between the terminals X and Y for the circuit shown in Fig. 2.82.* **[Dec.-2006]**



(All resistances in ohm)

Fig. 2.82

Solution : Converting inner delta to star.

$$\text{Each resistance} = \frac{3\times 3}{3+3+3} = 1\Omega$$

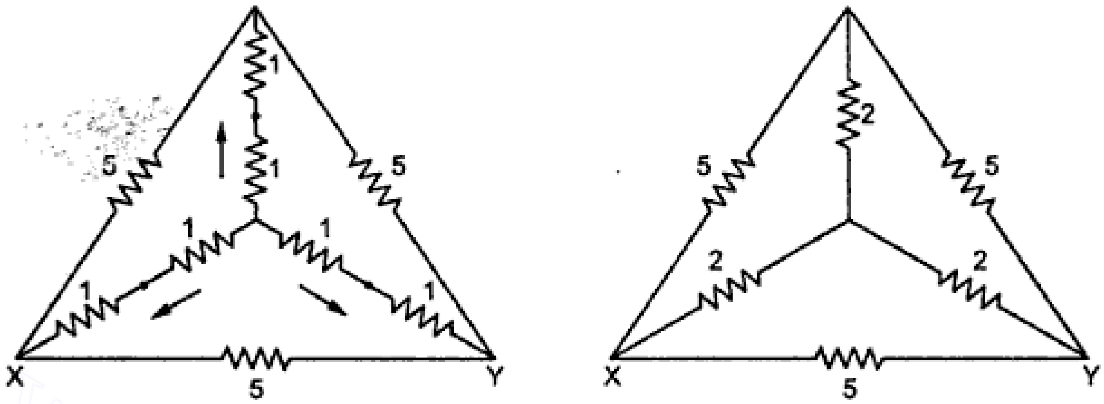
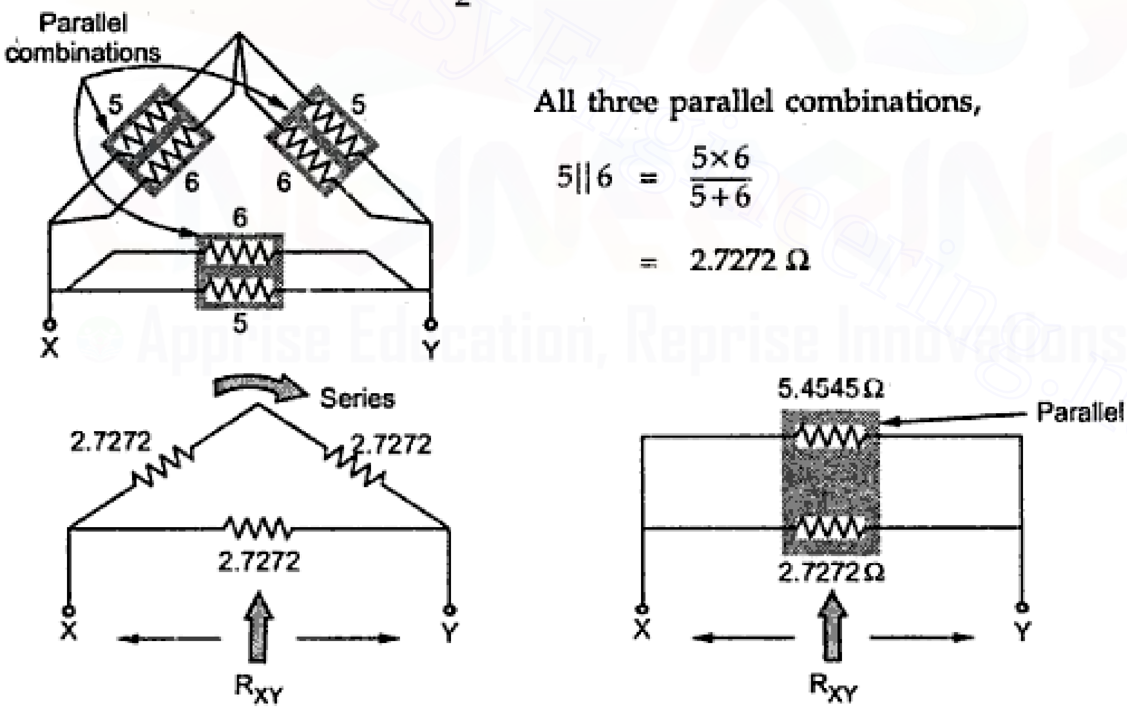


Fig. 2.82 (a)

Converting inner star to delta.

$$\text{Each resistance} = 2+2+\frac{2\times 2}{2} = 6\ \Omega$$



All three parallel combinations,

$$5||6 = \frac{5\times 6}{5+6}$$

$$= 2.7272\ \Omega$$

Fig. 2.82 (b)

$\therefore \quad R_{XY} = 5.4545||2.7272 = \mathbf{1.8181\ \Omega.}$

Example 2.34 : *Find the equivalent resistance across the terminals A and B shown in the Fig. 2.83.*

All resistances are in ohms.

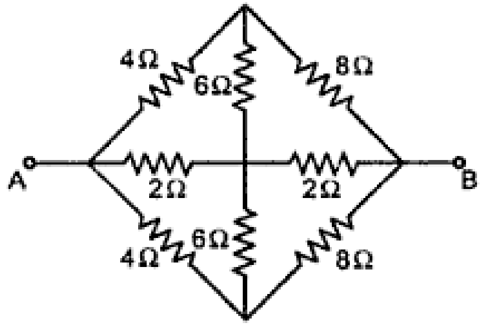


Fig. 2.83

Solution : Converting following Delta into Star.

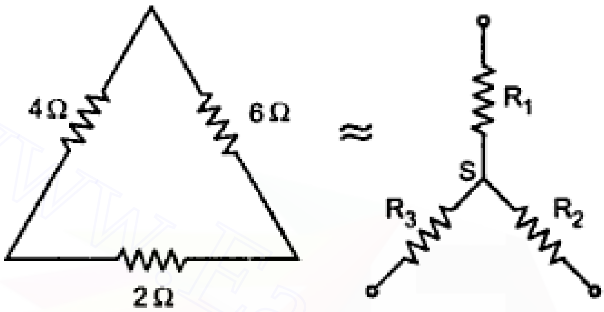


Fig. 2.83 (a)

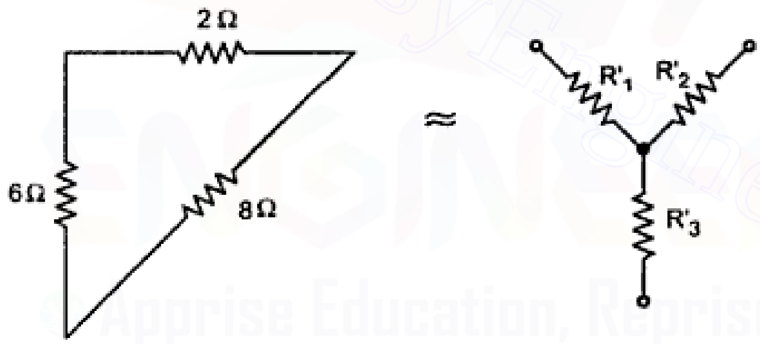


Fig. 2.83 (b)

$$R_1 = \frac{4\times6}{4+2+6} = 2\ \Omega,$$

$$R_2 = \frac{6\times2}{4+2+6} = 1\ \Omega,$$

$$R_3 = \frac{4\times2}{4+2+6} = 0.667\ \Omega$$

$$R_1' = \frac{6\times2}{6+2+8} = 0.75\ \Omega,$$

$$R_2' = \frac{2\times8}{6+2+8} = 1\ \Omega,$$

$$R_3' = \frac{6\times8}{6+2+8} = 3\ \Omega$$

Redrawing original network with above conversions.

Combining 4 and 3 which are in series to get 7 Ω and converting following Delta to Star, the circuit reduces as shown in the Fig. 2.83(e).

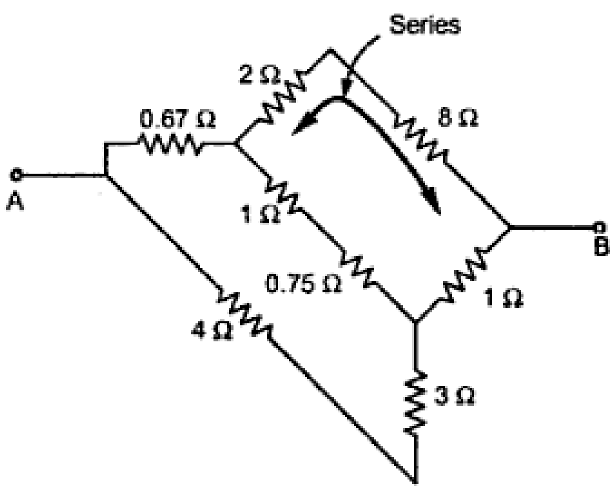


Fig. 2.83 (c)

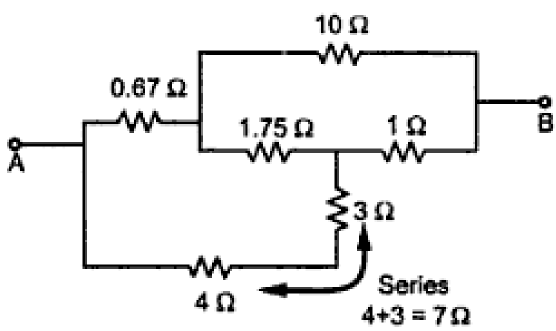


Fig. 2.83 (d)

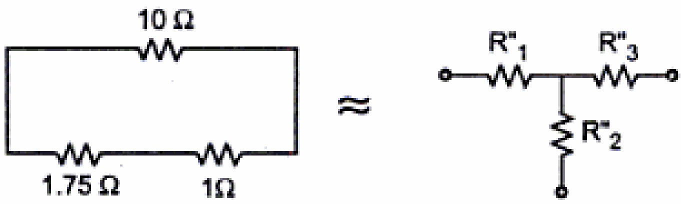


Fig. 2.83 (e)

$$R_1'' = \frac{1.75 \times 10}{1.75+10+1} = 1.3725\ \Omega$$

$$R_2'' = \frac{1.75 \times 1}{1.75+10+1} = 0.13725\ \Omega$$

$$R_3'' = \frac{10 \times 1}{1.75+10+1} = 0.7843\ \Omega$$

Redrawing the circuit,

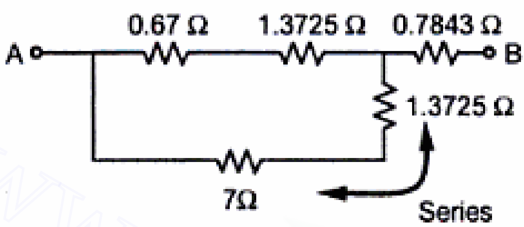


Fig. 2.83 (f)

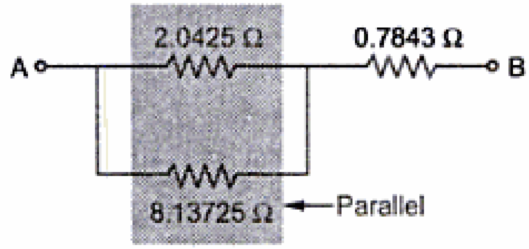


Fig. 2.83 (g)

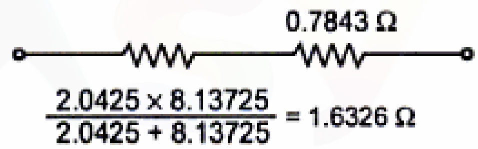


Fig. 2.83 (h)

$$\therefore \quad R_{AB} = 1.6326 + 0.7843$$

$$R_{AB} = 2.417\ \Omega$$

Example 2.35 : *Find the equivalent resistance across terminals X and Y.*

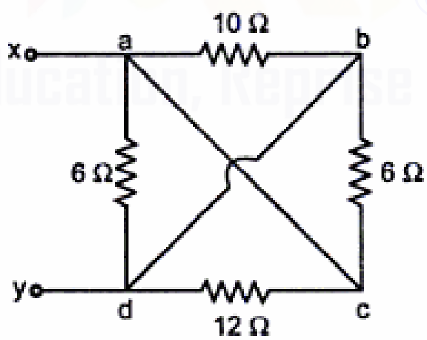


Fig. 2.84

Solution : Points 'X' and 'a' are at same potential.
Points 'Y' and 'd' are at same potential
Points 'a' and 'c' are at same potential.
Points 'd' and 'b' are at same potential.
All resistances are in parallel.

$$\therefore \quad R_{xy} = \frac{1}{\frac{1}{6}+\frac{1}{8}+\frac{1}{12}+\frac{1}{10}} = 2.1053\ \Omega$$

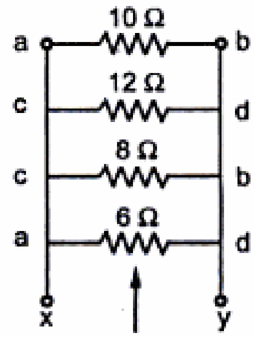


Fig. 2.84 (a)

Example 2.36 : *In a bridge circuit the resistance of branch AB = 30 Ω, BC = 41 Ω, AD = 6 Ω, while a 4 V battery is connected between points A and C. An ammeter with internal resistance of 10Ω, is connected between points B and D. The resistance of branch CD is 'R' ohms. If ammeter is showing a reading of 15 mA (↓), determine value of R.*

Solution : Step 1 : Draw the circuit diagram.

Step 2 and 3 : Assume the various branch currents applying KCL at various nodes. And mark polarities of all voltages due to these currents as shown in Fig. 2.85 (a).

Step 4 : Apply KVL to various loops.

Loop 1 : Loop A-D-C-A (through battery)

$$-6(i_1 - i_2) - R(i_1 - i_2 + 0.015) + 4 = 0$$

i.e $\quad -i_1(6+R) + i_2(6+R) = (0.015R - 4) \quad \dots (1)$

Loop 2 : Loop A-B-D-A

$$-30\, i_2 - 0.015 \times 10 + 6(i_1 - i_2) = 0$$

i.e. $\quad 6\, i_1 - 36\, i_2 = 0.15 \quad \dots (2)$

Loop 3 : Loop A-B-C-A (through battery)

$$-30\, i_2 - 41(i_2 - 0.015) + 4 = 0$$

i.e. $\quad -71\, i_2 = -4.615 \quad \dots (3)$

From (3) $\quad i_2 = 0.065$ A

From (2) $\quad i_1 = 0.415$ A

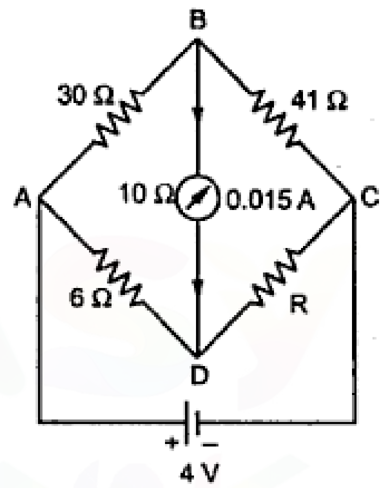


Fig. 2.85

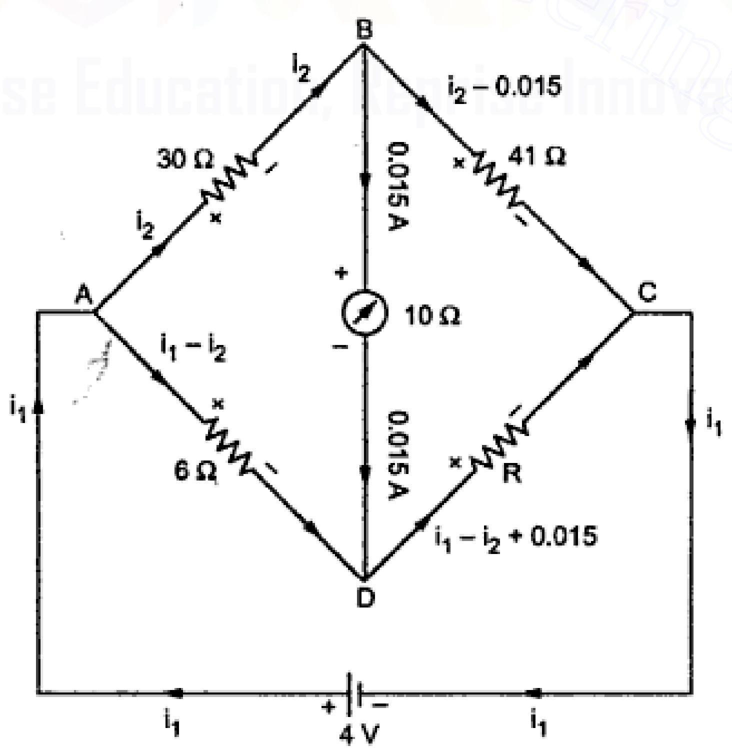


Fig. 2.85 (a)

From (1) $-0.415\,(6+R)+0.065\,(6+R) = 0.015\,R-4$

i.e. $-2.49-0.415\,R+0.39+0.065\,R = 0.015\,R-4$

$$\therefore \quad 0.365R = 1.9$$

$$\therefore \quad R = 5.205\ \Omega$$

Example 2.37 : *Two batteries A and B having e.m.fs of 209 V and 211 V having internal resistance 0.3 Ω and 0.8 Ω respectively are to be charged from a d.c. source of 225 V. If for that purpose they were connected in parallel and resistance of 4 Ω was connected between the supply and batteries to limit charging current, find*

i) Magnitude and direction of current through each battery.

ii) Power delivered by source.

Solution : The circuit diagram is as shown in Fig. 2.86.

We can use branch current method. Show the branch currents and polarities.

Apply KVL to different loops.

Loop 1 : Loop A-B-E-F-A

$$-0.3\,I_2-209+225-4\,I_1 = 0$$

i.e. $4\,I_1+0.3\,I_2 = 16$ ... (1)

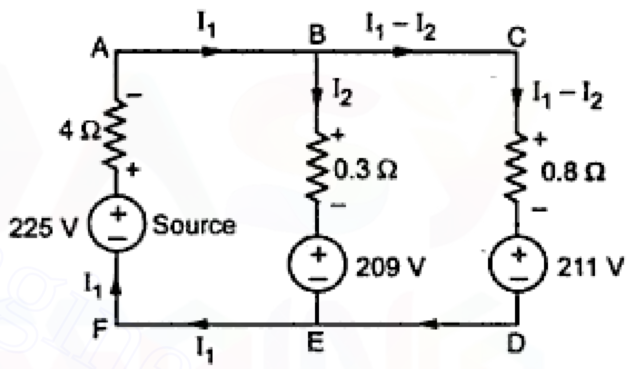


Fig. 2.86

Loop 2 : Loop A-B-C-D-E-F-A,

i.e. $-0.8\,(I_1-I_2)-211+225-4\,I_1 = 0$ i.e. $4.8\,I_1-0.8\,I_2 = 14$ ...(2)

Solving equations (1) and (2) simultaneously,

$$I_1 = 3.663\ \text{A} \quad \text{and} \quad I_2 = 4.482\ \text{A}$$

$\therefore$ $I_1 - I_2 = -0.8183$ A i.e. it is in opposite direction to what is assured.

i) Magnitude of current through source = **3.663 A** ↑

Magnitude of current through battery A = **4.482 A** ↓

Magnitude of current through battery B = **0.8183 A** ↑

ii) Power delivered by,

$$\text{Source} = 225 \times 3.663 = \mathbf{824.175\ watts}$$

Example 2.38 : *Find the current through branch AB, using Superposition theorem.*

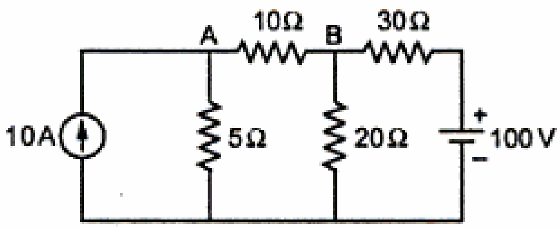


Fig. 2.87

Solution : **Step 1 :** Consider 10 A source alone, replacing 100 V source by short circuit

Resistance 20 and 30 are in parallel

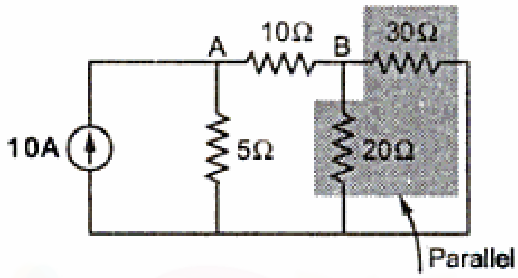


Fig. 2.87 (a)

$$\therefore \quad R_{eq} = \frac{20\times 30}{20+30}$$

$$= 12\ \Omega$$

By using current division rule,

$$I_{AB} = 10\times\frac{5}{5\times 22} = 1.85185\ \text{A} \rightarrow$$

$\therefore$ Current through AB is 1.85185 A from A to B due to 10 source.

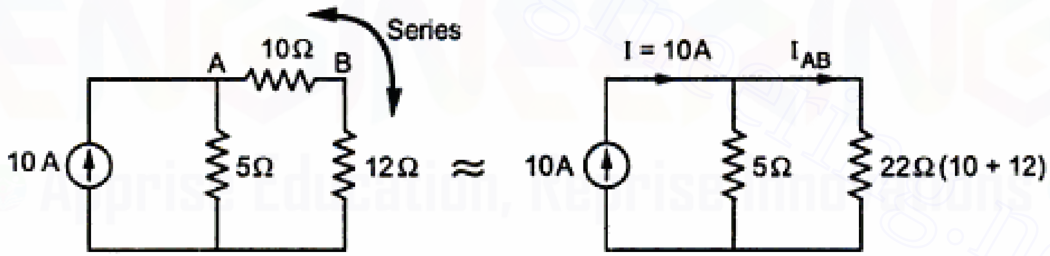


Fig. 2.87 (b)

Step 2 : Consider 100 V source alone, replacing current source by open circuit.

Note : Current source must be replaced by open circuit while voltage source must be replaced by short circuit if internal resistances are not given. Referring Fig. 2.87 (c) and (d),

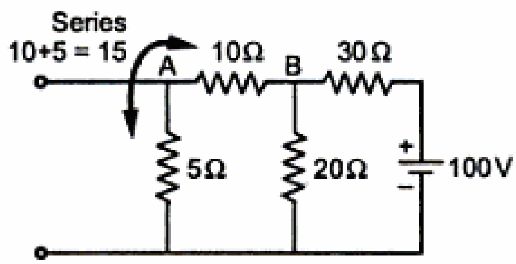


Fig. 2.87 (c)

$$\therefore \quad R_{eq} = \frac{20\times 15}{20+15}$$

$$= 8.571\ \Omega$$

$$\therefore \quad I = \frac{100}{30+8.571}$$

$$= 2.5925\ \text{A}$$

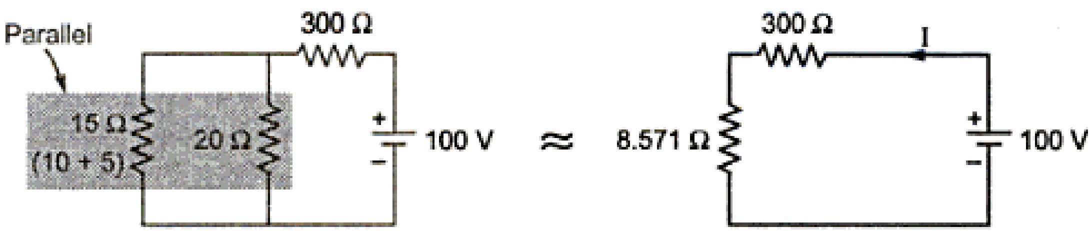


Fig. 2.87 (d)

∴ Current through 15 Ω using current division is

$$= 2.5925 \times \frac{20}{20+15}$$

$$I_{AB} = 1.48148 \text{ A}$$

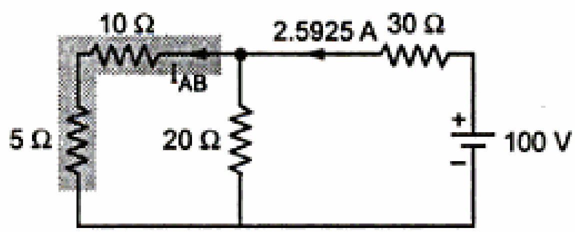


Fig. 2.87 (e)

∴ Current through AB is 1.48148 A from B to A due to 100 V source.

Step 3 : The current through branch AB due to both sources.

$$= 1.85185 \text{ A} \rightarrow + 1.48148 \text{ A} \leftarrow = \mathbf{0.3703 \text{ A}} \rightarrow$$

Both are in opposite direction so there will be subtraction of two currents.

∴ **Ans. : 0.3703 A from A to B**

➠ **Example 2.39 :** *Find the currents i_1, i_2, i_3 and powers delivered by the sources of the network in Fig. 2.88.* **(May - 2001)**

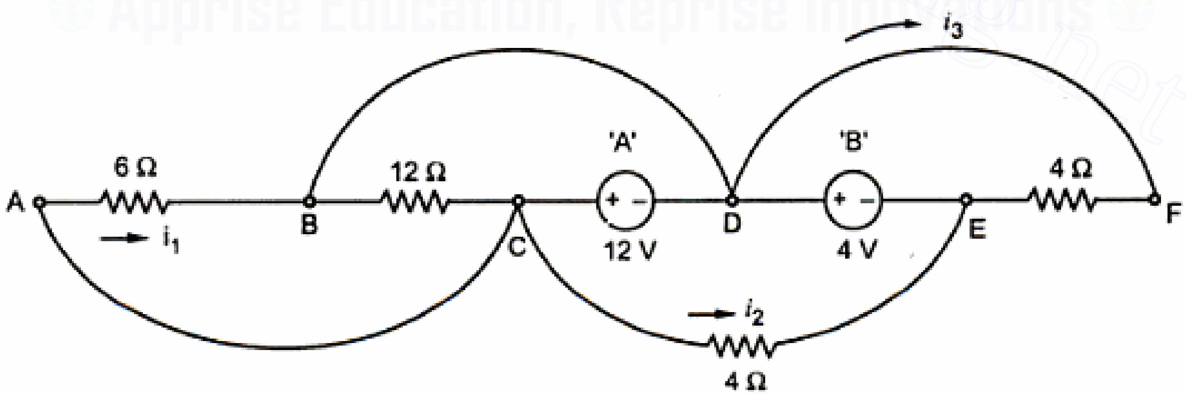


Fig. 2.88

Solution : The various branch currents are shown in the Fig. 2.88 (a), by applying KCL at various nodes interms of i_1, i_2, i_3 shown.

Loop ABCA, $\quad -6\, i_1 - 12\,(i_1 + i_2 - i_4) = 0 \quad$ i.e. $\quad -18\, i_1 - 12\, i_2 + 12\, i_4 = 0$ ...(1)

Loop CDEC, $\quad -4\, i_2 + 4 + 12 = 0 \quad$ i.e. $\quad i_2 = 4$ A ... (2)

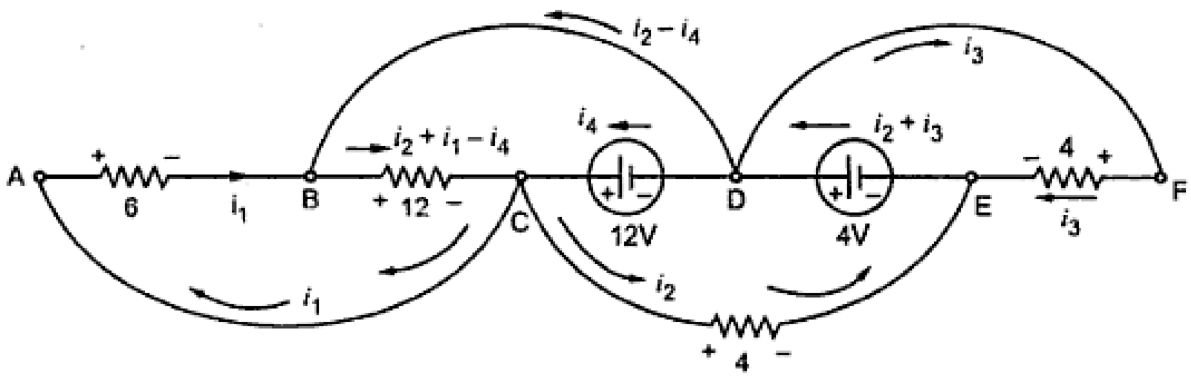


Fig. 2.88 (a)

Loop DEFD, $\quad -4 + 4\, i_3 = 0 \quad$ i.e. $\quad i_3 = 1$ A ... (3)

Loop BCDB, $\quad -12\,(i_1 + i_2 - i_4) - 12 = 0 \quad$ i.e. $\quad -i_1 - i_2 + i_4 = 1$ ... (4)

Substituting $i_2 = 4$ A in (1) and (4)

$$-3\, i_1 + 2\, i_4 = 8 \qquad \text{... (5)}$$

$$-i_1 + i_4 = 5 \qquad \text{... (6)}$$

$$-1.5\, i_1 + i_4 = 4 \qquad \text{... (7)}$$

$\therefore \quad 0.5\, i_1 = 1$

$\therefore \quad i_1 = 2$ **A** and $i_4 = 7$ **A**

$\therefore$ Power by 12 V source $= i_4 \times 12 = 84$ **W**

and Power by 4 V source $= 4 \times (i_2 + i_3) = 20$ **W**

Example 2.40 : *Find the value of 'R' so that 1 A would flow in it, for the network in the Fig. 2.89.* (May-2001)

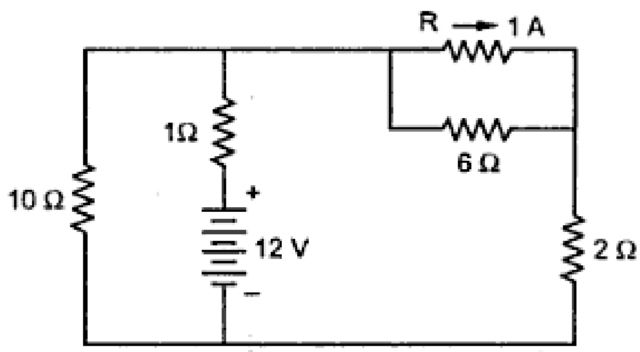


Fig. 2.89

Solution : The various branch currents are shown in the Fig. 2.89 (a).

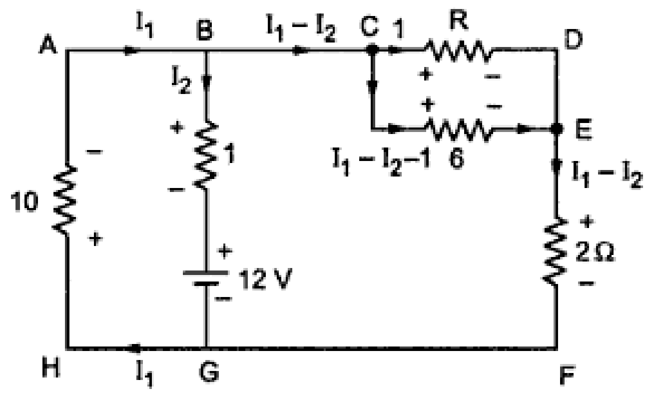


Fig. 2.89 (a)

Loop ABGH, $\quad -I_2 - 12 - 10\,I_1 = 0 \quad$ i.e. $10\,I_1 + I_2 = -12$ ... (1)

Loop BCEFGB, $\quad -6\,(I_1 - I_2 - 1) - 2\,(I_1 - I_2) + 12 + I_2 = 0$ i.e. $-8\,I_1 + 9\,I_2 = -18$ ... (2)

Multiplying equation (1) by 9 we get,

$\therefore \quad 90\,I_1 + 9\,I_2 = -108$ ... (3)

Subtracting equation (3) from (2) we get,

$\therefore \quad -98\,I_1 = +90$

$\therefore \quad I_1 = -0.9183$ A and $I_2 = -2.8163$ A

$\therefore$ Current through 6 Ω $= I_1 - I_2 - 1 = -0.9183 + 2.8163 - 1$

$= 0.898$ A

$\therefore$ Drop across 6 Ω $= 6 \times$ current through 6 Ω $= 6 \times 0.898$

$= 5.388$ V

Same is drop across R $= R \times 1 = 5.388$

$\therefore \quad R = 5.388\ \Omega$

Example 2.41 : *For a given circuit shown in Fig. 2.90, find out the equivalent resistance between terminals X and Y.* **(May - 2003)**

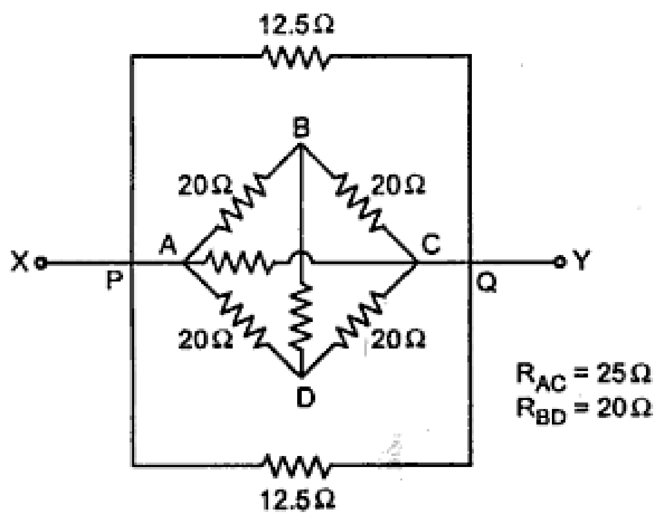


Fig. 2.90

Solution : The two 12.5 Ω resistances are in parallel so equivalent is (12.5/2) = 6.25 Ω. Converting star at point D to equivalent delta,

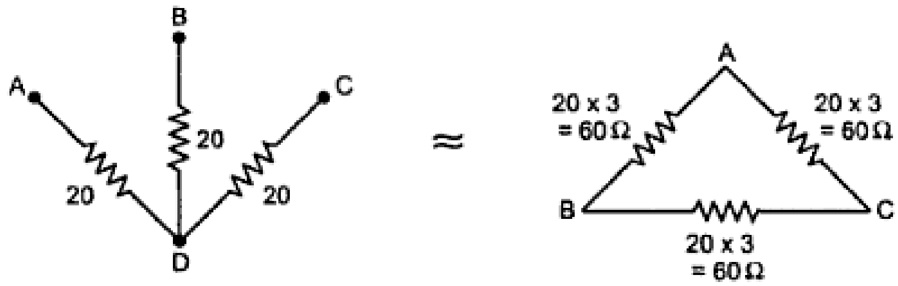


Fig. 2.90 (a)

Thus the circuit reduces as shown in the Fig. 2.90 (b).

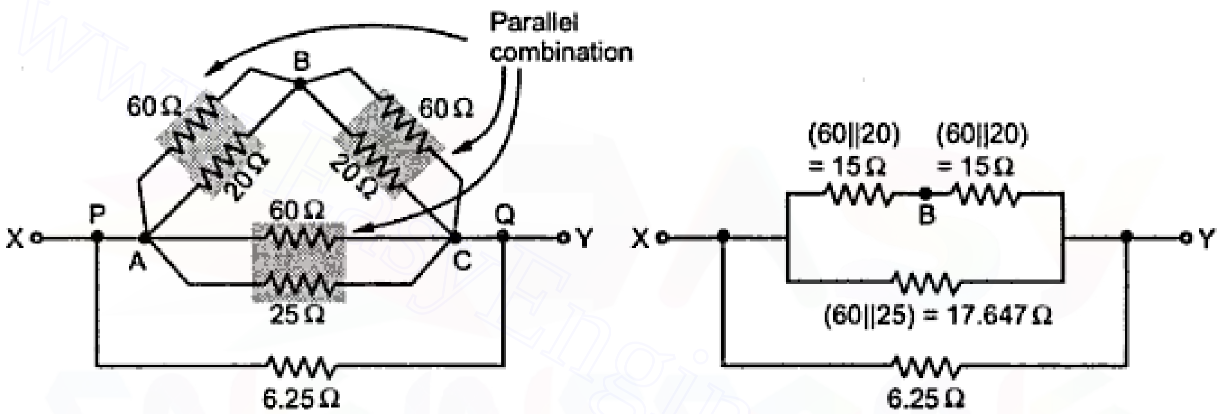


Fig. 2.90 (b) **Fig. 2.90 (c)**

$$\therefore \quad R_{XY} = (6.25) \parallel \{(15+15) \parallel (17.647)\} = (6.25) \parallel [30 \parallel 17.647]$$

$$= (6.25) \parallel [11.11] = 4\ \Omega$$

Example 2.42 : *For the network shown in Fig. 2.91, find the current in the 2-ohm resistance by using Superposition theorem.* **(Dec. - 2002)**

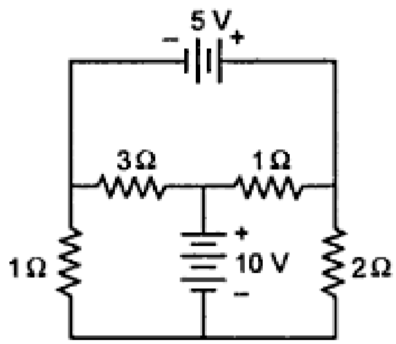


Fig. 2.91

Solution : Step 1 : Consider 5 V source acting, short the other.

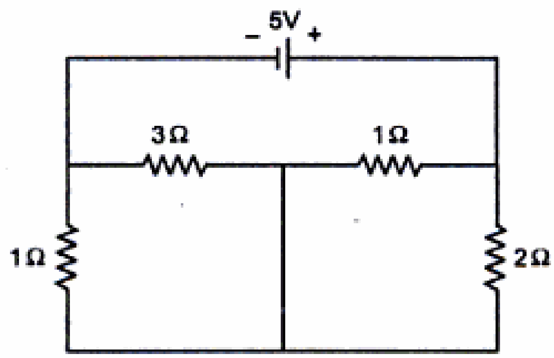


Fig. 2.91 (a)

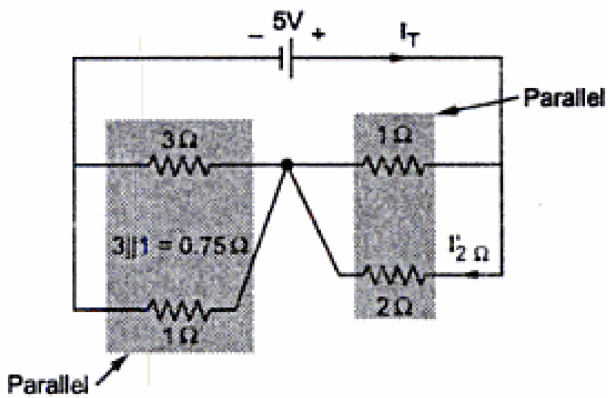


Fig. 2.91 (b)

$$\therefore \quad I_T = \frac{5}{0.75+[1||2]} = 3.5294 \text{ A}$$

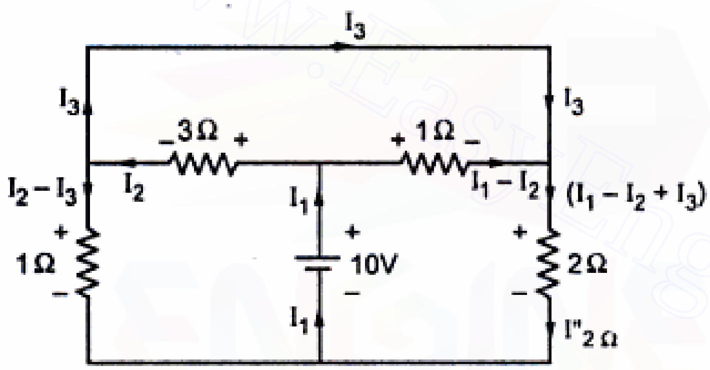


Fig. 2.91 (c)

Using current division,

$$I'_{2\Omega} = I_T \times \frac{1}{(1+2)} = 1.1764 \text{ A} \downarrow$$

Step 2 : Consider 10 V source acting, short the other.

Applying KVL to the various loops,

$$-3I_2 - (I_2 - I_3) + 10 = 0$$

$$\therefore \quad -4\, I_2 + I_3 = -10 \quad \text{... (1)}$$

$$-(I_1 - I_2) - 2\,(I_1 - I_2 + I_3) + 10 = 0$$

$$\therefore \quad -3I_1 + 3\, I_2 - 2I_3 = -10 \quad \text{... (2)}$$

$$-(I_1 - I_2) + 3\, I_2 = 0$$

$$\therefore \quad -I_1 + 4\, I_2 = 0 \quad \text{... (3)}$$

$$D = \begin{vmatrix} 0 & -4 & 1 \\ -3 & 3 & -2 \\ -1 & 4 & 0 \end{vmatrix} = -17, \quad D_1 = \begin{vmatrix} -10 & -4 & 1 \\ -10 & 3 & -2 \\ 0 & 4 & 0 \end{vmatrix} = -120$$

$$D_2 = \begin{vmatrix} 0 & -10 & 1 \\ -3 & -10 & -2 \\ -1 & 0 & 0 \end{vmatrix} = -30, \quad D_3 = \begin{vmatrix} 0 & -4 & -10 \\ -3 & 3 & -10 \\ -1 & 4 & 0 \end{vmatrix} = +50$$

$$\therefore \quad I_1 = \frac{D_1}{D} = 7.058 \text{ A}, \quad I_2 = \frac{D_2}{D} = 1.7647 \text{ A}, \quad I_3 = \frac{D_3}{D} = -2.9411 \text{ A}$$

$$\therefore \qquad I''_{2\Omega} = I_1 - I_2 + I_3 = 7.058 - 1.7647 + (-2.9411) = 2.3522 \text{ A} \downarrow$$

Step 3 : $I_{2\Omega} = I'_{2\Omega} + I''_{2\Omega} = 3.5286 \text{ A} \downarrow$ ... Both in same direction.

Example 2.43 : *Find the equivalent resistance of the network between the terminals A and B shown in the Fig. 2.92.* **(May - 91)**

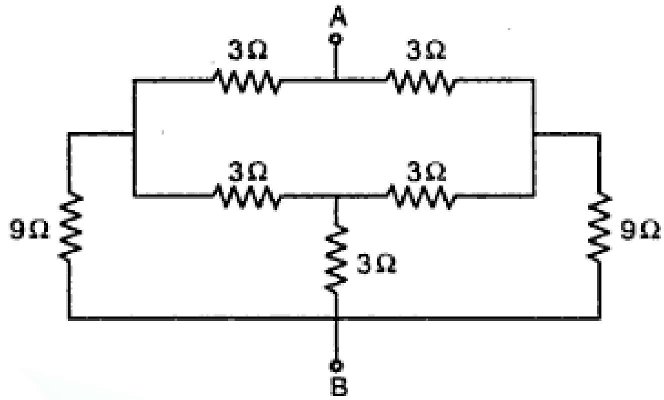


Fig. 2.92

Solution : Converting delta of 3 Ω, 3 Ω and 9 Ω to star,

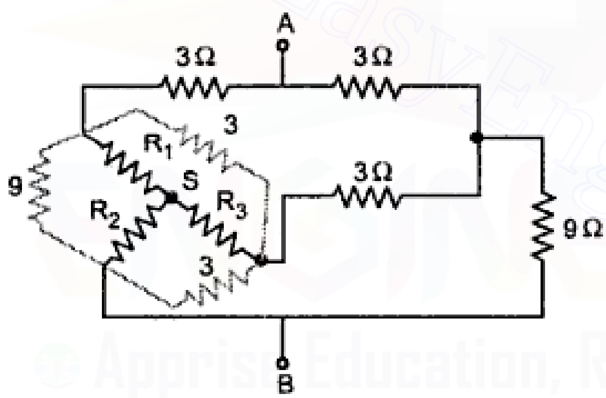


Fig. 2.92 (a)

$$R_1 = \frac{9 \times 3}{9+3+3} = 1.8 \ \Omega$$

$$R_2 = \frac{9 \times 3}{9+3+3} = 1.8 \ \Omega$$

$$R_3 = \frac{3 \times 3}{9+3+3} = 0.6 \ \Omega$$

$$R_1 + 3 = 4.8 \ \Omega$$

$$R_3 + 3 = 3.6 \ \Omega$$

Converting delta of 1.8 Ω, 3.6 Ω and 9 Ω to star,

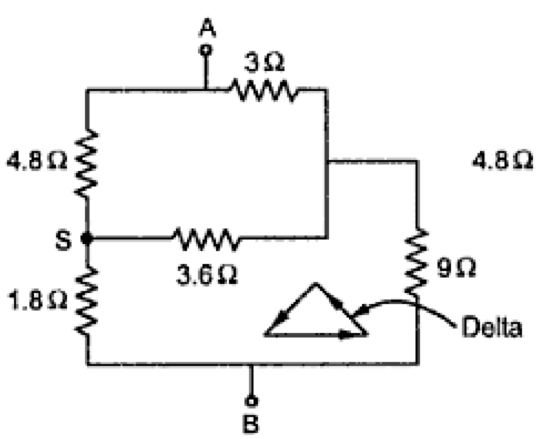


Fig. 2.92 (b)

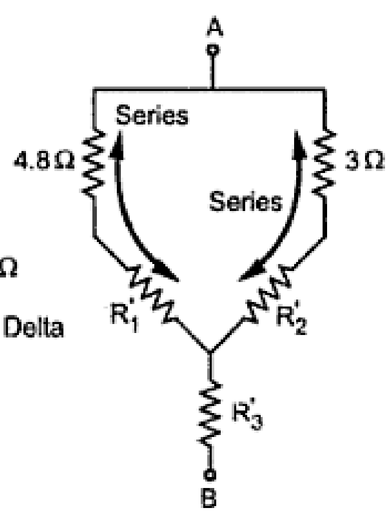


Fig. 2.92 (c)

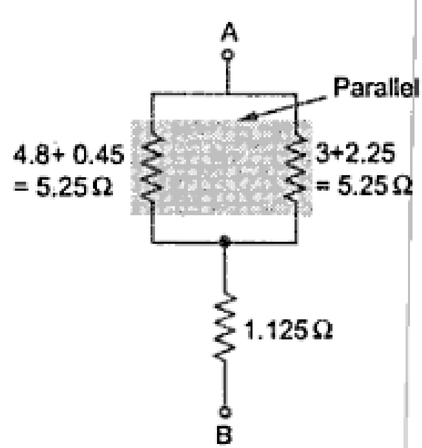


Fig. 2.92 (d)

$$R_1' = \frac{1.8 \times 3.6}{1.8+3.6+9} = 0.45\ \Omega$$

$$R_2' = \frac{3.6 \times 9}{1.8+3.6+9} = 2.25\ \Omega$$

$$R_3' = \frac{1.8 \times 9}{1.8+3.6+9} = 1.125\ \Omega$$

$$\therefore \quad R_{AB} = [(5.25) \parallel (5.25)] + 1.125 = 2.625 + 1.125 = \mathbf{3.75\ \Omega}$$

Example 2.44 : *For the circuit shown in the Fig. 2.93, find the current through 20 Ω using Thevenin's theorem.* **(Nov. - 87)**

Solution : Step 1 : Remove the 20 Ω resistance.

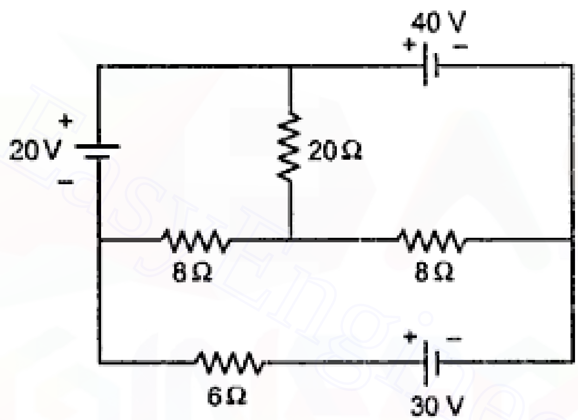


Fig. 2.93

Step 2 : Calculate the voltage across open circuit terminals.

Apply KVL to the two loops,

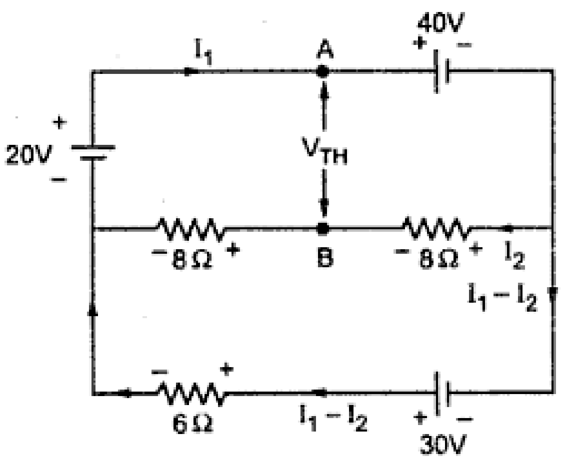


Fig. 2.93 (a)

$$-40 - 8\ I_2 - 8\ I_2 + 20 = 0$$

$$\therefore 16\ I_2 = -20$$

$$\therefore I_2 = -1.25\ A$$

$$-40 + 30 - 6\ (I_1 - I_2) + 20 = 0$$

$$\therefore I_1 = 0.4166\ A$$

Trace the path from A to B and show the various voltages and drops, as shown in the Fig. 2.93 (b).

$$\therefore \ V_{AB} = V_{TH} = 20 + 10 = \mathbf{30\ V}$$

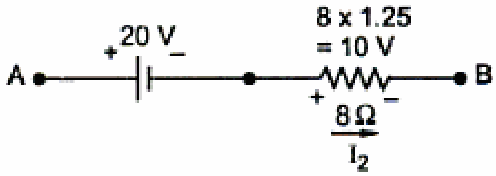


Fig. 2.93 (b)

Note that I_2 is flowing in opposite direction to what is assumed, as it is negative.

Step 3 : Find R_{eq} by replacing all voltage sources by short circuit.

> **Key Point :** *As there is direct short across 6 Ω resistance, it becomes redundant from the circuit.*

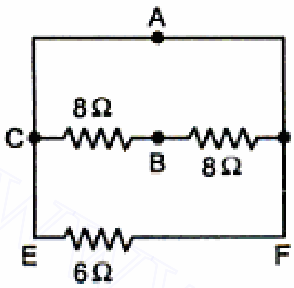


Fig. 2.93 (c)

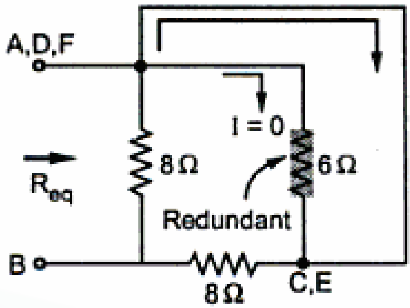


Fig. 2.93 (d)

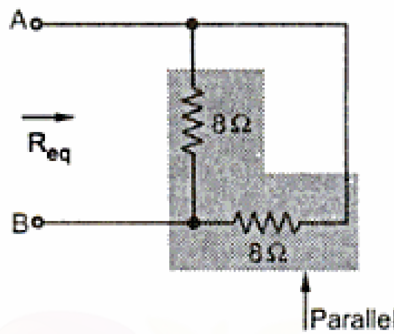


Fig. 2.93 (e)

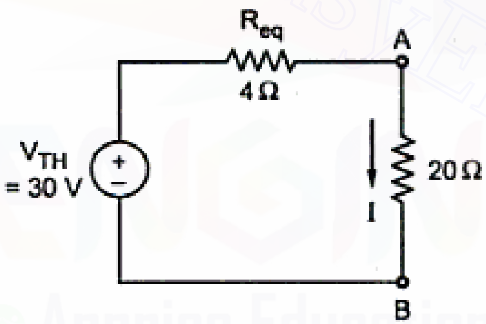


Fig. 2.93 (f)

$\therefore \quad R_{eq} = 8 \parallel 8 = 4\ \Omega$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.93 (f).

Step 5 : $I = \dfrac{30}{4+20}$

$= 1.25\ A \downarrow$

Example 2.45 : *In the circuit shown in the Fig. 2.94, calculate current through 1 Ω resistance connected between A-B, using Thevenin's theorem.* **(Nov. - 85)**

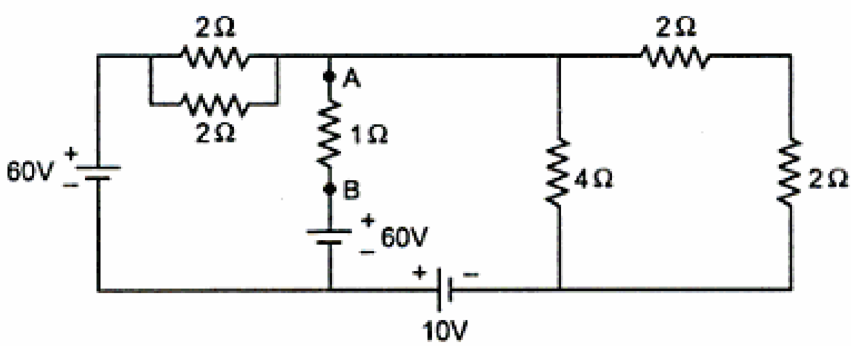


Fig. 2.94

Solution : **Step 1 :** Remove 1 Ω resistance and also combine 2 || 2 = 1 Ω.

Step 2 : Calculate voltage across open circuit.

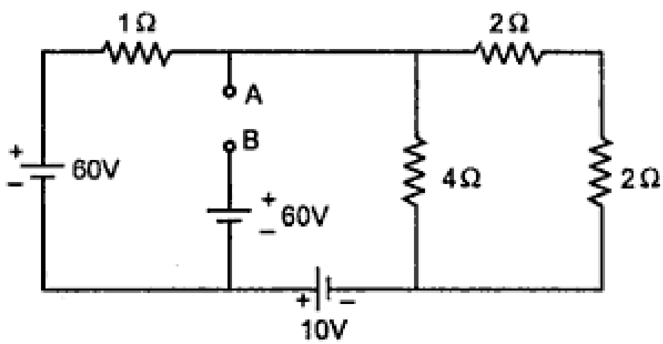


Fig. 2.94 (a)

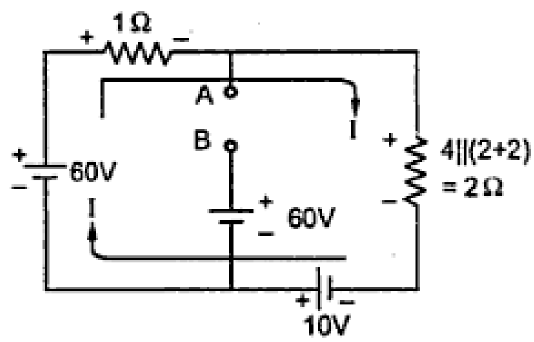


Fig. 2.94 (b)

$$-I - 2I + 10 + 60 = 0$$

$$\therefore \qquad I = \frac{70}{3} = 23.333 \text{ A}$$

So trace the path from A to B and show all voltage drops,

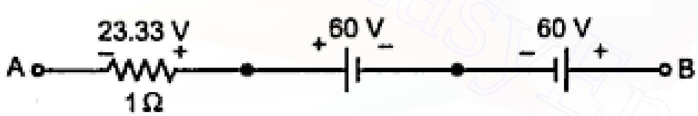


Fig. 2.94 (c)

$\therefore V_{AB} = V_{TH} = 23.33$ V

With A negative with respect to B

Step 3 : Calculate R_{eq}, by replacing voltage sources by short circuits.

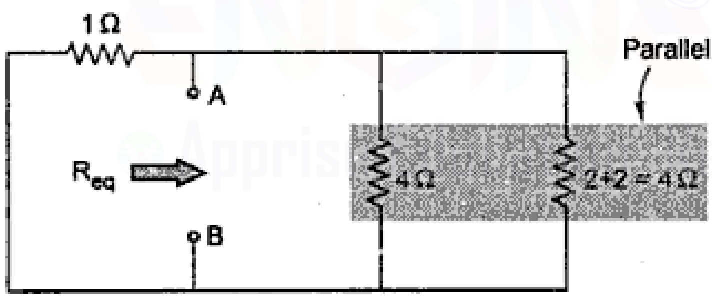


Fig. 2.94 (d)

$$\therefore \quad R_{eq} = R_{AB} = (1) \parallel (4) \parallel (4)$$

$$= (1) \parallel (2)$$

$$= 0.667 \ \Omega$$

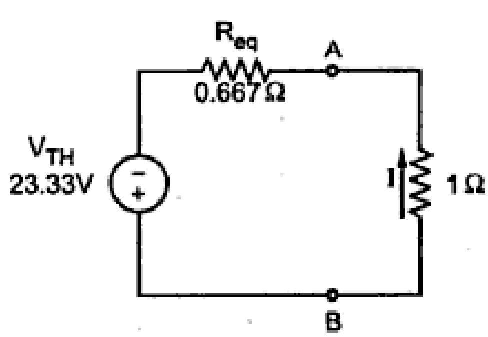


Fig. 2.94 (e)

Step 4 : Thevenin's equivalent is shown in the Fig. 2.94 (e).

Step 5 : $I = \dfrac{23.33}{1+0.667}$

$= 14 \text{ A} \uparrow$

Example 2.46 : *The network has following configuration, Arm AB = 10 Ω, Arm CD = 20 Ω, Arm BC = 30 Ω, Arm DA = 20 Ω, Arm DE = 5 Ω, Arm EC = 10 Ω and a galvanometer of 40 Ω is connected between B and E. Find by Thevenin's theorem, the current in the galvanometer if 2 V source is connected between A and C.*

(May - 86)

Solution : The network is shown in the Fig. 2.95.

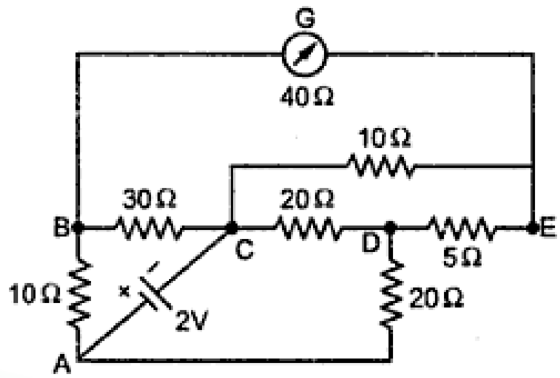


Fig. 2.95

Step 1 : Remove the galvanometer.

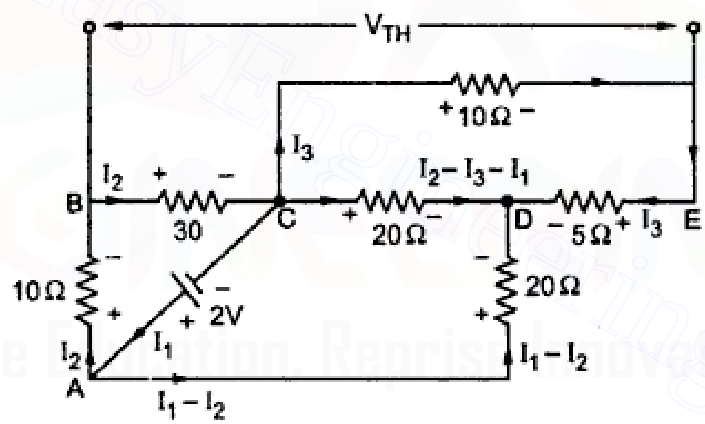


Fig. 2.95 (a)

Step 2 : Calculate voltage across open circuit between B and E. Let us use Kirchhoff's laws.

Apply KVL to different loops,

Loop BCAB, $\quad -30\ I_2 + 2 - 10\ I_2 = 0 \quad$ i.e. $I_2 = \frac{2}{40} = 0.05$ A ... (1)

Loop CDAC, $\quad -20\ (I_2 - I_3 - I_1) + 20\ (I_1 - I_2) - 2 = 0 \quad$ i.e. $40\ I_1 + 20\ I_3 = 4$ ... (2)

Loop CEDC, $\quad -10\ I_3 - 5I_3 + 20\ (I_2 - I_3 - I_1) = 0 \quad$ i.e. $-20\ I_1 - 35\ I_3 = -1$ ... (3)

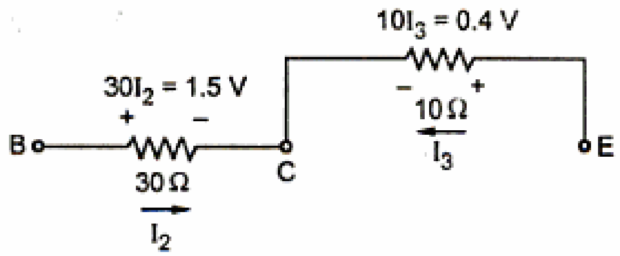


Fig. 2.95 (b)

Multiplying (3) by 2 and adding to (2) we get,

$$-50\ I_3 = 2$$

$$\therefore \quad I_3 = -0.04\ \text{A}$$

So direction of I_3 is opposite to what is assumed.

The various drops across the path BE are, as shown in the Fig. 2.95 (b).

$$V_{BE} = V_{TH} = 1.5 - 0.4 = 1.1 \text{ V with B +ve}$$

Step 3 : Calculate R_{eq}, replacing voltage source by short circuit

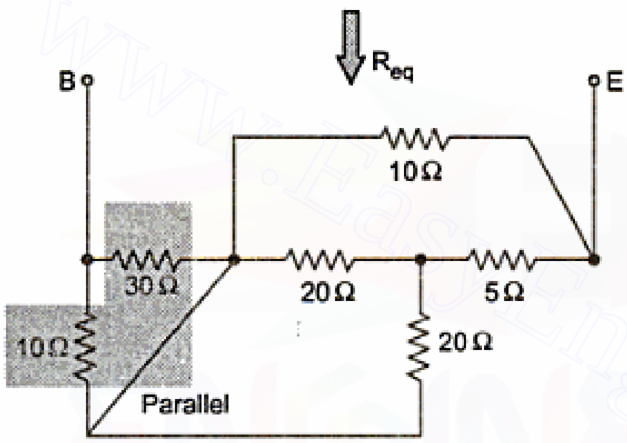


Fig. 2.95 (c)

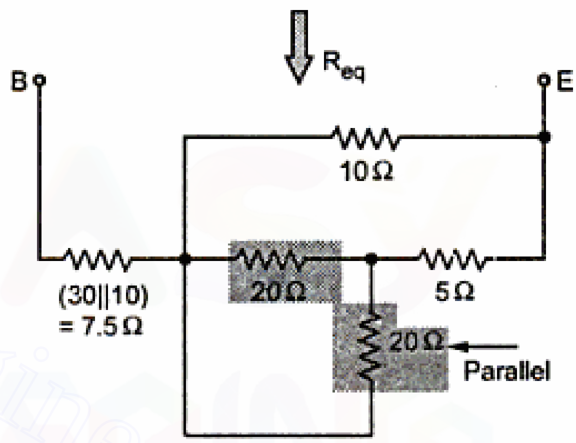


Fig. 2.95 (d)

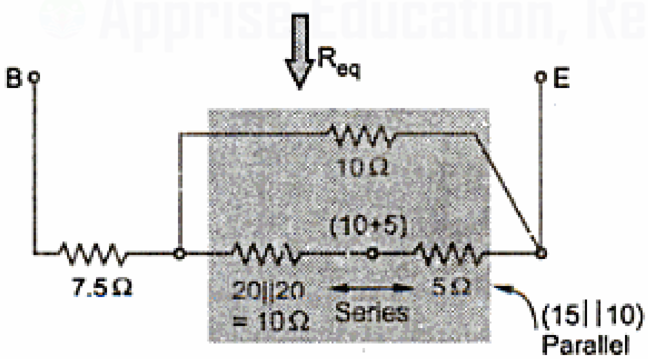


Fig. 2.95 (e)

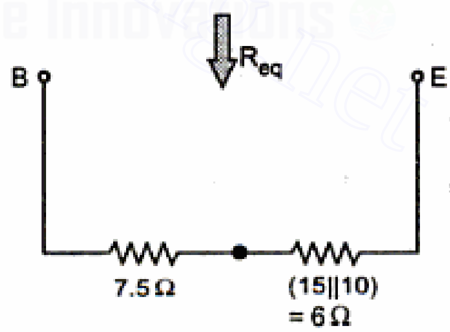


Fig. 2.95 (f)

$$\therefore \quad R_{eq} = R_{BE} = 7.5 + 6 = 13.5\ \Omega$$

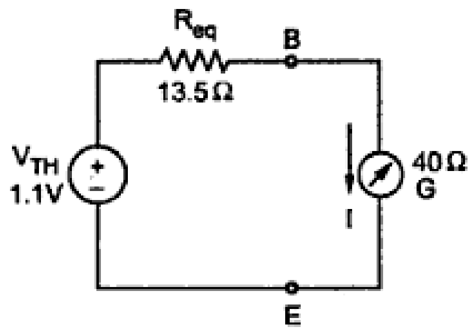


Fig. 2.95 (g)

Step 4 : Thevenin's equivalent is shown in the Fig. 2.95 (g).

Step 5 : Hence current through galvanometer is,

$$I = \frac{1.1}{13.5+40} = 0.02056 \text{ A}$$

$$= \mathbf{20.56 \text{ mA}}$$

Example 2.47 : *Find the current in 4 Ω resistance by Norton's theorem.*

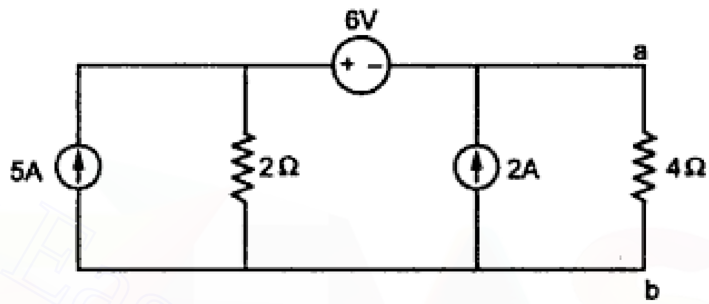


Fig. 2.96

Solution : Step 1 : Short the branch a-b.

Step 2 : Find the short circuit current I_N.

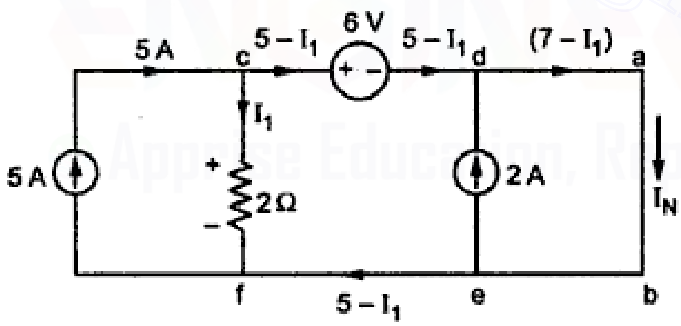


Fig. 2.96 (a)

From Fig. 2.96 (a),

$I_N = 7 - I_1$ A

Apply KVL to loop without any current source. i.e. cdabefc.

$-6 + 2I_1 = 0$

$\therefore I_1 = 3$ A

$\therefore I_N = 7 - 3$

$= \mathbf{4\ A} \downarrow$

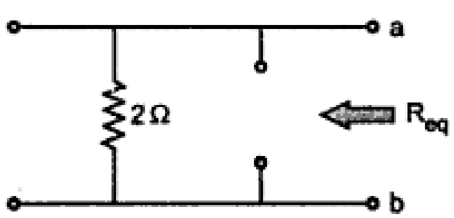


Fig. 2.96 (b)

Step 3 : Calculate R_{eq}, by opening current sources and shorting voltage source.

From the Fig. 2.96 (b)

$R_{eq} = 2\ \Omega$

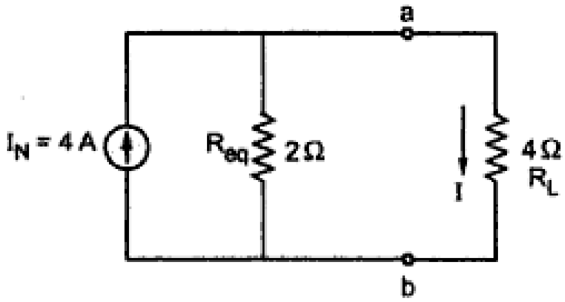


Fig. 2.96 (c)

Step 4 : Norton's equivalent across terminals a-b is as shown in the Fig. 2.96 (c).

Step 5 :

$$I = I_N \times \frac{R_{eq}}{R_L + R_{eq}}$$

$$= \frac{4 \times 2}{(2+4)}$$

$$= \mathbf{1.333\ A} \downarrow$$

Example 2.48 : *Find the magnitude of R_L for the maximum power transfer in the circuit shown in the Fig. 2.97. Also find out the maximum power.*

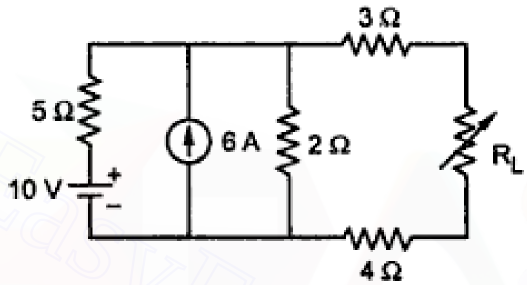


Fig. 2.97

Solution : Step 1 : Remove the load R_L.

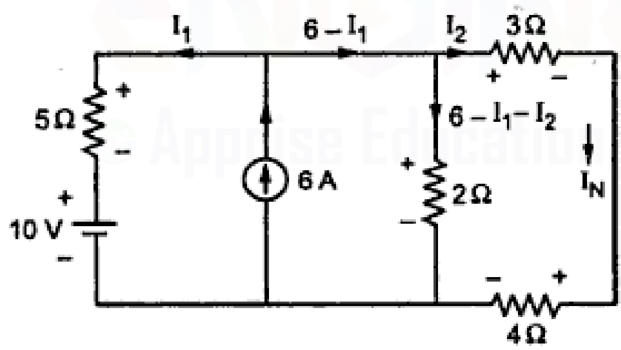


Fig. 2.97 (a)

Step 2 : Obtain V_{TH} or I_N by Kirchhoff's laws. Let us find I_N by shorting the load terminals.

Now $I_N = I_2$

Apply KVL to those loops which do not consist current source.

$$\therefore\ -2\,(6 - I_1 - I_2) + 10 + 5\,I_1 = 0$$

$$\therefore\ 7\,I_1 + 2\,I_2 = 2 \qquad \text{... (1)}$$

$$-3I_2 - 4I_2 + 2\,(6 - I_1 - I_2) = 0$$

$$\therefore\quad +2\,I_1 + 9\,I_2 = +12 \qquad \text{... (2)}$$

$$\therefore\quad D = \begin{vmatrix} 7 & 2 \\ 2 & 9 \end{vmatrix} = 59 \text{ and } D_2 = \begin{vmatrix} 7 & 2 \\ 2 & 12 \end{vmatrix} = 80$$

$$\therefore\quad I_N = I_2 = \frac{D_2}{D} = \frac{80}{59} = 1.3559\ \text{A}$$

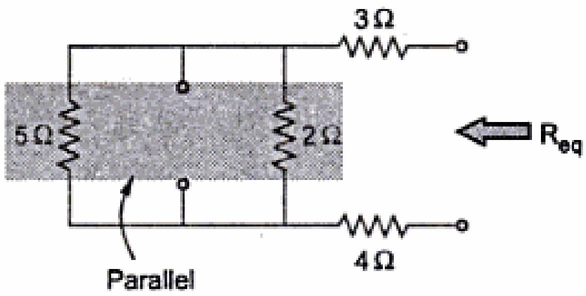


Fig. 2.97 (b)

Step 3 : Find R_{eq}, across load, opening current source and shorting voltage source.

$$\therefore \quad R_{eq} = 3 + (5 \parallel 2) + 4 = 3 + 1.4235 + 4 = \mathbf{8.4235\ \Omega}$$

Step 4 : For P_{max}, $R_L = R_{eq} = \mathbf{8.4235\ \Omega}$

And $\quad V_{TH} = I_N \times R_{eq} = 1.3559 \times 8.4235 = 11.4216\ V$

$$\therefore \quad P_{max} = \frac{V_{TH}^2}{4\,R_{eq}} = \frac{(11.4216)^2}{4 \times 8.4235} = \mathbf{3.8716\ W}$$

Example 2.49 : *Find the value R for maximum power to R and what is the value of maximum power ?*

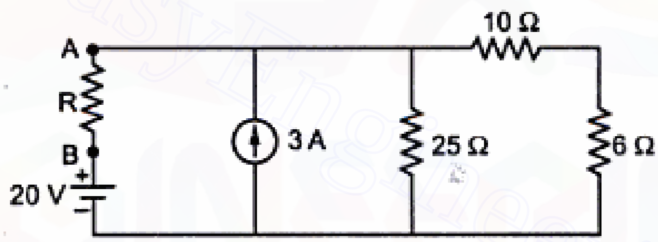


Fig. 2.98

Solution : Step 1 : Remove the resistance R.

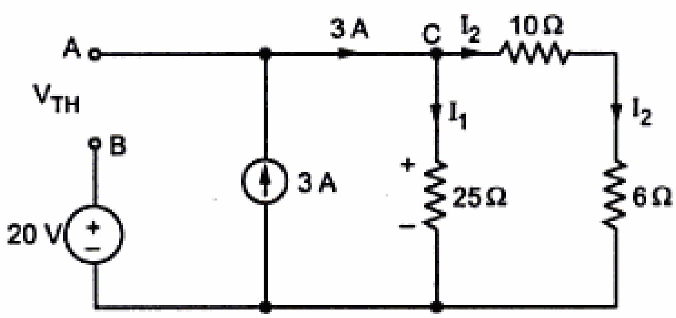


Fig. 2.98 (a)

Step 2 : Find the open circuit voltage V_{TH} across A-B, using Kirchhoff's laws.

I_1 and I_2 can be obtained by current division rule at node C.

$$I_1 = 3\,A \times \frac{16}{16+25} = 1.1707\ A$$

$$I_2 = 3\,A \times \frac{25}{16+25} = 1.8293\ A$$

$$\therefore \quad \text{Drop across } 25\ \Omega = 25\ I_1 = 29.2675\ V$$

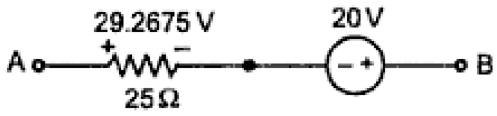


Fig. 2.98 (b)

Trace the path from A to B through 25Ω and show the various drops as shown in the Fig. 2.98 (b).

$$\therefore \quad V_{TH} = V_{AB} = 29.2675 - 20 = 9.2675 \text{ V with A positive}$$

Step 3 : To find R_{eq}, replace voltage source by short and current source by open circuit.

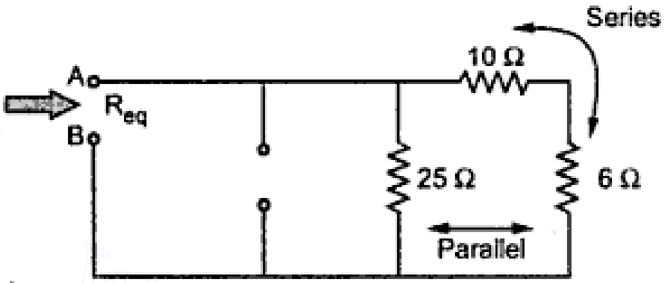


Fig. 2.98 (c)

$$\therefore R_{eq} = 25 \parallel 16 = \frac{25 \times 16}{25+16} = 9.756 \ \Omega$$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.98 (d).

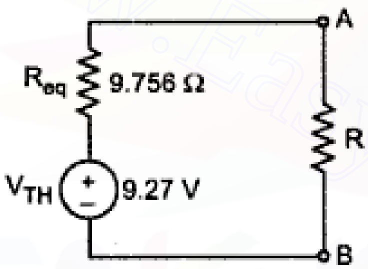


Fig. 2.98 (d)

Step 5 : For maximum power to R the value of R must be R.

$$\therefore \quad R = R_{eq} = 9.756 \ \Omega$$

$$\text{and } P_{max} = \frac{(V_{TH})^2}{4 R_{eq}} = \frac{(9.27)^2}{4 \times 9.756} = 2.202 \ \text{W}$$

Example 2.50 : *Find the current through the 30 Ω resistance by Thevenin's theorem.*

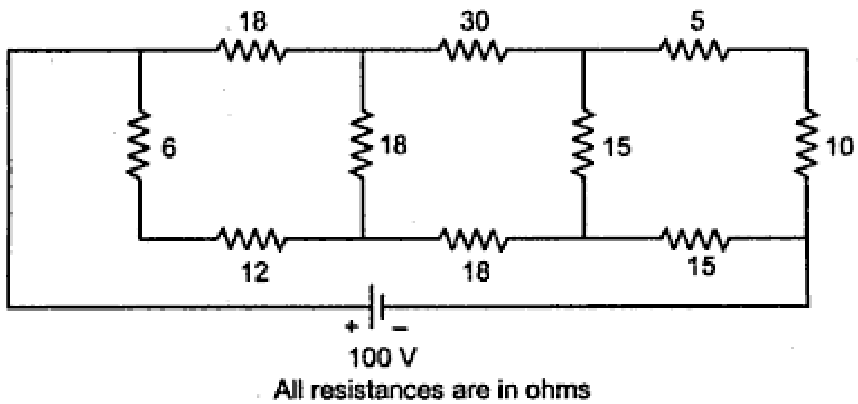


Fig. 2.99

Solution :

Step 1 : Remove 30 Ω resistance. Combine 6 and 12 Ω resistances as are in series. Similarly combine 5 and 10 Ω resistances which are in series.

Step 2 : Determine the voltage across terminals from where 30 Ω is removed.

Convert delta formed by 18 Ω to equivalent star. All resistances in delta are same so in star we get all resistances same as,

$$R = \frac{18 \times 18}{18+18+18} = 6\ \Omega$$

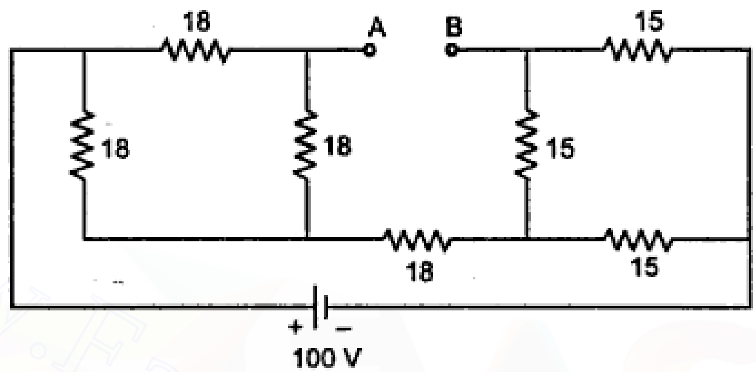


Fig. 2.99 (a)

Similarly convert delta of 15 Ω to equivalent star. We get all resistances in star equal as,

$$R = \frac{15 \times 15}{15+15+15} = 5\ \Omega$$

So replacing both delta by star we get the circuit as shown in the Fig. 2.99 (b).

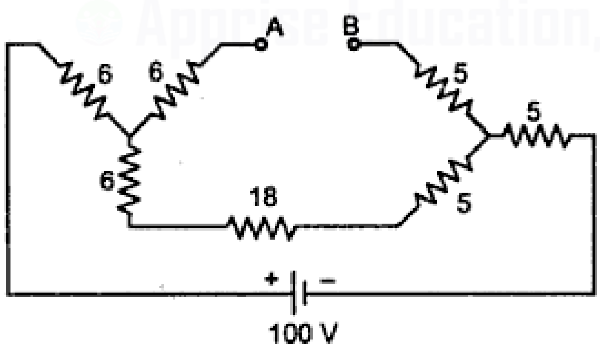


Fig. 2.99 (b)

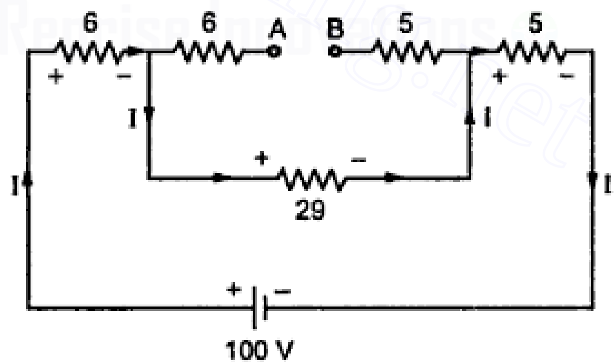


Fig. 2.99 (c)

From the Fig. 2.99 (c), the current I is,

$$I = \frac{100}{6+29+5} = 2.5\ A$$

$\therefore$ Drop across 29 Ω = 29 × I = 72.5 V

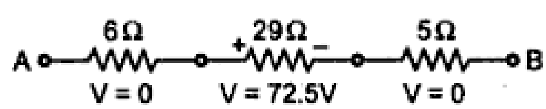


Fig. 2.99 (d)

As no current flows through 6 Ω and 5 Ω connected to A and B, the voltage V_{AB} is the drop across 29 Ω resistance.

$\therefore$ V_{AB} = 72.5 V = V_{TH} with A + ve w.r.t. B

Step 3 : Calculate R_{eq}, replacing voltage source by short circuit.

To calculate R_{eq}, use Fig. 2.99 (d) directly with voltage source shorted.

Rearranging we get the network as shown in the Fig. 2.99 (e).

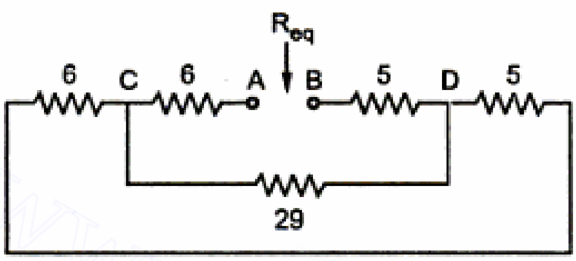


Fig. 2.99 (d)

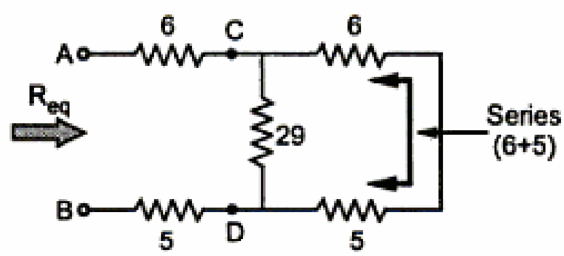


Fig. 2.99 (e)

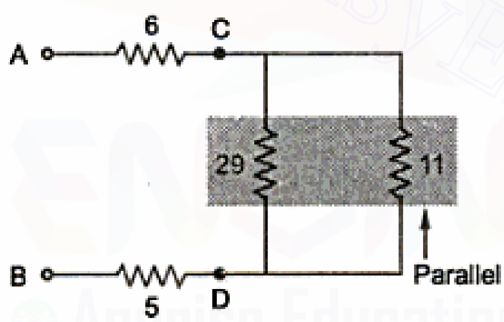


Fig. 2.99 (f)

$$\therefore R_{eq} = 6 + (29 \parallel 11) + 5 = 6 + \left(\frac{29 \times 11}{29+11}\right) + 5$$

$$= \mathbf{18.975\ \Omega}$$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.99 (g).

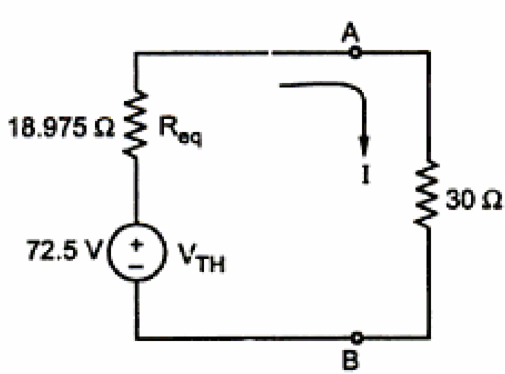


Fig. 2.99 (g)

Step 5 : So current through 30 Ω resistance now can be obtained as,

$$I_{30\Omega} = \frac{72.5}{18.975+30}$$

$$= \mathbf{1.4803\ A} \downarrow$$

Example 2.51 : *For the d.c. circuit shown in Fig. 2.100, write the Kirchhoff's law equations in the branch currents I_1, I_2 and I_3 as shown for loops ABGHA, BCFGB and CDEFC.*

Solve these equations to find current I_2. **[Dec-2003]**

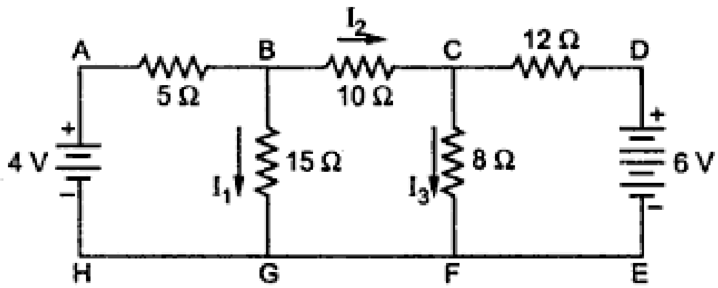


Fig. 2.100

Solution : The current distribution and voltage drops due to the branch currents is shown in the Fig. 2.100 (a). Applying KCL at node B, current through AB is $I_1 + I_2$.

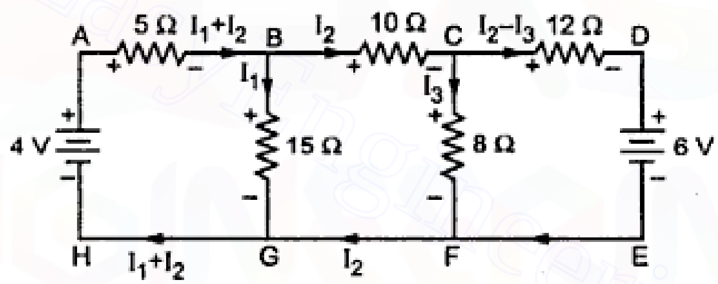


Fig. 2.100 (a)

Apply KVL with sign convention to the given loops.

Loop ABGHA, $\quad -5(I_1+I_2)-15I_1+4 = 0 \quad$ i.e. $20I_1+5I_2 = 4$ ... (1)

Loop BCFGB, $\quad -10I_2-8I_3+15I_1 = 0 \quad$ i.e. $15I_1-10I_2-8I_3 = 0$ ... (2)

Loop CDEFC, $\quad -12(I_2-I_3)-6+8I_3 = 0 \quad$ i.e. $-12I_2+20I_3 = 6$ ... (3)

Apply Cramer's rule to find I_2.

$$D = \begin{vmatrix} 20 & 5 & 0 \\ 15 & -10 & -8 \\ 0 & -12 & 20 \end{vmatrix} = -7420, \qquad D_2 = \begin{vmatrix} 20 & 4 & 0 \\ 15 & 0 & -8 \\ 0 & 6 & 20 \end{vmatrix} = -240$$

$$\therefore \quad I_2 = \frac{D_2}{D} = \frac{-240}{-7420} = \mathbf{32.345\ mA}$$

Example 2.52 : *Find, by Superposition theorem, the current I_3 in the 8 ohm resistance in the circuit shown in Fig. 2.100.* **[Dec-2003]**

Solution : Step 1 : Consider 4 V source alone, short 6 V source.

Applying KVL to the three loops,

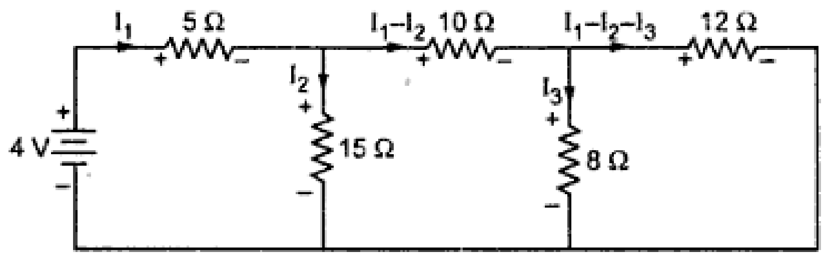


Fig. 2.101 (a)

Loop 1, $-5I_1-15I_2+4=0$ i.e. $5I_1+15I_2=4$ ... (1)

Loop 2, $-10(I_1-I_2)-8I_3+15I_2=0$ i.e. $-10I_1+25I_2-8I_3=0$ ... (2)

Loop 3, $-12(I_1-I_2-I_3)+8I_3=0$ i.e. $-12I_1+12I_2+20I_3=0$ ... (3)

$$\therefore \quad D = \begin{vmatrix} 5 & 15 & 0 \\ -10 & 25 & -8 \\ -12 & 12 & 20 \end{vmatrix} = 7420 \quad \text{and} \quad D_3 = \begin{vmatrix} 5 & 15 & 4 \\ -10 & 25 & 0 \\ -12 & 12 & 0 \end{vmatrix} = 720$$

$$\therefore \quad I_3 = \frac{D_3}{D} = 0.09703 \text{ A} \downarrow \quad \text{... current due to 4 V alone}$$

Step 2 : Consider 6 V source alone, short 4 V source.

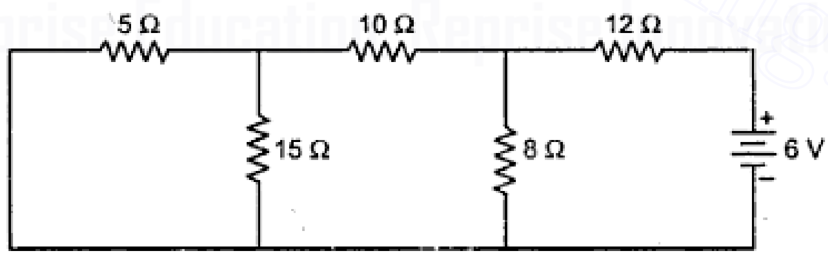


Fig. 2.101 (b)

Combine 5Ω and 15Ω in parallel (5||15) = 3.75 Ω

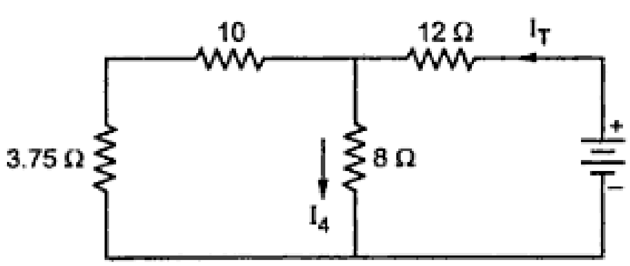


Fig. 2.101 (c)

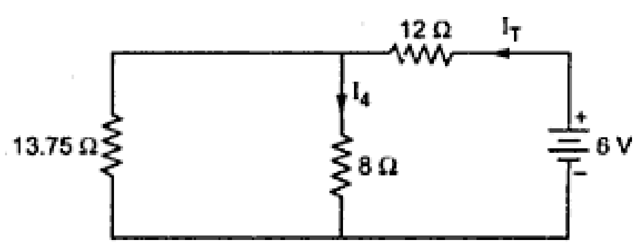


Fig. 2.101 (d)

Now $$I_T = \frac{6}{12+(13.75 \parallel 8)} = \frac{6}{12+5.0574} = 0.3517 \text{ A}$$

$$\therefore \quad I_4 = I_T \times \frac{13.75}{13.75+8} = 0.2223 \text{ A} \downarrow \quad \text{... Use of current division}$$

Thus total current through 8Ω due to both the sources is,

$$I_{8\Omega} = 0.09703 + 0.2223 = 0.31993 \text{ A} \downarrow$$

Example 2.53 : *Find the effective resistance across terminals M-N of the resistive network shown in Fig. 2.102.* **[Dec-2003]**

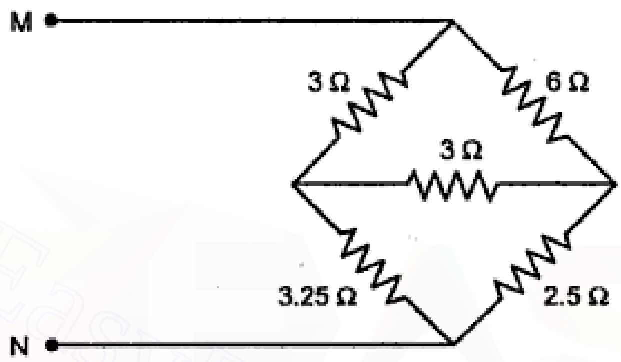


Fig. 2.102

Solution : Converting upper delta to star,

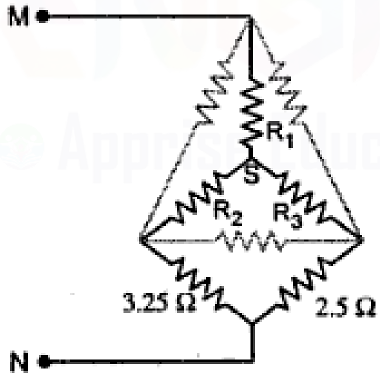


Fig. 2.102 (a)

$$R_1 = \frac{3\times 6}{3+3+6} = 1.5\ \Omega,$$

$$R_2 = \frac{3\times 3}{3+3+6} = 0.75\ \Omega,$$

$$R_3 = \frac{3\times 6}{3+3+6} = 1.5\ \Omega$$

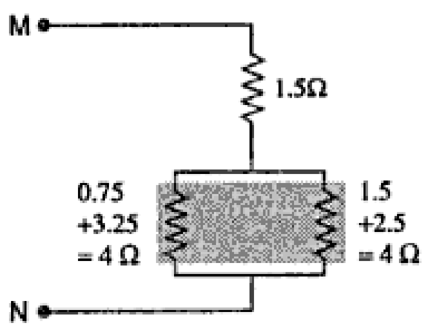


Fig. 2.102 (b)

$$\therefore \quad R_{MN} = 1.5 + (4 \| 4) = 1.5 + 2$$

$$= 3.5\ \Omega$$

Example 2.54 : *In the circuit shown in Fig. 2.103, find the source current by the method of simplification of network.* **[May-2004]**

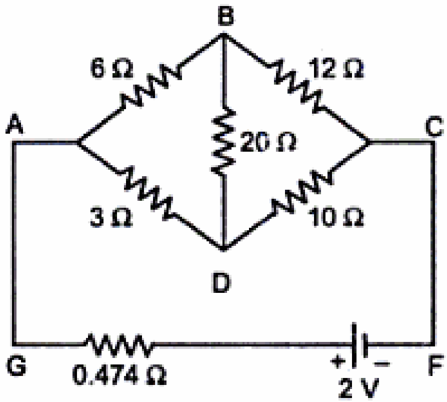


Fig. 2.103

Solution : Converting delta ABD to star we get,

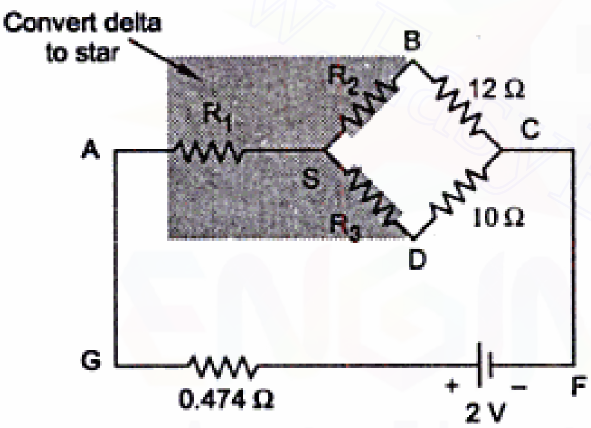


Fig. 2.103 (a)

$$R_1 = \frac{6\times 3}{6+3+20} = 0.6206\ \Omega$$

$$R_2 = \frac{6\times 20}{6+3+20} = 4.1379\ \Omega$$

$$R_3 = \frac{3\times 20}{6+3+20} = 2.0689\ \Omega$$

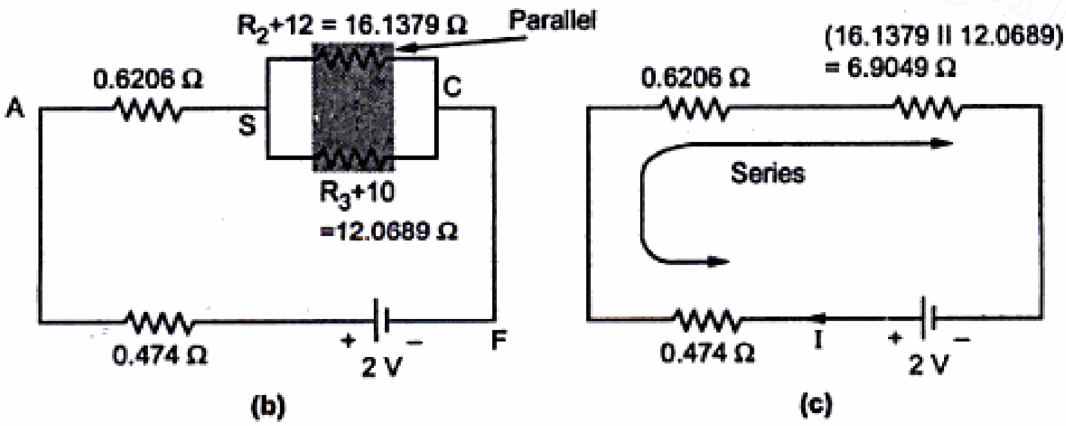


Fig. 2.103

$$\therefore \quad I = \frac{V_{total}}{R_{total}} = \frac{2}{0.6206+0.474+6.9049} = \mathbf{0.25\ A}$$

Example 2.55 : *For Fig. 2.104 shows a d.c. two-source network; the branch currents I_1 and I_2 are as marked in it. Write, using Kirchhoff's laws, two independent simultaneous equations in I_1 and I_2. Solve these to find I_1.* **[May-2004]**

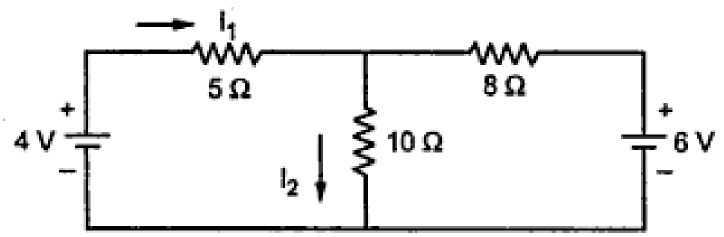


Fig. 2.104

Solution : The current distributions and voltage drops are shown in the Fig. 2.104 (a).

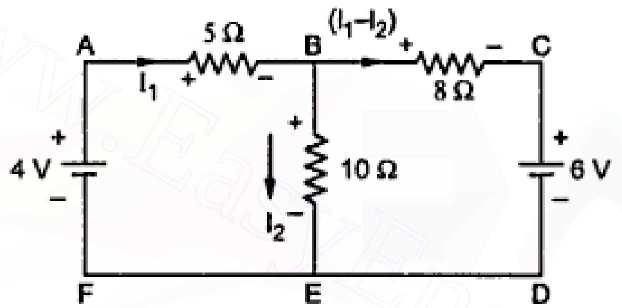


Fig. 2.104 (a)

Applying KCL at node B, current through BC is $(I_1 - I_2)$

Applying kVL to the two loops,

Loop ABEFA, $-5\,I_1 - 10\,I_2 + 4 = 0$ i.e. $I_1 + 2\,I_2 = 0.8$ ... (1)

Loop BCDEB, $-8(I_1 - I_2) - 6 + 10\,I_2 = 0$ i.e. $-8\,I_1 + 18\,I_2 = 6$ ... (2)

To solve for I_1, multiply (1) by 9 and subtract from (2),

$$\therefore \quad -17\,I_1 = 6 - (9 \times 0.8)$$

$$\therefore \quad \mathbf{I_1 = 0.0705\ A} \rightarrow$$

Example 2.56 : *Use Thevenin's theorem to find the current in the branch BD of the network shown in Fig. 2.105.* **[May-2004]**

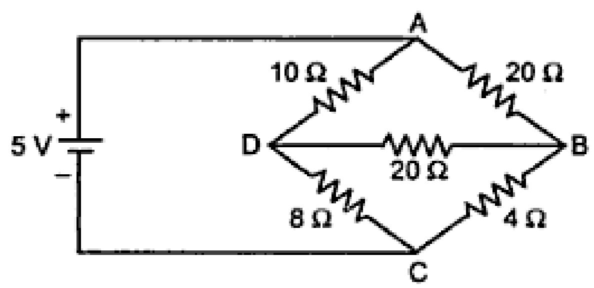


Fig. 2.105

Solution : For Thevenin's theorem.

Step 1 : Remove branch BD.

Step 2 : Find the open circuit voltage, $V_{TH} = V_{BD}$

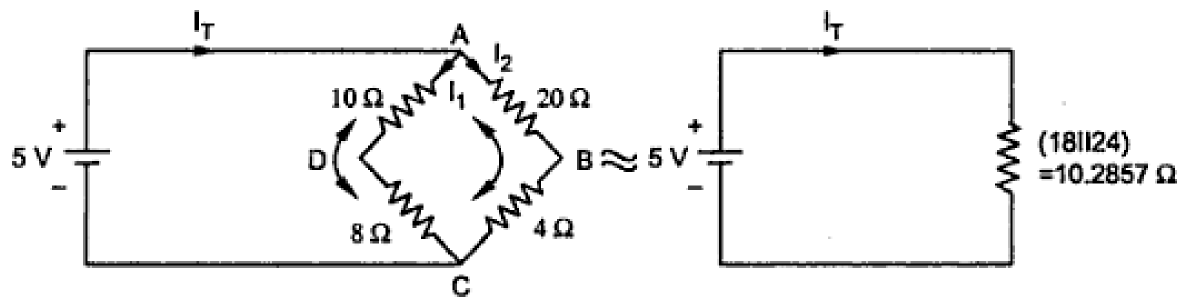


Fig. 2.105 (a)

$$I_T = \frac{5}{10.2857} = 0.48611 \text{ A}$$

Using current distribution,

$$I_1 = I_T \times \frac{24}{24+18} = 0.2778 \text{ A} \quad \text{and} \quad I_2 = I_T - I_1 = 0.20833 \text{ A}$$

The various drops due to I_1 and I_2 are as shown in the Fig. 2.105 (b). To find V_{BD}, trace the path BCD as shown as shown in the Fig. 2.105 (c).

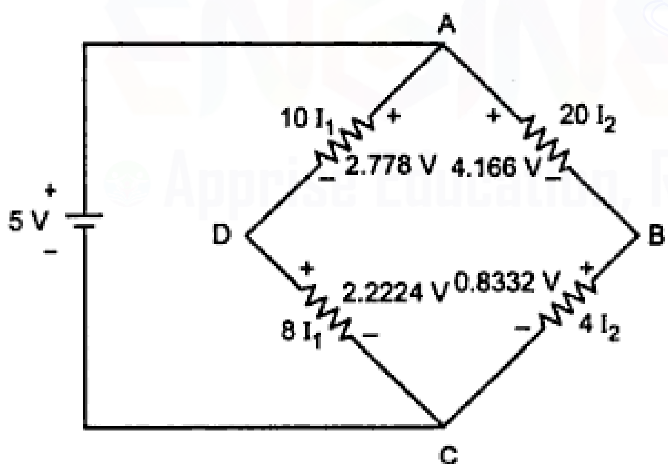


Fig. 2.105 (b)

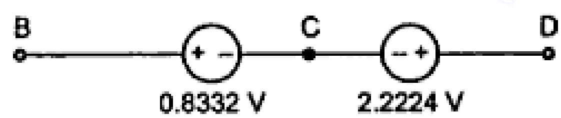


Fig. 2.105 (c)

$\therefore \quad V_{BD} = 2.2224 - 0.8332 = 1.3892 \text{ V}$ with B negative w.r.t. D.

$\therefore \quad V_{TH} = $ **1.3892 V with B negative**

Step 3 : Find R_{eq}, removing BD and shorting voltage source.

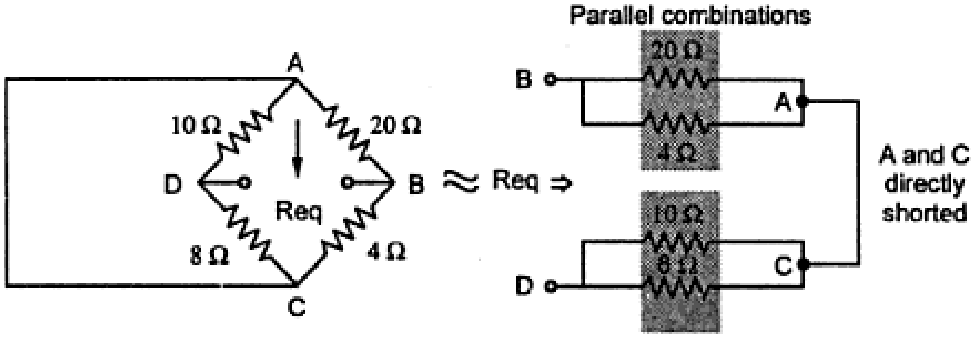


Fig. 2.105 (d)

$$\therefore \qquad R_{eq} = (20 \parallel 4)+(10 \parallel 8) = 3.333+4.444 = 7.7777\ \Omega$$

Step 4 : Thevenin's equivalent is shown.

Step 5 :

$$I = \frac{V_{TH}}{R_{eq}+20} = \frac{1.3892}{27.7777} = 0.05\ \text{A} \quad \text{from D to B}$$

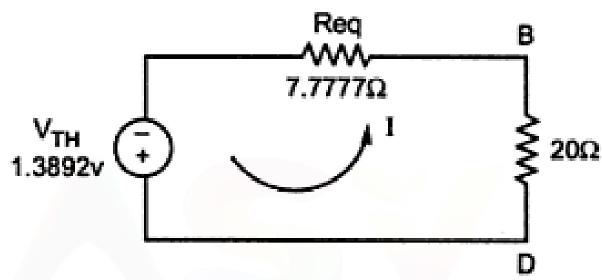


Fig. 2.105 (e)

Example 2.57 : *Write the Kirchhoff's voltage equations for the circuit shown in the Fig. 2.106 and hence find current flowing through 4 Ω resistance.* **(Dec. - 2004)**

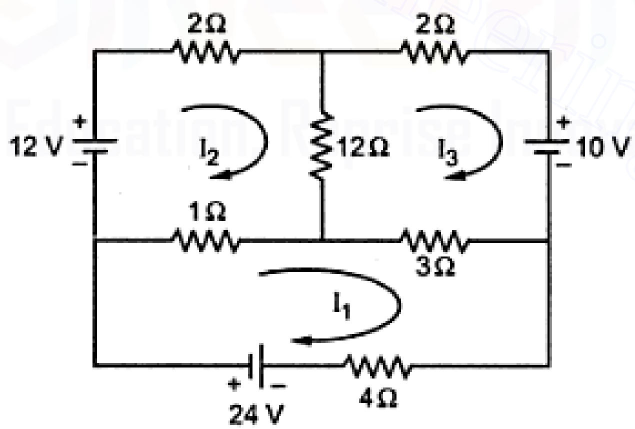


Fig. 2.106

Solution : The various branch currents are shown in the Fig. 2.106 (a).

Applying KVL to various loops,

Loop ABEDA, $-2I_2-12(I_2-I_3)+(I_1-I_2)+12 = 0$ i.e. $-I_1+15I_2-12I_3 = 12$ ... (1)

Loop BCFEB, $-2I_3-10+3(I_1-I_3)+12(I_2-I_3) = 0$ i.e. $3I_1+12I_2-17I_3 = 10$ ... (2)

Loop DEFHGD, $-(I_1-I_2)-3(I_1-I_3)-4I_1+24 = 0$ i.e. $8I_1-I_2-3I_3 = 24$ ... (3)

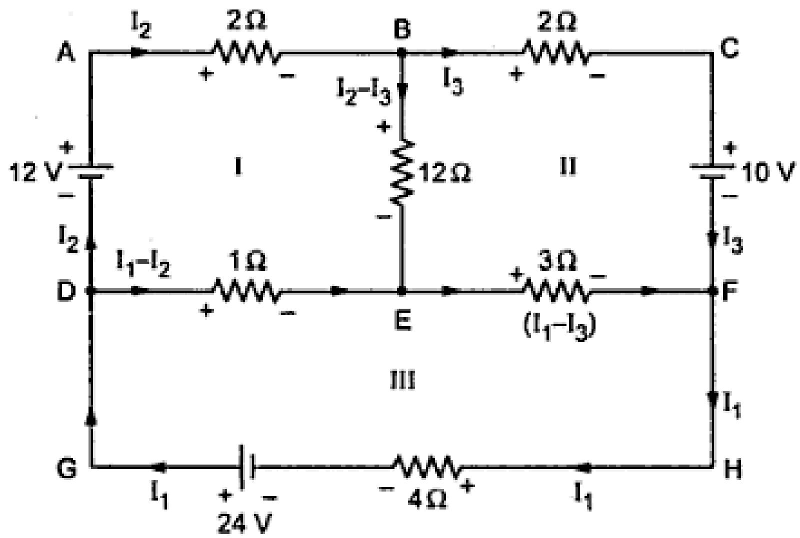


Fig. 2.106 (a)

To find current through 4Ω i.e. I_1.

$$\therefore \quad D = \begin{vmatrix} -1 & 15 & -12 \\ 3 & 12 & -17 \\ 8 & -1 & -3 \end{vmatrix} = -664, \quad D_1 = \begin{vmatrix} 12 & 15 & -12 \\ 10 & 12 & -17 \\ 24 & -1 & -3 \end{vmatrix} = -2730$$

$$\therefore \quad I_1 = \frac{D_1}{D} = \frac{-2730}{-664} = 4.111 \text{ A} \leftarrow \qquad \text{... Current through } 4\ \Omega$$

Example 2.58 : *For the network shown in the Fig. 2.107, find the current flowing through 5Ω resistance using Superposition theorem.* **(Dec. - 2004, May-2007)**

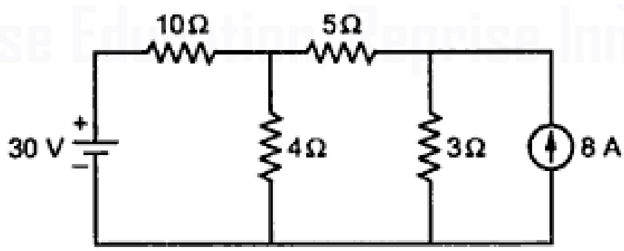


Fig. 2.107

Solution : Step 1 : Consider 30 V source alone, open 8 A source.

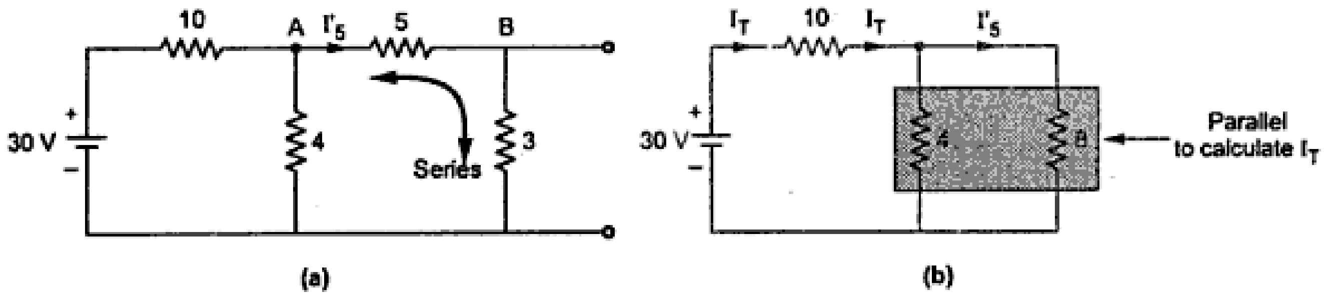


Fig. 2.107

$$I_T = \frac{30}{10+[4||8]} = \frac{30}{10+\left[\frac{4\times 8}{4+8}\right]} = \frac{30}{10+2.666} = 2.3684 \text{ A} \rightarrow$$

Using current division rule,

$$I_5' = I_T \times \frac{4}{4+8} = 2.3684 \times \frac{4}{12} = 0.78946 \text{ A} \rightarrow \quad \text{... Due to 30 V alone}$$

The direction is from A to B shown in the Fig. 2.107 (a).

Step 2 : Consider 8 A source alone, short 30 V source.

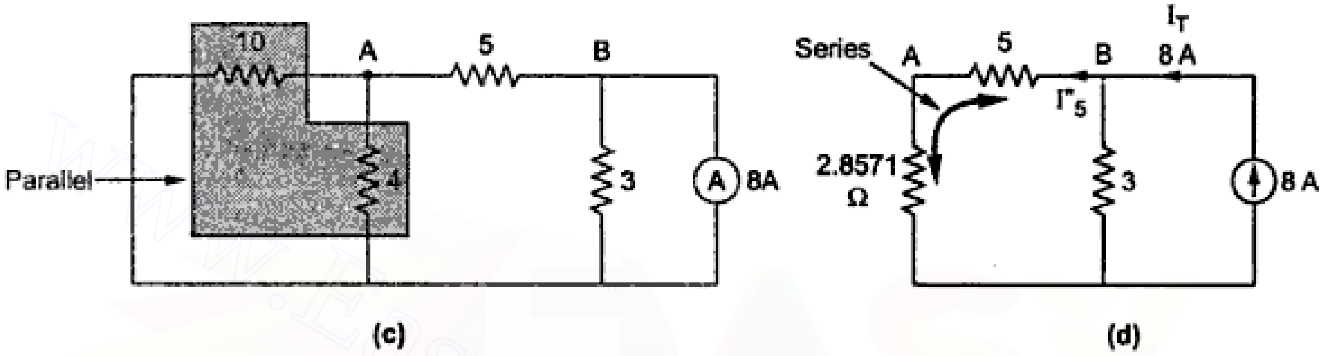


Fig. 2.107

Using current division rule,

$$I_5'' = 8 \times \frac{3}{5+2.8571+3} = 2.21053 \text{ A} \leftarrow \quad \text{... due to 8 A alone}$$

The direction is from B to A.

Step 3 : According superposition theorem.

$$I_5 = I_5' + I_5'' = 0.78946 \rightarrow + 2.21053 \leftarrow$$

$$= (2.21053 - 0.78946) \leftarrow = 1.42107 \text{ A} \leftarrow \text{from B to A}$$

Example 2.59 : *For the Fig. 2.108 of Ex. 2.58. find the current flowing through 5 Ω resistance using Thevenin's theorem.* (Dec. - 2004, May-2007)

Solution : Step 1 : Remove the branch of 5 Ω.

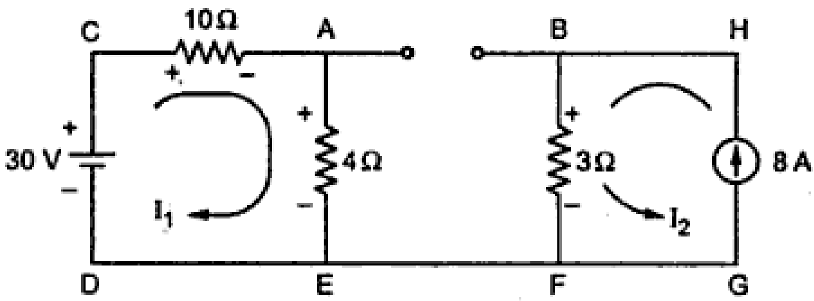


Fig. 2.108 (a)

Step 2 : Calculate the open circuit voltage across terminals AB from where 5 Ω is removed.

From loop CAEDC, $-10I_1 - 4I_1 + 30 = 0$ i.e. $I_1 = 2.1428$ A

From loop BHGFB, $I_2 = 8$ A ... as current source

∴ Drop across $4\ \Omega = I_1 \times 4 = 4 \times 2.1428 = 8.57142$ V

and Drop across $3\ \Omega = I_2 \times 3 = 8 \times 3 = 24$ V

Tracing the path from A to B as AEFB and arranging voltage drops as shown in the Fig. 2.108 (b)

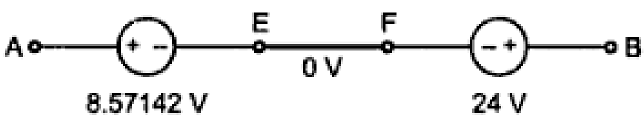


Fig. 2.108 (b)

$$\therefore \quad V_{AB} = 24 - 8.57142 = 15.4285 \text{ V with A –ve wrt B}$$

$$\therefore \quad V_{TH} = 15.4285 \text{ V with A –ve w.r.t. B}$$

Step 3 : Calculate R_{eq}, replacing voltage source by short and current source by open circuit.

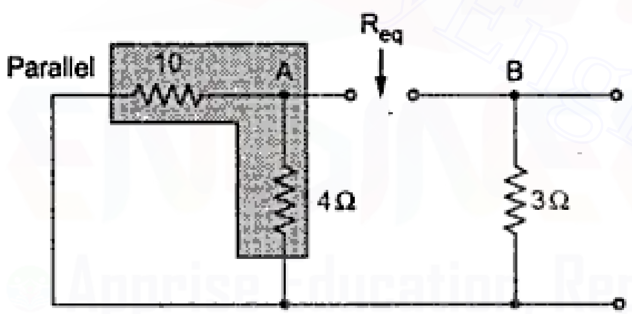


Fig. 2.108 (c)

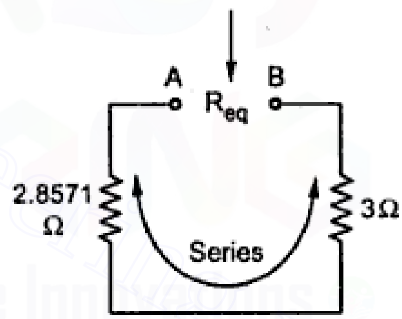


Fig. 2.108 (d)

$$\therefore \quad R_{eq} = R_{AB} = 2.8571 + 3 = 5.8571\ \Omega$$

Step 4 : The Thevenin's equivalent is shown in the Fig. 2.108 (e).

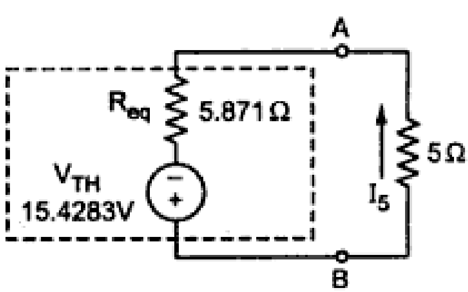


Fig. 2.108 (e)

Step 5 : Find the current through 5 Ω.

As polarity of V_{TH} is with A –ve, the current through 5 Ω flows from B to A.

$$I_{5\Omega} = \frac{V_{TH}}{R_{eq} + R_L} = \frac{15.4285}{5.8571 + 5}$$

$$= \mathbf{1.4210 \text{ A from B to A}}$$

This is same as obtained in earlier example.

Example 2.60 : *For the circuit shown in the Fig. 2.109, write the Kirchhoff's law equations for loops BCDB, CEDC and ABDEFA in terms of the branch currents I_1, I_2 and I_3 as shown. Find current I_1 by solving these equations.* **(May - 2005)**

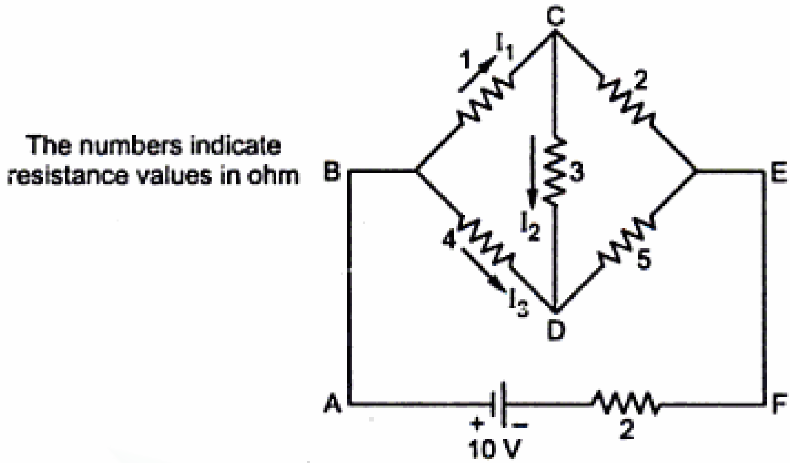


Fig. 2.109

Solution : The various other branch currents interms of the currents I_1, I_2, I_b and corresponding voltage drops are shown in the Fig. 2.109 (a).

> **Key Point :** *Apply KCL at various nodes and junction points to obtain current distribution.*

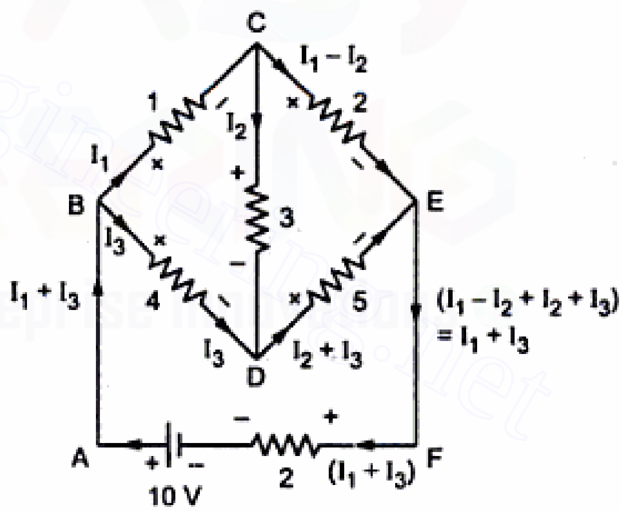


Fig. 2.109 (a)

Current entering at junction point B is $I_1 + I_3$.

Through branch CE it is $(I_1 - I_2)$.

Through branch DE it is $(I_2 + I_3)$.

Applying KVL to the various loops,

Loop BCDB, $-I_1 - 3I_2 + 4I_3 = 0$ ... (1)

Loop CEDC, $h - 2(I_1 - I_2) + 5(I_2 + I_3) + 3I_2 = 0$ i.e. $-2I_1 + 10I_2 + 5I_3 = 0$ ... (2)

Loop ABDEFA, $-4I_3 - 5(I_2 + I_3) - 2(I_1 + I_3) + 10 = 0$ i.e. $2I_1 + 5I_2 + 11I_3 = 10$ ... (3)

To find current I_1 using Cramer's rule,

$$D = \begin{vmatrix} -1 & -3 & 4 \\ -2 & 10 & 5 \\ 2 & 5 & 11 \end{vmatrix} = -301, \qquad D_1 = \begin{vmatrix} 0 & -3 & 4 \\ 0 & 10 & 5 \\ 10 & 5 & 11 \end{vmatrix} = -550$$

$$\therefore \qquad I_1 = \frac{D_1}{D} = \frac{-550}{-301} = 1.8272 \text{ A}$$

Key Point : *As answer is positive, assumed direction is correct, so I_1 flows from B to C.*

Example 2.61 : *For the network shown in the Fig. 2.110, find the current I_2 in the 3 ohm resistance, by applying Thevenin's Theorem.* **(May - 2005)**

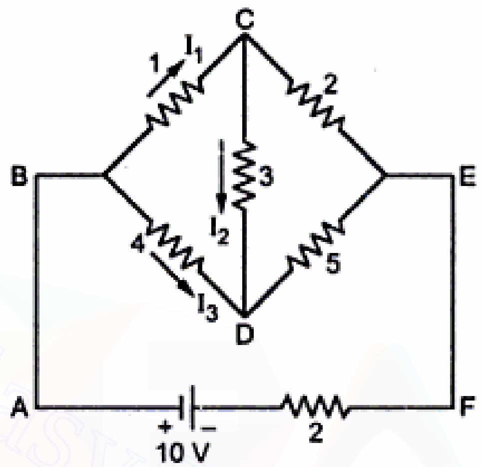


Fig. 2.110

Solution : Step 1 : Remove the branch through which I_2 is flowing i.e. CD.

Step 2 : Find the open circuit voltage across CD i.e. V_{TH}.

Let the branch currents are I_a and I_b. As CD is open, I_a flows from BD and DE while I_b flows from BC and CE.

Apply KVL to the two loops,

Loop BCEDB,

$$-I_b - 2I_b + 5I_a + 4I_a = 0$$

i.e. $\qquad 9I_a - 3I_b = 0 \qquad \dots (1)$

Loop ABDEFA,

$$-4I_a - 5I_a - 2(I_a + I_b) + 10 = 0$$

i.e. $\qquad 11I_a + 2I_b = 10 \qquad \dots (2)$

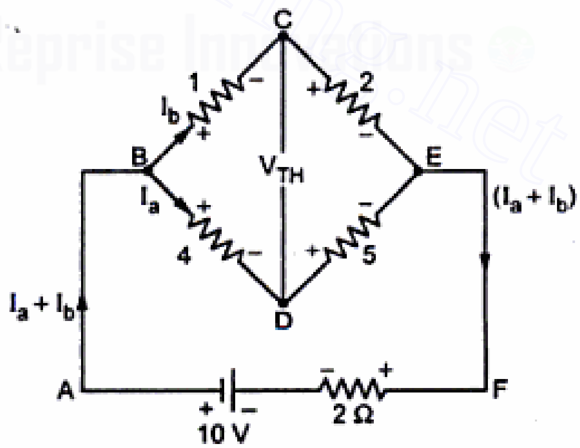


Fig. 2.110(a)

Solving (1) and (2), $\quad I_a = 0.5882$ A, $\quad I_b = 1.7647$ A

So, drop across BC = $I_b \times 1 = 1.7647$ V

drop across BD = $I_a \times 4 = 4 \times 0.5882 = 2.3528$ V

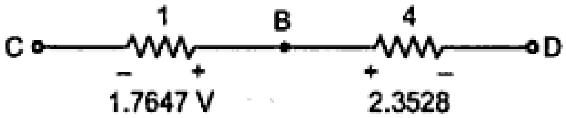


Fig. 2.110 (b)

To find V_{CD}, trace path from CBD and arrange all voltage drops as shown in the Fig. 2.110 (a).

$$\therefore \quad V_{CD} = 2.3528 - 1.7647 \quad \text{... as oppositive polarities}$$

$$= 0.5881 \text{ V with 'C' +ve w.r.t. 'D'}$$

$$\therefore \quad V_{TH} = V_{CD} = 0.5881 \text{ V with 'C' +ve w.r.t 'D'}$$

Step 3 : To find R_{eq}, replacing voltage source by short circuit.

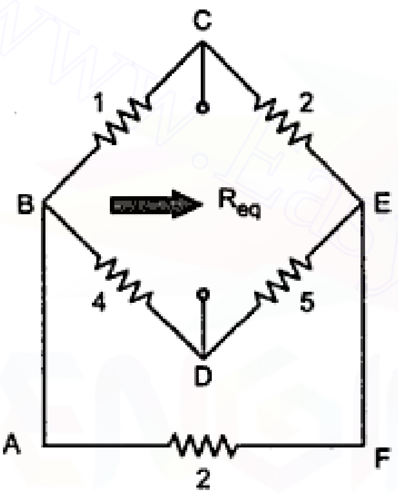


Fig. 2.110 (c)

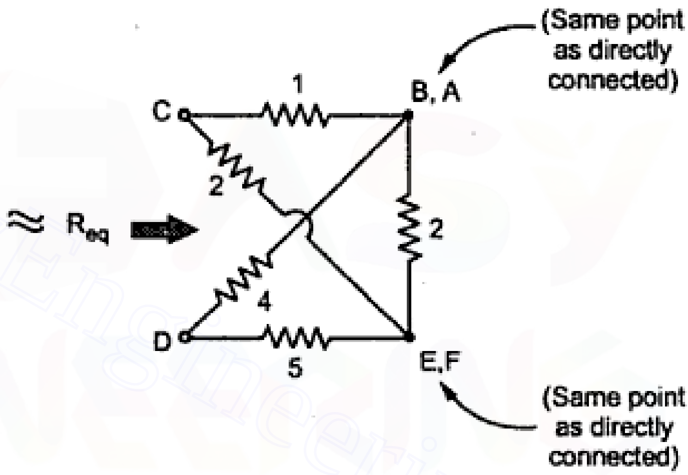


Fig. 2.110 (d)

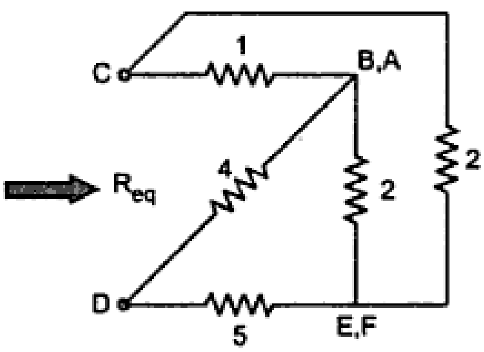


Fig. 2.110 (e)

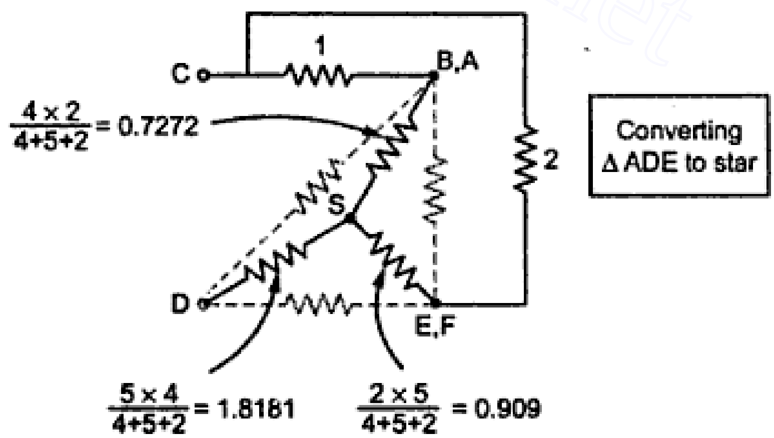


Fig. 2.110 (f)

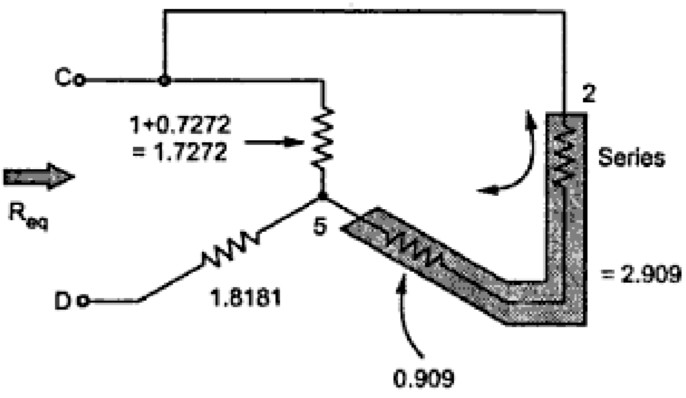


Fig. 2.110 (g)

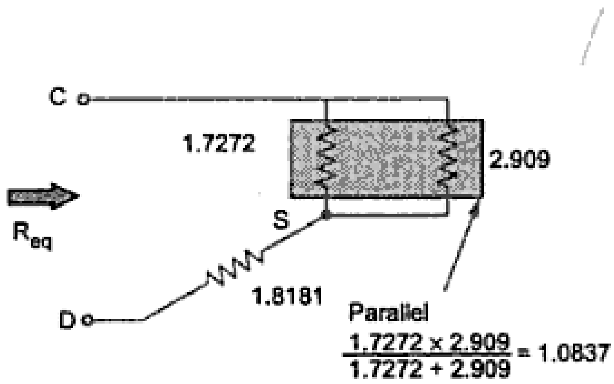


Fig. 2.110 (h)

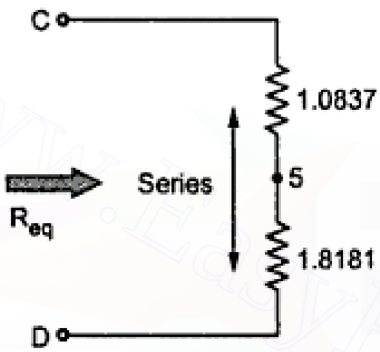


Fig. 2.110 (i)

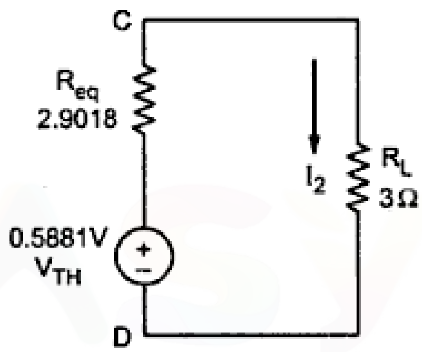


Fig. 2.110 (j)

$$\therefore \qquad R_{CD} = R_{eq} = 1.0837 + 1.8181 = 2.9018\ \Omega$$

Step 4 : The Thevenin's equivalent is shown in the Fig. 2.110 (i)

Step 5 : The current I_2 through 3 Ω resistance is,

$$\therefore \qquad I_2 = \frac{V_{TH}}{R_{eq} + R_L} = \frac{0.5881}{2.9018 + 3} = 0.0996\ \text{A} \downarrow$$

Example 2.62 : *Determine the current supplied by each battery in the circuit shown in Fig. 2.111 by using Kirchhoff's laws.* **[Dec.-2005]**

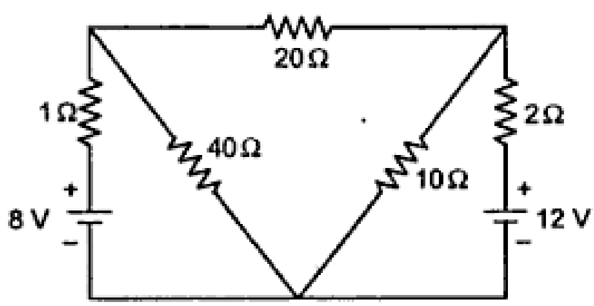


Fig. 2.111

Solution : The various branch currents are shown in the Fig. 2.111 (a).

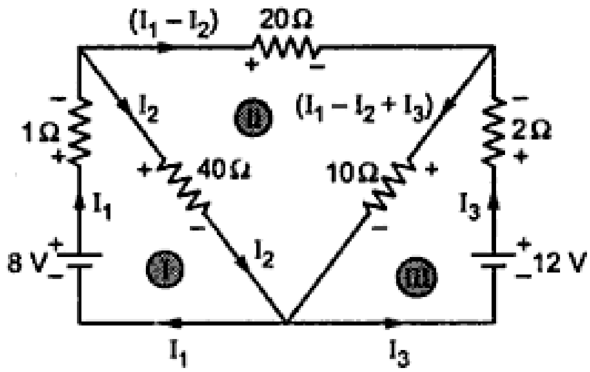


Fig. 2.111 (a)

Applying KVL to the three loops I, II and III,

$$-I_1 - 40\,I_2 + 8 = 0 \quad \text{i.e.} \quad I_1 + 40\,I_2 = 8 \qquad \ldots (1)$$

$$-20(I_1 - I_2) - 10(I_1 - I_2 + I_3) + 40\,I_2 = 0$$

i.e. $$-30\,I_1 + 70\,I_2 - 10\,I_3 = 0 \qquad \ldots (2)$$

$$+2I_3 - 12 + 10(I_1 - I_2 + I_3) = 0$$

i.e. $$10\,I_1 - 10\,I_2 + 12\,I_3 = 12 \qquad \ldots (3)$$

Solving (1), (2) and (3)

I_1 = **0.1005 A** ... current supplied by 8 V battery

I_3 = **1.0807 A** ... current supplied by 12 V battery

Example 2.63 : *Using Superposition theorem, calculate the current flowing in 1 Ω resistance for the network shown in Fig. 2.112.* **[8]**

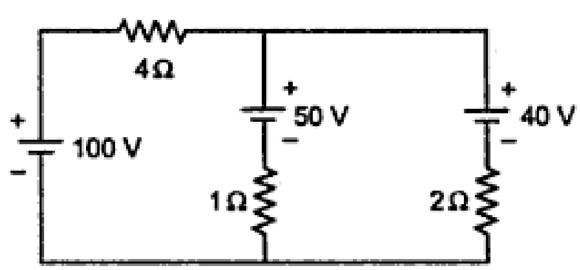
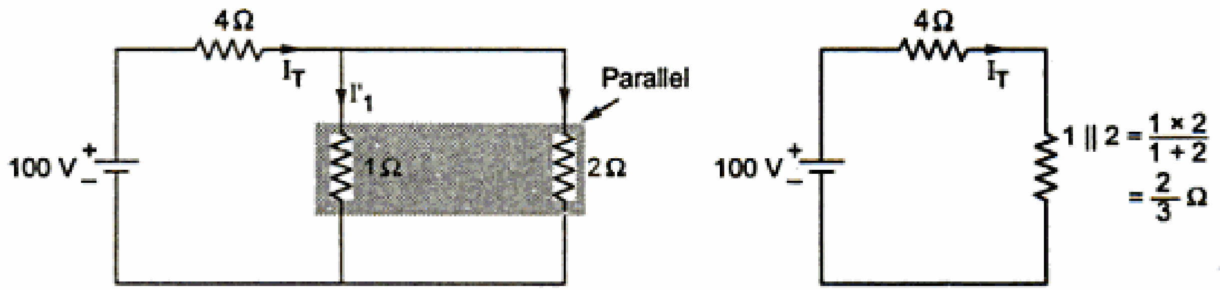


Fig. 2.112

Solution : **Step 1 :** Consider 100 V battery alone, short other sources.



(a) (b)

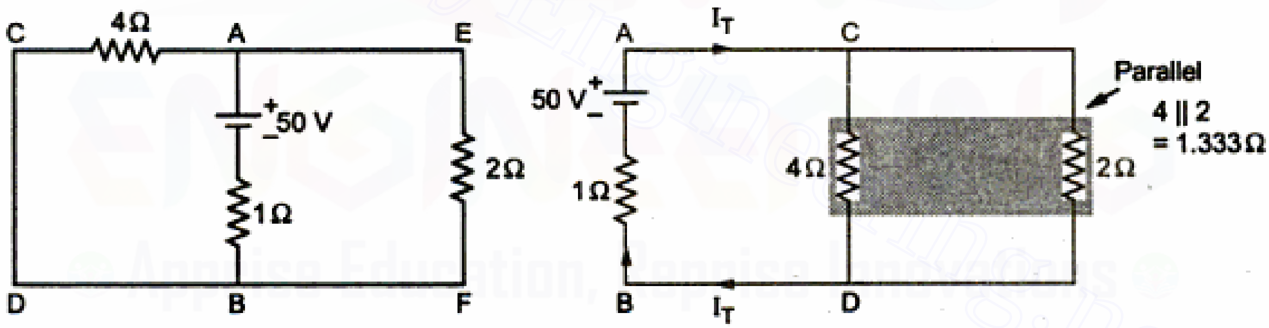
Fig. 2.112

$$\therefore \qquad I_T = \frac{100}{4+\frac{2}{3}} = 21.4285 \text{ A}$$

Using current division rule,

$$I'_1 = I_T \times \frac{2}{(2+1)} = 14.2857 \text{ A} \downarrow \qquad \text{... Due to 100 V alone}$$

Step 2 : Consider 50 V alone, shorting other sources.



(c) (d)

Fig. 2.112

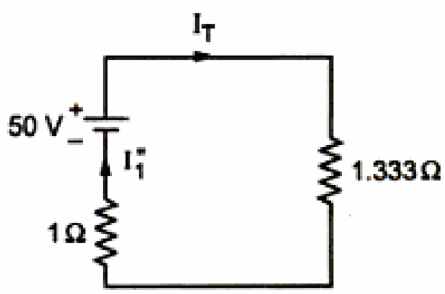


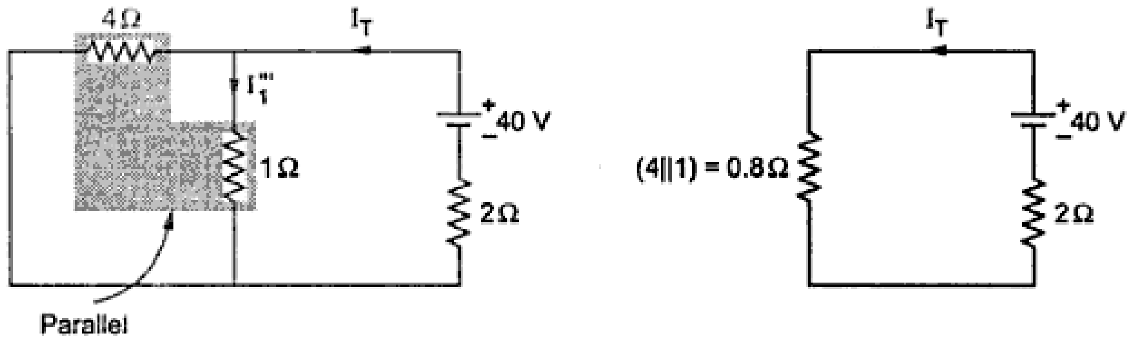
Fig. 2.112 (e)

$$I_T = \frac{50}{1+1.333} = 21.4316 \text{ A}$$

i.e. $\quad I''_1 = I_T = 21.4316 \text{ A} \uparrow$

... Due to 50 V alone

Step 3 : Consider 40 V alone, shorting other sources.



(f) (g)

Fig. 2.112

$$\therefore \quad I_T = \frac{40}{0.8+2} = 14.2857 \text{ A}$$

Using current division rule,

$$I_1''' = I_T \times \frac{4}{4+1} = 14.2857 \times \frac{4}{5} = \mathbf{11.4285 \text{ A}} \downarrow$$

... Due to 40 V alone

Using superposition principle,

$$I_{1\Omega} = I_1' + I_1'' + I_1''' = 14.2857 \text{ A} \downarrow + 21.4316 \text{ A} \uparrow + 11.4285 \text{ A} \downarrow$$

$$= \mathbf{4.2826 \text{ A}} \downarrow \quad \text{... Current through } 1\Omega$$

Explain 2.64 : *Use Kirchhoff's Law to find current supplied by the battery for the circuit shown in Fig. 2.113.* **[May-2006]**

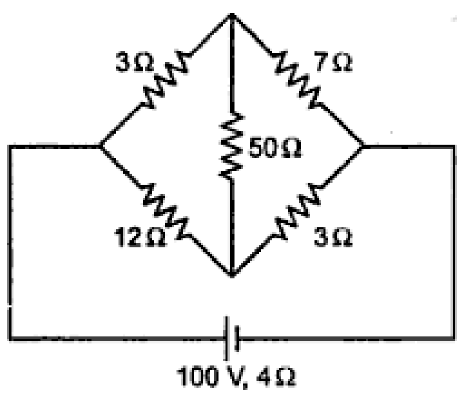


Fig. 2.113

Solution : The branch currents are shown in the Fig. 2.113 (a).

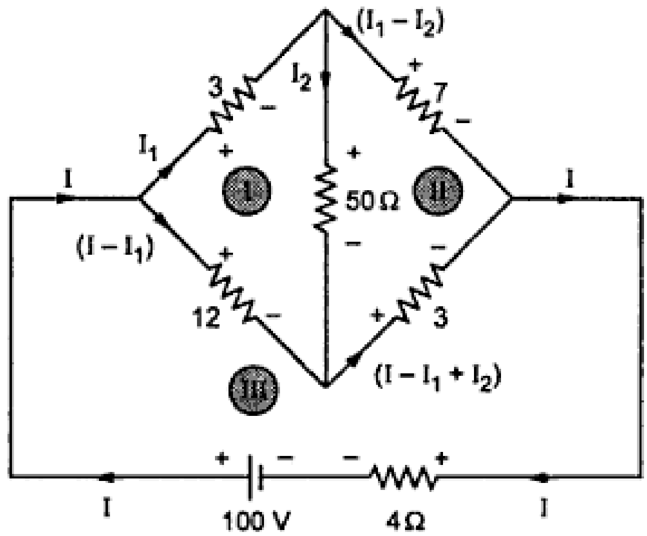


Fig. 2.113 (a)

Apply KVL to the three loops I, II and III.

$$-3I_1 - 50I_2 + 12(I - I_1) = 0 \quad \text{i.e.} \quad 12I - 15I_1 - 50I_2 = 0 \quad \text{... (1)}$$

$$-7(I_1 - I_2) + 3(I - I_1 + I_2) + 50I_2 = 0 \quad \text{i.e.} \quad 3I - 10I_1 + 60I_2 = 0 \quad \text{... (2)}$$

$$-12(I - I_1) - 3(I - I_1 + I_2) - 4I + 100 = 0 \quad \text{i.e.} \quad -19I + 15I_1 - 3I_2 = -100 \quad \text{... (3)}$$

Solving (1), (2) and (3),

$$I = 10.1634 \text{ A} \quad \text{... Current supplied by battery}$$

Example 2.65 : *Find current flowing through 3 Ω resistance by Superposition Theorem for the circuit shown in the Fig. 2.114.* **[May-2006]**

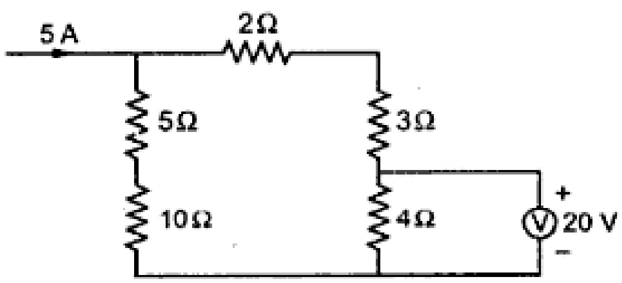


Fig. 2.114

Solution : **Step 1 :** Consider 5 A alone, short 20 V source.

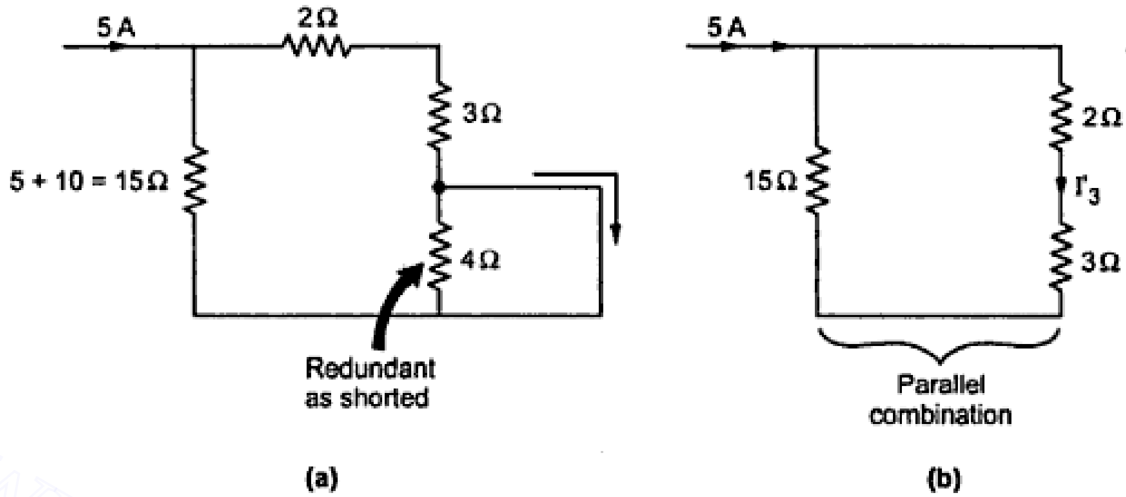


Fig. 2.114

Using current division rule,

$$I'_3 = 5 \times \frac{15}{15+2+3} = 3.75 \text{ A} \downarrow \qquad \text{... Due to 5 A alone}$$

Step 2 : Consider 20 V alone, open 5 A i.e. treat it zero.

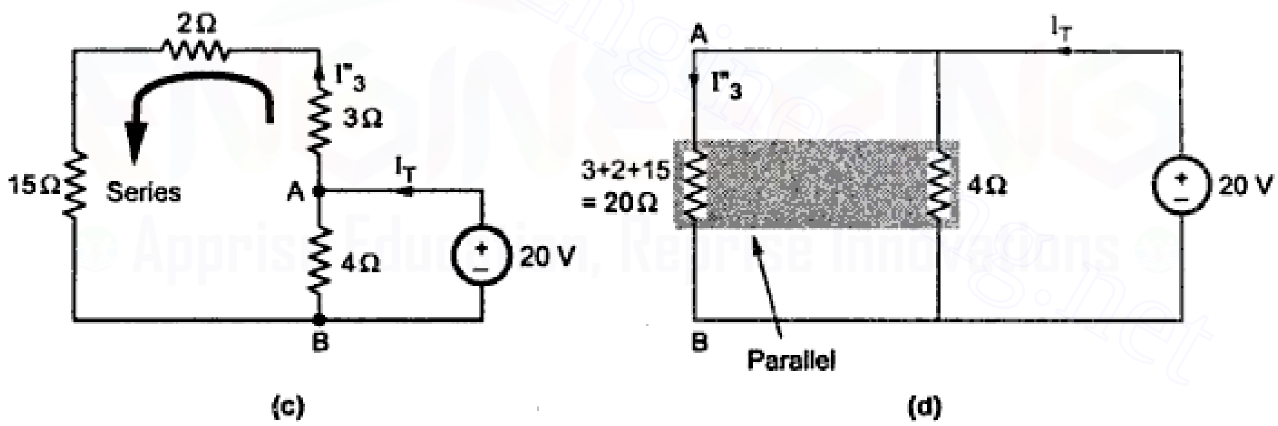


Fig. 2.114

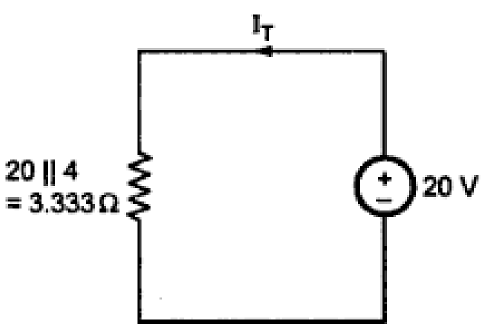


Fig. 2.114 (e)

$$\therefore I_T = \frac{20}{3.333} = 6 \text{ A}$$

Using current division rule,

$$I''_3 = 6 \times \frac{4}{20+4} = 1 \text{ A} \uparrow \qquad \text{... Due to 20 V alone}$$

$$\therefore I_{3\Omega} = 3.75 \text{ A} \downarrow + 1 \text{ A} \uparrow = 2.75 \text{ A} \downarrow$$

Example 2.66 : *Use Thevenin's Theorem to find current in 1 Ω resistance for the circuit shown in Fig. 2.115.* **[May-2006]**

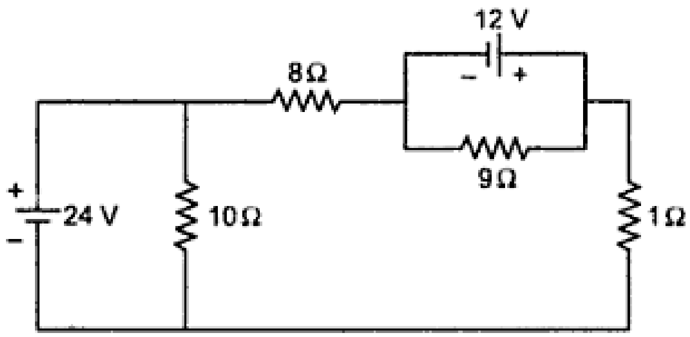


Fig. 2.115

Solution : Step 1 : Remove the 1Ω resistance.

Step 2 : Find open circuit voltage V_{TH}.

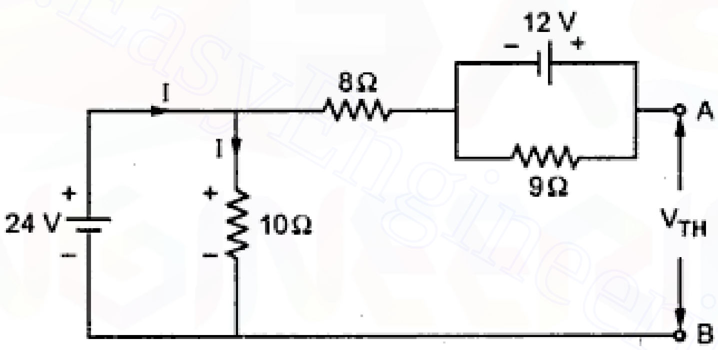


Fig. 2.115 (a)

$$I = \frac{24}{10} = 2.4 \text{ A} \quad \text{and drop across } 10\ \Omega \text{ is } 24 \text{ V.}$$

No current can flow through 8 Ω so drop across it is zero.

Tracing path from A to B as shown in the Fig. 2.115 (b).

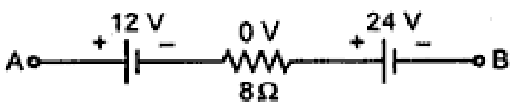
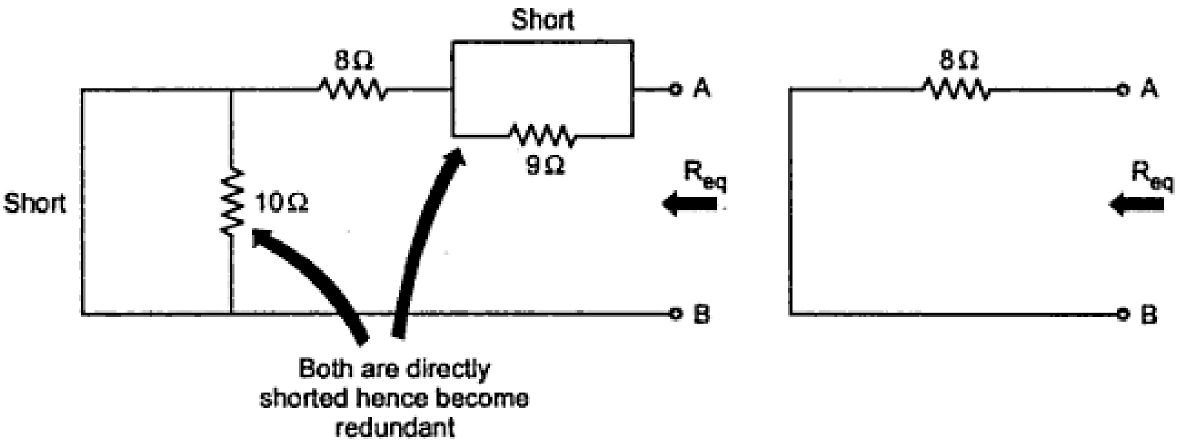


Fig. 2.115 (b)

$\therefore$ V_{TH} = 12 + 24 = 36 V with A positive

Step 3 : Find R_{eq}, shorting voltage sources.



(c) (d)

Fig. 2.115

$$\therefore \qquad R_{eq} = 8\,\Omega$$

Step 4 : The thevenin's equivalent is shown in the Fig. 2.115 (e)

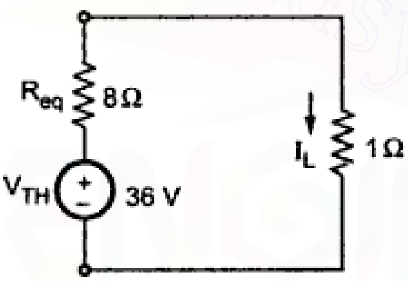


Fig. 2.115 (e)

Step 5 : $I_L = \dfrac{V_{TH}}{R_{eq}+1} = \dfrac{36}{8+1}$

$= 4\ \text{A} \downarrow$ ... Current through 1Ω

Example 2.67 : *Apply Thevenin's theorem to calculate current flowing in branch AB for the circuit shown in Fig. 2.116.* **[Dec.-2006]**

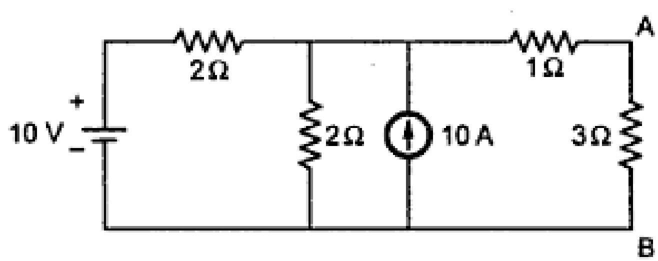


Fig. 2.116

Solution : Step 1 : Remove branch AB.

Step 2 : Calculate $V_{AB} = V_{TH}$, open circuit voltage.

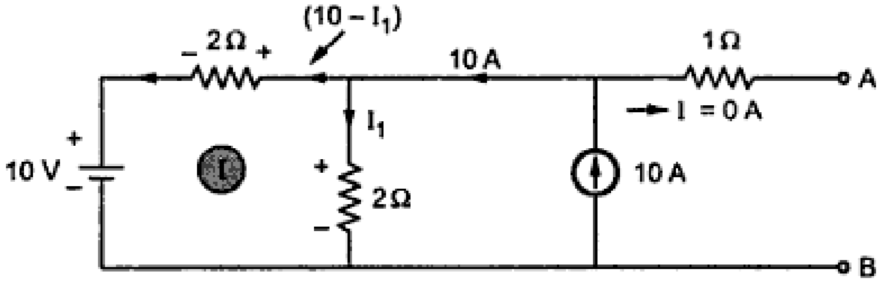


Fig. 2.116 (a)

Loop I : $-2(10-I_1)-10+2I_1 = 0$ i.e. $I_1 = 7.5$ A ... KVL

As 2 Ω and 10 A are in parallel across A-B,

$$V_{AB} = \text{Drop across } 2\Omega = 2\ I_1 = 2\times 7.5 = 15\ V$$

$$\therefore \quad V_{TH} = \mathbf{15\ V} \text{ with A positive}$$

Step 3 : Calculate R_{eq}, opening 10 A and shorting 10 V source.

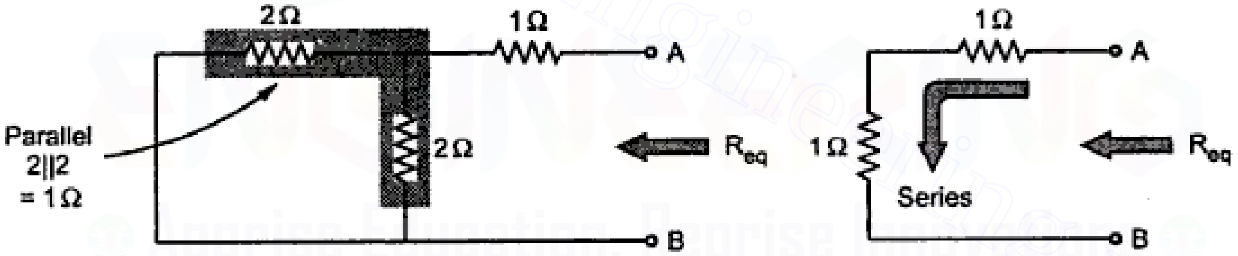


Fig. 2.116 (b)

$$\therefore \quad R_{eq} = 1+1 = 2\ \Omega$$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.116 (c).

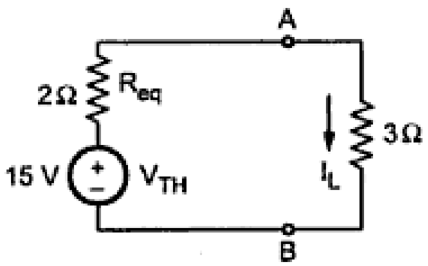


Fig. 2.116 (c)

Step 5 :

$$I_L = \frac{V_{TH}}{R_{eq}+R_L} = \frac{15}{2+3} = \mathbf{3\ A} \downarrow$$

Example 2.68 : *Determine the resistance between the terminals X and Y for the circuit shown in Fig. 2.117.* **[Dec.-2006]**

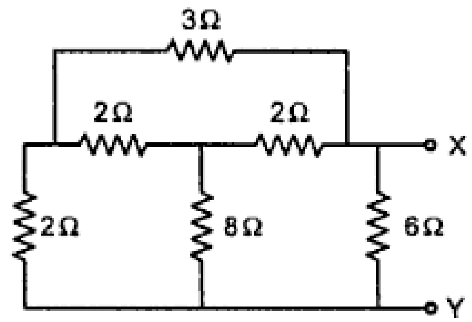
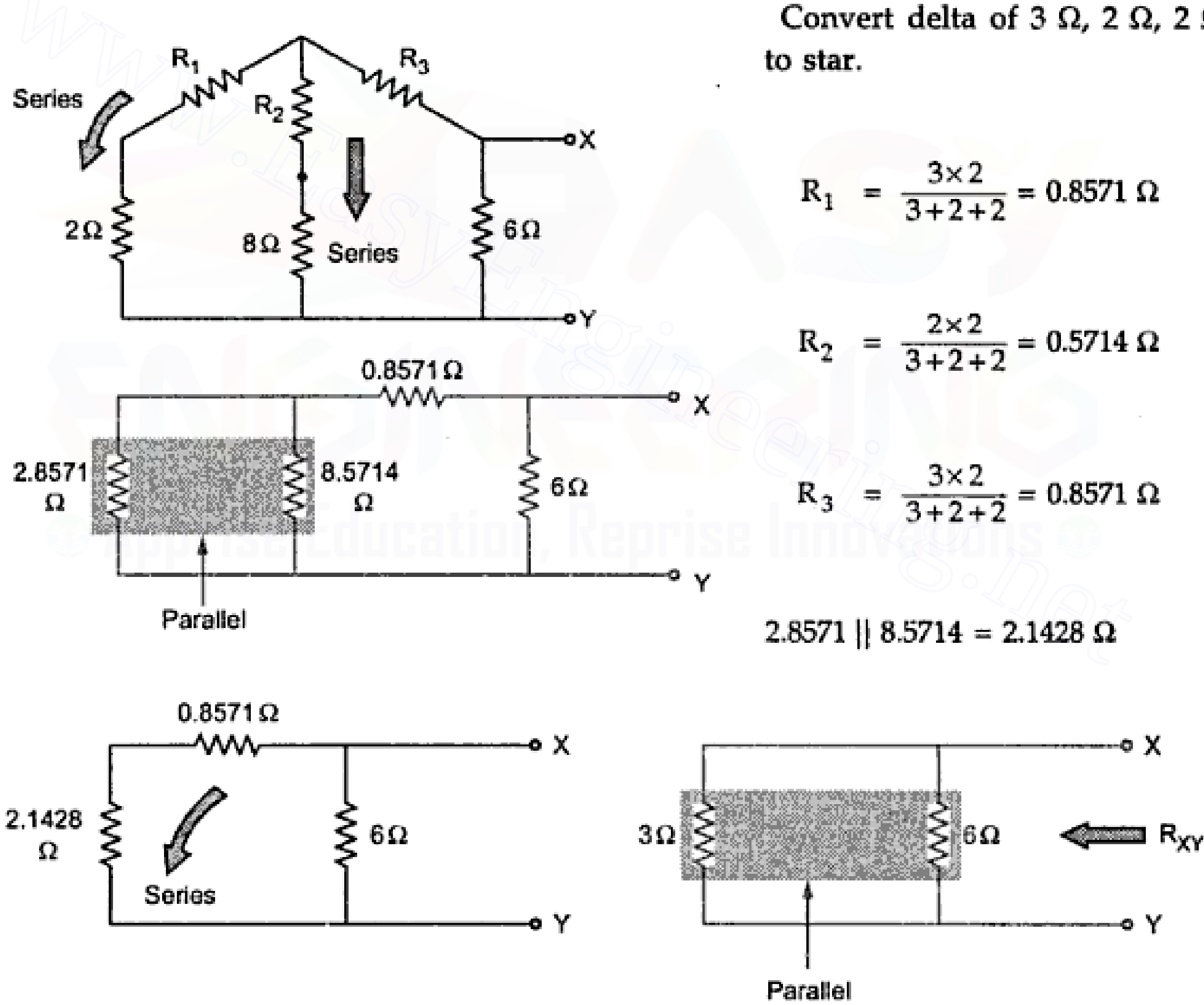


Fig. 2.117

Solution :

Convert delta of 3 Ω, 2 Ω, 2 Ω to star.



$$R_1 = \frac{3 \times 2}{3+2+2} = 0.8571\ \Omega$$

$$R_2 = \frac{2 \times 2}{3+2+2} = 0.5714\ \Omega$$

$$R_3 = \frac{3 \times 2}{3+2+2} = 0.8571\ \Omega$$

$$2.8571 \parallel 8.5714 = 2.1428\ \Omega$$

Fig. 2.117 (a)

$$\therefore \quad R_{XY} = 3 \parallel 6 = \frac{6 \times 3}{6+3} = 2\ \Omega$$

Example 2.69 : *Using source transformations, determine the voltage across 5 ohm resistance for the circuit shown in Fig. 2.118.* **[Dec.-2006]**

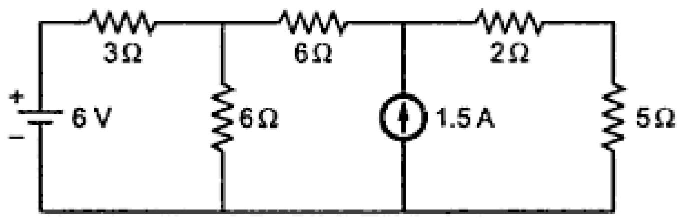


Fig. 2.118

Solution : Converting 6 V voltage source to current source.

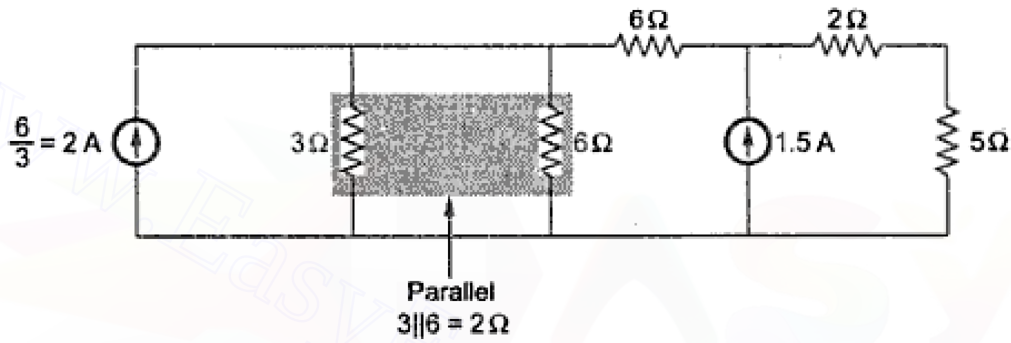
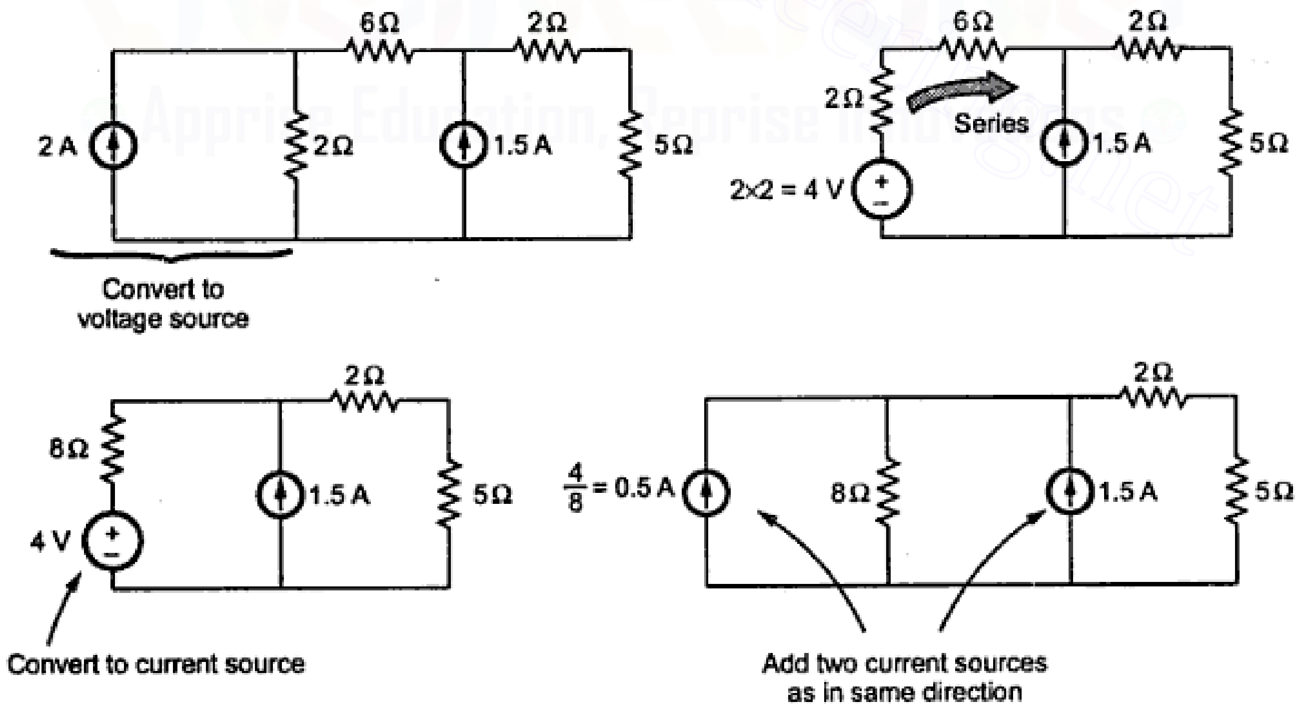


Fig. 2.118 (a)



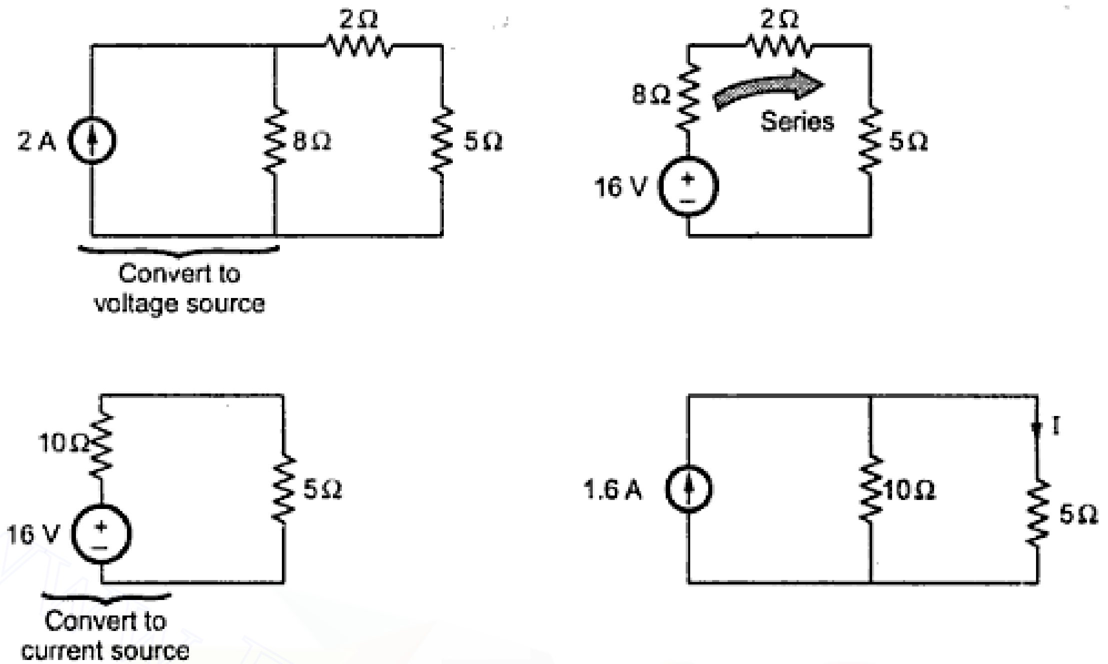


Fig. 2.118 (b)

Using current division rule,

$$I = 1.6 \times \frac{10}{5+10} = 1.0667 \text{ A}$$

$$\therefore \quad V_{5\Omega} = 5 \times I = 5.333 \text{ V}$$

➡ **Example 2.70 :** *Using Kirchhoff's laws, find the current flowing in 2 ohm resistance for the circuit shown in Fig. 2.119.* **[May-2007]**

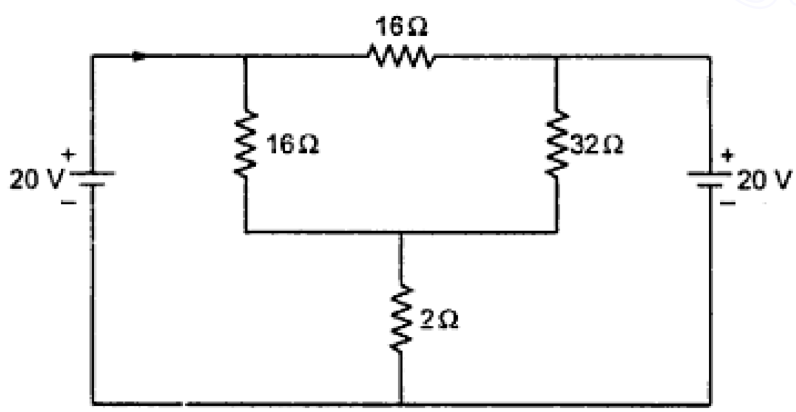


Fig. 2.119

Ans. : The various branch currents are as shown.

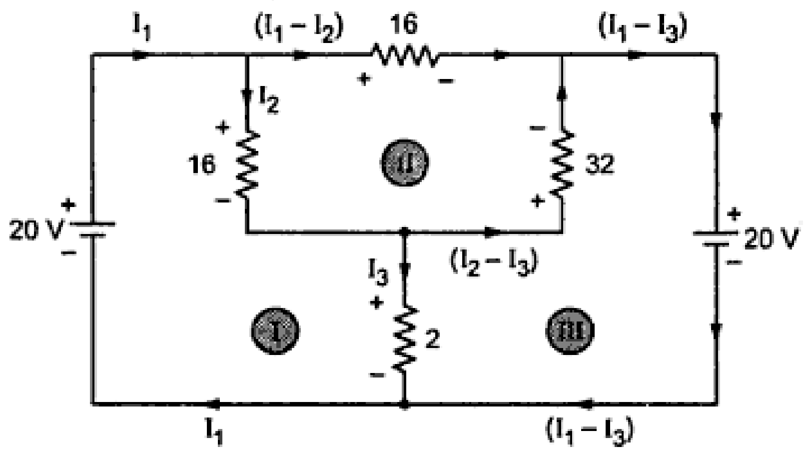


Fig. 2.119 (a)

Loop I : $-16\,I_2 - 2I_3 + 20 = 0$ i.e. $+16\,I_2 + 2I_3 = 20$ ...(1)

Loop II : $-16(I_1 - I_2) + 32(I_2 - I_3) + 16I_2 = 0$ i.e. $-16\,I_1 + 64I_2 - 32I_3 = 0$ ...(2)

Loop III : $-20 + 2I_3 - 32(I_2 - I_3) = 0$ i.e. $-32I_2 + 34I_3 = 20$ ...(3)

Solving for I_3, $I_3 = 1.5789$ A $\downarrow$ ... Use Cramer's rule

Example 2.71 : *Apply Thevenin's theorem to calculate the current in 6 Ω resistance for the circuit shown in the Fig. 2.120. Also use superposition theorem to calculate the current in 10 Ω resistance.* (Dec. - 2007)

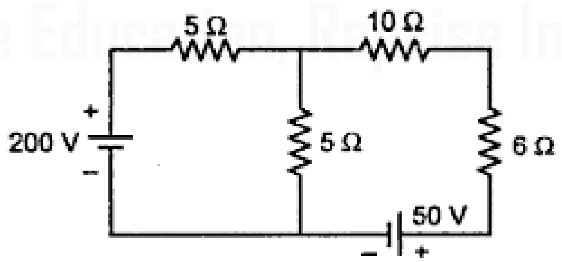


Fig. 2.120

Soution : Use of Thevenin's theorem to find $I_{6\,\Omega}$

Step 1 : Remove 6 Ω branch.

Step 2 : Calculate V_{TH} across open circuit

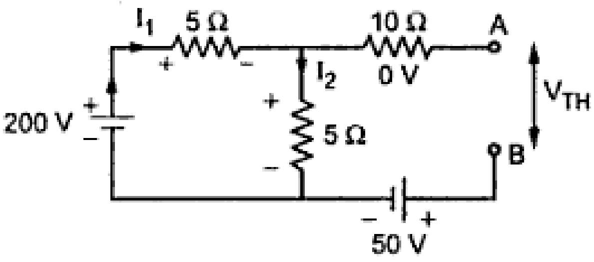


Fig. 2.120 (a)

Applying KVL to the loop,

$$-5I_1 - 5I_1 + 200 = 0$$

$$\therefore \quad I_1 = 20 \text{ A}$$

$\therefore$Drop across 5 Ω = 100 V

Trace the path from a to B as shown in the Fig. 2.120 (b).

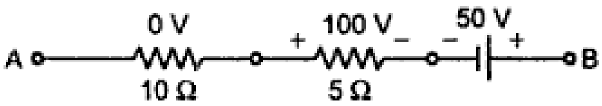


Fig. 2.120 (b)

$$\therefore \quad V_{TH} = V_{AB} = 100 - 50$$

$$= 50 \text{ V with A positive}$$

Step 3 : Calculate R_{eq}, shorting voltage soures.

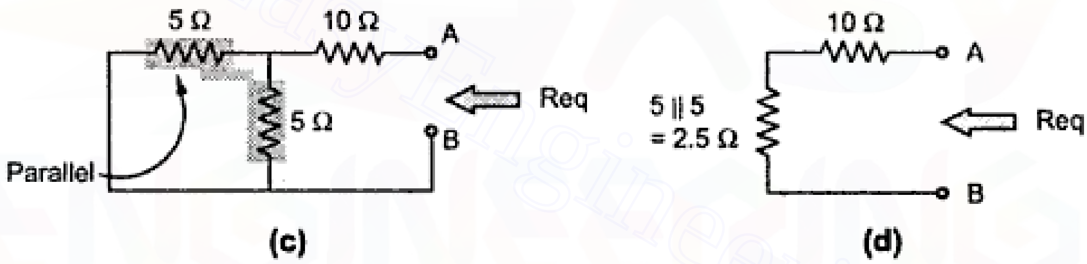


Fig. 2.120

$$\therefore \quad R_{eq} = 10 + 2.5 = 12.5 \ \Omega$$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.120 (e).

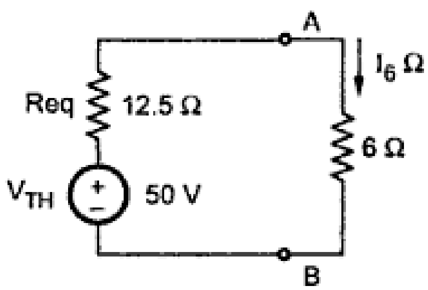


Fig. 2.120 (e)

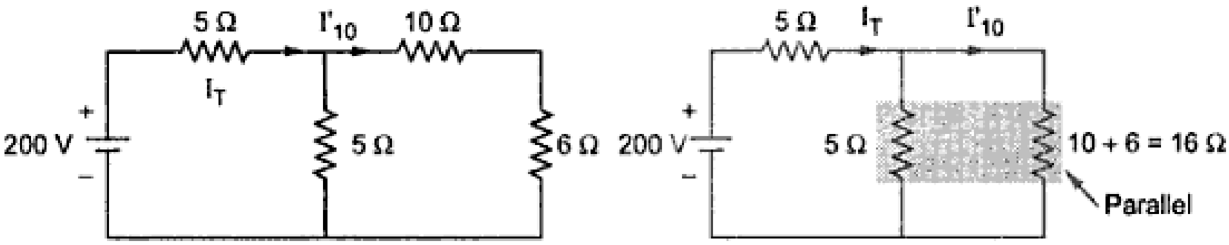
Step 5 : Hence the current through 6Ω is,

$$I_{6\,\Omega} = \frac{V_{TH}}{R_{eq}+6} = \frac{50}{12.5+6}$$

$$= 2.7027 \text{ A} \downarrow$$

Use of superposition theorem to find $I_{10\,\Omega}$.

Step 1 : Consider 200 V source alone, shorting 50 V source.



(f) (g)

Fig. 2.120

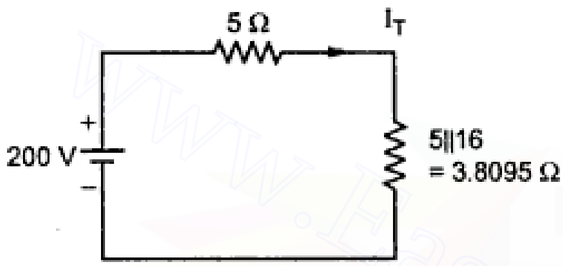


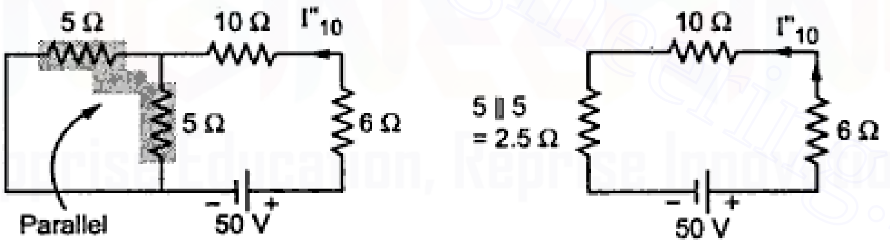
Fig. 2.120 (h)

$$\therefore \qquad I_T = \frac{200}{5+3.8095} = 22.7027 \text{ A}$$

Using current division rule,

$$I'_{10} = I_T \times \frac{5}{5+16} = \mathbf{5.4054\ A} \rightarrow$$

Step 2 : Consider 50 V source, short 20 V source.



(i) (j)

Fig. 2.120

$$\therefore \qquad I''_{10} = \frac{50}{10+6+2.5} = \mathbf{2.7027\ A} \leftarrow$$

Step 3 : $I_{10\Omega} = I'_{10} + I''_{10} = 5.4054 \rightarrow 2.7027 \leftarrow = \mathbf{2.7027\ A} \rightarrow$

Note that 10 Ω and 6 Ω are in series, hence carry same current hence the answer through 6 Ω by Thevenin's theorem and through 10 Ω by superposition theorem, matches with each other.

Example 2.72 : *Calculate the current delivered by each of the voltage source for the circuit shown in Fig. 2.121.* **(Dec.-2007)**

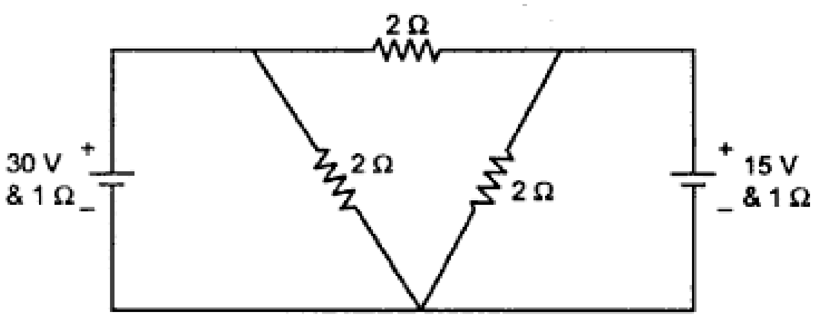


Fig. 2.121

Solution : 1 Ω and 1 Ω of each voltage source are the internal resistances and appear in series with the respective source in the circuit.

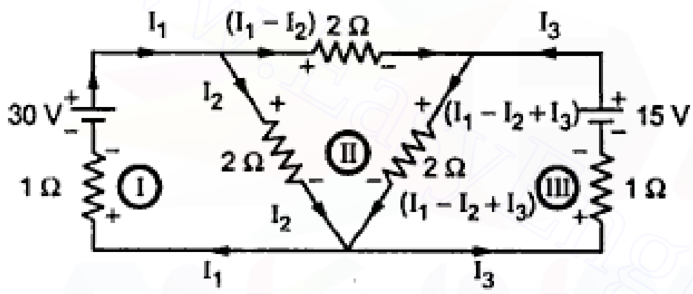


Fig. 2.121 (a)

The current distribution using KCL at the various nodes is shown in the Fig. 2.121 (a).

Applying KVL to the three loops I, II and III.

$$-2I_2 - I_1 + 30 = 0 \quad \text{i.e.} \quad I_1 + 2I_2 = 30 \quad \quad ...(1)$$

$$-2(I_1 - I_2) - 2(I_1 - I_2 + I_3) + 2I_2 = 0 \quad \text{i.e.} \quad -4I_1 + 6I_2 - 2I_3 = 0 \quad \quad ...(2)$$

$$-2(I_1 - I_2 + I_3) - I_3 + 15 = 0 \quad \text{i.e.} \quad +2I_1 - 2I_2 + 3I_3 = 15 \quad \quad ...(3)$$

Solving (1), (2) and (3),

$I_1 = 12$ A, $\quad I_2 = 9$ A, $\quad I_3 = 3$ A

Thus the current delivered by source 30 V is **12 A** while that by source 15 V is **3 A.**

Example 2.73 : *For the circuit shown in Fig. 2.122, find the voltage across 4 Ω resistance by source transformation.* **(May-2008)**

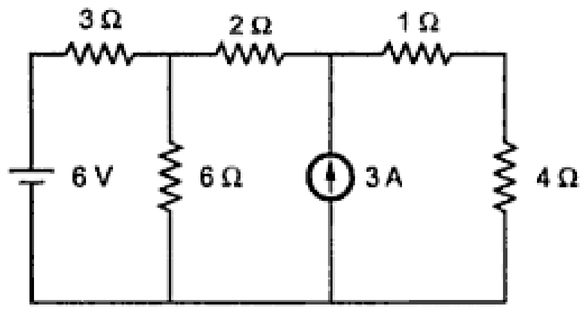
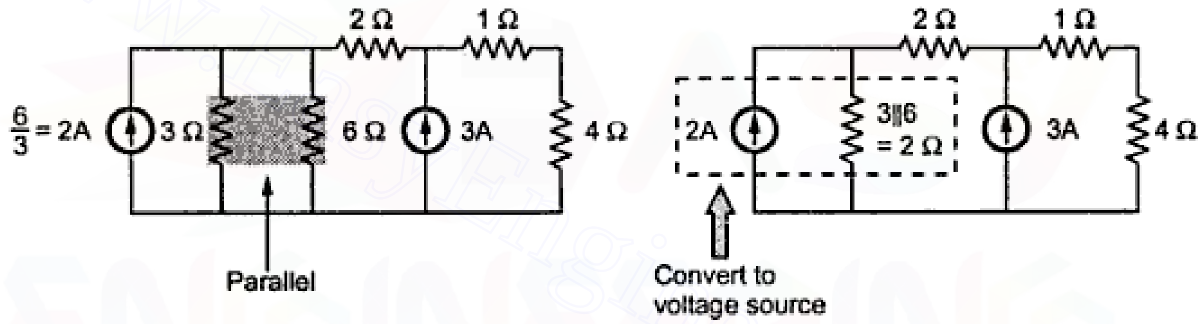


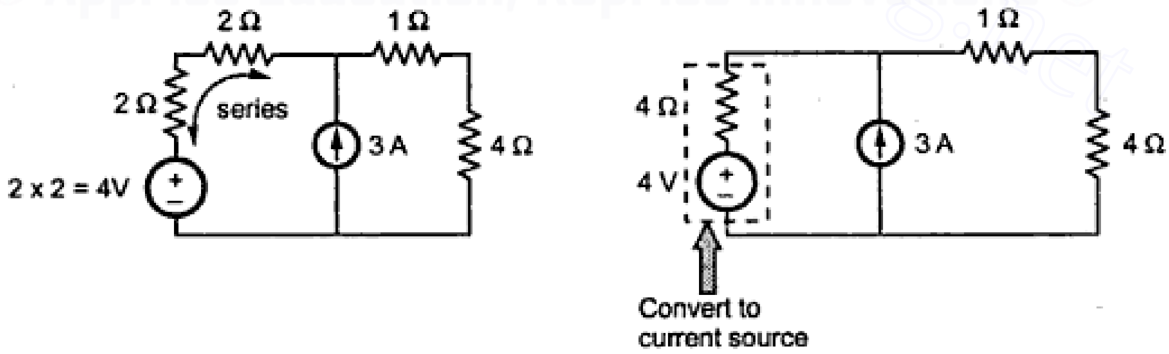
Fig. 2.122

Solution : Convert 6 V voltage source to equivalent current source,



(a) (b)

Fig. 2.122



(c) (d)

Fig. 2.122

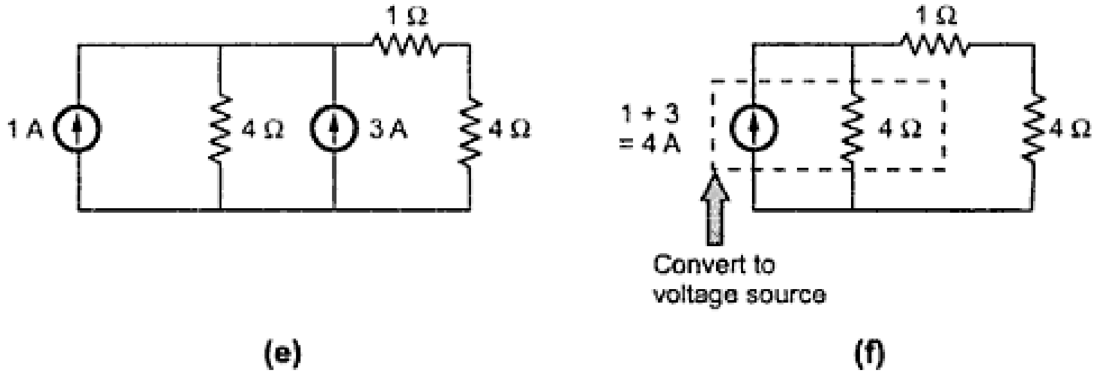


Fig. 2.122

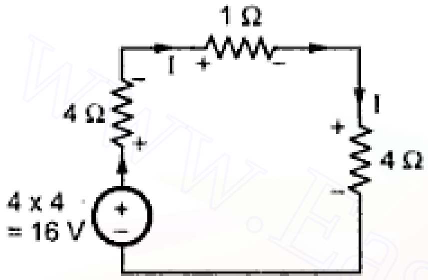


Fig. 2.122 (g)

Once a single loop is obtained, KVL can be applied.

$\therefore \quad -1-4I+16-4I = 0$

$\therefore \quad I = \frac{16}{9} = 1.778 \text{ A}\downarrow$

Example 2.74 : *For the circuit shown in Fig. 2.123, find the vlaue of unknown resistance 'R' so that maximum power will be transferred to load. Hence find maximum power.*

(May-2008)

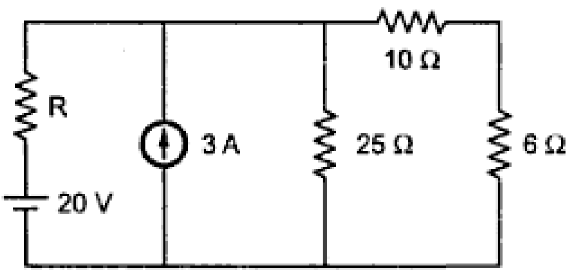
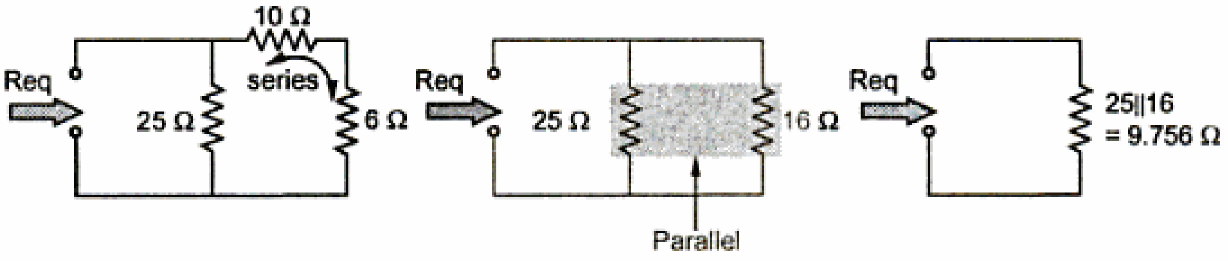


Fig. 2.123

Solution : R for P_{max} is the R_{eq}, opening current source and shorting voltage source.

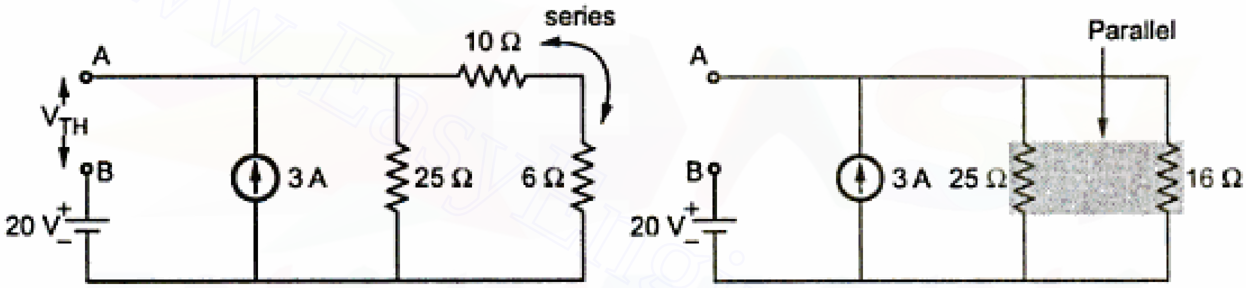


(a) (b) (c)

Fig. 2.123

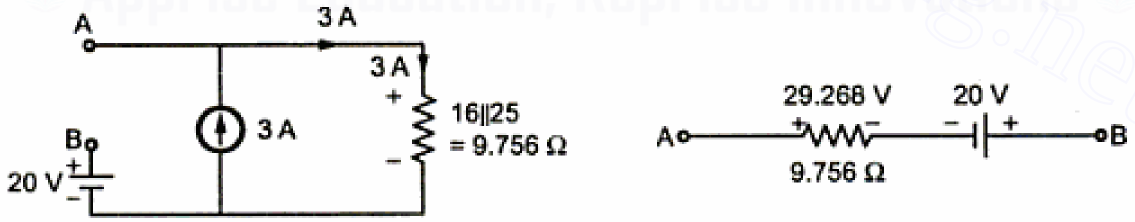
$$\therefore \quad R_{eq} = 9.756\ \Omega = R \text{ for } P_{max}$$

To find P_{max}, obtain the value of V_{TH}.



(d) (e)

Fig. 2.123



(f) (g)

Fig. 2.123

$$V_{AB} = V_{TH} = 9.268 \text{ V with A positive}$$

$$\therefore \quad P_{max} = \frac{V_{TH}^2}{4R} = \frac{(9.268)^2}{4 \times 9.756} = 2.2011 \text{ W}$$

Review Questions

1. *Explain the various types of sources used in d.c. circuits.*
2. *Explains the concept of source transformation with suitable example.*
3. *Derive the relationship to express three star connected resistances into equivalent delta.*
4. *Derive the relationship to express three delta connected resistances into equivalent star.*
5. *State and exp;ain Superposition theorem.*
6. *Two voltmeters A and B, having resistances of 5.2 k Ω and 15 k Ω respectively are connected in series across 240 V supply. What is the reading on each voltmeter ?* **(Ans. : 61.78 V, 178.21 V)**
7. *Two resistances 15 Ω and 20 Ω are connected in parallel. A resistance of 12 Ω is connected in series with the combination . A voltage of 120 V is applied across the entire circuit. Find the current in each resistance, voltage across 12 Ω resistance and power consumed in all the resistances.* **(Ans. : 3.33 A, 2.5 A, 70 V)**
8. *A resistance R is connected in series with a parallel circuit comprising two resistances of 12 and 8 Ω. The total power dissipated in the circuit is 700 watts when the applied voltage is 200 V. Calculate the value of R..* **(Ans. : 52.3428 Ω)**
9. *In the series parallel circuit shown in the Fig. 2.124, find the*

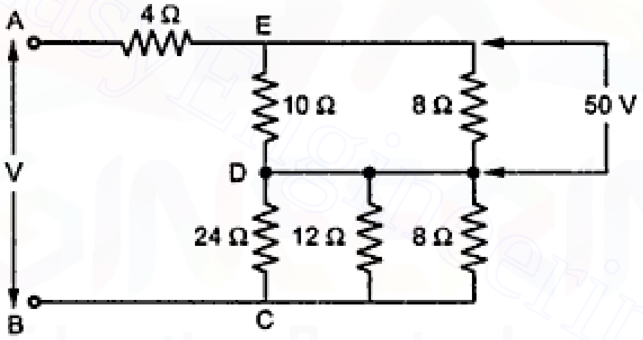


Fig. 2.124

i) voltage drop across the 4 Ω resistance *ii) the supply voltage V* **(Ans. : 45 V, 140V)**

10. *Find the current in all the branches of the network shown in the Fig. 2.125.*
(Ans. : 39 A, 21 A, 39 A, 81 A, 11 A, 41 A)

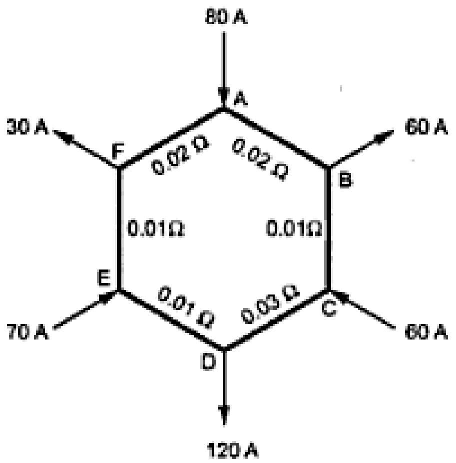


Fig. 2.125

11. *If the total power dissipated in the circuit shown in the Fig. 2.126 is 18 watts, find the value of R and current through it.* **(Ans. : 12 Ω, 0.6 A)**

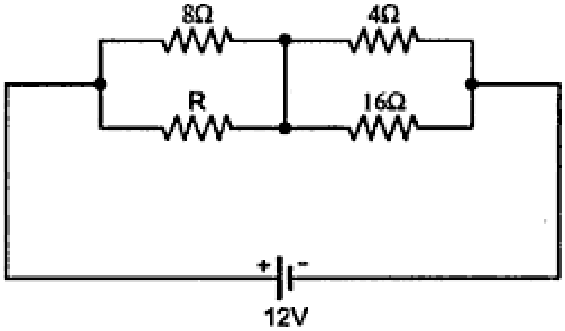


Fig. 2.126

12. *The current in the 6 Ω resista nce of the network shown in the Fig. 2.127 is 2A. Determine the currents in all the other resistances and the supply voltage V.*

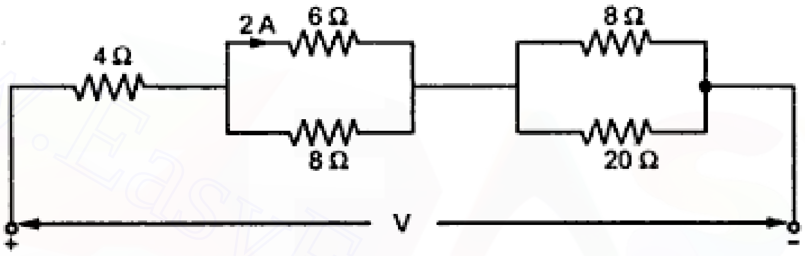


Fig. 2.127

(Ans. : 1.5 A, 2.5 A, 1A, 46 V)

13. *A particular battery when loaded by a resistance of 50 Ω gives the terminal voltage of 48.6 V. If the load resistance is increased to 100 Ω, the terminal voltage is observed to be 49.2 V.*

Determine, *i) E.M.F. of battery*

ii) Internal resistance of battery

Also calculate the load resistance required to be connected to get the terminal voltage of 49.5 V
(Ans. : 49.815 V, 196.42 Ω)

14. *Determine the value of R shown in the Fig. 2.128, if the power dissipated in 10 Ω resistance is 90 W.* **(Ans. : 100 Ω)**

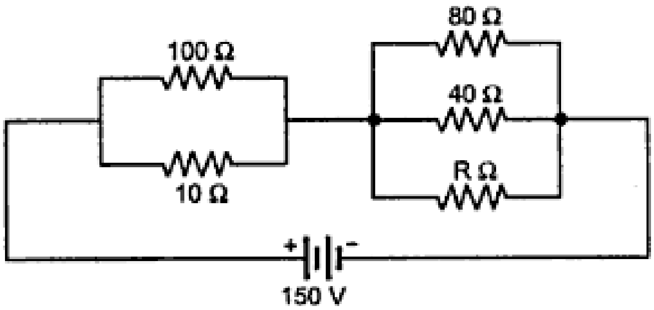


Fig. 2.128

15. *A resistance of 10 Ω is connected in series with the two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken will be 1.5 A from 20 V supply applied ?* **(Ans. : 6 Ω)**

16. *Two coils are connected in parallel and a voltage of 200 V is applied between the terminals. The total current taken is 25 A and power dissipated in one of the resistances is 1500 W. Calculate the resistances of two coils.* **(Ans. : 26.67 Ω, 11.43 Ω)**

17. *Two storage batteries A and B are connected in parallel to supply a load of 0.3 Ω. The open circuit e.m.f. of A is 11.7 V and that of B is 12.3 V. The internal resistances are 0.06 Ω and 0.05Ω respectively. Find the current supplied to the load.* **(Ans. : 36.778 A)**

18. *Using Kirchhoff's laws, find the current flowing through the galvanometer G in the Wheatstone bridge network shown in the Fig. 2.129.* **(Ans. : 48.746 mA)**

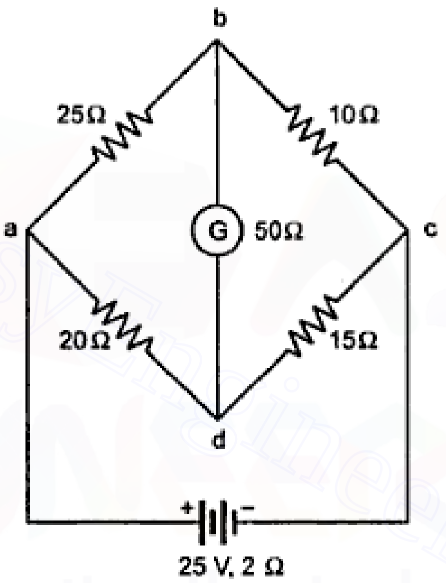


Fig. 2.129

19. *A network ABCD is made up as follows :*

    AB has a cell of 2 V and negligible resistance, with the positive terminal connected to A; BC is a resistor of 25Ω ; CD is a resistor of 100 Ω ; DA is a battery of 4 V and negligible resistance with positive terminal connected to D; AC is a milliammeter of resistance 10 Ω. Calculate the reading on the milliammeter. **(Ans. : 26.67 mA)**

20. *State and explain Thevenin's theorem.*
21. *State and explain Norton's theorem.*
22. *State and explain Maximum power transfer theorem.*
23. *Use Thevenin's theorem to calculate current through branch A-B.* **(Ans. : 2.4857 A from A to B)**

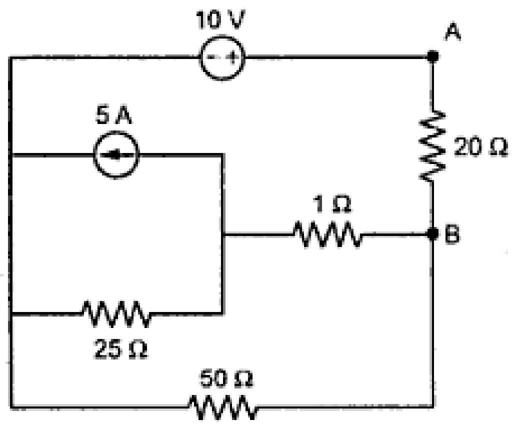


Fig. 2.130

24. *Find current through 8 Ω resistance by Norton's theorem.* **(Ans. : 1.7954 A from A to B)**

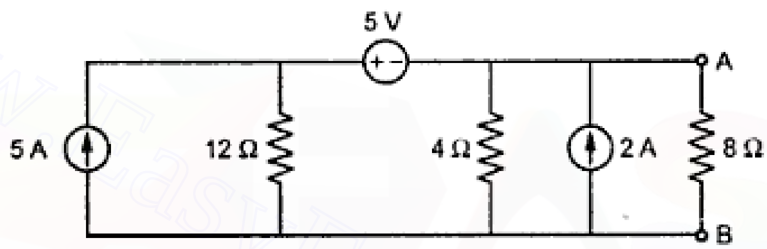


Fig. 2.131

25. *Calculate current through 10 Ω resistance by*

i) Superposition Theorem

ii) Thevenin's Theorem **(Ans. : 1.6208 A from A to B)**

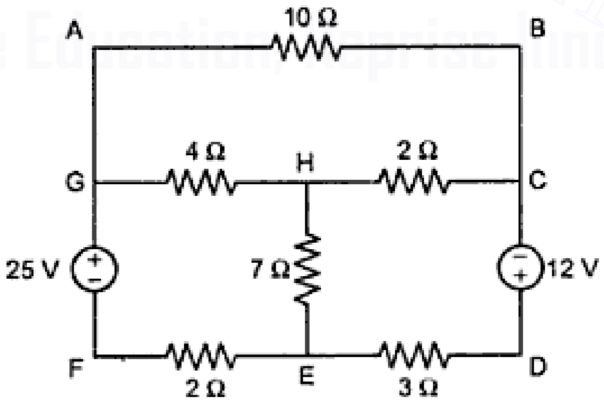


Fig. 2.132

26. *Find the current through branch A-B by using*

i) Thevenin's Theorem

ii) Norton's Theorem **(Ans. : 0.2273 A from A to B)**

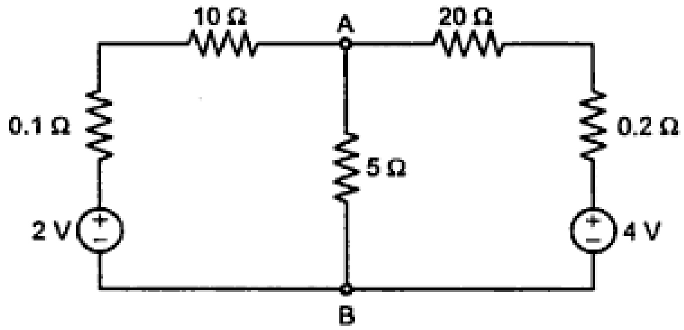


Fig. 2.133

□□□

3 Magnetic Circuits

3.1 Introduction

All of us are familiar with a magnet. It is a piece of solid body which possesses a property of attracting iron pieces and pieces of some other metals. This is called a **natural magnet** . While as per the discovery of Scientist **Oersted** we can have an electromagnet. Scientist **Oersted** stated that any current carrying conductor is always surrounded by a magnetic field. The property of such current is called **magnetic effect of an electric current**. Natural magnet or an electromagnet, both have close relation with electromotive force (e.m.f.), mechanical force experienced by conductor, electric current etc. To understand this relationship it is necessary to study the fundamental concepts of magnetic circuits. In this chapter we shall study laws of magnetism, magnetic field due to current carrying conductor, magnetomotive force, simple series and parallel magnetic circuits .

3.2 Magnet and its Properties

As stated earlier, magnet is a piece of solid body which possesses property of attracting iron and some other metal pieces.

i) When such a magnet is rolled into iron pieces it will be observed that iron pieces cling to it as shown in Fig. 3.1

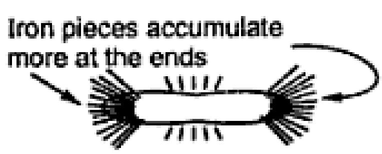


Fig. 3.1 Natural magnet

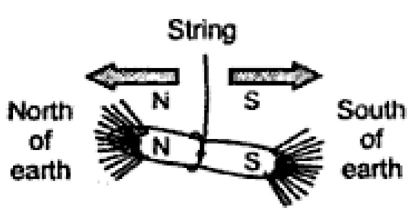


Fig. 3.2 Freely suspended

The maximum iron pieces accumulate at the two ends of the magnet while very few accumulate at the centre of the magnet.

The points at which the iron pieces accumulate maximum are called **Poles** of the magnet while imaginary line joining these poles is called **Axis** of the magnet.

ii) When such magnet is suspended freely by a piece of silk fibre, it turns and always adjusts itself in the direction of North and South of the earth.

The pole which adjusts itself in the direction of North is called North seeking or **North (N) pole**, while the pole which points in the direction of South is called South seeking or **South (S) pole**. Such freely suspended magnet is shown in the Fig. 3.2

This is the property due to which it is used in the compass needle which is used by navigators to find the directions.

iii) When a magnet is placed near an iron or steel piece, its property of attraction gets transferred to iron or steel piece. Such transfer of property of attraction is also possible by actually rubbing the pole of magnet on an iron or steel piece. Such property is called **magnetic induction.**

Magnetic Induction : The phenomenon due to which a magnet can induce magnetism in a (iron or steel) piece of magnetic material placed near it without actual physical contact is called magnetic induction.

iv) An ordinary piece of magnetic material when brought near to any pole N or S gets attracted towards the pole. But if another magnet is brought near the magnet such that two like poles ('N' and 'N' or 'S' and 'S'), it shows a repulsion in between them while if two unlike poles are brought near, it shows a force of attraction.

Key Point : *Like poles repel each other and the unlike poles attract each other. Repulsion is the sure test of magnetism as ordinary piece of magnetic material always shows attraction towards both the poles.*

Let us see the molecular theory behind this magnetism.

3.3 Molecular Theory of Magnetization

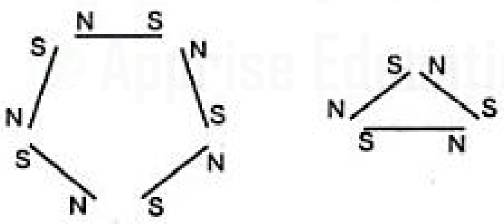


Fig. 3.3 Molecular magnets in unmagnetised material

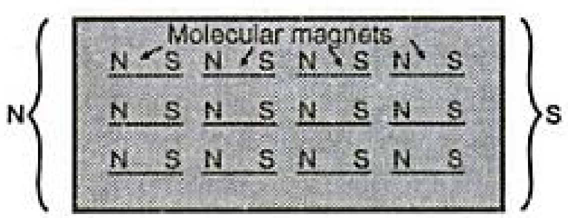


Fig. 3.4 Magnetised piece of material

Not only magnetized but materials like iron, steel are also complete magnets according to molecular theory. All materials consist of small magnets internally called **molecular magnets.** In unmagnetised materials such magnets arrange themselves in closed loops as shown in the Fig. 3.3

So at any joint, effective strength at a point is zero, due to presence of two unlike poles. Such poles cancel each other's effect. But if magnetized material is considered or unmagnetized material subjected to magnetizing force is considered, then such small molecular magnets arrange themselves in the direction of magnetizing force, as shown in the Fig. 3.4

Unlike poles of these small magnets in the middle are touching each other and hence neutralizing the effect.

But on one end 'N' poles of such magnets exist without neutralizing effect. Similarly on other end 'S' poles of such magnets exist. Thus one end behaves as 'N' pole while other as 'S' pole. So most of the iron particles get attracted towards end and not in the middle.

From this theory, we can note down the following points :

1) When magnetizing force is applied, immediately it is not possible to have alignment of all such small magnets, exactly horizontal as shown in the Fig. 3.4. There is always some limiting magnetizing force exists for which all such magnets align exactly in horizontal position .

Key Point: *Though magnetizing force is increased beyond certain value, there is no chance for further alignment of molecular magnets hence further magnetization is not possible. Such condition or phenomenon is called* ***Saturation.***

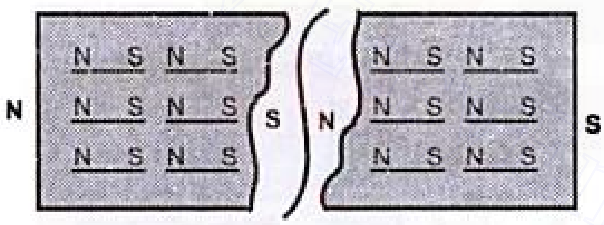


Fig. 3.5 Breaking of magnet

2) If the magnet is broken at any point, each piece behaves like an independent magnet with two poles to each, 'N' and 'S'.

3) The piece of soft iron gets magnetized more rapidly than hard steel. This is because alignment of molecular magnets in soft iron takes place quickly for less magnetizing force than in hard steel.

4) If unmagnetised piece is subjected to alternating magnetizing force i.e. changing magnetizing force, then heat is produced. This is because molecular magnets try to change themselves as per change in magnetizing force. So due to molecular friction heat is generated.

5) If a magnet is heated and allowed to cool, it demagnetizes. This is because heat sets molecular magnets into motion so that the molecules again form a closed loop, neutralizing the magnetism.

6) **Retentivity** : When a soft iron piece is magnetized by external magnetizing force due to magnetic induction, it loses its magnetism immediately if such force is removed. As against this hard steel continues to show magnetism though such force is removed. It retains magnetism for some time.

Key Point : *The power of retaining magnetism after the magnetizing force is removed is called* ***Retentivity.*** *The time for which material retains such magnetism in absence of magnetizing force depends on its retentivity.*

3.4 Laws of Magnetism

There are two fundamental laws of magnetism which are as follows :

Law 1 : *It states that 'Like magnetic poles repel and unlike poles attract each other'*

This is already mentioned in the properties of magnet.

Law 2 : *This law is experimentally proved by Scientist Coulomb and hence also known as Coulomb's Law.*

The force (F) exerted by one pole on the other pole is,

a) directly proportional to the product of the pole strengths,

b) inversely proportional to the square of the distance between them, and

c) nature of medium surrounding the poles.

Mathematically this law can be expressed as,

$$F \propto \frac{M_1 M_2}{d^2}$$

where M_1 and M_2 are pole strengths of the poles while d is distance between the poles.

$$\therefore \qquad \boxed{F = \frac{K M_1 M_2}{d^2}}$$

where K depends on the nature of the surroundings and called permeability.

3.5 Magnetic Field

We have seen that magnet has its influence on the surrounding medium. 'The region around a magnet within which the influence of the magnet can be experienced is called **magnetic field**. Existence of such field can be experienced with the help of compass needle, iron or pieces of metals or by bringing another magnet in vicinity of a magnet.

3.5.1 Magnetic Lines of Force

The magnetic field of magnet is represented by imaginary lines around it which are called **magnetic lines of force.** Note that these lines have no physical existence, these are purely imaginary and were introduced by **Michael Faraday** to get the visualization of distribution of such lines of force.

3.5.2 Direction of Magnetic Field

The direction of magnetic field can be obtained by conducting small experiment.

Let us place a permanent magnet on table and cover it with a sheet of cardboard. Sprinkle steel or iron fillings uniformly over the sheet. Slight tapping of cardboard causes fillings to adjust themselves in a particular pattern as shown in the Fig. 3.6

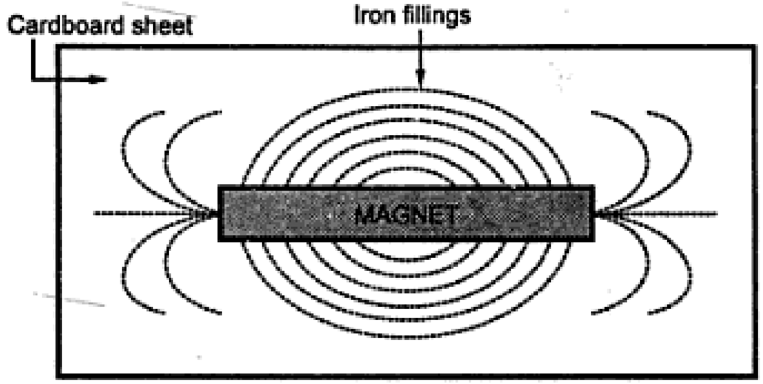


Fig. 3.6 Magnetic lines of force

The shape of this pattern projects a mental picture of the magnetic field present around a magnet.

A line of force can be defined as,

Consider the isolated N pole (we cannot separate the pole but imagine to explain line of force) and it is allowed to move freely, in a magnetic field. Then path along which it moves is called **line of force**. Its shape is as shown in the Fig. 3.6 and direction always from N-pole towards S-pole.

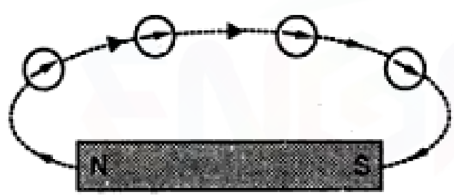


Fig. 3.7 Compass needle experiment

The direction of lines of force can be understood with the help of small compass needle. If magnet is placed with compass needles around it, then needles will take positions as shown in the Fig. 3.7. The tangent drawn at any point, of the dotted curve shown, gives direction of resultant force at that point. The N poles are all pointing along the dotted line shown,from N- pole to its S-pole.

The lines of force for a bar magnet and U-shaped magnet are shown in the Fig. 3.8.

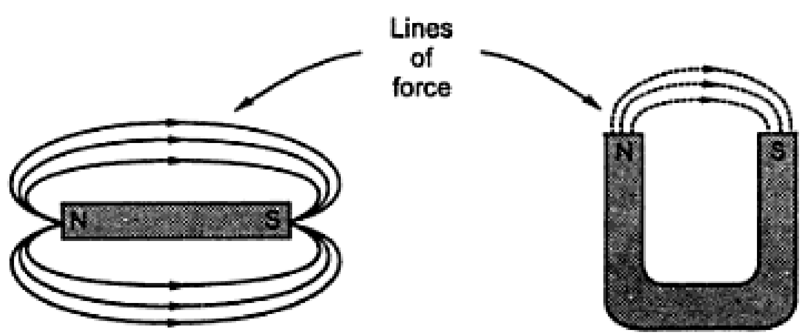


Fig. 3.8 (a) Bar magnet **Fig. 3.8 (b) U-shaped magnet**

Attraction between the unlike poles and repulsion between the like poles of two magnets can be easily understood from the direction of magnetic lines of force. This is shown in the Fig. 3.9 (a) and (b).

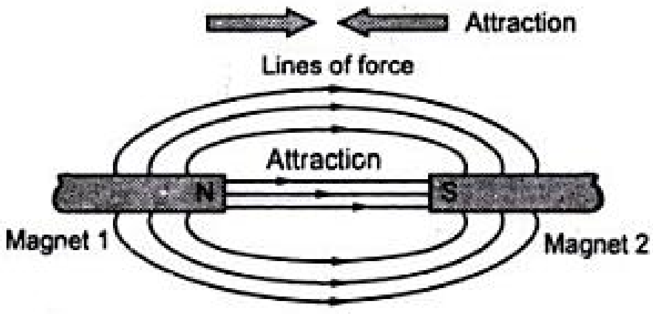


Fig. 3.9 (a) Force of attraction

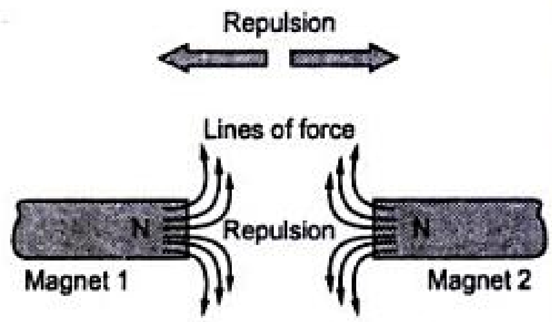


Fig. 3.9 (b) Force of repulsion

3.5.3 Properties of Lines of Force

Though the lines of force are imaginary, with the help of them various magnetic effects can be explained very conveniently. Let us see the various properties of these lines of force.

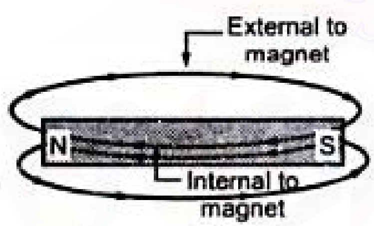


Fig. 3.10 Lines of force complete the closed path

1) Lines of force are always originating on a N-pole and terminating on a S-pole, external to the magnet.
2) Each line forms a **closed loop** as shown in the Fig. 3.10.

> **Key Point** : *This means that a line emerging from N-pole, continues upto S-pole* ***external to the magnet*** *while it is assumed to continue from S-pole to N-pole* ***internal to the magnet*** *completing a closed loop. Such lines internal to the magnet are called as* ***lines of induction.***

3) Lines of force never intersect each other.
4) The lines of force, are like stretched rubberbands and always try to contract in length.
5) The lines of force, which are parallel and travelling in the same direction repel each other.
6) Magnetic lines of force always prefer a path offering least opposition.

> **Key Point** : *The opposition by the material to the flow of lines of force is called reluctance. Air has more reluctance while magnetic materials like iron, steel etc. have low reluctance. Thus magnetic lines of force can easily pass through iron or steel but cannot pass easily through air.*

3.6 Magnetic Flux (ϕ)

The total number of lines of force existing in a particular magnetic field is called **magnetic flux**. Lines of force can be called **lines of magnetic flux**. The unit of flux is weber and flux is denoted by symbol (ϕ) . The unit weber is denoted as Wb.

$$1 \text{ weber} = 10^8 \text{ lines of force.}$$

3.7 Pole Strength

We have seen earlier that force between the poles depends on the pole strengths. As we are now familiar with flux, we can have idea of pole strength. Every pole has a capacity to radiate or accept certain number of magnetic lines of force i.e. magnetic flux which is called its **strength**. Pole strength is measurable quantity assigned to poles which depends on the force between the poles. If two poles are exerting equal force on one other, they are said to have equal pole strengths.

Unit of pole strength is **weber** as pole strength is directly related to flux i.e. lines of force

Key Point : *A unit pole may be defined as that pole which when placed from an identical pole at a distance of 1 metre in free space experiences a force of* $\dfrac{10^7}{16\pi^2}$ *newtons.*

So when we say Unit N-pole, it means a pole is having a pole strength of 1 weber.

3.8 Magnetic Flux Density (B)

It can be defined as 'The flux per unit area (a) in a plane at right angles to the flux is known as '**flux density**'. Mathematically,

$$B = \frac{\phi}{a} \quad \frac{Wb}{m^2} \text{ or Tesla}$$

It is shown in the Fig. 3.11.

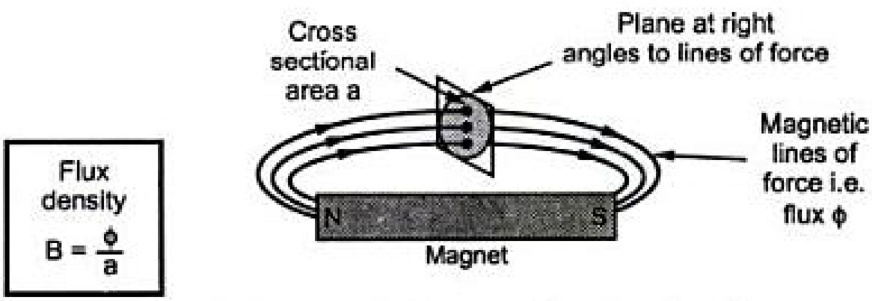


Fig. 3.11 Concept of magnetic flux density

Key Point: *The unit of flux density is Wb/m², also called tesla denoted as T.*

3.9 Magnetic Field Strength (H)

This gives quantitative measure of strongness or weakness of the magnetic field. Note that pole strength and magnetic field strength are different. This can be defined as 'The force experienced by a unit N-pole (i.e. N pole with 1 Wb of pole strength) when placed at any point in a magnetic field is known as **magnetic field strength** at that point.

It is denoted by H and its unit is newtons per weber i.e. (N/Wb) or amperes per metre (A/m.) or ampere turns per metre (AT/m). The mathematical expression for calculating magnetic field strength is,

$$H = \frac{\text{ampere turns}}{\text{length}}$$

$$\therefore \quad \boxed{H = \frac{NI}{l} \text{ AT/m}}$$

> **Key Point:** *More the value of 'H', more stronger is the magnetic field. This is also called magnetic field intensity.*

Example 3.1 : *A pole having strength of 0.5×10^{-3} Wb is placed in a magnetic field at a distance of 25 cm from another pole. It is experiencing a force of 0.5 N. Assume constant of medium $\left(\frac{1}{36\pi^2\times10^{-7}}\right)$. Determine,*

a) magnetic field strength at the point. b) the strength of other pole.

c) distance at which force experienced will be doubled.

Solution : The given values are,

$$M_1 = 0.5\times10^{-3} \text{ Wb}, \quad d = 25 \text{ cm} = 0.25 \text{ m}, \quad F = 0.5 \text{ N}$$

$$K = \left(\frac{1}{36\pi^2\times10^{-7}}\right) = 28144.773$$

(a) Magnetic field strength,

$$H = \frac{\text{Newton}}{\text{Wb}} = \frac{\text{Force experienced}}{\text{Pole strength}} = \frac{0.5}{0.5\times10^{-3}}$$

=1000 N / Wb.

b) According to Coulomb's law,

$$F = \frac{KM_1M_2}{d^2}$$

$$\therefore \quad 0.5 = \frac{0.5\times10^{-3}\times28144.773\times M_2}{(0.25)^2}$$

$$\therefore \quad M_2 = 2.22\times10^{-3} \text{ Wb} \qquad \text{... pole strength of other pole}$$

c) $F = 1\text{ N}$

$$\therefore \quad 1 = \frac{28144.773 \times 0.5 \times 10^{-3} \times 2.22 \times 10^{-3}}{d^2}$$

$$\therefore \quad d = 0.1767\text{ m} = 17.67\text{ cm}$$

At a distance of 17.67 cm from another pole, the first pole will experience a force 1 N.

Key Point: *When poles are brought nearer and nearer, force experienced by them increases.*

3.10 Magnetic Effect of an Electric Current (Electromagnets)

When a coil or a conductor carries a current, it produces the magnetic flux around it. Then it starts behaving as a magnet. Such a current carrying coil or conductor is called an electromagnet. This is due to magnetic effect of an electric current.

If such a coil is wound around a piece of magnetic material like iron or steel and carries current then piece of material around which the coil is wound, starts behaving as a magnet, which is called an electromagnet.

The flux produced and the flux density can be controlled by controlling the magnitude the current.

The direction and shape of the magnetic field around the coil or conductor depends on the direction of current and shape of the conductor through which it is passing. The magnetic field produced can be experienced with the help of iron fillings or compass needle.

Let us study two different types of electromagnets,

1) Electromagnet due to straight current carrying conductor
2) Electromagnet due to circular current carrying coil

3.10.1 Magnetic Field due to Straight Conductor

When a straight conductor carries a current, it produces a magnetic field all along its length. The lines of force are in the form of concentric circles in the planes right angles to the conductor. This can be demonstrated by a small experiment.

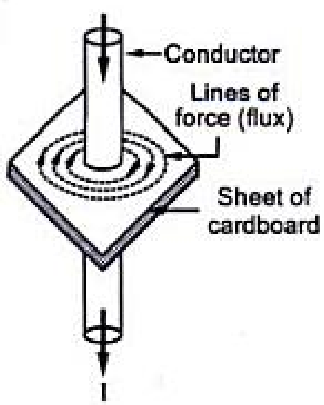


Fig. 3.12 Magnetic field due to a straight conductor

Consider a straight conductor carrying a current, passing through a sheet of cardboard as shown in the Fig. 3.12. Sprinkle iron fillings on the cardboard. Small tapping on the cardboard causes the iron filling to set themselves, in the concentric circular pattern. The direction of the magnetic flux can be determined by placing compass needle near the conductor. This direction depends on the direction of the current passing through the conductor. For the current direction shown in the Fig. 3.12 i.e. from top to bottom the direction of flux is clockwise around the conductor.

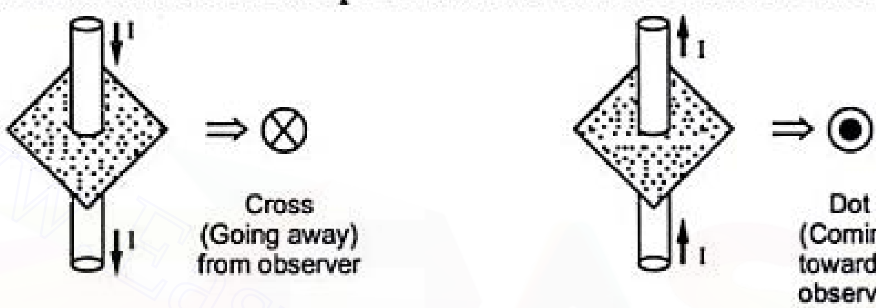
Conventionally such current carrying conductor is represented by small circle, (top view of conductor shown in the Fig. 3.12). Then current through such conductor will either come out of paper or will go into the plane of the paper.

> **Key Point :** *When current is going into the plane of the paper, i.e. away from observer, it is represented by a 'cross', inside the circle indicating the conductors.*

The cross indicates rear view of feathered end of an arrow.

> **Key Point:** *The current flowing towards the observer i.e. coming out of the plane of the paper is represented by a 'dot' inside the circle.*

The dot indicates front view i.e. tip of an arrow. This is shown in the Fig. 3.13.



(a) Current into the paper **(b) Current out of the paper**

Fig. 3.13 Cross and Dot convention

3.10.1.1 Rules to Determine Direction of Flux Around Conductor

1) Right Hand Thumb Rule : It states that, hold the current carrying conductor in the right hand such that the thumb pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of the magnetic field or flux around it. The Fig. 3.14 explains the rule.

Let us apply this rule to the conductor passing through card sheet considered earlier. This can be explained by the Fig. 3.15.

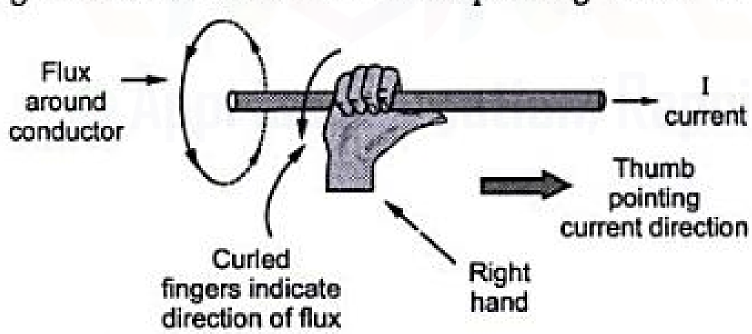


Fig. 3.14 Right hand thumb rule

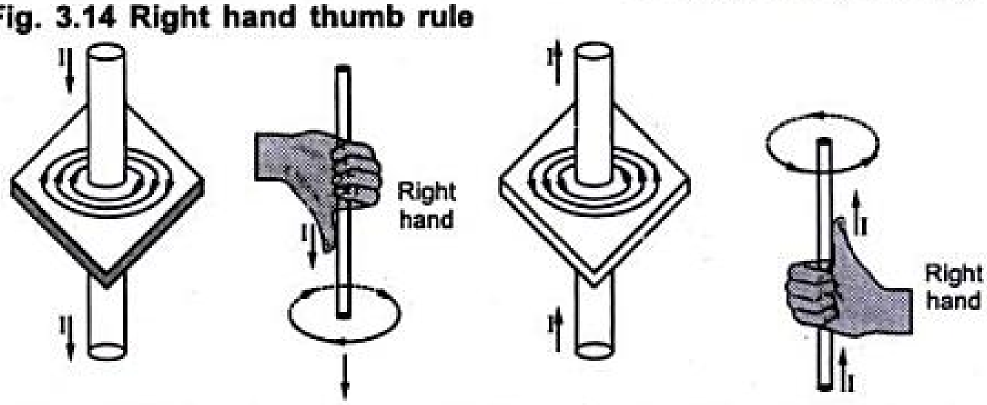


Fig. 3.15 Direction of magnetic lines by Right hand thumb rule

Conventionally it is shown as in the Fig. 3.16.

Fig. 3.16 Representation of direction of flux

2) Corkscrew Rule : Imagine a right handed screw to be along the conductor carrying current with its axis parallel to the conductor and tip pointing in the direction of the current flow.

Then the direction of the magnetic field is given by the direction in which the screw must be turned so as to advance in the direction of the current.

This is shown in the Fig. 3.17.

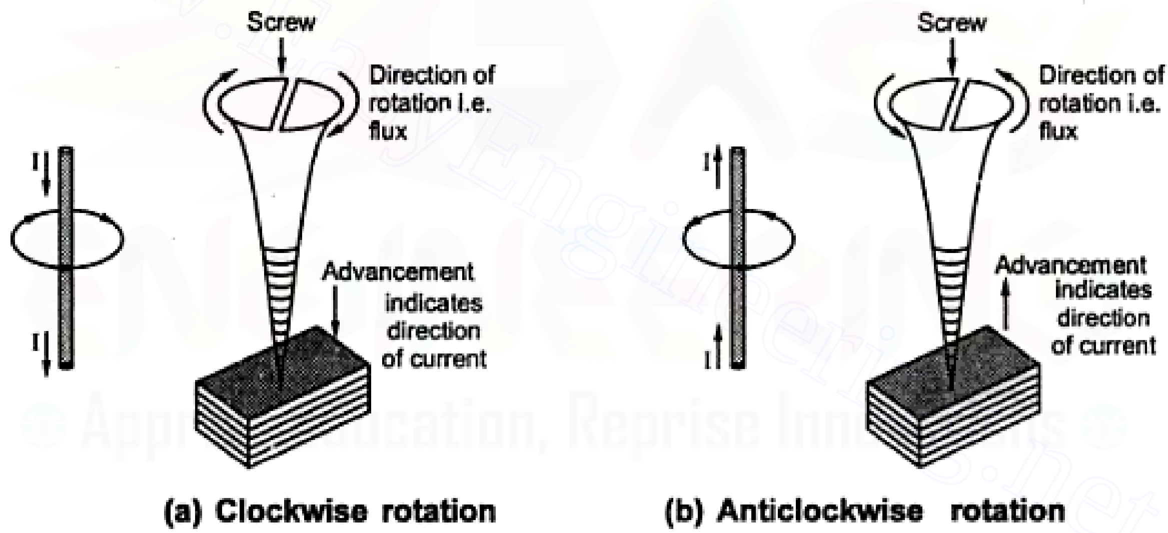


Fig. 3.17 Corkscrew rule

3.10.2 Magnetic Field due to Circular Conductor i.e. Solenoid

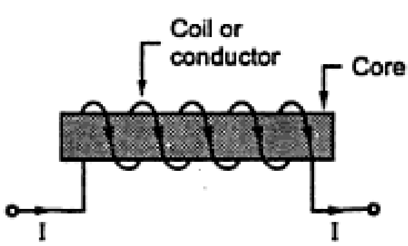


Fig. 3.18(a) Solenoid

A solenoid is an arrangement in which long conductor is wound with number of turns close together to form a coil. The axial length of conductor is much more than the diameter of turns. The part or element around which the conductor is wound is called as **core of the solenoid.** Core may be air or may be some magnetic material. Solenoid with a steel or iron core in shown in Fig. 3.18(a).

When such conductor is excited by the supply so that it carries a current then it produces a magnetic field which acts through the coil along its axis and also around the solenoid. Instead of using a straight core to wound the

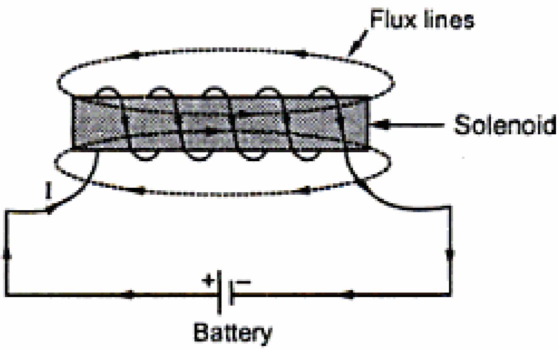


Fig. 3.18(b) Flux around a solenoid

conductor, a circular core also can be used to wound the conductor. In such case the resulting solenoid is called **Toroid.** Use of magnetic material for the core produces strong magnet. This is because current carrying conductor produces its own flux. In addition to this, the core behaves like a magnet due to magnetic induction, producing its own flux. The direction of two fluxes is same due to which resultant magnetic field becomes more strong.

The pattern of the flux around the solenoid is shown in the Fig. 3.18(b).

The rules to determine the direction of flux and poles of the magnet formed:

1) The right hand thumb rule :

Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid point to the **North pole** of the solenoid or point the direction of flux lines **inside the core.**

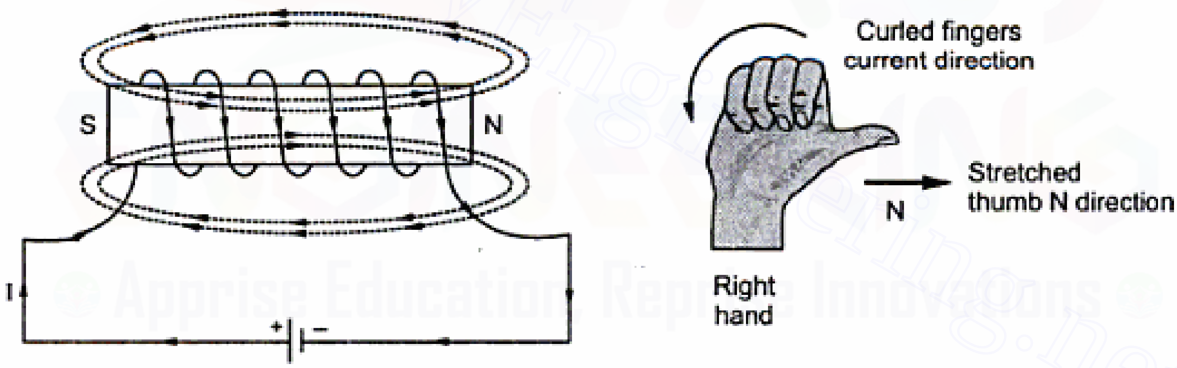


Fig. 3.19 (a) Direction of flux around a solenoid

This is shown in Fig. 3.19 (a) and (b).

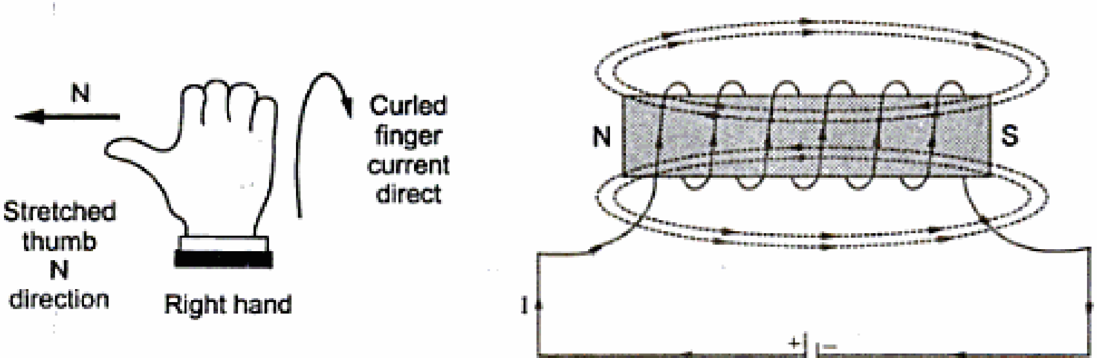


Fig. 3.19 (b) Direction of flux around a solenoid

In case of toroid, the core is circular and hence using right hand thumb rule, the direction of flux in the core, due to current carrying conductor can be determined. This is shown in the Fig. 3.20(a) and (b). In the Fig. 3.20 (a), corresponding to direction of winding, the flux set in the core is anticlockwise while in the Fig. 3.20 (b) due to direction of winding, the direction of flux set in the core is clockwise. The winding is also called **magnetising winding** or **magnetising coil** as it magnetises the core.

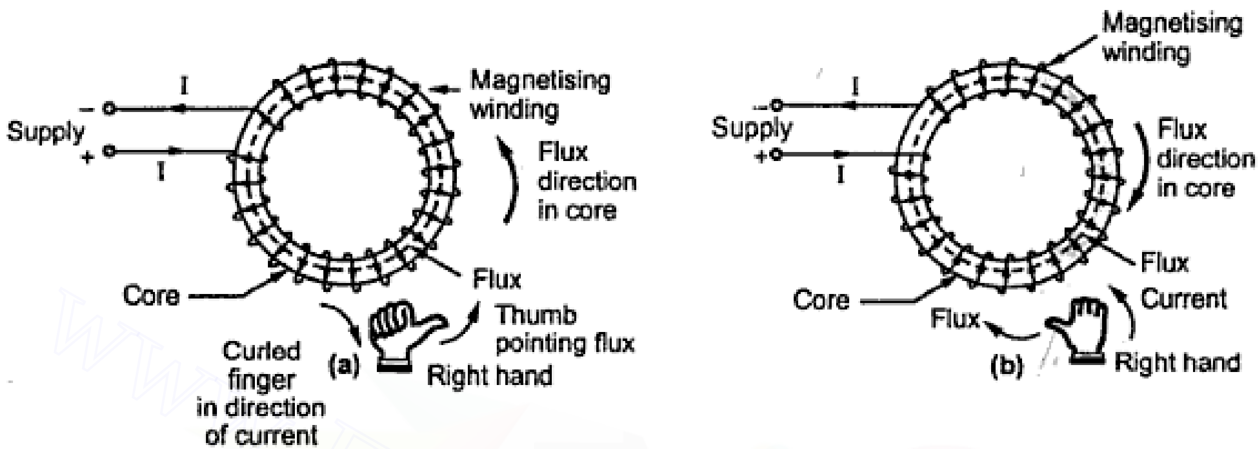


Fig. 3.20

2) Corkscrew rule : If axis of the screw is placed along the axis of the solenoid and if screw is turned in the direction of the current, then it travels towards the **N-pole** or in the direction of the magnetic field **inside** the solenoid.

3) End rule : If solenoid is observed from any one end then its polarity can be decided by noting direction of the current.

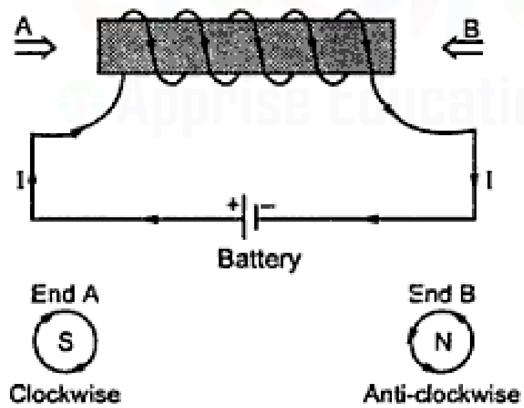


Fig. 3.21 End rule

Consider solenoid shown in the Fig. 3.21.

If it is seen from the end A, current will appear to flow in clockwise direction, so that end behaves as S-pole of the magnet. While as seen from the end B, current appears to flow in anticlockwise direction then that end which is B, behaves as N-pole of the magnet.

Generally right hand thumb rule is used to determine direction of flux and nature of the poles formed. Using such concept of an electromagnet, various magnetic circuits can be obtained.

3.11 Nature of Magnetic Field of Long Straight Conductor

We have seen that any current carrying conductor produces magnetic field around it and behaves like a permanent magnet with its field around.

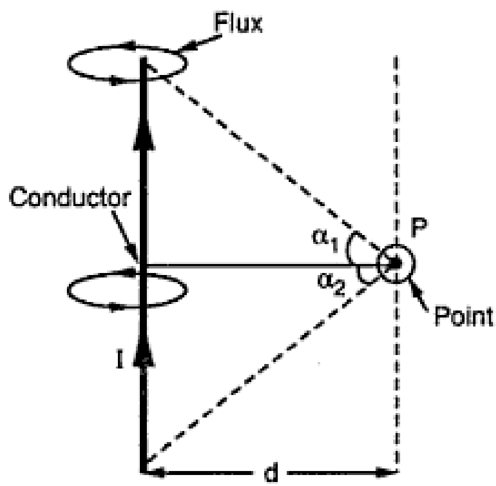


Fig. 3.22 (a) Field strength due to

Consider a conductor carrying current I amperes of length 'l' meters. Consider point P in the vicinity of such conductor. There will be influence of magnetic field on point P which can be quantified by magnetic field strength H at point P. This is definitely proportional to current I and inversely proportional to distance of point P from the conductor.

The magnitude of such magnetic field strength 'H' can be calculated by using the expression

$$\boxed{H = \frac{I}{4\pi d}(\sin\alpha_1 + \sin\alpha_2)}$$

The proof of this is out of the scope of this book.

For infinitely long conductor i.e. length 'l' is very very large then α_1 and α_2 tend to 90°.

$$H = \frac{I}{4\pi d}[\sin 90 + \sin 90] = \frac{2I}{4\pi d}$$

$$\boxed{H = \frac{I}{2\pi d} \quad A/m}$$

This unit of magnetic field strength A/m is mentioned earlier when field strength is defined.

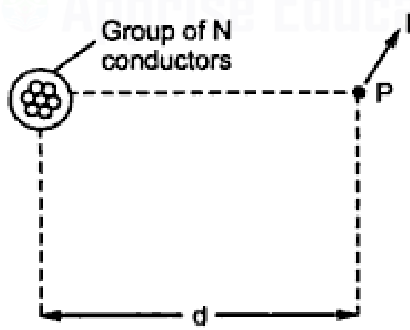


Fig. 3.22(b)

If such 'N' conductors are grouped together to form a coil or a cable then field strength due to current I passing through each conductor of the group can also be calculated by using same expression. The only change will be the field strength calculated above will get multiplied by 'N'.

So magnetic field strength at a point 'd' metres away from centre of such group of 'N' conductors each carrying current I amperes is

$$\boxed{H = \frac{NI}{2\pi d} \quad AT/m}$$

where AT/m means ampere turns per metre.

3.11.1 Magnetic Field Strength due to a Long Solenoid

Similar to the case of long straight conductor, we can decide field strength along the axis of a long solenoid. Such field strength depends on the number of turns of the conductor around the core and magnitude of current I passing through the conductor. If 'l' is the length of the solenoid in metres then H can be determined by the expression :

$$\boxed{H = \frac{NI}{l} \text{ AT/m}}$$

Key Point : *The expression is applicable for solenoids which are very very long but in practice the expression is used for all types of solenoids.*

➡ **Example 3.2 :** *A current of 2 amp is flowing through each of the conductors in a coil containing 15 such conductors. If a point pole of unit strength is placed at a perpendicular distance of 10 cm from the coil, determine the field intensity at that point.*

Solution : I = 2 A, N = 15, d = 10 cm = 0.1 m.

$$H = \frac{NI}{2\pi d} = \frac{15\times 2}{2\times\pi\times 0.1} = \textbf{47.74 AT/m}$$

➡ **Example 3.3 :** *A solenoid of 100 cm is wound on a brass tube. If the current through the coil is 0.5 A, calculate the number of turns necessary over the solenoid to produce a field strength of 500 AT/m at the center of the coil.*

Solution : The field strength on the axis of a long solenoid is given by

$$H = \frac{NI}{l} \text{ AT/m}$$

$$l = \text{Length of coil} = 100 \text{ cm} = 1 \text{ m} , \; N = \text{Number of turns}$$

$$I = \text{Current} = 0.5 \text{ A}$$

$$\therefore \quad 500 = \frac{N\times 0.5}{1}$$

$$N = \mathbf{1000}$$

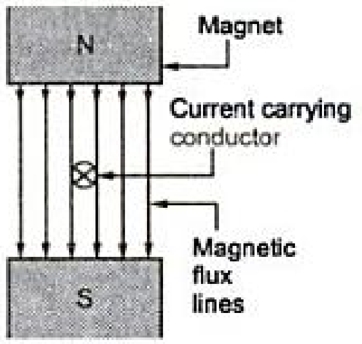
So 1000 turns on solenoid are necessary to produce the required field strength.

3.12 Force on a Current Carrying Conductor in a Magnetic Field

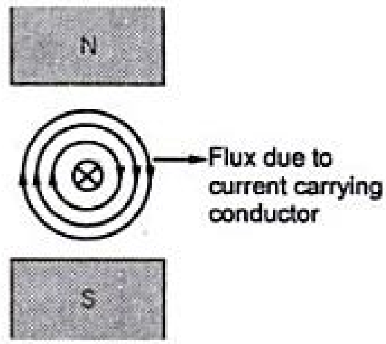
We have already mentioned that magnetic effects of electric current are very useful in analysing various practical applications like generators, motors etc. One of such important effects is force experienced by a current carrying conducto. in a magnetic field.

Let a straight conductor, carrying a current is placed in a magnetic field as shown in the Fig. 3.23 (a).

The magnetic field in which it is placed has a flux pattern as shown in the Fig. 3.23 (a).



(a) Flux due to magnet



(b) Flux due to current carrying conductor

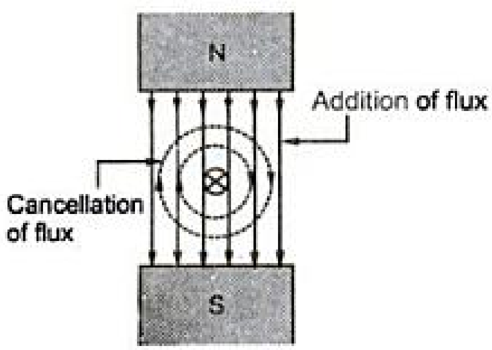
Fig. 3.23 Current carrying conductor in a magnetic field

Now current carrying conductor also produces its own magnetic field around it. Assuming current direction away from observer i.e. into the paper, the direction of its flux can be determined by right hand thumb rule. This is clockwise as shown in the Fig. 3.23 (b). [For simplicity, flux only due to current carrying conductor is shown in the Fig. 3.23 (b).]

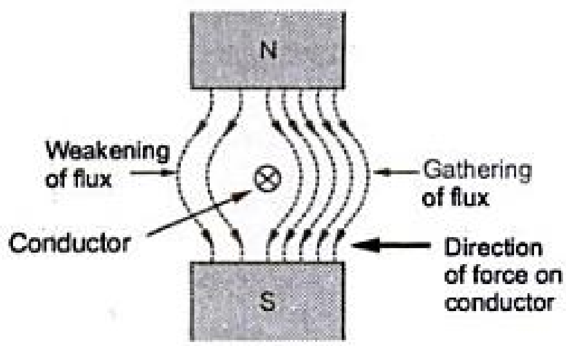
Now there is presence of two magnetic fields namely due to permanent magnet and due to current carrying conductor. These two fluxes interact with each other. Such interaction is shown in the Fig. 3.24 (a).

This interaction as seen is in such a way that on one side of the conductor the two lines help each other, while on other side the two try to cancel each other. This means on left hand side of the conductor shown in the Fig. 3.24 the two fluxes are in the same direction and hence assisting each other . As against this, on the right hand side of the conductor the two fluxes are in opposite direction hence trying to cancel each other. Due to such interaction on one side of the conductor, there is **accumulation** of flux lines (gathering of the flux lines) while on the other side there is **weakening** of the flux lines.

The resultant flux pattern around the conductor is shown in the Fig. 3.24 (b).



(a) Presence of the two fluxes



(b) Resultant flux pattern

Fig. 3.24 Interaction of the two flux lines

According to properties of the flux lines, these flux lines will try to shorten themselves. While doing so, flux lines which are gathered will exert force on the conductor. So conductor experiences a mechanical force from high flux lines area towards low flux lines area i.e. from left to right for a conductor shown in the Fig. 3.24.

Key Point: *Thus we can conclude that current carrying conductor placed in the magnetic field, experiences a mechanical force, due to interaction of two fluxes.*

This is the basic principle on which D.C. electric motors work and hence also called **motoring action.**

3.12.1 Fleming's Left Hand Rule

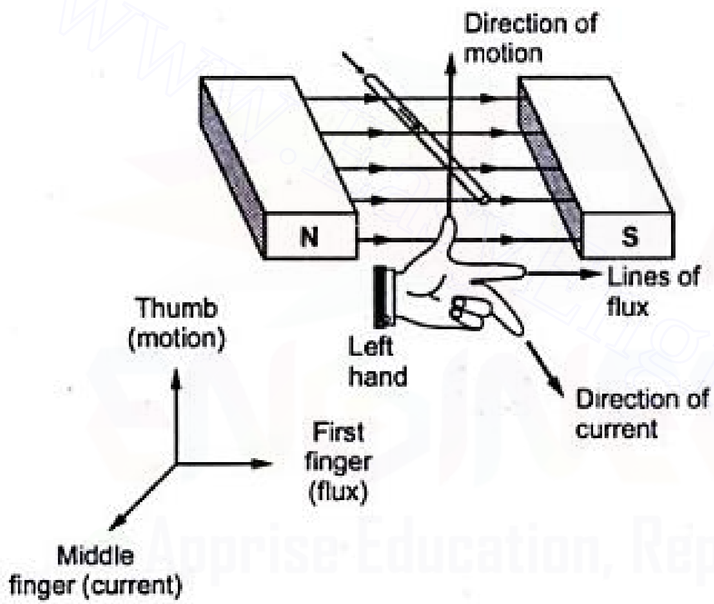


Fig. 3.25 Fleming's left hand rule

The direction of the force experienced by the current carrying conductor placed in magnetic field can be determined by a rule called **'Fleming's Left Hand Rule'**. The rule states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor'.

The rule is explained in the diagrammatic form in the Fig. 3.25.

Apply the rule to crosscheck the direction of force experienced by a single conductor, placed in the magnetic field, shown in the Fig. 3.26 (a), (b), (c) and (d).

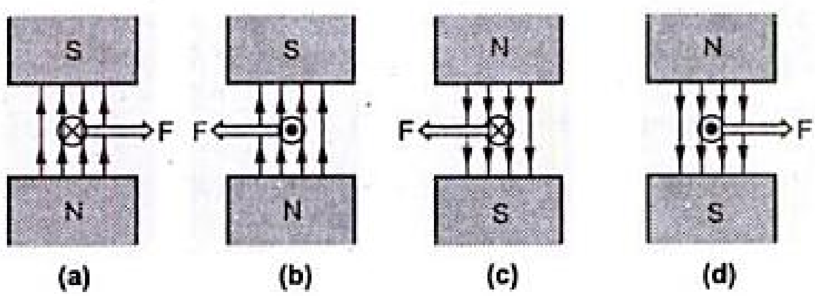


Fig. 3.26 Direction of force experienced by conductor

3.12.2 Magnitude of Force Experienced by the Conductor

The magnitude of the force experienced by the conductor depends on the following factors,

1) Flux density (B) of the magnetic field in which the conductor is placed measured in Wb/m^2 i.e. Tesla.
2) Magnitude of the current I passing through the conductor in Amperes.
3) Active length 'l' of the conductor in metres.

The **active length** of the conductor is that part of the conductor which is actually under the influence of magnetic field.

If the conductor is at right angles to the magnetic field as shown in Fig. 3.27 (a) then force F is given by,

$$F = B I l \text{ Newtons}$$

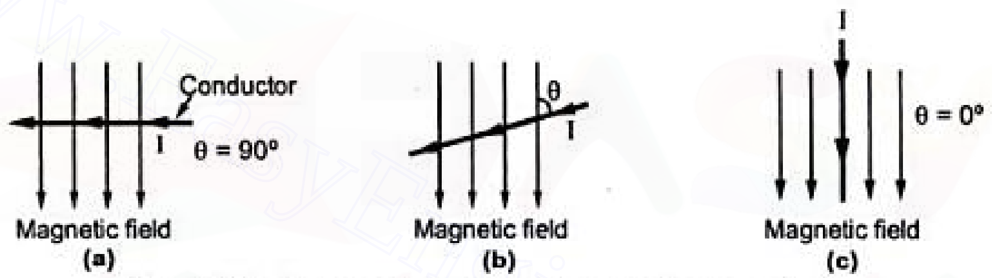


Fig. 3.27 Force on a current carrying conductor

But if the conductor is not exactly at right angles, but inclined at angle θ degrees with respect to axis of magnetic field as shown in the Fig. 3.27 (b) then force F is given by,

$$F = B I l \sin\theta \text{ Newtons}$$

As shown in the Fig. 3.27 (c), if conductor is kept along the lines of magnetic field then $\theta = 0^\circ$ and as $\sin 0^\circ = 0$, the force experienced by the conductor is also zero.

> **Key Point :** *The direction of such force can be reversed either by changing the direction of current or by changing the direction of the flux lines in which it is kept. If both are reversed, the direction of force remains same.*

➡ **Example 3.4 :** *Calculate the force experienced by the conductor of 20 cm long, carrying 50 amperes, placed at right angles to the lines of force of flux density 10×10^{-3} Wb/m^2.*

Solution : The force experienced is given by,

$$F = B I l \sin\theta \quad \text{where} \quad \sin(\theta) = 1 \quad \text{as } \theta = 90 \text{ degrees}$$

$$B = \text{Flux density} = 10\times10^{-3} \text{ Wb/m}^2$$

$$l = \text{Active length} = 20 \text{ cm} = 0.2 \text{ m}$$

$$I = \text{current} = 50 \text{ A}$$

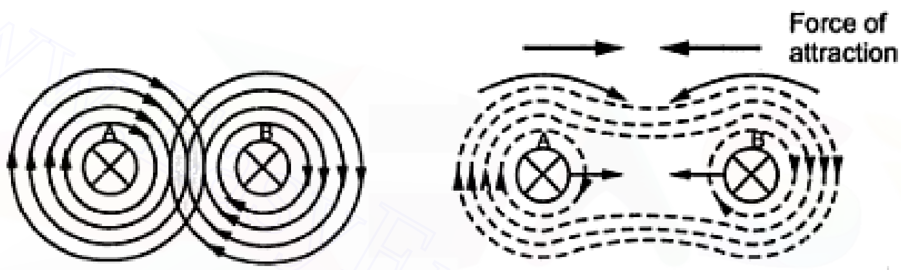
$$F = 10 \times 10^{-3} \times 50 \times 0.2 = \mathbf{0.1 \text{ N}}$$

3.13 Force between Two Parallel Current Carrying Conductors

The force between two parallel current carrying conductors depends on the directions of these two currents. We have seen that whenever there is interaction of two fluxes, the force gets generated. In this case each current carrying conductor produces its own flux around it. So when such two conductors are placed nearby, due to interaction of two fluxes there exists a force between them.

3.13.1 Direction of Both the Currents Same

Consider two parallel conductors A and B which are carrying current in the same direction as shown in the Fig. 3.28 (a).



(a) Current carrying conductors (b) Flux pattern

Fig. 3.28 Force of attraction

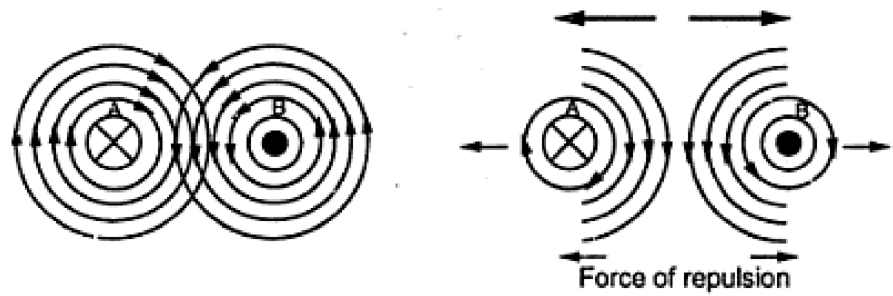
Then the direction of the two radial fields around them can be decided by right hand thumb rule. When such two conductors are placed parallel to each other and nearby the two fields interact. In space between the conductors, the two fluxes are in opposite direction and cancel each other's effect and get neutralised while in space outside the conductors two fields help or assist each other, producing high flux area around the conductors. The resultant flux pattern is shown in the Fig. 3.28 (b).

As flux lines outside try to shorten as per their property, they exert force on the conductors. Hence conductors experience a force of attraction in between them.

3.13.2 Directions of Two Currents Opposite to Each Other

Consider the conductors A and B which are carrying currents in the directions opposite to each other. Then the directions of the two radial fields can be shown in the Fig. 3.29 (a)

When such two conductors are placed nearby, parallel to each other then these two fields interact with each other. Now in space between the conductors two fluxes assist each other producing high flux zone. While surrounding the conductors, two fluxes oppose each other and cancel each other. The resultant flux pattern is shown in the Fig. 3.29 (b).



(a) Current carrying conductors (b) Flux pattern

Fig. 3.29 Force of repulsion

As flux lines within the space try to shorten as per their property and due to this they exert force on the conductors in directions opposite to each other. Due to this conductors experience a force of repulsion in between them.

3.13.3 Magnitude of Force between Two Parallel Conductors

We have seen that two current carrying conductors when placed nearby, parallel to each other experience a force. The direction of such a force depends on the directions of the flow of currents. Let us derive expression for its magnitude which requires the understanding of the permeability.

Let the two parallel conductors be 'A' and 'B' carrying currents I_1 and I_2 amperes respectively, placed in vacuum.

Now the magnetic field strength at a point 'd' metres from the axis of the current carrying conductor is given by

$$H = \frac{I}{2\pi d} \quad \text{AT/m or A/m}$$

where I is current through the conductor.

Now let distance between the centres of the conductors be 'r' metres. So magnetic field strength due to conductor A at a centre of B which is 'r' metres away is

$$H = \frac{I_1}{2\pi r}$$

Now let conductors are placed in vacuum then

$$B = \mu_0 H$$

∴ Flux density due to conductor A at center of B is

$$B = \frac{\mu_0 I_1}{2\pi r} \text{ Wb/m}^2$$

Now force experienced by conductor B is

$$F = B I l$$

when I is current through conductor B = I_2

$$\therefore \quad \boxed{F = \frac{\mu_0 I_1}{2\pi r} \cdot I_2 \cdot l \text{ Newtons.}}$$

Now $\quad \mu_0 = 4\pi \times 10^{-7}$ for vacuum

$$\therefore \quad F = \frac{4\pi \times 10^{-7} I_1 I_2 \times l}{2\pi r}$$

$$\therefore \quad \boxed{F = 2 \times 10^{-7} I_1 I_2 \frac{l}{r} \text{ Newtons}}$$

Similarly due to magnetic field of B conductor A also experiences a force of same magnitude but in opposite direction.

Example 3.5 : *Two long parallel conductors carry currents of 70 A and 120 A in opposite directions. The perpendicular distance between the conductors is 15 cm. Calculate the force per metre which one conductor will exert on the other. Also calculate the field strength at a point which is 60 mm from conductor A and 90 mm from conductor B.*

Solution : I_1 = 70 A, I_2 = 120 A, R = 15 cm = 0.15 m.

Force per metre is required i.e. l = 1m

$$\therefore \quad F = 2 \times 10^{-7} I_1 I_2 \frac{l}{r} = 2 \times 10^{-7} \times 70 \times 120 \times \frac{1}{0.15}$$

$$= 0.0112 \text{ N/m}$$

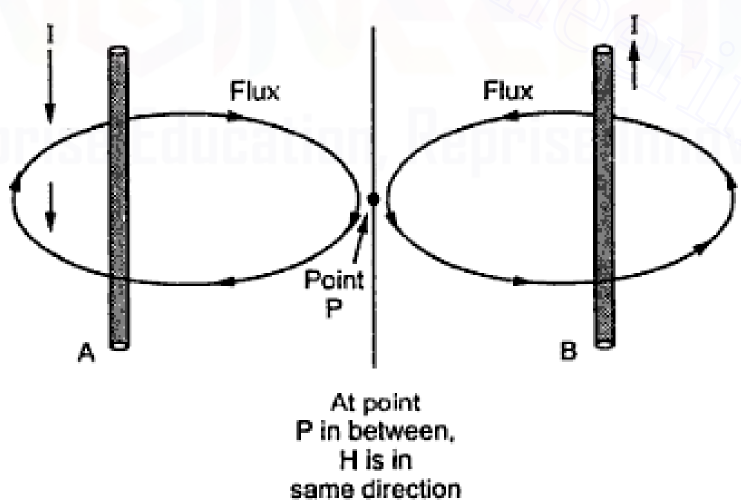


Fig. 3.30

Now magnetic field strength at a point 60 mm away from A is given by

$$H_A = \frac{I}{2\pi d} = \frac{70}{2\pi \times (60 \times 10^{-3})} = 185.68 \text{ A/m}$$

Magnetic field strength at a point 90 mm away from conductor B is given by

$$H_B = \frac{I}{2\pi d} = \frac{120}{2\pi\times(90\times10^{-3})} = 212.20 \text{ A/m}$$

> **Key Point:** *Since the currents in the two conductors are in opposite direction, at a point in between the two, magnetic field strengths will be in same direction, as shown.*

∴ Resultant magnetic field strength at a point 60 mm from A and 90 mm from B is

$$H = H_A + H_B = 185.68 + 212.20 = \mathbf{397.88\ A/m}$$

3.13.4 Unit of Ampere

If the current through each conductor is 1 A while the distance between their centers is 1 m and length l of each conductor which is under the influence of flux density B is 1 m then,

$$I_1 = I_2 = 1 \text{ A} \quad \text{and } l = r = 1 \text{ m}$$

$$\therefore \quad F = 2\times10^{-7}\times1\times1\times\frac{1}{1} = \mathbf{2\times10^{-7}\ N.}$$

In such a case, force experienced by each conductor is 2×10^{-7} N. From this, unit of ampere can be defined.

> **Key Point:** *A current, which when flowing through each of the two long parallel straight conductors having infinite length and negligible cross-section, separated from each other by a distance of one meter in vacuum, producing a force 2×10^{-7} N per meter length in between the conductors is called **one ampere current**.*

3.14 Permeability

The flow of flux produced by the magnet not only depends on the magnetic field strength but also on one important property of the magnetic material called permeability. It is related to the medium in which magnet is placed. The force exerted by one magnetic pole on other depends on the medium in which magnets are placed.

> **Key Point:** *The **permeability** is defined as the ability or ease with which the magnetic material forces the magnetic flux through a given medium.*

For any magnetic material, there are two permeabilities,
i) Absolute permeability ii) Relative permeability.

3.14.1 Absolute Permeability (μ)

The magnetic field strength (H) decides the flux density (B) to be produced by the magnet around it, in a given medium. The ratio of magnetic flux density B in a particular medium (other than vacuum or air) to the magnetic field strength H producing that flux density is called **absolute permeability** of that medium.

It is denoted by μ and mathematically can be expressed as,

$$\boxed{\begin{aligned} \mu &= \frac{B}{H} \\ \text{i.e.} \quad B &= \mu H \end{aligned}}$$

The permeability is measured in units henries per metre denoted as H/m.

3.14.2 Permeability of Free Space or Vacuum (μ_0)

If the magnet is placed in a free space or vacuum or in air then the ratio of flux density B and magnetic field strength H is called **Permeability of free space** or **Vacuum** or **air.**

It is denoted as μ_0 and measured in H/m. It denotes the ease with which the magnetic flux permeates the free space or vacuum or air.

It is experimentally found that this μ_0 i.e. ratio of B and H in vacuum remains constant every where in the vacuum and its value is $4\pi\times10^{-7}$ H/m.

$$\therefore \quad \boxed{\mu_0 = \frac{B}{H} \text{ in vacuum } = 4\pi\times10^{-7} \text{ H/m}}$$

> **Key Point** : *For a magnetic material, the absolute permeability μ is not constant. This is because B and H bears a nonlinear relation in case of magnetic materials. If magnetic field strength is increased, there is change in flux density B but not exactly proportional to the increase in H.*

The ratio B to H is constant only for free space, vacuum or air which is $\mu_0 = 4\pi\times10^{-7}$ H/m.

3.14.3 Relative Permeability (μ_r)

Generally the permeability of different magnetic materials is defined relative to the permeability of free space (μ_0). The **relative permeability** is defined as the ratio of flux density produced in a medium (other than free space) to the flux density produced in free space, under the influence of same magnetic field strength and under identical conditions.

Thus if the magnetic field strength is H which is producing flux density B in the medium while flux density B_0 in free space then the relative permeability is defined as,

$$\mu_r = \frac{B}{B_0} \quad \text{where H is same.}$$

It is dimensionless and has no units.

$\boxed{\text{For free space, vacuum or air, } \mu_r = 1}$

According to definition of absolute permeability we can write for given H,

$$\mu = \frac{B}{H} \quad \text{in medium} \qquad \ldots(1)$$

$$\mu_0 = \frac{B_0}{H} \quad \text{in free space} \qquad ...(2)$$

Dividing (1) and (2) , $\frac{\mu}{\mu_0} = \frac{B}{B_0}$

but $\frac{B}{B_0} = \mu_r$

$\therefore$ $\frac{\mu}{\mu_0} = \mu_r$

$\therefore$ $\boxed{\mu = \mu_0 \mu_r \quad H/m}$

The relative permeability of metals like iron, steel varies from 100 to 100,000

Key Point : *If we require maximum flux production for the lesser magnetic field strength then the value of the relative permeability of the core material should be as high as possible.*

For example if relative permeability of the iron is 1000 means it is 1000 times more magnetic than the free space or air.

3.15 Magnetomotive Force (M.M.F.or F)

The flow of electrons is current which is basically due to electromotive force (e.m.f.). Similarly the force behind the flow of flux or production of flux in a magnetic circuit is called magnetomotive force (m.m.f.) The m.m.f. determines the magnetic field strength.

It is the driving force behind the magnetic circuit. It is given by the product of the number of turns of the magnetizing coil and the current passing through it.

Mathematically it can be expressed as,

$$\boxed{\text{m.m.f.} = N\,I \quad \text{ampere turns}}$$

where N = Number of turns of magnetising coil and I = Current through coil

Its unit is ampere turns (AT) or amperes (A).

It is also defined as the work done in joules on a unit magnetic pole in taking it once round a closed magnetic circuit.

3.16 Reluctance (S)

In an electric circuit, current flow is opposed by the resistance of the material, similarly there is opposition by the material to the flow of flux which is called **reluctance**

It is defined as the resistance offered by the material to the flow of magnetic flux through it. It is denoted by 'S'. It is directly proportional to the length of the magnetic circuit while inversely proportional to the area of cross-section.

$$S \propto \frac{l}{a} \quad \text{where '}l\text{' in 'm' while 'a' in 'm}^2\text{'}$$

$$\therefore \quad S = \frac{Kl}{a}$$

where K = Constant of proportionality

= Reciprocal of absolute permeability of material $= \frac{1}{\mu}$

$$\therefore \quad \boxed{S = \frac{l}{\mu a} = \frac{l}{\mu_0 \mu_r a} \ \text{A / Wb}}$$

It is measured in amperes per weber (A/Wb).

The reluctance can be also expressed as the ratio of magnetomotive force to the flux produced.

$$\text{i.e.} \quad \text{Reluctance} = \frac{\text{m.m.f}}{\text{flux}}$$

$$\therefore \quad \boxed{S = \frac{NI}{\phi} \quad \text{AT / Wb or A / Wb}}$$

3.17 Permeance

The **permeance** of the magnetic circuit is defined as the reciprocal of the reluctance.

It is defined as the property of the magnetic circuit due to which it allows flow of the magnetic flux through it.

$$\therefore \quad \boxed{\text{Permeance} = \frac{1}{\text{Reluctance}}}$$

It is measured in weber per amperes (Wb/A).

3.18 Magnetic Circuits

The **magnetic circuit** can be defined as, the closed path traced by the magnetic lines of force i.e. flux. Such a magnetic circuit is associated with different magnetic quantities as m.m.f., flux reluctance, permeability etc.

Consider simple magnetic circuit shown in the Fig. 3.31 (a). This circuit consists of an iron core with cross-sectional area of 'a' m^2 with a mean length of 'l' m. (This is mean length of the magnetic path which flux is going to trace.) A coil of N turns is wound on one of the sides of the square core which is excited by a supply. This supply drives a current I through the coil. This current carrying coil produces the flux (ϕ) which completes its path through the core as shown in the Fig. 3.31 (a).

This is analogous to simple electric circuit in which a supply i.e. e.m.f. of E volts drives a current I which completes its path through a closed conductor having resistance R. This analogous electrical circuit is shown in the Fig. 3.31 (b).

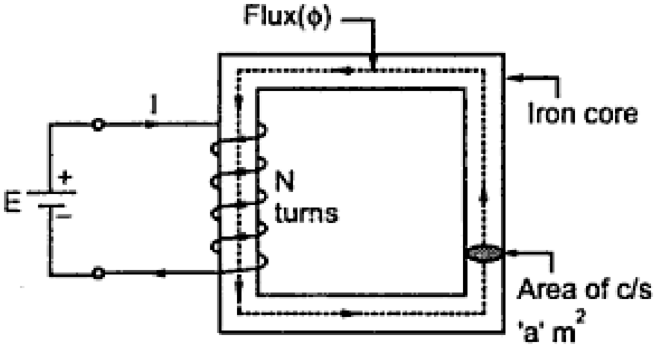


Fig. 3.31 (a) Magnetic circuit

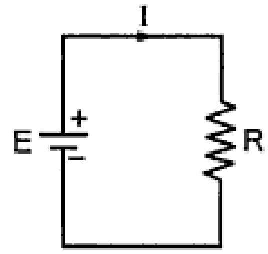


Fig. 3.31 (b) Electrical equivalent

Let us derive relationship between m.m.f., flux and reluctance.

I = Current flowing through the coil.

N = Number of turns.

ϕ = Flux in webers.

B = Flux density in the core.

μ = Absolute permeability of the magnetic material

μ_r = Relative permeability of the magnetic material

Magnetic field strength inside the solenoid is given by,

$$H = \frac{NI}{l} \quad \text{AT/m} \qquad \ldots(1)$$

Now flux density is,

$$B = \mu H$$

$$B = \frac{\mu_0 \mu_r N I}{l} \quad \text{Wb/m}^2 \qquad \ldots(2)$$

Now as area of cross-section is 'a' m^2, total flux in core is,

$$\phi = B\,a = \frac{\mu_0 \mu_r N I a}{l} \quad \text{Wb} \qquad \ldots(3)$$

i.e.

$$\phi = \frac{\frac{NI}{l}}{\mu_0 \mu_r a}$$

$\therefore$

$$\boxed{\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{F}{S}}$$

where NI = Magnetomotive force m.m.f. in AT

$$S = \frac{l}{\mu_0 \mu_r a}$$

= Reluctance offered by the magnetic path.

This expression of the flux is very much similar to expression for current in electric circuit.

$$I = \frac{\text{e.m.f.}}{\text{resistance}}$$

Key Point : *So current is analogous to the flux, e.m.f. is analogous to the m.m.f. and resistance is analogous to the reluctance.*

Example 3.6 : *An iron ring of circular cross sectional area of 3.0 cm² and mean diameter of 20 cm is wound with 500 turns of wire and carries a current of 2.09 A to produce the magnetic flux of 0.5 m Wb in the ring. Determine the permeability of the material.*

(May - 2000)

Solution : The given values are :

$a = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2$, $d = 20$ cm, $N = 500$, $I = 2$ A, $\phi = 0.5$ m Wb

Now, $l = \pi \times d = \pi \times 20 = 62.8318 \text{ cm} = 0.628318 \text{ m}$

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{0.628313}{4\pi \times 10^{-7} \times \mu_r \times 3 \times 10^{-4}} = \frac{1.6667 \times 10^9}{\mu_r} \quad \text{... (1)}$$

$$f = \frac{\text{m.m.f.}}{S} = \frac{NI}{S}$$

$$\therefore \quad S = \frac{NI}{\phi} = \frac{500 \times 2}{0.5 \times 10^{-3}} = 2 \times 10^6 \text{ AT / Wb} \quad \text{... (2)}$$

Equating (1) and (2),

$$\therefore \quad 2 \times 10^6 = \frac{1.6667 \times 10^9}{\mu_r}$$

$$\therefore \quad \mu_r = 833.334$$

3.18.1 Series Magnetic Circuits

In practice magnetic circuit may be composed of various materials of different permeabilities, of different lengths and of different cross-sectional areas. Such a circuit is called **composite** magnetic circuit.When such parts are connected one after the other the circuit is called **series magnetic circuit.**

Consider a circular ring made up of different materials of lengths l_1, l_2 and l_3 and with cross-sectional areas a_1 ,a_2 and a_3 with absolute permeabilities μ_1, μ_2 and μ_3 as shown in the Fig. 3.32.

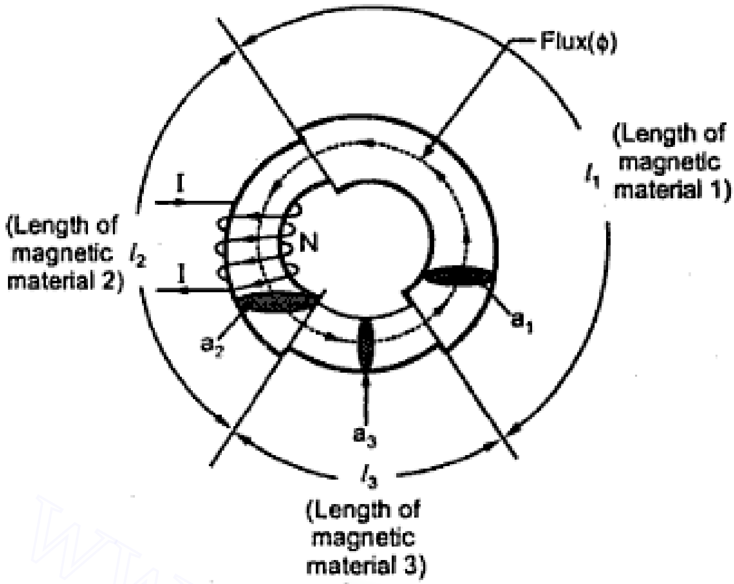


Fig. 3.32 A series magnetic circuit

Let coil wound on ring has N turns carrying a current of I amperes.

The total m.m.f. available is = NI AT

This will set the flux 'ϕ' which is same through all the three elements of the circuit.

This is similar to three resistances connected in series in electrical circuit and connected to e.m.f. carrying same current 'I' through all of them.

Its analogous electric circuit can be shown as in the Fig. 3.33.

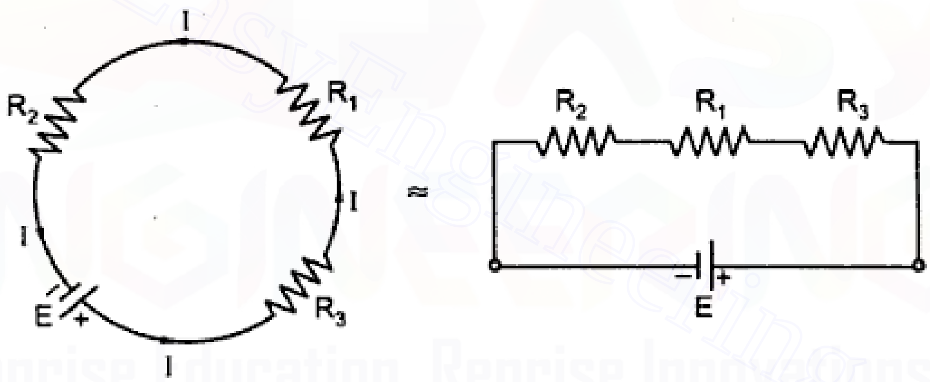


Fig. 3.33 Equivalent electrical circuit

The total resistance of the electric circuit is $R_1 + R_2 + R_3$. Similarly the total reluctance of the magnetic circuit is,

$$\text{Total } S_T = S_1 + S_2 + S_3 = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}$$

$$\therefore \quad \text{Total } \phi = \frac{\text{Total m.m.f.}}{\text{Total reluctance}} = \frac{NI}{S_T} = \frac{NI}{(S_1 + S_2 + S_3)}$$

$$\therefore \quad NI = S_T \phi = (S_1 + S_2 + S_3)\phi$$

$$NI = S_1\phi + S_2\phi + S_3\phi$$

$$\therefore \quad (\text{m.m.f.})T = (\text{m.m.f.})_1 + (\text{m.m.f.})_2 + (\text{m.m.f.})_3$$

The total m.m.f. also can be expressed as,

$$(\text{m.m.f.})T = H_1 l_1 + H_2 l_2 + H_3 l_3$$

where $H_1 = \frac{B_1}{\mu_1}, \quad H_2 = \frac{B_2}{\mu_2}, \quad H_3 = \frac{B_3}{\mu_3}$

So for a **series magnetic** circuit we can remember,

1) The magnetic flux through all the parts is same.
2) The equivalent reluctance is sum of the reluctance of different parts.
3) The resultant m.m.f. necessary is sum of the m.m.f.s in each individual part.

3.18.2 Series Circuit with Air Gap

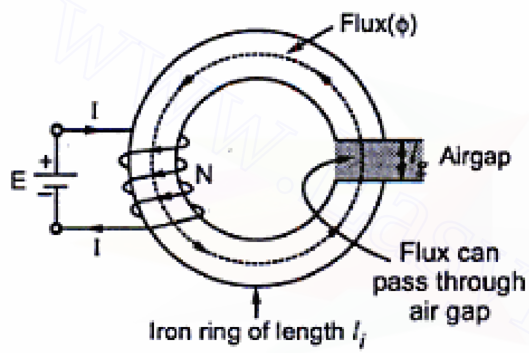


Fig. 3.34 A ring with an air gap

The series magnetic circuit can also have a short air gap.

Key Point : *This is possible because we have seen earlier that flux can pass through air also.*

Such air gap is not possible in case of electric circuit.

Consider a ring having mean length of iron part as 'l_i' as shown in the Fig. 3.34.

Total m.m.f = N I AT

Total reluctance $\boxed{S_T = S_i + S_g}$

where S_i = Reluctance of iron path

S_g = Reluctance of air gap

$$\therefore \quad S_i = \frac{l_i}{\mu a_i}$$

$$S_g = \frac{l_g}{\mu_0 a_i}$$

Key Point: *The absolute permeability of air* $\mu = \mu_0$

The cross-sectional area of air gap is assumed to be equal to area of the iron ring.

$$\therefore \quad S_T = \frac{l_i}{\mu a_i} + \frac{l_g}{\mu_0 a_i}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S_T}$$

or Total m.m.f. = m.m.f. for iron + m.m.f for air gap

$$\therefore \quad \boxed{NI = S_i\phi + S_g\phi \quad \text{AT for ring.}}$$

Example 3.7 : *An iron ring 8 cm. mean diameter is made up of round iron of diameter 1 cm and permeability of 900, has an air gap of 2mm wide. It consists of winding with 400 turns carrying a current of 3.5A. Determine,*

i) m.m.f. ii) total reluctance iii) the flux iv) flux density in ring **(May - 98, Dec - 99)**

Solution : The ring and the winding is shown in the Fig. 3.35.

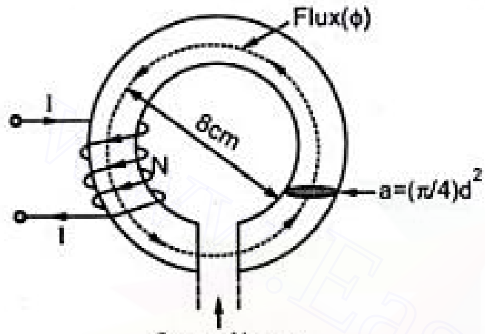


Fig. 3.35

Diameter of ring d = 8 cm,

∴ length of iron = πd − length of air gap

$$l_i = \pi\times\left(8\times10^{-2}\right)-2\times10^{-3}$$

$$= 0.2493 \text{ m.}$$

> **Key Point :** *While calculating iron length, do not forget to subtract length of air gap from total mean length.*

$$l_g = \text{Length of air gap}$$

$$= 2 \text{ mm} = 2\times10^{-3} \text{ m}$$

diameter of iron = 1 cm

$$\therefore \quad \text{area of cross section} \quad a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(1\times10^{-2}\right)^2$$

$$a = 7.853\times10^{-5} \text{ m}^2$$

Area of cross section of air gap and ring is to be assumed same.

i) Total m.m.f. produced $= N\,I = 400\times3.5$

$$= \textbf{1400 AT} \text{ (ampere turns)}$$

ii) Total reluctance $S_T = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} \qquad \text{...Given } \mu_r = 900$$

$$= \frac{0.2493}{4\pi\times10^{-7}\times900\times7.853\times10^{-5}}$$

$$= 2806947.615 \text{ AT/ Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} \qquad \text{as } \mu_r = 1 \text{ for air}$$

$$\therefore \quad S_g = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times7.853\times10^{-5}} = 20.2667\times10^6 \quad \text{AT / Wb}$$

$$\therefore \quad S_T = 2806947.615 + 20.2667\times10^6 = 23.0737\times10^6 \quad \text{AT / Wb}$$

ii) $$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S_T} = \frac{1400}{23.0737\times10^6}$$

$$= 6.067\times10^{-5} \quad \text{Wb}$$

iv) $$\text{Flux density} = \frac{\phi}{a} = \frac{6.067\times10^{-5}}{7.853\times10^{-5}} = 0.7725 \quad \text{Wb / m}^2$$

3.18.3 Parallel Magnetic Circuits

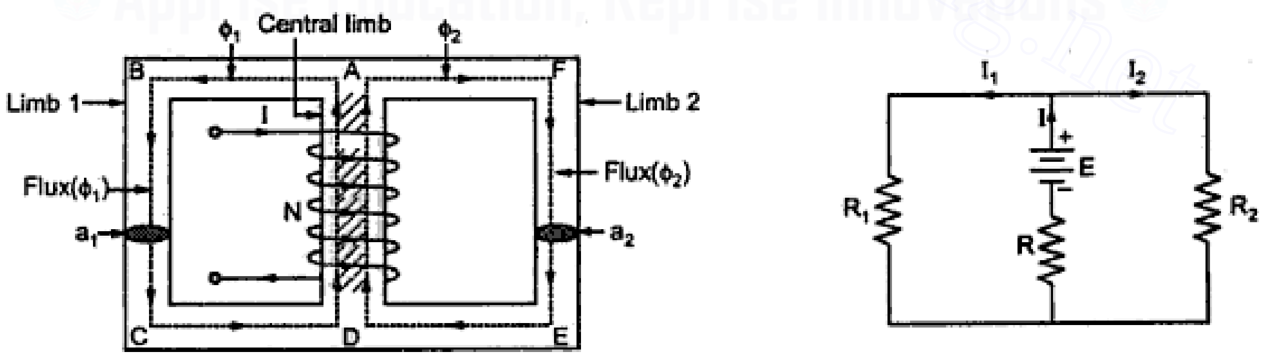
In case of electric circuits, resistances can be connected in parallel. Current through each of such resistances is different while voltage across all of them is same. Similarly different reluctances may be in parallel in case of magnetic circuits. A magnetic circuit which has more than one path for the flux is known as a **parallel magnetic circuit.**

Consider a magnetic circuit shown in the Fig. 3.36 (a). At point A the total flux ϕ, divides into two parts ϕ_1 and ϕ_2.

$$\therefore \quad \boxed{\phi = \phi_1 + \phi_2}$$

The fluxes ϕ_1 and ϕ_2 have their paths completed through ABCD and AFED respectively.

This is similar to division of current in case of parallel connection of two resistances in an electric circuit. The analogous electric circuit is shown in the Fig. 3.36 (b).



(a) Magnetic circuit (b) Equivalent electrical circuit

Fig. 3.36 A parallel magnetic circuit

The mean length of path ABCD = l_1 m

The mean length of the path AFED = l_2 m

The mean length of the path AD = l_c m

The reluctance of the path ABCD = S_1

The reluctance of path AFED = S_2

The reluctance of path AD = S_c

The total m.m.f. produced = N I AT

$$\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$$

$$\therefore \quad \text{m.m.f.} = \phi \times S$$

$$\therefore \quad \text{For path ABCDA, NI} = \phi_1 S_1 + \phi S_c$$

For path AFEDA, $NI = \phi_2 S_2 + \phi S_c$

where $S_1 = \dfrac{l_1}{\mu a_1}$, $S_2 = \dfrac{l_2}{\mu a_2}$ and $S_c = \dfrac{l_c}{\mu a_c}$

Generally $a_1 = a_2 = a_c$ = Area of cross-section

For parallel circuit,

$$\boxed{\text{Total m.m.f.} = \begin{array}{c}\text{m.m.f. required} \\ \text{by central limb}\end{array} + \begin{array}{c}\text{m.m.f. required by} \\ \text{any one of outer limbs}\end{array}}$$

$$NI = (NI)_{AD} + (NI)_{ABCD} \text{ or } (NI)_{AFED}$$

$$NI = \phi S_c + [\phi_1 S_1 \text{ or } \phi_2 S_2]$$

As in the electric circuit e.m.f. across parallel branches is same, in the magnetic circuit the m.m.f. across parallel branches is same.

Thus same m.m.f. produces different fluxes in the two parallel branches. For such parallel branches,

$$\boxed{\phi_1 S_1 = \phi_2 S_2}$$

Hence while calculating total m.m.f., the m.m.f. of **only one** of the two parallel branches must be considered.

3.18.4 Parallel Magnetic Circuit with Air Gap

Consider a parallel magnetic circuit with air gap in the central limb as shown in the Fig. 3.37.

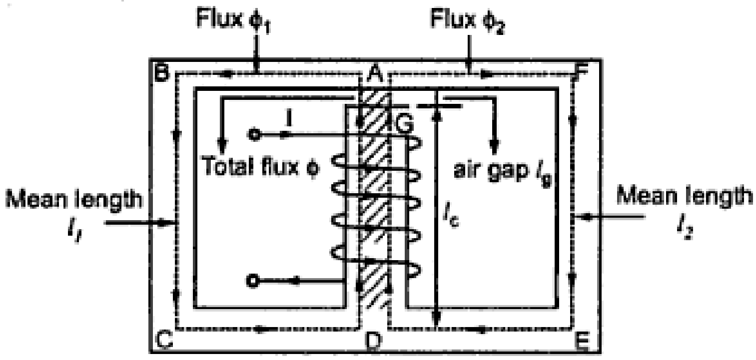


Fig. 3.37 Parallel circuit with air gap

The analysis of this circuit is exactly similar to the parallel circuit discussed above. The only change is the analysis of central limb. The central limb is series combination of iron path and air gap. The central limb is made up of,

$$\text{path GD} = \text{iron path} = l_c$$

$$\text{path GA} = \text{air gap} = l_g$$

The total flux produced is ϕ. It gets divided at A into ϕ_1 and ϕ_2.

$$\therefore \quad \phi = \phi_1 + \phi_2$$

The reluctance of central limb is now,

$$S_c = S_i + S_g = \frac{l_c}{\mu a_c} + \frac{l_g}{\mu_0 a_c}$$

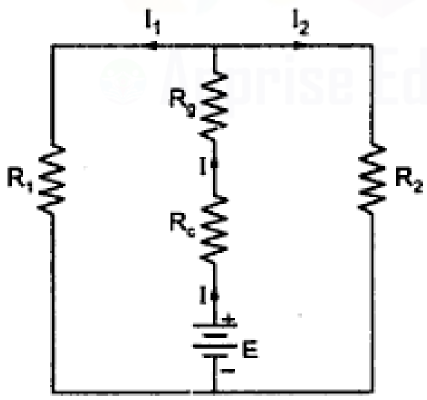


Fig. 3.38 Electrical equivalent circuit

Hence m.m.f. of central limb is now,

$$\boxed{(\text{m.m.f.})_{AD} = (\text{m.m.f.})_{GD} + (\text{m.m.f.})_{GA}}$$

Hence the total m.m.f. can be expressed as,

$$\boxed{(NI)_{total} = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD} \text{ or } (NI)_{AFED}}$$

Thus the electrical equivalent circuit for such case becomes as shown in the Fig. 3.38.

Similarly there may be air gaps in the side limbs but the method of analysis remains the same.

Example 3.8 : *The magnetic circuit shown in Fig. 3.39 is constructed of wrought iron. The cross-section of the centre limb is 8 cm^2 and of each other limb, 5 cm^2. If the coil on centre limb is wound with 1000 turns, calculate the exciting current required to set up a flux of 1.2 mWb in the centre limb. Width of each air gap is 1 mm. Points on the B/H curve of wrought iron are as follows -* **(May - 2002)**

B (in Tesla)	1.2	1.35	1.5
H (in AT/m)	625	1100	2000

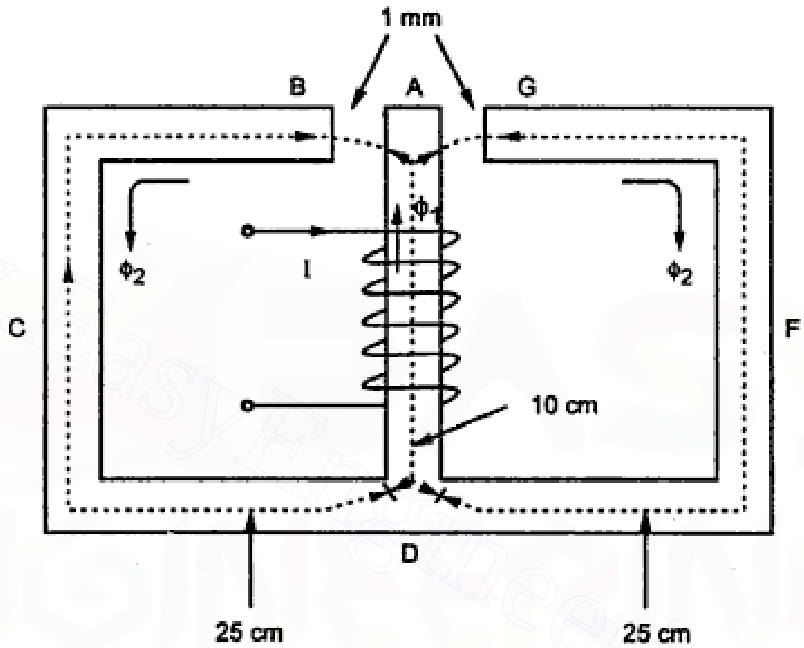


Fig. 3.39

Solution : Given; l_c = length of central limb = 10 cm = 0.1 m

$$a_c = 8 \text{ cm}^2 = 8\times10^{-4} \text{ m}^2, \phi_c = 1.2 \text{ m Wb} = 1.2\times10^{-3} \text{ Wb}$$

$$l_i = \text{Length of iron path of side limb} = 25 \text{ cm}$$

$$= 0.25 \text{ m (on each side)}$$

$$l_g = \text{Length of air gap} = 1 \text{ mm} = 1\times10^{-3} \text{ m}$$

$$a_i = 5 \text{ cm}^2 = 5\times10^{-4} \text{ m}^2$$

This is the example of parallel magnetic circuit.

The flux in central limb 1.2 mWb gets divided into two equal paths as shown in Fig. 3.39.

$$\text{Flux in side limbs} = \frac{1.2}{2} \text{ i.e. } \therefore \phi_i = 0.6 \text{ mWb}$$

Flux density in central limb is,

$$B_c = \frac{\phi_c}{a_c} = \frac{1.2\times10^{-3}}{8\times10^{-4}} = 1.5 \text{ Tesla}$$

Flux density in air gap is,

$$B_g = \frac{\phi_i}{a_i} = \frac{0.6\times10^{-3}}{5\times10^{-4}} = 1.2 \text{ Wb/m}^2 \text{ i.e. Tesla}$$

Flux density in side limb is,

$$B_i = \frac{\phi_i}{a_i} = \frac{0.6\times10^{-3}}{5\times10^{-4}} = 1.2 \text{ Wb/m}^2 \text{ i.e. Tesla}$$

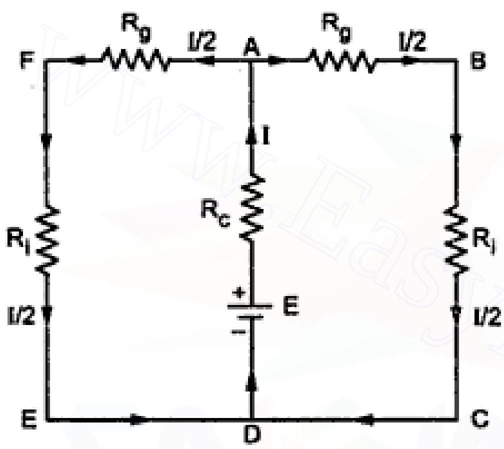


Fig. 3.40

The equivalent circuit in electrical form is shown in Fig. 3.40 (a).

Applying KVL to loop ABCD,

$$E - IR_c - \frac{I}{2} R_g - \frac{I}{2} R_i = 0$$

$$\therefore \quad E = IR_c + \frac{I}{2} R_g + \frac{I}{2} R_i$$

Similarly applying Kirchhoff's mmf law to the loop,

$$\text{mmf} = H_c l_c + H_g l_g + H_i l_i$$

where

$H_c l_c$ = m.m.f. required by central limb

$H_g l_g$ = m.m.f. required by air gap

$H_i l_i$ = m.m.f. required by iron path on any one side

I) Central limb

$$B_c = 1.5 \text{ Tesla}$$

From B-H table given, corresponding, $H_c = 2000$

$$H_c l_c = 2000\times0.1 = 200 \text{ AT}$$

II) Side limb

$$B_i = 1.2 \text{ Tesla}$$

From B-H table, given corresponding $H_i = 625$

$$H_i l_i = 625\times0.25 = 156.25 \text{ AT}$$

III) The air gap

> **Key Point:** *For air gap, B–H table should not be referred but value of field strength H_g for air gap is to be calculated as,*

$$B_g = \mu_0 H_g$$

$$H_g = \frac{B_g}{\mu_0} = \frac{1.2}{4\pi\times10^{-7}}$$

$$H_g = 954929.65 \text{ AT}$$

$$\therefore \quad H_g\, l_g = 954929.65 \times (1\times 10^{-3})$$

$$\therefore \quad H_g\, l_g = 954.9296 \text{ AT}$$

$$\text{Total m.m.f. required} = H_c\, l_c + H_i\, l_i + H_g\, l_g$$

$$\therefore \quad N\,I = 200 + 156.25 + 954.9296$$

$$\therefore \quad N\,I = 1311.17 \text{ AT}$$

$$\therefore \quad \text{Current } I = \frac{1311.17}{\text{No. of turns}} = \frac{1311.17}{1000}$$

$$\therefore \quad I = 1.31 \text{ A}$$

Example 3.9 : *A cast steel structure is made of a rod of square section 2.5 cm × 2.5 cm as shown in the Fig. 3.41. What is the current that should be passed in a 500 turn coil on the left limb so that a flux of 2.5 mWb is made to pass in the right limb. Assume permeability as 750 and neglect leakage.*

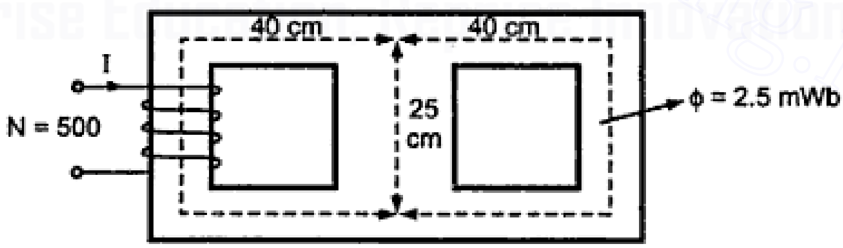


Fig. 3.41

Solution : This is parallel magnetic circuit. Its electrical equivalent is shown in the Fig. 3.41 (a).

The total flux produced gets distributed into two parts having reluctances S_1 and S_2.

S_1 = Reluctance of centre limb

S_2 = Reluctance of right side

$$S_1 = \frac{l_1}{\mu_0 \mu_r a_1} = \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 750 \times 2.5 \times 2.5 \times 10^{-4}}$$

$$= 424.413 \times 10^3 \text{ AT/Wb}$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r a_1} = \frac{40 \times 10^{-2}}{4\pi \times 10^{-7} \times 750 \times 2.5 \times 2.5 \times 10^{-4}}$$

$$= 679.061 \times 10^3 \text{ AT/Wb}$$

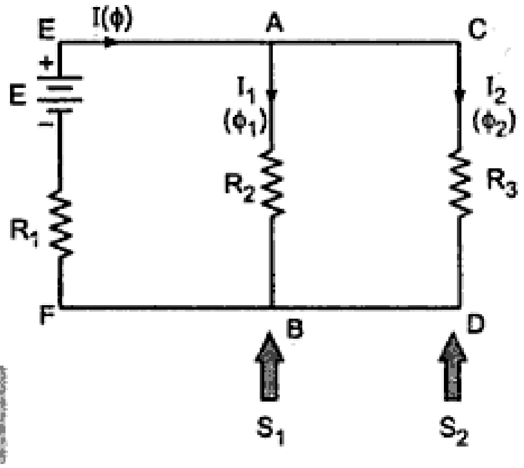


Fig. 3.41 (a)

> **Key Point :** *For parallel branches, m.m.f. remains same.*

For branch AB and CD, m.m.f. is same.

$$\therefore \quad \text{m.m.f.} = \phi_1 S_1 = \phi_2 S_2$$

And $\quad \phi_2 = 2.5 \text{ mWb}$ ... given

$$\therefore \quad \phi_1 = \frac{\phi_2 S_2}{S_1} = \frac{2.5 \times 10^{-3} \times 679.061 \times 10^3}{424.413 \times 10^3} = 4 \text{ mWb}$$

$$\therefore \quad \phi = \phi_1 + \phi_2 = 2.5 + 4 = 6.5 \text{ mWb}$$

Total m.m.f. required is sum of the m.m.f. required for AEFB and that for either central or side limb.

$$S_{AEFB} = S_2 = 679.061 \times 10^3 \text{ AT/Wb}$$

$$\therefore \quad \text{m.m.f. for AEFB} = S_{AEFB} \times \phi = 679.061 \times 10^3 \times 6.5 \times 10^{-3}$$

$$= 4413.8965 \text{ AT}$$

$$\therefore \quad \text{Total m.m.f.} = 4413.8965 + \phi_1 S_1$$

$$= 4413.8965 + 4 \times 10^{-3} \times 424.413 \times 10^3 = 6111.548 \text{ AT}$$

But $\quad NI = \text{total m.m.f.}$

$$\therefore \quad I = \frac{6111.548}{500} = 12.223 \text{ A}$$

3.19 Kirchhoff's Laws for Magnetic Circuit

Similar to the electrical circuit Kirchhoff's Laws can be used to analyse complex magnetic circuit. The laws can be stated as below :

3.19.1 Kirchhoff's Flux Law

The total magnetic flux arriving at any junction in a magnetic circuit is equal to the total magnetic flux leaving that junction.

At a junction, $$\boxed{\sum \phi = 0}$$

The law infact is used earlier to analyse parallel magnetic circuit at a junction A shown in the Fig.3.36 (a), where

$$\phi = \phi_1 + \phi_2$$

3.19.2 Kirchhoff's M.M.F. Law

The resultant m.m.f. around a closed magnetic circuit is equal to the algebraic sum of the products of the flux and the reluctance of each part of the closed circuit i.e. for a closed magnetic circuit.

$$\boxed{\sum \text{m.m.f.} = \sum \phi S}$$

As $\phi \times S = \text{flux} \times \text{reluctance} = \text{m.m.f.}$

M.M.F. also can be calculated as $H \times l$ where H is field strength and 'l' is mean length

$\therefore \quad \text{m.m.f.} = Hl$

Alternatively the same law can be stated as :

The resultant m.m.f. around any closed loop of a magnetic circuit is equal to the algebraic sum of the products of the magnetic field strength and the length of each part of the circuit i.e. for a closed magnetic circuit

$$\boxed{\sum \text{m.m.f.} = \sum H.l}$$

3.20 Comparison of Magnetic and Electric Circuits

Similarities between electric and magnetic circuits are listed below :

Sr. No.	Electric Circuit	Magnetic Circuit
1.	Path traced by the current is called electric circuit.	Path traced by the magnetic flux is defined as magnetic circuit.
2.	E.M.F. is the driving force in electric circuit, the unit is volts.	M.M.F. is the driving force in the magnetic circuit , the unit of which is ampere turns.
3.	There is current I in the electric circuit measured in amperes.	There is flux ϕ in the magnetic circuit measured in webers.
4.	The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
5.	Resistance oppose the flow of the current. Unit is ohm.	Reluctance is opposed by magnetic path to the flux. Unit is ampere turn/weber.

6.	$R = \rho \frac{l}{a}$. Directly proportional to l. Inversely proportional to 'a'. Depends on nature of material.	$S = \frac{l}{\mu_0 \mu_r a}$. Directly proportional to l. Inversely proportional to $\mu = \mu_0 \mu_r$. Inversely proportional to area 'a'.
7.	The current $I = \frac{\text{e.m.f.}}{\text{resistance}}$	The flux $\phi = \frac{\text{m.m.f.}}{\text{reluctance}}$
8.	The current density $\delta = \frac{I}{a}$ A/m^2	The flux density $B = \frac{\phi}{a}$ Wb/m^2
9.	Conductivity is reciprocal of the resistivity. Conductance $= \frac{1}{R}$	Permeance is reciprocal of the reluctance. Permeance $= \frac{1}{S}$
10.	Kirchhoff's current and voltage law is applicable to the electric circuit.	Kirchhoff's m.m.f. law and flux law is applicable to the magnetic circuit.

There are few dissimilarities between the two which are listed below :

Sr. No.	Electric Circuit	Magnetic Circuit
1.	In the electric circuit the current actually flows i.e. there is movement of electrons.	Due to m.m.f. flux gets established and does not flow in the sense in which current flows.
2.	There are many materials which can be used as insulators i.e. air, P.V.C., synthetic resin etc, from which current can not pass.	There is no magnetic insulator as flux can pass through all the materials, even through the air as well.
3.	Energy must be supplied to the electric circuit to maintain the flow of current.	Energy is required to create the magnetic flux, but is not required to maintain it.
4.	The resistance and the conductivity are independent of current density (δ) under constant temperature. But may change due to the temperature.	The reluctance, permeance and permeability are dependent on the flux density.
5.	Electric lines of flux are not closed. They start from positive charge and end on negative charge.	Magnetic lines of flux are closed lines. They flow from N pole to S pole externally while S pole to N pole internally.
6.	There is continuous consumption of electrical energy.	Energy is required to create the magnetic flux and not to maintain it.

3.21 Magnetic Leakage and Fringing

Most of the applications which are using magnetic effects of an electric current, are using flux in air gap for their operation. Such devices are generators, motors, measuring instruments like ammeter, voltmeter etc. Such devices consist of magnetic circuit with an air gap and flux in air gap is used to produce the required effect.

Such flux which is available in air gap and is utilised to produce the desired effect is called **useful flux** denoted by ϕ_u .

It is expected that whatever is the flux produced by the magnetizing coil, it should complete its path through the iron and air gap. So all the flux will be available in air gap. In actual practice it is not possible to have entire flux available in air gap. This is because, we have already seen that there is no perfect insulator for the flux. So part of the flux completes its path through the air or medium in which coil and magnetic circuit is placed.

Key Point: *Such flux which leaks and completes its path through surrounding air or medium instead of the desired path is called the leakage flux.*

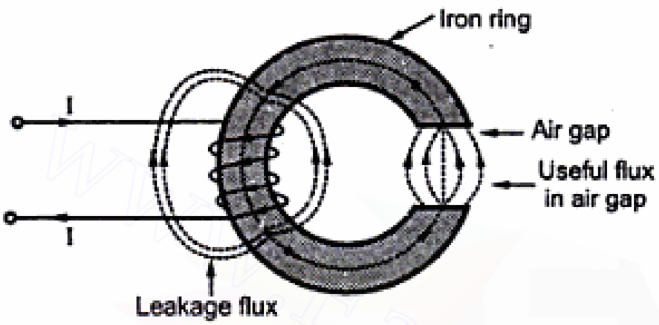


Fig. 3.42 Leakage and useful flux

The Fig. 3.42 shows the useful and leakage flux.

3.21.1 Leakage Coefficient or Hopkinson's Coefficient

The ratio of the total flux (ϕ_T) to the useful flux (ϕ_u) is defined as the **leakage coefficient of Hopkinson's coefficient** or **leakage factor** of that magnetic circuit.

It is denoted by λ .

$$\therefore \quad \boxed{\lambda = \frac{\text{total flux}}{\text{useful flux}} = \frac{\phi_T}{\phi_u}}$$

The value of 'λ' is always greater than 1 as ϕ_T is always more than ϕ_u. It generally varies between 1.1 and 1.25. Ideally its value should be 1.

3.21.2 Magnetic Fringing

When flux enters into the air gap, it passes through the air gap in terms of parallel flux lines. There exists a force of repulsion between the magnetic lines of force which are parallel and having same direction. Due to this repulsive force there is tendency of the magnetic flux to bulge out (spread out) at the edge of the air gap. This tendency of flux to bulge out at the edges of the air gap is called **magnetic fringing**.

It has following two effects :

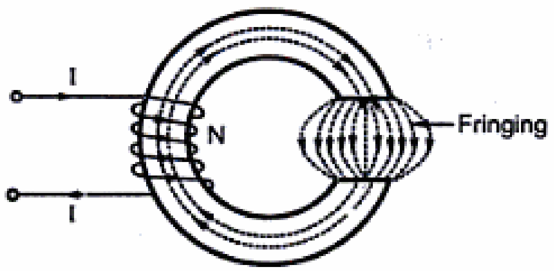


Fig. 3.43 Magnetic fringing

1) It increases the effective cross-sectional area of the air gap.

2) It reduces the flux density in the air gap.

So leakage, fringing and reluctance, in practice should be as small as possible.

> **Key Point :** *This is possible by choosing good magnetic material and making the air gap as narrow as possible.*

Example 3.10 : *A cast iron ring of 40 cm mean length and circular cross section of 5 cm diameter is wound with a coil. The coil carries a current of 3 A and produces a flux of 3 mWb in the air gap. The length of the air gap is 2 mm. The relative permeability of the cast iron is 800. The leakage coefficient is 1.2. Calculate number of turns of the coil.*

Solution : Given, $l_t = 40 \text{ cm} = 0.4 \text{ m}, \quad l_g = 2\times10^{-3} \text{ m}$

$$l_i = l_t - l_{gap} = 0.4 - 2\times10^{-3} = 0.398 \text{ m}$$

$$I = 3 \text{ A}, \quad \phi_g = 2\times10^{-3} \text{ Wb}, \mu_r = 800, \quad \lambda = 1.2$$

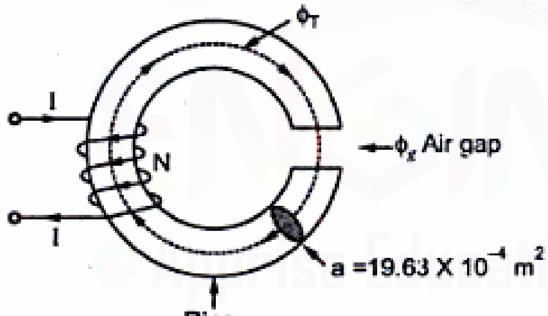


Fig. 3.44

$$\lambda = \frac{\phi_T}{\phi_g} \quad \text{... Leakage coefficient}$$

$$\therefore \quad 1.2 = \frac{\phi_T}{2\times10^{-3}}$$

$$\therefore \quad \phi_T = 2.4\times10^{-3} \text{ Wb}$$

Now reluctance of iron path $S_i = \dfrac{l_i}{\mu_0 \mu_r a}$

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times5^2 = 19.6349 \text{ cm}^2 = 19.634\times10^{-4} \text{m}^2$$

$$\therefore \quad S_i = \frac{0.398}{4\pi\times10^{-7}\times800\times19.63\times10^{-4}} = 201629.16 \text{ AT/Wb}$$

$$\phi_T = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S_i}$$

$$\therefore \quad 2.4\times10^{-3} = \frac{NI}{201629.16}$$

$$\therefore \quad \text{NI for iron path} = 483.909 \text{ AT}$$

Reluctance of air gap $\quad S_g = \dfrac{l_g}{\mu_0 a}$

$\therefore \quad S_g = \dfrac{2\times10^{-3}}{4\pi\times10^{-7}\times19.634\times10^{-4}}$

$= 810608.86$ AT/Wb

Now $\quad \phi_g = \dfrac{\text{m.m.f.}}{S_g}$

$\therefore \quad 2\times10^{-3} = \dfrac{N.I}{810608.86}$

$\therefore$ NI for air gap $= 1621.2177$ AT

$\therefore$ Total m.m.f. required $= (NI)_{iron} + (NI)_{air\ gap}$ $\quad NI = 483.909 + 1621.2177$

$\therefore \quad NI = 2105.1267$ i.e. $N\times3 = 2105.1267$

$\therefore \quad N = 701.7 \approx 702$ **turns.**

Examples with Solutions

Example 3.11 : *An iron ring has circular cross-section 4 cm in radius and the average circumference of 100 cm. The ring is uniformly wound with a coil of 700 turns. Calculate,*

i) Current required to produce a flux of 2 mWb in the ring, if relative permeability of the iron is 900.

ii) If a saw cut of 1mm wide is made in the ring, calculate the current which will give same flux as in part (i). Neglect leakage and fringing. **(Dec. - 2001, May - 2003, May-2006)**

Solution : Given, $l = 100$ cm $= 1$ m, $N = 700$, $\phi = 2$ mWb $= 2\times10^{-3}$ Wb, $\mu_r = 900$

i) Radius r = 4 cm

$\therefore \quad a = \pi r^2 = \pi\times(4)^2 = 50.2654\ \text{cm}^2 = 50.2654\times10^{-4}\ \text{m}^2$

$\phi = \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{NI}{S}$

and $\quad S = \dfrac{l}{\mu_0\mu_r a}$

$\therefore \quad \phi = \dfrac{NI\mu_0\mu_r a}{l}$

$\therefore \quad 2\times10^{-3} = \dfrac{700\times I\times4\pi\times10^{-7}\times900\times50.2654\times10^{-4}}{1}$

$\therefore \qquad I = 0.5025$ A

ii) Air gap of 1 mm is cut in the ring. So length of iron = 100 cm – 1 mm

$$l_i = 99.9 \text{ cm}$$

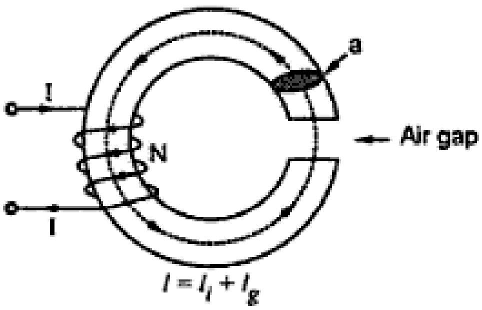


Fig. 3.45

Length of air gap $l_g = 1$ mm

$= 1 \times 10^{-3}$ m.

$\therefore$ Reluctance $S = S_i + S_g$

$$= \frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a}$$

as for air gap $\mu_r = 1$

$$S = \frac{99.9 \times 10^{-2}}{4\pi \times 10^{-7} \times 900 \times 50.26 \times 10^{-4}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 50.26 \times 10^{-4}}$$

$$\therefore \qquad S = 175748.1 + 158331.62 = 334079.72 \text{ AT/Wb}$$

$$\therefore \qquad \phi = \frac{NI}{S} \text{ i.e. } 2 \times 10^{-3} = \frac{700 \ I}{334079.72}$$

$$\therefore \qquad I = 0.9545 \text{ A}$$

Example 3.12 : *An iron ring has a mean diameter of 20 cm and a uniform circular cross section of 2.5232 cm diameter with a small brass piece fitted of 1 mm length. Three coils are wound on the ring as shown in the Fig. 3.46 and carry identical d.c. current of 2 A. If the relative permeability of iron is 800, estimate :- i) the magnetic flux produced in air-gap, ii) self-inductance of the arrangement. iii) net m.m.f. in the ring.* **(May - 2001)**

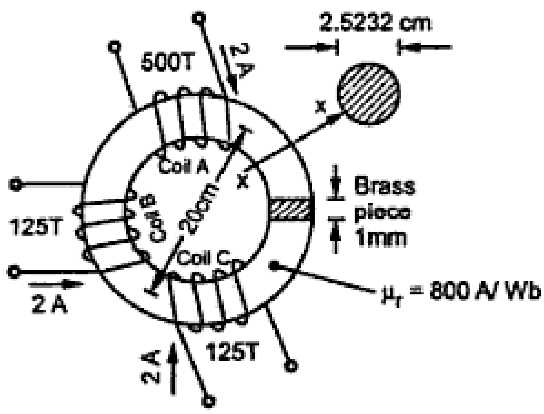


Fig. 3.46

Solution : From the various values given,

$$a = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(2.5232)^2 = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

$$l_T = \pi \times d_{mean} = \pi \times 20 = 62.8318 \text{ cm} = 0.6283 \text{ m}$$

$$\therefore \quad l_i = l_T - l_g = 0.6283 - 1 \times 10^{-3} = 0.6273 \text{ m}$$

and $$l_g = 1 \text{ mm} = 1 \times 10^{-3} \text{m}$$

$$\mu_r = 800 \quad \text{and} \quad I = 2 \text{ A}$$

$$S_T = S_i + S_g = \frac{l_i}{\mu_o \mu_r a} + \frac{l_g}{\mu_o a}$$

$$= \frac{0.6273}{4\pi \times 10^{-7} \times 800 \times 5 \times 10^{-4}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}}$$

$$= 1247973.698 + 1591549.431 = 2839523.129 \text{ AT/Wb}$$

$$\text{Net m.m.f.} = N_1 I_1 + N_2 I_2 + N_3 I_3 \quad \text{... all produce flux in same direction}$$

$$= 2\,[500 + 125 + 125] = \mathbf{1500 \text{ AT}} \quad \text{...} I_1 = I_2 = I_3 = 2 \text{ A}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{1500}{2839523.129} = 5.282 \times 10^{-4} \text{ Wb}$$

$$L = \frac{N^2}{S} = \frac{(500+125+125)^2}{2839523.129} = 0.198 \text{ H}$$

Example 3.13 : *A magnetic circuit is excited by three coils as shown in the Fig. 3.47. Calculate the flux produced in the air gap. The material used for core is iron having relative permeability of 800. The length of the magnetic circuit is 100 cm with an air gap of 2 mm in it. The core has uniform cross-section of 6 cm².* **(Dec. - 2002)**

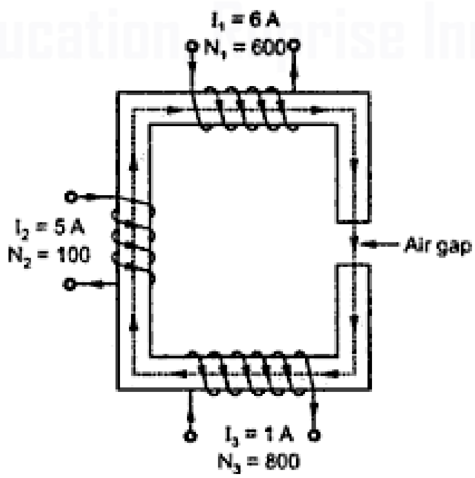


Fig. 3.47

Solution : Given, $N_1 = 600, \quad I_1 = 6 \text{ A}, \quad N_2 = 100, \quad I_2 = 5 \text{ A}$

$$N_3 = 800, \quad I_3 = 1 \text{ A}, \qquad l_T = 100 \text{ cm} = 1 \text{ m}$$

$$l_i = l_T - l_g = 1 \text{ m} - 2 \times 10^{-3} = 0.998 \text{ m}$$

$$l_g = 2 \times 10^{-3} \text{ m}, \qquad \mu_r = 800, \; a = 6 \text{ cm}^2 = 6 \times 10^{-4} \text{ m}^2$$

Now total reluctance $S = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.998}{4\pi \times 10^{-7} \times 800 \times 6 \times 10^{-4}} = 1654548.263 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = 2652582.385 \text{ AT/Wb}$$

$$\therefore \quad S = 4307130.648 \text{ AT/Wb.}$$

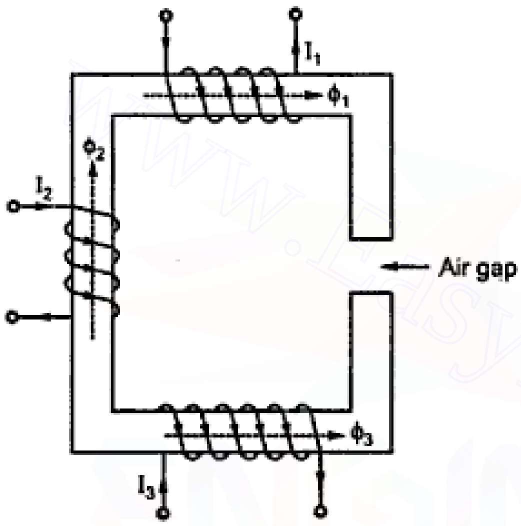


Fig. 3.47 (a)

Let us find the direction of flux due to various coils using **right hand thumb rule.**

As shown in the Fig. 3.47(a) m.m.f of coil (1) and (2) are in same direction while m.m.f. of coil (3) is in opposite direction.

$$\therefore \quad \text{Net m.m.f.} = (N_1 I_1) + (N_2 I_2) - (N_3 I_3)$$

$$= (600 \times 6) + (100 \times 5) - (1 \times 800)$$

$$NI = 3300 \text{ AT}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S} = \frac{3300}{4307130.648}$$

$\therefore$ Flux in air gap ϕ = **0.7661 mWb**

Example 3.14 : *A magnetic circuit consists of two materials as shown in Fig. 3.48. The core has uniform cross-section of 6 cm². The core carries a winding with 900 turns. The current in the coil is 3 A. Calculate the flux produced in the air gap if the length of the air gap is 1mm. Relative permeability of material A is 1000 and that for B is 1500. The length of the magnetic circuit for A is 80 cm and for B it is 50 cm.*

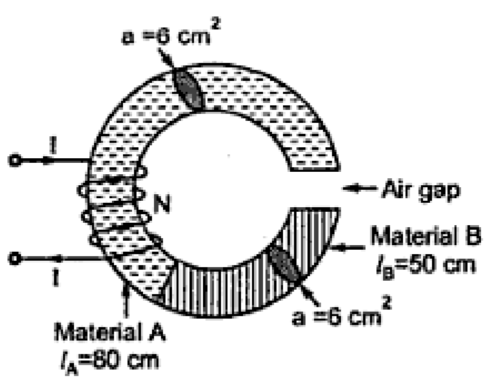


Fig. 3.48

Solution : $l_A = 80$ cm $= 0.8$ m, $l_B = 50$ cm $= 0.5$ m

$$a = 6 \text{ cm}^2 = 6 \times 10^{-4} \text{ m}^2, \quad l_g = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$N = 900, \quad I = 3 \text{ A}, \quad (\mu_r)_A = 1000, \quad (\mu_r)_B = 1500$$

The total reluctance $S = S_A + S_B + S_g$

$$S_A = \frac{l_A}{\mu_0 (\mu_r)_A a} = \frac{0.8}{4\pi \times 10^{-7} \times 1000 \times 6 \times 10^{-4}} = 1061033 \text{ AT/Wb}$$

$$S_B = \frac{l_B}{\mu_0 (\mu_r)_B a} = \frac{0.5}{4\pi \times 10^{-7} \times 1500 \times 6 \times 10^{-4}} = 442097 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = 1326291.2 \text{ AT/Wb}$$

$$\therefore \quad S = 2894221.2 \text{ AT/Wb}$$

$$\therefore \quad \text{The flux } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S}$$

$$\therefore \quad \phi = \frac{3 \times 900}{2829421.2} = 9.542 \times 10^{-4} \text{ Wb} = \mathbf{0.9542 \text{ mWb}}$$

Example 3.15 : *A ring of cast steel has an external diameter of 24 cm and internal diameter of 18 cm. The area of cross-section is 3 cm × 3 cm. Inside and across the ring an ordinary steel bar 18 cm × 3 cm × 0.4 cm is fitted with negligible air gap. Calculate the m.m.f. required to produce a flux density of 1 Wb/m² in the other half ABD. Neglect leakage. The B-H curves are given below in table form. Refer the Fig. 3.49.*

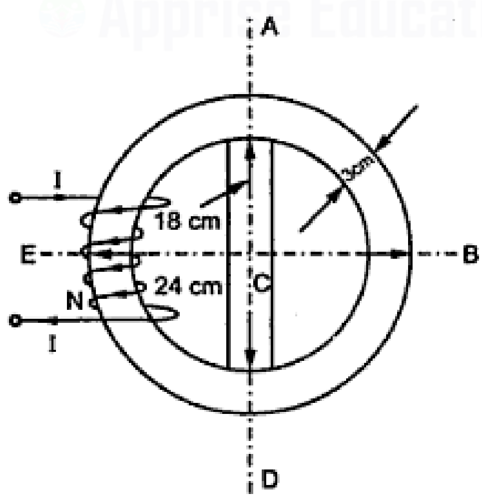


Fig. 3.49

For cast steel ring

B Wb/m²	1	1.193	1.4	1.6
H AT/m	900	800	1750	2000

For steel strip

B Wb/m²	1.4	1.45	1.5	1.6
H AT/m	1200	1650	1700	2000

Solution : Mean diameter of ring $= \dfrac{18 + 24}{2} = 21$ cm

$\therefore$ Mean circumference $= \pi D = \pi \times 21 = 65.9732$ cm

$= 0.659734$ m.

Length of path AED = Length of path ABD

$= \dfrac{0.659734}{2} = 0.32986$ m

For required flux density of 1 Wb/m^2 in path ABD, referring to corresponding B-H curve, value of H is 900 AT/m.

$\therefore$ m.m.f. for path ABD $= H \times l = 900 \times 0.32986$

$= 296.874$ AT

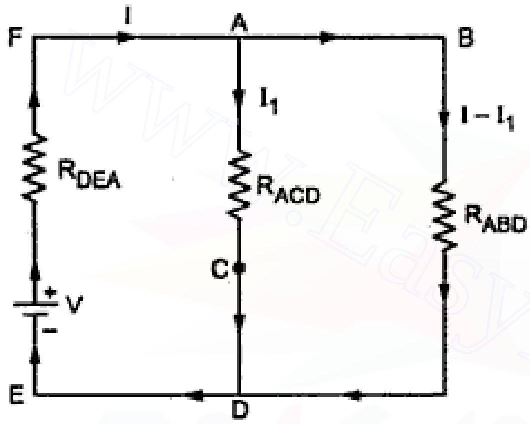


Fig. 3.49 (a)

Now the circuit is **parallel magnetic circuit**, the corresponding equivalent electric circuit is shown in the Fig. 3.49(a)

The m.m.f. across path ACD is same as across path ABD as both paths are parallel to each other, across the path DEA which is supplying m.m.f.

$\therefore$ m.m.f. across ACD = 296.874 AT

$\therefore$ H for path ACD $= \dfrac{\text{m.m.f}}{l_{ACD}}$

$= \dfrac{296.874}{0.18} = 1650$

The corresponding B = 1.45 from B-H curve given for the steel strip,

$\therefore$ Flux through path ACD $= B \times a$

Cross sectional area of steel $= 3 \times 0.4 = 1.2 \text{ cm}^2 = 1.2 \times 10^{-4} \text{ m}^2$

$\phi_{ACD} = 1.45 \times 1.2 \times 10^{-4} = 1.74 \times 10^{-4}$ Wb

Similarly flux through path ABD $= B \times a$

Cross sectional area of ABC $= 3 \times 3 = 9 \text{ cm}^2 = 9 \times 10^{-4} \text{ m}^2$

$\phi_{ABD} = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4}$ Wb

$\therefore$ Total flux supplied by path AED $= \phi_{ACD} + \phi_{ABD}$

$\phi_{AED} = 1.74 \times 10^{-4} + 9 \times 10^{-4} = 1.074 \times 10^{-3}$ Wb.

$\therefore$ Flux density for path AED, $$B = \frac{\phi_{AED}}{a}$$

$$\therefore \quad B = \frac{1.074\times10^{-3}}{9\times10^{-4}} = 1.1933 \text{ Wb/m}^2$$

The corresponding 'H' value required, from the table given is 1200.

$\therefore$ m.m.f. for path AED $= H \times l = 1200 \times 0.32986 = 395.832$ AT

$\therefore$ Total m.m.f. required = m.m.f. for path AED + m.m.f for path ACD or ABD

Key Point: *The m.m.f. for one path ACD or ABD is to be added, as the two paths in parallel, same m.m.f. is required for both paths.*

$\therefore$ Total m.m.f $= 395.832 + 296.874 =$ **692.706 AT**

Example 3.16 : *A soft iron ring of 20 cm mean diameter and circular cross-section of 4 cm diameter is wound with a magnetising coil. A current of 5 A flowing in the coil produces flux of 2.5 mWb in the air gap which is 2.2 mm wide. Taking relative permeability to be 1000 at this flux density and allowing for a leakage coefficient of 1.2, find the number of the turns on the coil.* **(Dec. - 97)**

Solution : $d_{mean} = 20$ cm, $d = 4$ cm, $I = 5$ A, $\phi_g = 2.5$ mWb, $l_g = 2.2$ mm, $\lambda = 1.2$

$\therefore$ mean length $l = \pi \times d_{mean} = \pi \times 20 \times 10^{-2} = 0.6283$ m

Cross section diameter = 4 cm

$$\therefore \quad a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(4)^2 = 12.566 \text{ cm}^2 = 12.566\times10^{-4} \text{ m}^2$$

l_g = length of air gap = 2.2 mm = 2.2×10^{-3} m

$\therefore$ l_i = length of iron path = $l - l_g = 0.6261$ m

Now $$\lambda = \frac{\text{total flux}}{\text{air gap flux}} = \frac{\phi}{\phi_g} \quad \text{i.e. } 1.2 = \frac{\phi}{2.5\times10^{-3}}$$

$\therefore$ $\phi = 3 \times 10^{-3}$ Wb.

The total reluctance of the magnetic circuit,

$$S = S_i + S_g$$

Now $$S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.6261}{4\pi\times10^{-7}\times1000\times12.566\times10^{-4}} = 396494.15 \text{ AT/Wb}$$

While $$S_g = \frac{l_g}{\mu_0 a} = \frac{2.2\times10^{-3}}{4\pi\times10^{-7}\times12.566\times10^{-4}} = 1393207.4 \text{ AT/Wb}$$

Now $\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{Si+Sg}$

$$\phi_g = \frac{\text{m.m.f. for air gap}}{S_g} \text{ i.e. } 2.5\times10^{-3} = \frac{\text{m.m.f. for air gap}}{1393207.4}$$

$\therefore$ m.m.f. for air gap = 3483.01

$$\phi = \frac{\text{m.m.f. for iron}}{S_i} \text{ i.e. } 3\times10^{-3} = \frac{\text{m.m.f. for iron}}{396494.15}$$

$\therefore$ m.m.f. for iron = 1189.4825

Hence the total m.m.f. can be obtained as :

Total = m.m.f. for air gap + m.m.f. for iron

$\therefore$ m.m.f. = 3483.01 + 1189.48 = **4672.501 AT/Wb**

Now m.m.f = N × I i.e. 4672.501 = N × 5

$$\therefore \quad N = \frac{4672.501}{5} = 934.5$$

Hence the number of turns on the coil required is approximately **935.**

Example 3.17 : *An iron ring, cross sectional area of 5 cm² and mean length of 100 cm, has an air gap of 2 mm cut in it. Three separate coils having 100, 200 and 300 turns are wound on the ring and carry currents of 1 A, 2.5 A and 3 A respectively such that they produce additive fluxes in the ring. Relative permeability of the ring material is 1000. Calculate the flux in the air gap.* **(Dec. - 98)**

Solution : $a = 5 \text{ cm}^2 = 5\times10^{-4} \text{ m}^2$, $l_T = 100 \text{ cm} = 1 \text{ m}$, $l_g = 2 \text{ mm} = 2\times10^{-3}$ m

$$l_i = l_T - l_g = 1 - 2\times10^{-3} = 0.998 \text{ m}$$

Total reluctance $S = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.998}{4\pi\times10^{-7}\times1000\times5\times10^{-4}} = 1588366.332 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times5\times10^{-4}} = 3183098.862 \text{ AT/Wb}$$

$$\therefore \quad S = 1588366.332 + 3183098.862 = 4771465.194 \text{ AT/Wb}$$

$$\text{Net m.m.f.} = N_1I_1 + N_2I_2 + N_3I_3 = 100\times1 + 200\times2.5 + 300\times3 = 1500 \text{ AT}$$

All m.m.f.s help each other as they produce **additive fluxes** in the ring.

$$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{1500}{4771465.194} = 0.0003143 \text{ Wb}$$

$$= 0.3143 \text{ mWb}$$

Example 3.18 : *The Fig. 3.50 shows a magnetic circuit with two similar branches and an exciting coil of 1500 turns on central limb. The flux density in the air gap is 1 Wb/m² and leakage coefficient 1.2. Determine exciting current through the coil. Assume relative permeability of the iron constant equal to 600.*

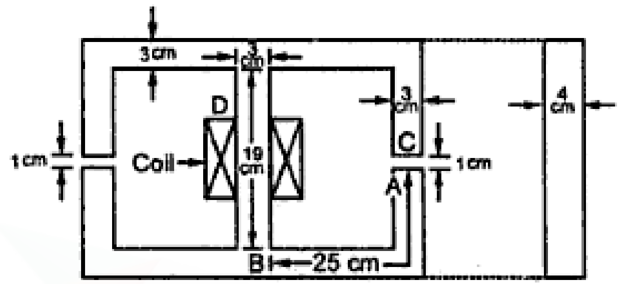


Fig. 3.50

Solution : Flux density in air gap $B_g = 1 \text{ Wb/m}^2$

$$a = 3 \times 4 = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad \phi_g = B_g \times a = 1 \times 12 \times 10^{-4} = 12 \times 10^{-4} \text{ Wb.}$$

$$\lambda = \frac{\phi_T}{\phi_g} = \frac{\phi_T}{12 \times 10^{-4}} = 1.2$$

$$\phi_T = \phi_{side} = 1.2 \times 12 \times 10^{-4}$$

$$\phi_{side} = 1.44 \times 10^{-3} \text{ Wb}$$

As the circuit is parallel magnetic circuit,

$$\phi_{coil} = \phi_{side1} + \phi_{side\,2} = 2 \times 1.44 \times 10^{-3}$$

$$= 2.88 \times 10^{-3} \text{ (as sides are similar)}$$

Section I] Central limb

$$l_c = 19 \text{ cm} = 0.19 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_c = \frac{l_c}{\mu_0 \mu_r a} = \frac{0.19}{4\pi \times 10^{-7} \times 600 \times 12 \times 10^{-4}}$$

$$= 209996.11 \text{ AT/Wb}$$

Section II] One side limb

$$l_i = 25 + 25 = 50 \text{ cm} = 0.5 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.5}{4\pi \times 10^{-7} \times 600 \times 12 \times 10^{-4}}$$

$$= 552621.33 \text{ AT/Wb}$$

Section III] Air gap

$$l_g = 1 \text{ cm} = 0.01 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{0.01}{4\pi \times 10^{-7} \times 12 \times 10^{-4}}$$

$$= 6631456 \text{ AT/Wb}$$

$\therefore$ m.m.f. for central limb $= \phi_{coil} \times S_c = 2.88 \times 10^{-3} \times 209996.11$

$= 604.78$ AT

$\therefore$ m.m.f. for one side limb $= \phi_{side} \times S_i = 1.44 \times 10^{-3} \times 552621.33$

$= 795.774$ AT

$\therefore$ m.m.f. for air gap $= \phi_g \times S_g = 12 \times 10^{-4} \times 6631456$

$= 7957.7472$ AT

$\therefore$ Total m.m.f. = m.m.f. for central limb + m.m.f. for one side + m.m.f. for air gap

$$NI = 604.78 + 795.774 + 7957.7452$$

$\therefore \quad 1500 \text{ I} = 9358.3107$

$\therefore \quad \text{I} = \mathbf{6.2388 \text{ A}}$

Example 3.19 : *A magnetic circuit has the mean length of flux path of 20 cm, and cross sectional area of 1cm². Relative permeability of its material is 2400. Find the m.m.f. required to produce a flux density of 2tesla in it. If an airgap of 1mm is introduced in it, find the m.m.f. required for the air gap as a fraction of the total m.m.f. to maintain the same flux density.* **(Dec. - 2003)**

Solution : $l_i = 20$ cm, $a = 1 \text{ cm}^2$, $\mu_r = 2400$, $B = 2$ T

$$S = \frac{l_i}{\mu_0 \mu_r a} = \frac{20 \times 10^{-2}}{4\pi \times 10^{-7} \times 2400 \times 1 \times 10^{-4}}$$

$$= 663.145\times10^{3} \quad AT/Wb$$

$$\phi = B\times a = 2\times1\times10^{-4} \quad Wb$$

Now $\quad \phi = \dfrac{m.m.f}{S}$

$\therefore \quad m.m.f. = \phi\times S = 2\times10^{-4}\times663.145\times10^{3} = \mathbf{132.6291 \quad AT}$

Now $\quad l_g = 1$ mm is introduced in it.

$\therefore \quad l_i = 20$ cm $- 1$ mm $= 0.199$ m

$\therefore \quad S_i = \dfrac{l_i}{\mu_0\mu_r a} = 659.829\times10^{3} \quad AT/Wb$

and $\quad S_g = \dfrac{l_g}{\mu_0 a} = 7.9577\times10^{6} \quad AT/Wb \quad$ ... $\mu_r = 1$ for air

$\phi = B\times a = 2\times10^{-4} \quad Wb \quad$ ... same as B is same

$\therefore \quad (m.m.f)_{iron\ path} = S_i\times\phi = 131.9658 \quad AT$

and $\quad (m.m.f)_{air\ gap} = S_g\times\phi = 1591.5494 \quad AT$

$\therefore \quad$ Total m.m.f $=$ **1723.5152 AT**

$\therefore \quad (m.m.f)_{air\ gap} =$ **0.9234** times total m.m.f

Example 3.20 : *A coil is wound uniformly with 300 turns over a steel of relative permiability 900, having a mean circumference of 40 mm and corss-sectional area of 50 mm^2. If a current of 5 A is passed through the coil, find*

i) m.m.f. ii) reluctance of the ring and iii) flux **(Dec. - 2004)**

Solution : Given : $N = 300,\ \mu_r = 900,\ l = 40$ mm $= 40\times10^{-3}$ m,

$a = 50$ mm^2 $= 50\times10^{-6}$ m^2, $I = 5$ A

i) $\quad m.m.f. = NI = 300\times5 = \mathbf{1500\ AT}$

ii) $$S = \frac{l}{\mu_0\mu_r a} = \frac{40\times10^{-3}}{4\pi\times10^{-7}\times900\times50\times10^{-6}} = \mathbf{70.7355\times10^{3}\ AT/Wb}$$

This is reluctance of the ring.

iii) $$S = \frac{m.m.f.}{\phi}$$

$\because \quad \phi = \dfrac{m.m.f}{S} = \dfrac{1500}{70.7355\times10^{3}} = \mathbf{21.2057\ mWb} \quad$ ... Flux

Example 3.21 : *An iron ring has its mean length of flux path as 60 cm and its cross-sectional areas as 15 cm^2. Its relative permeability is 500. Find the current required to be passed, through a coil of 300 turns wound uniformly around it, to produce a flux density of 1.2 tesla. What would be the flux density with the same current, if the iron ring is replaced by air-core ?* **(May - 2005)**

Solution : Given : l = 60 cm = 60×10^{-2} m, a = 15 cm^2 = 15×10^{-4} m^2, $\mu_r = 500$

N = 300, B = 1.2 T, I = ?

Case 1 :
$$S = \frac{l}{\mu_o \mu_r a} = \frac{60\times10^{-2}}{4\pi\times10^{-7}\times500\times15\times10^{-4}} = 636.6197 \times 10^3 \text{ AT/Wb}$$

$$B = \frac{\phi}{a} \text{ i.e. } \phi = B \times a = 1.2 \times 15 \times 10^{-4} = 1.8 \times 10^{-3} \text{ Wb}$$

Now
$$S = \frac{\text{m.m.f.}}{\phi} = \frac{NI}{\phi}$$

$$\therefore \quad 636.6197 \times 10^3 = \frac{300\text{ I}}{1.8\times10^{-3}}$$

$$\therefore \quad I = \mathbf{3.8197\ A}$$ ... Current required

Case 2 : Ring replaced by air core for which $\mu_r = 1$ hence $\mu = \mu_o$

$$\therefore \quad S = \frac{l}{\mu_o a} = \frac{60\times10^{-2}}{4\pi\times10^{-7}\times15\times10^{-4}} = 318.3098 \times 10^6 \text{ AT/Wb}$$

I is same as calculated above i.e. I = 3.8197 A

$$\therefore \quad \text{m.m.f} = NI = 300 \times 3.8197 = 1145.91 \text{ AT}$$

$$\therefore \quad \phi = \frac{\text{m.m.f}}{S} = \frac{1145.91}{318.3098\times10^6} = 3.6 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad B = \frac{\phi}{a} = \frac{3.6\times10^{-6}}{15\times10^{-4}} = \mathbf{2.4 \times 10^{-3} \text{ T or Wb/m}^2}$$ ... New flux density

Example 3.22 : *A conductor of length 10 cm carrying 5 A is placed in a uniform magnetic field of flux density 1.25 tesla. Find the force acting on the conductor, if it is placed (i) along the lines of magnetic flux, (ii) perpendicular to the lines of flux, and (ii) at 30° to the flux.* **(May - 2005)**

Solution : l = 10 cm = 10×10^{-2} m, I = 5 A, B = 1.25 T

Case 1 : Along lines of magnetic flux

θ = Angle between conductor and axis of magnetic field

= 0°

$\therefore$ F = B.I.l. sin θ = $1.25 \times 5 \times 10 \times 10^{-2}$ sin 0° = 0 N

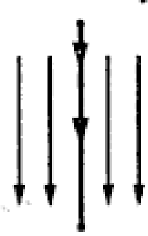


Fig. 3.51

Case 2 : Perpendicular to lines of flux i.e. $\theta = 90°$

$$\therefore \quad F = B.I.l.\ \sin 90 = B.I.l. = 1.25 \times 5 \times 10 \times 10^{-2} = 0.625\ N$$

Case 3 : At 30° to the flux i.e. $\theta = 30°$

$$\therefore \quad F = B.I.l.\ \sin 30° = 1.25 \times 5 \times 10 \times 10^{-2} \times \frac{1}{2} = 0.3125\ N.$$

Example 3.23 : *A ring shaped core is made up of two parts of same material. Part one is a magnetic path of length 25 cm and with cross sectional area 4 cm^2, whereas part two is of length 10 cm and cross sectional area of 6 cm^2. The flux density in part two is 1.5 Tesla. If the current through the coil, wound over core, is 0.5 Amp., calculate the number of turns of coil. Assume μ_r is 1000 for material.* **[Dec.-2005]**

Solution : The arrangement is shown in the Fig. 3.52.

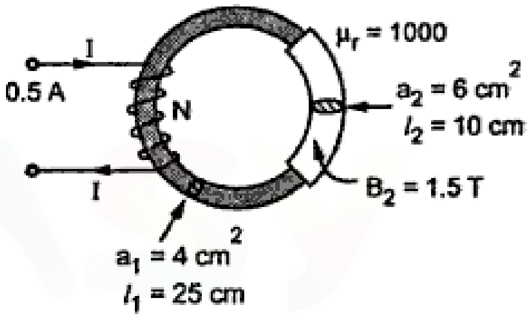


Fig. 3.52

$$B_2 = \frac{\phi}{a_2}$$

$$\therefore \quad \phi = 1.5 \times 6 \times 10^{-4}$$

$$= 9 \times 10^{-4}\ Wb$$

Key Point : *The flux ϕ is same through both the parts, as series circuit.*

$$S = S_I + S_{II} = \frac{l_1}{\mu_0 \mu_r a_1} + \frac{l_2}{\mu_0 \mu_r a_2}$$

$$= \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 4 \times 10^{-4}} + \frac{10 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 6 \times 10^{-4}}$$

$$= 629988.3164\ AT/Wb$$

$$\phi = \frac{m.m.f}{S} = \frac{NI}{S}$$

$$\therefore \quad 9 \times 10^{-4} = \frac{N \times 0.5}{629988.3164}$$

$$\therefore \quad N = 1133.979 \approx 1134 \qquad \text{... Number of turns}$$

Example 3.24 : *The mean diameter of steel ring is 40 cm and flux density of 0.9 Wb/m^2 is produced by 3500 Aturns/meter. If the cross-section of the ring is 15 cm^2 and number of turns 440, calculate:*
i) the exciting current ii) the self inductance in henry and iii) exciting current and inductance when air gap of 1 cm is cut in the ring. **[Dec.-2006]**

Solution : d_{mean} = 40 cm, B = 0.9 Wb/m^2, H = 3500 AT/m, a = 15 cm^2, N = 440

$$l_T = \pi \times d_{mean} = \pi \times 40 = 125.6637 \text{ cm}$$

$$B = \mu_0 \mu_r H = \mu H$$

$$\therefore \quad \mu = \frac{B}{H} = \frac{0.9}{3500} = 2.5714 \times 10^{-4}$$

$$S_T = \frac{l_T}{\mu a}$$

$$\therefore \quad S_T = \frac{125.6637 \times 10^{-2}}{2.5714 \times 10^{-4} \times 15 \times 10^{-4}} = 3.2579 \times 10^6 \text{ AT/Wb}$$

$$\text{m.m.f.} = NI = H \times l_T = 3500 \times 125.6637 \times 10^{-2} = 4398.2295 \text{ AT}$$

i) $$I = \frac{4398.2295}{N} = \frac{4398.2295}{440} = 9.99 \approx \mathbf{10\ A}$$

ii) $$L = \frac{N^2}{S_T} = \frac{(440)^2}{3.2579 \times 10^6} = \mathbf{0.0594\ H}$$

iii) Now air gap l_g = 1 cm is cut.

$$\therefore \quad l_i = l_T - l_g = 124.6637 \text{ cm}$$

$$\therefore \quad S_T = S_i + S_g = \frac{l_i}{\mu a} + \frac{l_g}{\mu_0 a}$$

$$= \frac{124.6637 \times 10^{-2}}{2.5714 \times 10^{-4} \times 15 \times 10^{-4}} + \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 15 \times 10^{-4}}$$

$$= 3.232 \times 10^6 + 5.3051 \times 10^6 = 8.5371 \times 10^6 \text{ AT/Wb}$$

$$\phi = B \times a = 0.9 \times 15 \times 10^{-4} = 1.35 \times 10^{-3} \text{ Wb}$$

Now $$\phi = \frac{\text{m.m.f}}{\text{Reluctance}}$$

$$\therefore \quad 1.35 \times 10^{-3} = \frac{\text{m.m.f}}{8.5371 \times 10^6}$$

$$\therefore \quad \text{m.m.f} = NI = 11525.085$$

$$\therefore \quad I = \frac{11525.1494}{N} = \mathbf{26.1935\ A}$$

and $$L = \frac{N^2}{S_T} = \frac{(440)^2}{8.5371 \times 10^6} = \mathbf{0.0226\ H}$$

Example 3.25 : *An iron ring of mean length 50 cm has air gap of 1mm and a winding of 200 turns. If the relative permeability of iron is 300, find the flux density when a current of 1 amp flows through the coil.* **[Dec.-2007]**

Solution : The total m.m.f. is,

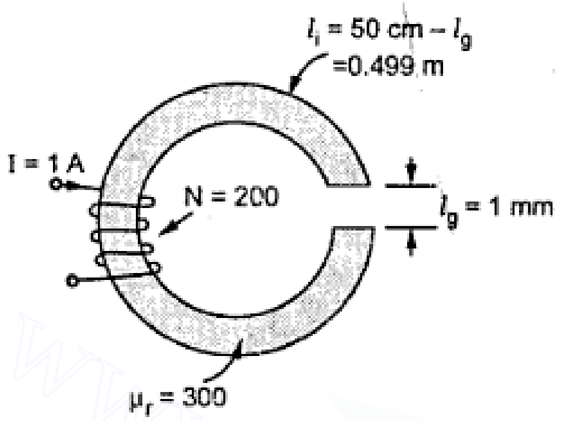


Fig. 3.53

$$\text{m.m.f.} = NI = 200 \times 1 = 200 \text{ AT}$$

The total reluctance is,

$$S_T = S_i + S_g$$

$$= \text{iron path} + \text{air gap}$$

$$= \frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a} \qquad \ldots \mu_r = 1 \text{ for air}$$

$$= \frac{1}{a}\left[\frac{0.499}{4\pi \times 10^{-7} \times 300} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7}}\right]$$

$$= \frac{2119.4133}{a}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S_T} = \frac{200}{\left(\frac{2119.4133}{a}\right)}$$

$$\therefore \quad \frac{\phi}{a} = \frac{200}{2119.4133} = 0.0943 \quad \text{but } \frac{\phi}{a} = B = \text{flux density}$$

$$\therefore \quad B = 0.0943 \text{ Wb/m}^2$$

Example 3.26 : *A ring has diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semi-circular sections of cast iron and cast steel with each joint having a reluctance equal to an air gap of 0.2 mm. Find the Amp-Turns required to produce a flux of 8 × 10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively.* **[May-2008]**

Solution : The arrangement is shown in the Fig. 3.54.

At each joint there is an air gap. This is a series magnetic cirucit.

The total reactance is,

$$S_T = S_{iron} + S_{steel} + S_g + S_g$$

d = 21 cm = total diameter of ring

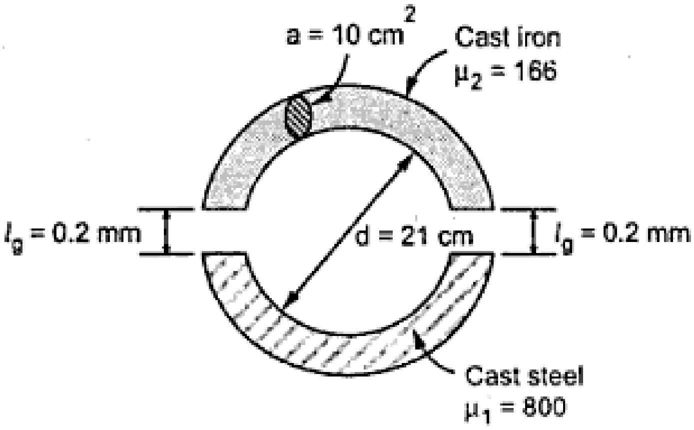


Fig. 3.54

$\therefore$ Total circumference = $\pi \times d$ = 65.973445 cm = 0.659734 m

Total air gap length = $2 \times l_g$ = 0.4 mm = 0.4×10^{-3} m

$$\therefore \quad l_{iron} = \frac{\pi d - 2 l_g}{2} = 0.329667 \text{ m} = l_{steel}$$

$$\therefore \quad S_T = \frac{l_{steel}}{\mu_0 \mu_{r1} a} + \frac{l_{iron}}{\mu_0 \mu_{r2} a} + 2 \times \frac{l_g}{\mu_0 a}$$

$$= \frac{1}{4\pi \times 10^{-7} \times 10 \times 10^{-4}} \left[\frac{0.329667}{800} + \frac{0.329667}{166} + \frac{2 \times 0.2 \times 10^{-3}}{1} \right]$$

$$= 2226601.156 \text{ AT/Wb}$$

and $\quad \phi = 8 \times 10^{-4} \text{ Wb} = \dfrac{\text{m.m.f.}}{S_T}$

$$\therefore \quad \text{m.m.f.} = \phi \times S_T = 8 \times 10^{-4} \times 2226601.156 = \mathbf{1781.281 \text{ AT}}$$

Review Questions

1. *State and explain the laws of magnetism.*
2. *What is magnetic field and magnetic lines of force? State the properties of lines of force.*
3. *Define and state the units of following parameters:*
   i) magnetic flux *ii) magnetic pole strength*
   iii) magnetic flux density *iv) magnetic field strength*
   v) absolute permeability *vi) relative permeability*
   vii) m.m.f. *viii) reluctance*
4. *Derive the relation between m.m.f., reluctance and the flux.*
5. *State and explain the following rules:*
   i) Right hand thumb rule
   ii) Fleming's left hand rule
   iii) Fleming's right hand rule
   iv) Lenz's law
   v) Kirchhoff's laws for magnetic circuits
6. *Explain the procedure to analyse following circuit, with suitable example :*
   i) Series magnetic circuit
   ii) Series magnetic circuit with air gap
   iii) Parallel magnetic circuit
7. *What is an electromagnet ? What is solenoid?*
8. *Point out the analogy between electric and magnetic circuits.*
9. *Explain the magnetic leakage and magnetic fringing.*

10. *Define leakage coefficient*
11. *Explain how current carrying conductor when placed in a magnetic field experiences a force.*
12. *A steel ring of 180 cm mean diameter has a cross sectional area of 250 mm^2. Flux developed in the ring is 500 μWb when a 4000 turns coil carries certain current. Find*

    i) m.m.f. required ii) reluctance iii) current in the coil

    Given that the relative permeability of the steel is 1100.

    (Ans. : 8181.72 AT, 1.6363×10^7, 2.045 A)
13. *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 25 A is passed through the coil, determine*

    i) m.m.f. ii) reluctance of ring and

    iii) flux **(Ans. : 7500 AT, 707355.3 AT/Wb, 0.0106 Wb)**
14. *Find the number of ampere turns required to produce a flux of 0.44 milli-weber in an iron ring of 100 cm mean circumference and 4 cm^2 in cross-section. B Vs μ_r test for the iron gives the following result :*

B in Wb/m^2	0.8	1.0	1.1	1.2	1.4
μ_r	2300	2000	1800	1600	1000

If a saw cut of 2 mm wide is made in the above ring, how many extra ampere turns are required to maintain same flux ? **(Ans. : 486.307 AT, 1744 AT)**

15. *An iron ring of 20 cm mean diameter and 10 cm^2 cross-section is magnetised by a coil of 500 turns. The current through the coil is 8 A. The relative permeability of iron is 500. Find the flux density inside the ring .* **(Ans. : 4 Wb/m^2)**
16. *An iron ring of 100 cm mean circumference is made from round iron of cross-section 10cm^2, it relative permeability is 800. If it is wound with 300 turns, what current is required to produce a flux of 1.1×10^{-3} Wb ?* **(Ans. 3.647 A)**
17. *A coil of 300 turns and of resistance 10 Ω is wound uniformly over a steel ring of mean circumference 30 cm, and cross-sectional area 9 cm^2. It is connected to a supply at 20 V d.c. If the relative permeability of the ring is 1500, find : (i) the magnetising force ; (ii) the reluctance ; (iii) the m.m.f. ; and (iv) the flux.*

    (Ans. : 600 AT, 176838.82 AT/Wb, 2000 AT/m, 3.3929 mWb)
18. *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900 having a mean circumference of 400 mm and cross-sectional area of 500 mm^2. If a current of 25 A is passed through the coil find*

    i) m.m.f. ii) reluctance and iii) flux **(Ans. : 7500 AT, 707355.3 AT/Wb, 10.6 mWb)**

4 Electromagnetic Induction

4.1 Introduction

Uptill now we have discussed the basic properties, concepts of magnetism and magnetic circuits. Similarly we have studied, the magnetic effects of an electric current. But we have not seen the generation of e.m.f. with the help of magnetism. The e.m.f. can be generated by different ways, by chemical action, by heating thermocouples etc. But the most popular and extensively used method of generating an e.m.f. is based on electromagnetism.

After the magnetic effects of an electric current, attempts were made to produce electric current with the help of magnetism rather than getting magnetism due to current carrying conductor. In 1831, an English Physicist, **Michael Faraday** succeeded in getting e.m.f. from magnetic flux. The phenomenon by which e.m.f. is obtained from flux is called **electromagnetic induction**. Let us discuss, what is electromagnetic induction and its effect on the electrical engineering branch, in brief.

4.2 Faraday's Experiment

Let us study first the experiment conducted by **Faraday** to get understanding of electromagnetic induction.

Consider a coil having 'N' turns connected to a galvanometer as shown in the Fig. 4.1. Galvanometer indicates flow of current in the circuit, if any. A permanent magnet is moved relative to coil, such that magnetic lines of force associated with coil get changed. Whenever, there is motion of permanent magnet, galvanometer deflects indicating flow of current through the circuit.

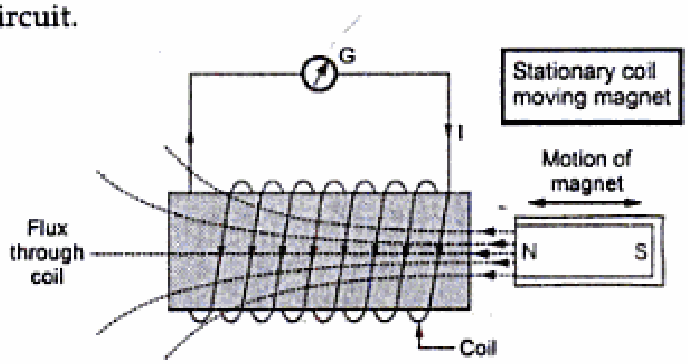


Fig. 4.1 Faraday's experiment

Key Point : *The galvanometer deflects in one direction, when magnet is moved towards a coil. It deflects in other direction, when moved away from the coil.*

The deflection continues as long as motion of magnet exists. More quickly the magnet is moved, the greater is the deflection. Now deflection of galvanometer indicates flow of current. But to exist flow of current there must be presence of e.m.f. Hence such movement of flux lines with respect to coil generates an e.m.f. which drives current through the coil. This is the situation where coil in which e.m.f. is generated is fixed and magnet is moved to create relative motion of flux with respect to coil.

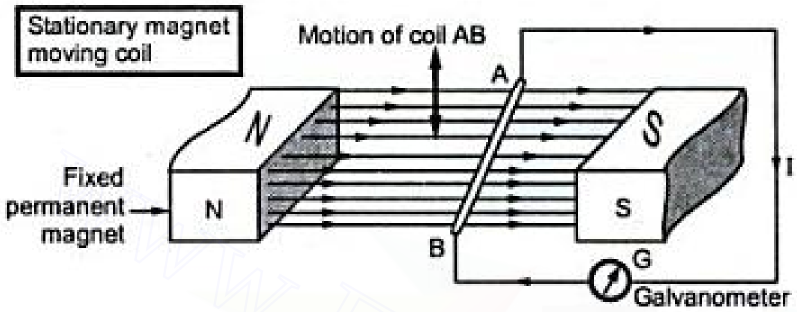


Fig. 4.2 Another form of Faraday's experiment

Similar observations can be made by moving a coil in the magnetic field of fixed permanent magnet, creating relative motion between flux and coil. This arrangement is shown in the Fig. 4.2. The coil AB is moved by some external means in the magnetic field of fixed permanent magnet. Coil is connected to galvanometer.

Whenever conductor AB is moved in the direction shown in the Fig. 4.2 the galvanometer deflects indicating flow of current through coil AB.

Key Point: *The deflection is on one side when conductor is moved up. While it is in other direction, when it is moved down.*

Similarly greater is the deflection if conductor is moved quickly in magnetic field.

In both cases, basically there is **change of flux lines with respect to conductor** i.e there is cutting of the flux lines by the conductor in which e.m.f. induced.

With this experiment Faraday stated laws called **Faraday's Laws of Electromagnetic Induction.**

This phenomenon of cutting of flux lines by the conductor to get the induced e.m.f. in the conductor or coil is called ***electromagnetic induction.***

Thus, to have induced e.m.f. there must exist,

1) A coil or conductor.
2) A magnetic field (permanent magnet or electromagnet).
3) Relative motion between conductor and magnetic flux (achieved by moving conductor with respect to flux or moving with respect to conductor.)

Key Point: *The e.m.f. exists as long as relative motion persists.*

4.3 Faraday's Laws of Electromagnetic Induction

From the experiment discussed above, Michael Faraday a British scientist stated two laws of electromagnetic induction.

4.3.1 First Law

Whenever the number of magnetic lines of force (flux) linking with a coil or circuit changes, an e.m.f. gets induced in that coil or circuit.

4.3.2 Second Law

The magnitude of the induced e.m.f. is directly proportional to the rate of change of flux linkages (flux × turns of coil).

$$\boxed{\text{Flux linkages} = \text{Flux} \times \text{Number of turns of coil}}$$

The law can be explained as below.

Consider a coil having N turns. The initial flux linking with a coil is ϕ_1

$$\therefore \quad \text{Initial flux linkages} = N\phi_1$$

In time interval t, the flux linking with the coil changes from ϕ_1 to ϕ_2.

$$\therefore \quad \text{Final flux linkages} = N\phi_2$$

$$\therefore \quad \text{Rate of change of flux linkages} = \frac{N\phi_2 - N\phi_1}{t}$$

Now as per the first law, e.m.f. will get induced in the coil and as per second law the magnitude of e.m.f. is proportional to the rate of change of flux linkages.

$$\therefore \quad e \propto \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \quad e = K \times \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \quad e = N\frac{d\phi}{dt}$$

With K as unity to get units of e as volts, $d\phi$ is change in flux, dt is change in time hence $(d\phi / dt)$ is rate of change of flux.

Now as per Lenz's law (discussed later), the induced e.m.f. sets up a current in such a direction so as to oppose the very cause producing it. Mathematically this opposition is expressed by a negative sign.

Thus such an induced e.m.f. is mathematically expressed alongwith its sign as,

$$\therefore \quad \boxed{e = -N\frac{d\phi}{dt} \quad \text{volts}}$$

4.4 Nature of the Induced E.M.F.

E.M.F. gets induced in a conductor, whenever there exists change in flux with that conductor, according to Faraday's Law. Such change in flux can be brought about by different methods.

Depending upon the nature of methods, the induced e.m.f. is classified as,

1) Dynamically induced e.m.f. and
2) Statically induced e.m.f.

4.5 Dynamically Induced E.M.F.

The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux. Both these methods are discussed earlier in discussion of Faraday's experiment .

> **Key Point** : *Such an induced e.m.f. which is due to **physical movement** of coil, conductor with respect to flux or movement of magnet with respect to stationary coil, conductor is called **dynamically induced e.m.f. or motional induced e.m.f.***

4.5.1 Magnitude of Dynamically Induced E.M.F.

Consider a conductor of length l metres moving in the air gap between the poles of the magnet.

If plane of the motion of the conductor is parallel to the plane of the magnetic field then there is no cutting of flux lines and there can not be any induced e.m.f. in the conductor such condition is shown in the Fig. 4.3(a).

> **Key Point** : *When plane of the flux is parallel to the plane of the motion of conductors then there is no cutting of flux, hence no induced e.m.f.*

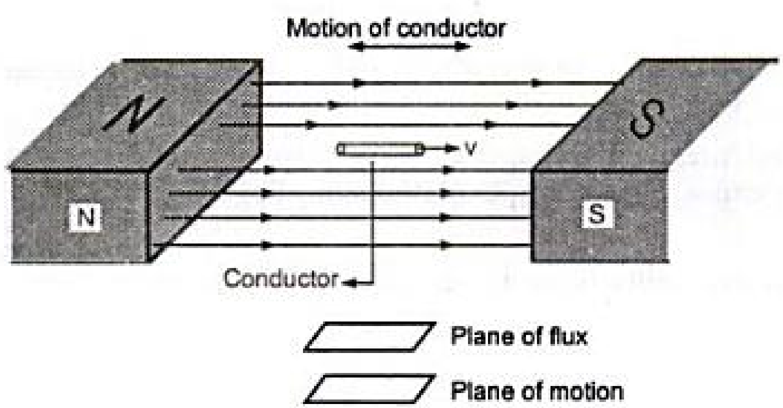


Fig. 4.3 (a) No cutting of flux

In second case as shown in the Fig. 4.3(b), the velocity direction i.e. motion of conductor is perpendicular to the flux. Hence whole length of conductor is cutting the flux line hence there is maximum possible induced e.m.f. in the conductor. **Under such condition plane of flux and plane of motion are perpendicular to each other.**

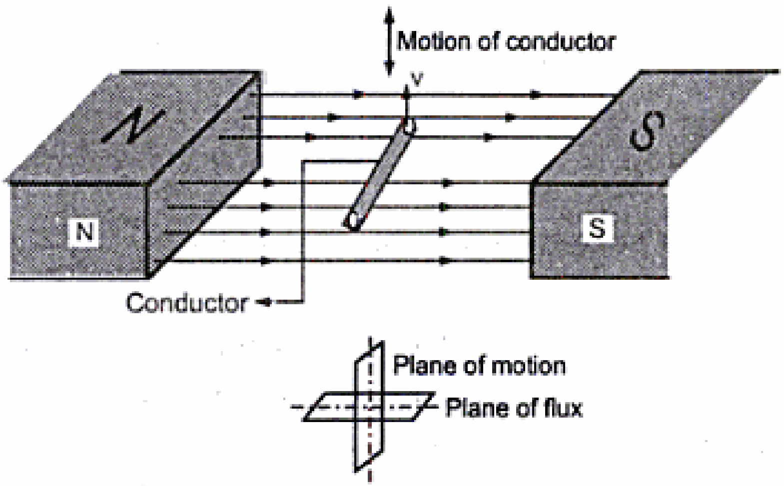
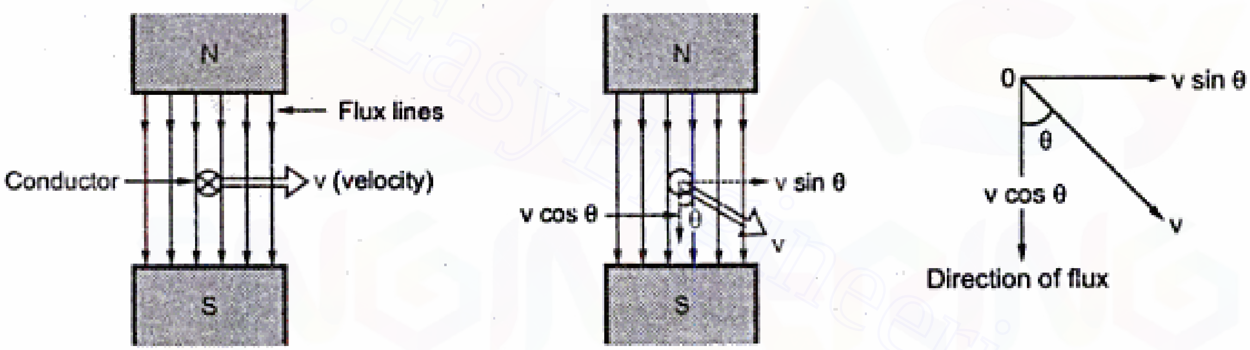


Fig. 4.3 (b) Maximum cutting of flux

> **Key Point :** *When plane of the flux is perpendicular to the plane of the motion of the conductors then the cutting of flux is maximum and hence induced e.m.f. is also maximum.*

Consider a conductor moving with velocity v m/s such that its plane of motion or direction of velocity is perpendicular to the direction of flux lines as shown in Fig. 4.4 (a).



(a) (b) (c)

Fig. 4.4

B = Flux density in Wb/m^2

l = Active length of conductor in metres.

(This is the length of conductor which is actually responsible for cutting of flux lines.)

v = Velocity in m/sec.

Let this conductor is moved through distance dx in a small time interval dt, then

$$\text{Area swept by conductor} = l \times dx \quad m^2$$

$$\therefore \quad \text{Flux cut by conductor} = \text{Flux density} \times \text{Area swept}$$

$$d\phi = B \times l \times dx \quad Wb$$

According to Faraday's law, magnitude of induced e.m.f. is proportional to the rate of change of flux.

$$\therefore \quad e = \frac{\text{Flux cut}}{\text{Time}}$$

$$= \frac{d\phi}{dt} \quad \text{[Here N = 1 as single conductor]}$$

$$= \frac{B\, l\, dx}{dt}$$

But $\frac{dx}{dt}$ = Rate of change of displacement

= Velocity of the conductor

= v

$$\therefore \quad \boxed{e = B\, l\, v \quad \text{volts}}$$

This is the induced e.m.f. when plane of motion is exactly perpendicular to the plane of flux. This is maximum possible e.m.f. as plane of motion is at right angles to plane of the flux.

But if conductor is moving with a velocity v but at a certain angle θ measured with respect to direction of the field (plane of the flux) as shown in the Fig. 4.4 (b) then component of velocity which is $v \sin\theta$ is perpendicular to the direction of flux and hence responsible for the induced e.m.f.. The other component $v \cos\theta$ is parallel to the plane of the flux and hence will not contribute to the dynamically induced e.m.f.

Under this condition magnitude of induced e.m.f. is given by,

$$\boxed{e = B\, l\, v\, \sin\theta \quad \text{volts}}$$

where θ is measured with respect to plane of the flux.

Example 4.1 : *A conductor of 2 m length moves with a uniform velocity of 1.27 m/sec under a magnetic field having a flux density of 1.2 Wb/m² (tesla). Calculate the magnitude of induced e.m.f. if conductor moves,*

i) at right angles to axis of field.

ii) at an angle of 60° to the direction of field.

Solution : i) The magnitude of induced e.m.f.

$$e = B\, l\, v \quad \text{for } \theta = 90°$$

$$\therefore \quad e = 1.2 \times 2 \times 1.27 = \mathbf{3.048 \text{ volts}}$$

ii) $$e = B\, l\, v\, \sin\theta \quad \text{where } \theta = 60°$$

$$e = 1.2 \times 2 \times 1.27 \times \sin 60 = \mathbf{2.6397} \text{ volts.}$$

Example 4.2 : *A coil carries 200 turns gives rise a flux of 500 μWb when carrying a certain current. If this current is reversed in $\frac{1}{10}$ th of a second. Find the average e.m.f. induced in the coil.*

Solution : The magnitude of induced e.m.f. is,

$$= N\frac{d\phi}{dt}$$

where $d\phi$ is change in flux linkages i.e. change in $N\phi$. Now in this problem flux is 500×10^{-6} for given current. After reversing this current, flux will reverse its direction. So flux becomes $\left(-500\times10^{-6}\right)$.

$$\therefore \quad d\phi = \phi_2-\phi_1 = -500\times10^{-6}-\left(+500\times10^{-6}\right).$$

This happens in time dt = 0.1 sec.

$$\therefore \quad \text{Average e.m.f.} = -N\frac{d\phi}{dt} = -200\times\frac{\left(-1\times10^{3}\right)}{0.1} = \mathbf{2\ volts}$$

4.5.2 Direction of Dynamically Induced E.M.F.

The direction of induced e.m.f. can be decided by using two rules.

1) Fleming's Right Hand Rule

As discussed earlier, the Fleming's Left Hand Rule is used to get direction of force experienced by conductor carrying current, placed in a magnetic field while Fleming's Right Hand Rule is to be used to get direction of induced e.m.f. when conductor is moving in a magnetic field.

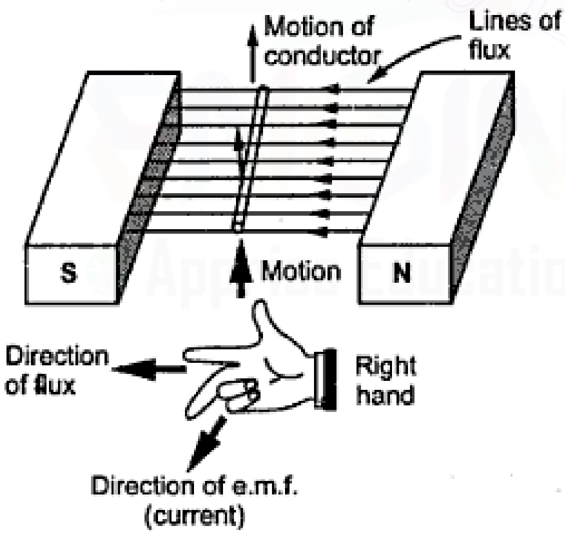


Fig. 4.5 (a)

According to Fleming's right hand rule, outstretch the three fingers of right hand namely the thumb, fore finger and the middle finger, perpendicular to each other. Arrange the right hand so that first finger point in the direction of flux lines (from N to S) and thumb in the direction of motion of conductor with respect to the flux then the middle finger will point in the direction of the induced e.m.f. (or current).

Consider the conductor moving in a magnetic field as shown in the Fig. 4.5 (a). It can be verified using Fleming's right hand rule that the direction of the current due to the induced e.m.f. is coming out. Symbolically this is shown in the Fig. 4.5 (b).

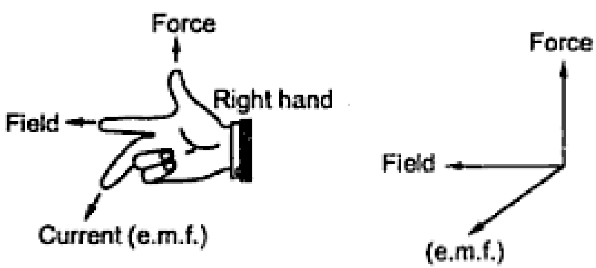


Fig. 4.5 (b)

Key Point : *In practice though magnet is moved keeping the conductor stationary, while application of rule, thumb should point in the direction of relative motion of conductor with respect to flux, assuming the flux stationary.*

This rule mainly gives direction of current which induced e.m.f. in conductor will set up when closed path is provided to it.

Verify the direction of the current through conductor in the four cases shown in the Fig. 4.6 by the use of Fleming's right hand rule.

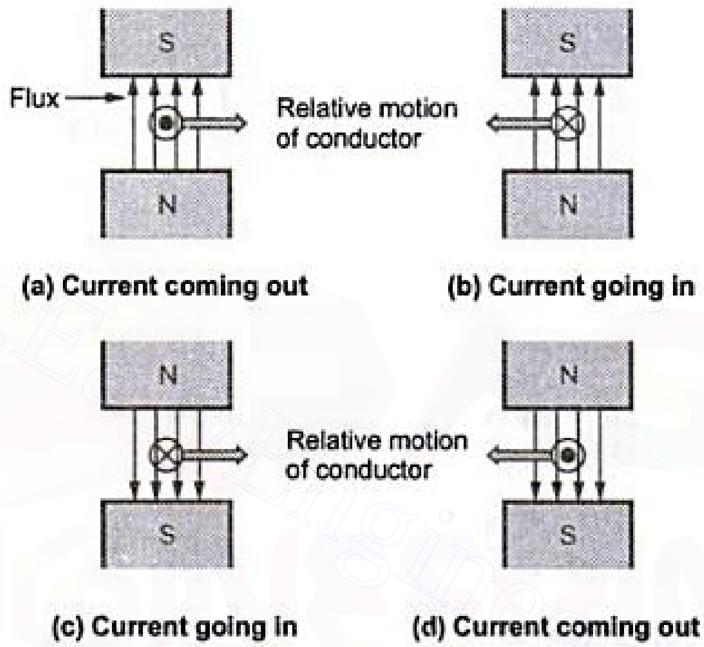


Fig. 4.6 Verifying Fleming's right hand rule

2) Lenz's Law

This rule is based on the principles derived by German Physicist Heinrich Lenz.

The Lenz's law states that, **'The direction of an induced e.m.f. produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the e.m.f.'**

In short the induced e.m.f. always opposes the cause producing it, which is represented by a negative sign, mathematically in its expression.

$$\therefore \qquad e = -N\frac{d\phi}{dt}$$

The explanation can be given as below :

Consider a solenoid as shown in the Fig. 4.7. Let a bar magnet is moved towards coil such that N-pole of magnet is facing a coil which will circulate the current through the coil.

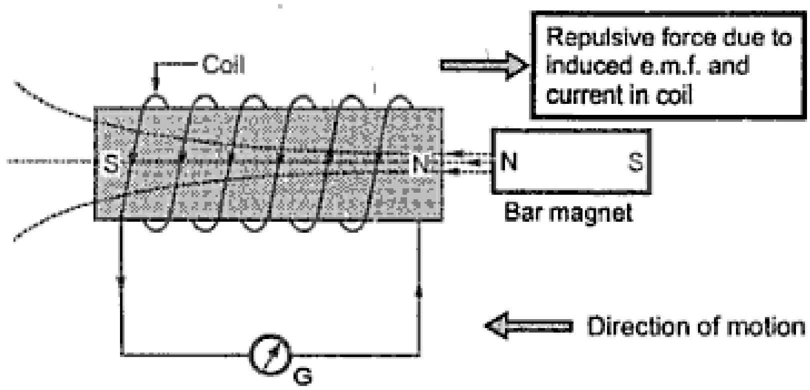


Fig. 4.7 Lenz's law

According to Lenz's Law, the direction of current due to induced e.m.f. is so as to oppose the cause. The cause is motion of bar magnet towards coil. So e.m.f. will set up a current through coil in such a way that the end of solenoid facing bar magnet will become N-pole. Hence two like poles will face each other experiencing force of repulsion which is opposite to the motion of bar magnet as shown in the Fig. 4.7.

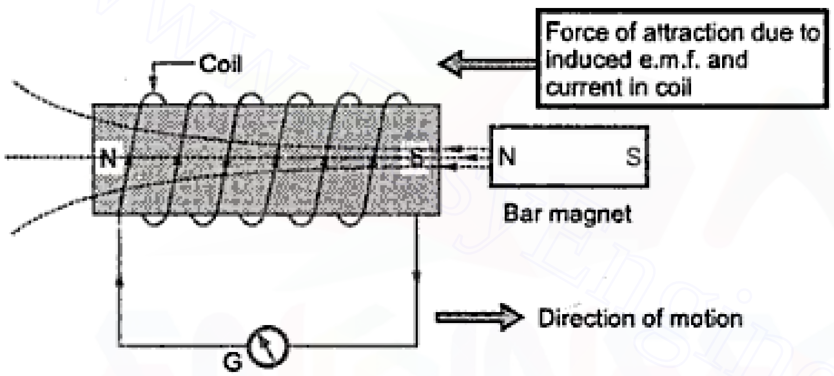


Fig. 4.8 Lenz's law

If the same bar magnet is moved away from the coil, then induced e.m.f. will set up a current in the direction which will cause, the end of solenoid facing bar magnet to behave as S-pole. Because of this two unlike poles face each other and there will be force of attraction which is direction of magnet, away from the coil. The galvanometer shows deflection in other direction as shown in the Fig. 4.8.

The Lenz's law can be summarized as,

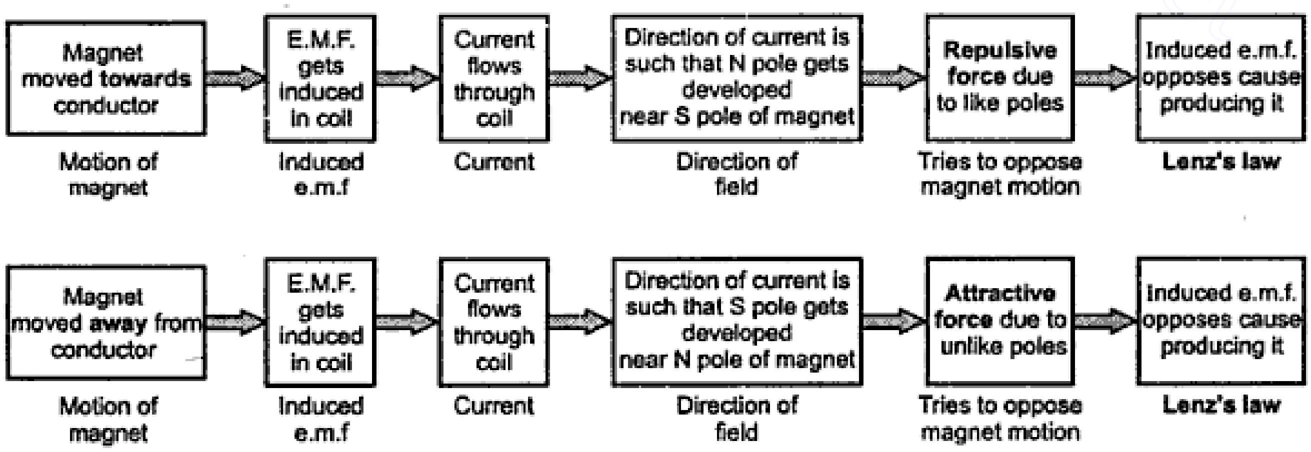


Fig. 4.9 Concept of Lenz's law

4.6 Statically Induced E.M.F.

Key Point : *The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f. in a coil which is without physical movement of coil or a magnet is called* ***statically induced e.m.f.***

Explanation : To have an induced e.m.f. there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.

Consider an electromagnet which is producing the necessary flux for producing e.m.f. Now let current through the coil of an electromagnet be an alternating one. Such alternating current means **it changes its magnitude periodically with time.** This produces the flux which is also alternating i.e. changing with time. Thus there exists $d\phi/dt$ associated with coil placed in the viscinity of an electromagnet. This is responsible for producing an e.m.f. in the coil. This is called statically induced e.m.f.

Key Point : *It can be noted that there is no physical movement of magnet or conductor, it is the alternating supply which is responsible for such an induced e.m.f.*

The concept of statically induced e.m.f. is shown in the Fig. 4.10.

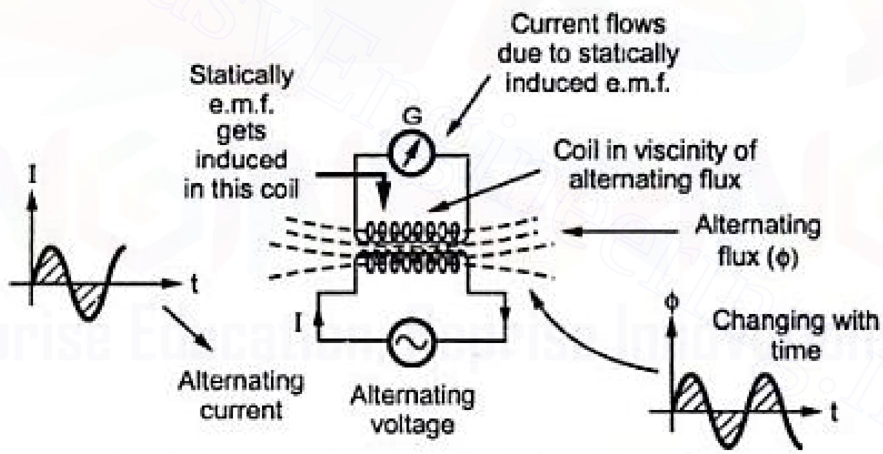


Fig. 4.10 Concept of statically induced e.m.f.

Such an induced e.m.f. can be observed in case of a device known as transformer.

Note : Due to alternating flux linking with the coil itself, the e.m.f. gets induced in that coil itself which carries an alternating current.

The statically induced e.m.f. is further classified as,

1) Self induced e.m.f. and 2) Mutually induced e.m.f.

We shall study now these two types of statically induced e.m.f.s.

4.7 Self Induced E.M.F.

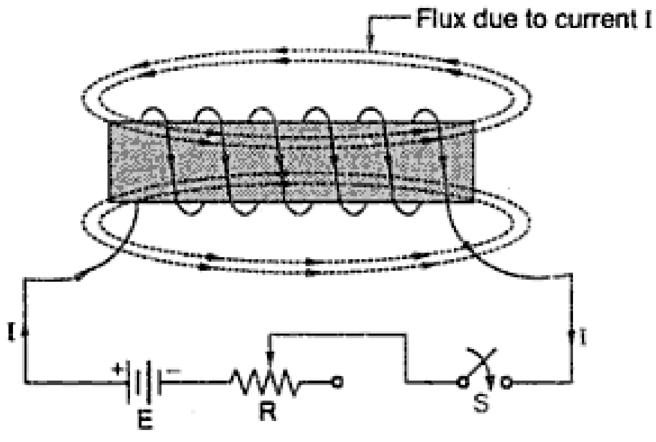


Fig. 4.11

Consider a coil having 'N' turns and carrying current 'I' when switch 'S' is in closed position. The current magnitude can be varied with the help of variable resistance connected in series with battery, coil and switch as shown in the Fig. 4.11.

The flux produced by the coil links with the coil itself. The total flux linkages of coil will be N ϕ Wb-turns. Now if the current 'I' is changed with the help of variable resistance, then flux produced will also change, due to which flux linkages will also change.

Hence according to Faraday's law, due to rate of change of flux linkages there will be induced e.m.f. in the coil. So without physically moving coil or flux there is induced e.m.f. in the coil. The phenomenon is called **self induction.**

The e.m.f. induced in a coil due to the change of its own flux linked with it is called **self induced e.m.f.**

Key Point: *The self induced e.m.f. lasts till the current in the coil is changing. The direction of such induced e.m.f. can be obtained by Lenz's law.*

4.7.1 Self Inductance

According to Lenz's law the direction of this induced e.m.f. will be so as to oppose the cause producing it. The cause is the current I hence the self induced e.m.f. will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced e.m.f. reduces the current tries to keep it to its original value. If current is decreased, self induced e.m.f. increases the current and tries to maintain it back to its original value. **So any change in current through coil is opposed by the coil.**

This property of the coil which opposes any change in the current passing through it is called **Self Inductance or Only Inductance.**

It is analogous to electrical inertia or electromagnetic inertia.

4.7.2 Magnitude of Self Induced E.M.F.

From the Faraday's law of electromagnetic induction, self induced e.m.f. can be expressed as

$$e = -N\frac{d\phi}{dt}$$

Negative sign indicates that direction of this e.m.f. is opposing change in current due to which it exists.

The flux can be expressed as,

$$\phi = (\text{Flux / Ampere}) \times \text{Ampere} = \frac{\phi}{I} \times I$$

Now for a circuit, as long as permeability 'μ' is constant, ratio of flux to current (i.e. B/H) remains constant.

$\therefore$ Rate of change of flux $= \frac{\phi}{I} \times$ rate of change of current

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -N.\frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$$

The constant $\frac{N\phi}{I}$ in this expression is nothing but the quantitative measure of the property due to which coil opposes any change in current.

So this constant $\frac{N\phi}{I}$ **is called coefficient of self inductance and denoted by 'L'.**

$$\therefore \quad \boxed{L = \frac{N\phi}{I}}$$

It can be defined as flux linkages per ampere current in it. Its unit is **henry (H).**

A circuit possesses a self inductance of 1 H when a current of 1 A through it produces flux linkages of 1 Wb-turn in it.

$$\therefore \quad \boxed{e = -L\frac{dI}{dt} \quad \text{volts}}$$

From this equation , the unit henry of self inductance can be defined as below.

Key Point: *A circuit possesses an inductance of 1 H when a current through coil is changing uniformly at the rate of one ampere per second inducing an opposing e.m.f. 1 volt in it.*

The coefficient of self inductance is also defined as the e.m.f. induced in volts when the current in the circuit changes uniformly at the rate of one ampere per second.

4.7.3 Expressions for Coefficient of Self Inductance (L)

$$L = \frac{N\phi}{I} \qquad \dots (1)$$

But $$\phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S}$$

$\therefore$ $$L = \frac{N.NI}{I.S}$$

$\therefore$ $$\boxed{L = \frac{N^2}{S} \text{ henries}} \qquad \dots (2)$$

Now $$S = \frac{l}{\mu a}$$

$$L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$\therefore$ $$\boxed{L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l} \text{ henries}} \qquad \dots (3)$$

Where l = length of magnetic circuit

a = area of cross-section of magnetic circuit through which flux is passing.

4.7.4 Factors Affecting Self Inductance of a Coil

Now as defined in last section,

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

We can define factors on which self inductance of a coil depends as,

1) It is directly proportional to the square of number of turns of a coil. This means for same length, if number of turns are more then self inductance of coil will be more.
2) It is directly proportional to the cross-sectional area of the magnetic circuit.
3) It is inversely proportional to the length of the magnetic circuit.
4) It is directly proportional to the relative permeability of the core. So for iron and other magnetic materials inductance is high as their relative permeabilities are high.

5) For air cored or non magnetic cored magnetic circuits, $\mu_r=1$ and constant, hence self inductance coefficient is also small and always constant.

As against this for magnetic materials, as current i.e. magnetic field strength H (NI/l) is changed, μ_r also changes. Due to this change in current, cause change in value of self inductance. So for magnetic materials it is not constant but varies with current.

Key Point: *For magnetic materials, L changes as the current I.*

6) Since the relative permeability of iron varies with respect to flux density, the coefficient of self inductance varies with respect to flux density.

7) If the conductor is bent back on itself, then magnetic fields produced by current through it will be opposite to each other and hence will neutralize each other. Hence **inductance will be zero** under such condition.

Example 4.3 : *If a coil has 500 turns is linked with a flux of 50 mWb, when carrying a current of 125 A. Calculate the inductance of the coil. If this current is reduced to zero uniformly in 0.1 sec, calculate the self induced e.m.f. in the coil.*

Solution : The inductance is given by,

$$L = \frac{N\phi}{I}$$

Where $N = 500, \quad \phi = 50 \text{ mWb} = 50\times10^{-3} \text{ Wb}, \quad I = 25 \text{ A}$

$$\therefore \quad L = \frac{500\times50\times10^{-3}}{125} = 0.2 \text{ H}$$

$$e = -L\frac{dI}{dt} = -L\left[\frac{\text{Final value of I} - \text{Initial value of I}}{\text{Time}}\right]$$

$$= -0.2\times\left(\frac{0-125}{0.1}\right) = \mathbf{250 \text{ volts}}$$

This is positive because current is decreased. So this 'e' will try to oppose this decrease, means will try to increase current and will help the growth of the current.

Example 4.4 : *A coil is wound uniformly on an iron core. The relative permeability of the iron is 1400. The length of the magnetic circuit is 70 cm. The cross-sectional area of the core is 5 cm^2. The coil has 1000 turns. Calculate,*

i) Reluctance of magnetic circuit ii) Inductance of coil in henries.

iii) E.M.F. induced in coil if a current of 10 A is uniformly reversed in 0.2 seconds.

Solution : $\mu_r = 1400, \quad L = 70 \text{ cm} = 0.7 \text{ m}, \quad N = 1000$

$$A = 5 \text{ cm}^2 = 5\times10^{-4} \text{ m}^2, \quad \mu_0 = 4\pi\times10^{-7}$$

i) $$S = \frac{l}{\mu_0\mu_r a} = \frac{0.7}{4\pi\times10^{-7}\times1400\times5\times10^{-4}} = 7.957\times10^5 \text{ AT/Wb}$$

ii) $$L = \frac{N^2}{S} = \frac{(1000)^2}{7.957 \times 10^5} = 1.2566 \quad H$$

iii) A current of + 10 A is made – 10 A in 0.2 sec.

$$\therefore \qquad \frac{dI}{dt} = \frac{-10-10}{0.2} = -100$$

$$e = -L\frac{dI}{dt} = -1.2566 \times (-100) = 125.66 \text{ volts}$$

Again it is positive indicating that this e.m.f. opposes the reversal i.e. decrease of current from +10 towards –10 A.

4.8 Mutually Induced E.M.F.

If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced e.m.f. in the second coil, then such an e.m.f. is called **mutually induced e.m.f.**

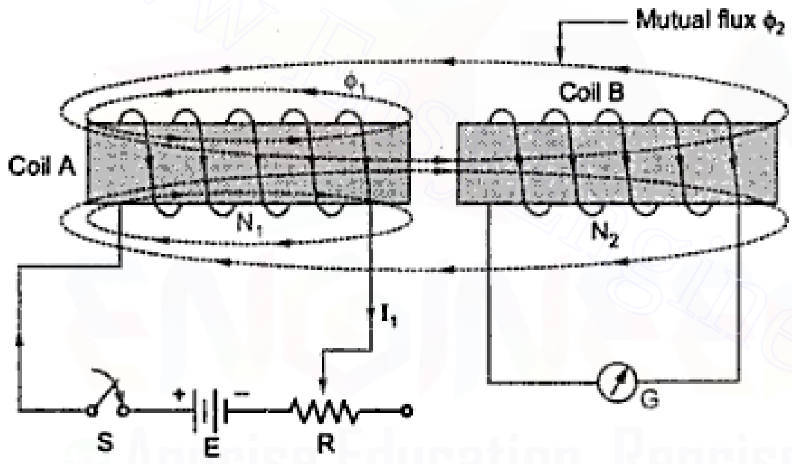


Fig. 4.12 Mutually induced e.m.f.

Consider two coils which are placed adjacent to each other as shown in the Fig. 4.12. The coil A has N_1 turns while coil B has N_2 number of turns. The coil A has switch S, variable resistance R and battery of 'E' volts in series with it. A galvanometer is connected across coil B to sense induced e.m.f. and current because of it.

Current through coil A is I_1 producing flux ϕ_1. Part of this flux will link with coil B i.e. will complete its path through coil B as shown in the Fig. 4.12. This is the mutual flux ϕ_2.

Now if current through coil A is changed by means of variable resistance R, then flux ϕ_1 changes. Due to this, flux associated with coil B, which is mutual flux ϕ_2 also changes. Due to Faraday's law there will be induced e.m.f. in coil B which will set up a current through coil B, which will be detected by galvanometer G.

Key Point : *Any change in current through coil A produces e.m.f. in coil B, this phenomenon is called mutual induction and e.m.f. is called mutually induced e.m.f.*

4.8.1 Magnitude of Mutually Induced E.M.F.

Let N_1 = Number of turns of coil A

N_2 = Number of turns of coil B

I_1 = Current flowing through coil A

ϕ_1 = Flux produced due to current I_1 in webers.

ϕ_2 = Flux linking with coil B

According to Faraday's law, the induced e.m.f. in coil B is,

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

Negative sign indicates that this e.m.f. will set up a current which will oppose the change of flux linking with it.

Now $$\phi_2 = \frac{\phi_2}{I_1} \times I_1$$

If permeability of the surroundings is assumed constant then $\phi_2 \propto I_1$ and hence ϕ_2 / I_1 is constant.

$$\therefore \quad \text{Rate of change of } \phi_2 = \frac{\phi_2}{I_1} \times \text{Rate of change of current } I_1$$

$$\therefore \quad \frac{d\phi_2}{dt} = \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -N_2 \cdot \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -\left(\frac{N_2 \phi_2}{I_1}\right) \frac{dI_1}{dt}$$

Here $\left(\frac{N_2 \phi_2}{I_1}\right)$ **is called coefficient of mutual inductance denoted by M.**

$$\therefore \quad \boxed{e_2 = -M \frac{dI_1}{dt} \quad \text{volts}}$$

Coefficient of mutual inductance is defined as the property by which e.m.f. gets induced in the second coil because of change in current through first coil.

Coefficient of mutual inductance is also called mutual inductance. It is measured in henries.

4.8.2 Definitions of Mutual Inductance and its Unit

1) The coefficient of mutual inductance is defined as the flux linkages of the coil per ampere current in other coil.

2) It can also be defined as equal to e.m.f. induced in volts in one coil when current in other coil changes uniformly at a rate of one ampere per second.

Similarly its unit can be defined as follows :

1. Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current of one ampere flowing through one coil produces a flux linkage of one weber turn in the other coil.

2. Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current changing uniformly at the rate of one ampere per second in one coil, induces as e.m.f. of one volt in the other coil.

4.8.3 Expressions of the Mutual Inductance (M)

1) $$\boxed{M = \frac{N_2 \phi_2}{I_1}}$$

2) ϕ_2 is the part of the flux ϕ_1 produced due to I_1. Let K_1 be the fraction of ϕ_1 which is linking with coil B.

$$\therefore \quad \phi_2 = K_1 \phi_1$$

$$\therefore \quad \boxed{M = \frac{N_2 K_1 \phi_1}{I_1}}$$

3) The flux ϕ_1 can be expressed as,

$$\phi_1 = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{N_1 I_1}{S}$$

$$\therefore \quad M = \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)$$

$$\boxed{M = \frac{K_1 N_1 N_2}{S}}$$

If all the flux produced by coil A links with coil B then $K_1 = 1$.

$$M = \frac{N_1 N_2}{S}$$

4) Now $\quad S = \dfrac{l}{\mu a}$ and $K_1 = 1$

Then $$M = \frac{N_1 N_2}{\left(\dfrac{l}{\mu a}\right)} = \frac{N_1 N_2 a \mu}{l}$$

$$\therefore \quad \boxed{M = \frac{N_1 N_2 a \mu_0 \mu_r}{l}}$$

5) If second coil carries current I_2, producing flux ϕ_2, the part of which links with coil A i.e. ϕ_1 then,

$$\phi_1 = K_2 \phi_2 \quad \text{and} \quad M = \frac{N_1 \phi_1}{I_2}$$

$$M = \frac{N_1 K_2 \phi_2}{I_2}$$

Now $$\phi_2 = \frac{N_2 I_2}{S}$$

$$\therefore \quad M = \frac{N_1 K_2 N_2 I_2}{I_2 S}$$

$$\therefore \quad \boxed{M = \frac{K_2 N_1 N_2}{S}}$$

If entire flux produced by coil B_2 links with coil 1, $K_2 = 1$ hence,

$$M = \frac{N_1 N_2}{S}$$

4.8.4 Coefficient of Coupling or Magnetic Coupling Coefficient

We know that, $M = \frac{N_2 K_1 \phi_1}{I_1}$ and $M = \frac{N_1 K_2 \phi_2}{I_2}$

Multiplying the two expressions of M,

$$M \times M = \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2}$$

$$\therefore \quad M^2 = K_1 K_2 \left(\frac{N_1 \phi_1}{I_1}\right)\left(\frac{N_2 \phi_2}{I_2}\right)$$

But $$\frac{N_1 \phi_1}{I_1} = \text{Self inductance of coil 1} = L_1$$

$$\frac{N_2 \phi_2}{I_2} = \text{Self inductance of coil 2} = L_2$$

$$\therefore \quad M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \cdot \sqrt{L_1 L_2} = K\sqrt{L_1 L_2}$$

where $$\boxed{K = \sqrt{K_1 K_2}}$$

The K is called **coefficient of coupling.**

If entire flux produced by one coil links with other then $K = K_1 = K_2 = 1$ and maximum mutual inductance existing between the coil is $M = K\sqrt{L_1 L_2}$.

This gives an idea about magnetic coupling between the two coils. When entire flux produced by one coil links with other, this coefficient is maximum i.e. Unity.

It can be defined as the ratio of the actual mutual inductance present between the two coils to the maximum possible value of the mutual inductance.

The expression for K is,

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

Key Point: *When K = 1 coils are said to be* ***tightly coupled*** *and if K is a fraction the coils are said to be* ***loosely coupled.***

4.9 Effective Inductance of Series Connection

Similar to the resistances, the two inductances can be coupled in series. The inductances can be connected in series either in series aiding mode called **cumulatively coupled connection** or series opposition mode called **differentially coupled connection.**

4.9.1 Series Aiding or Cumulatively Coupled Connection

Two coils are said to be cumulatively coupled if their fluxes are always in the same direction at any instant.

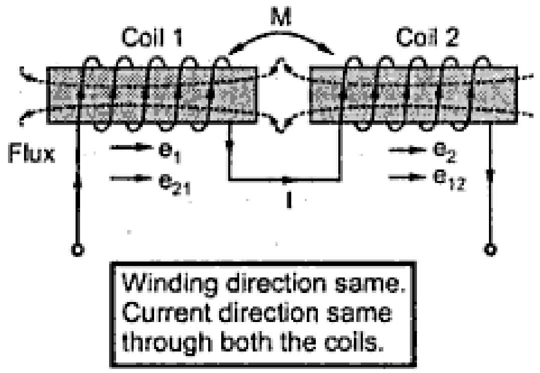


Fig. 4.13 Series aiding

For this, winding direction of the two coils on the core must be the same so that both will carry current in same direction. The Fig. 4.13 shows cumulatively coupled connection.

Coil 1 has self inductance L_1 and Coil 2 has self inductance L_2.

While both have a mutual inductance of M.

4.9.2 Equivalent Inductance of Series Aiding Connection

Refer to Fig. 4.13 which shows two coil of self inductances L_1 and L_2 connected in series aiding mode. The mutual inductance between the two is M.

If current flow through the circuit is changing at the rate of $\frac{di}{dt}$ then total e.m.f. induced will be due to self induced e.m.f.s and due to mutually induced e.m.f.s.

Due to flux linking with coil 1 itself, there is self induced e.m.f.,

$$e_1 = -L_1 \frac{di}{dt}$$

Due to flux produced by coil 2 linking with coil 1 there is mutually induced e.m.f.,

$$e_{21} = -M\frac{di}{dt}$$

Due to flux produced by coil 1 linking with coil 2 there is mutually induced e.m.f.,

$$e_{12} = -M\frac{di}{dt}$$

Due to flux produced by coil 2 linking with itself there is self induced e.m.f.

$$e_2 = -L_2 \frac{di}{dt}$$

The total induced e.m.f. is addition of these e.m.f.s as all are in the same direction,

$$e = e_1 + e_{21} + e_{12} + e_2 = -L_1 \frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} - L_2 \frac{di}{dt}$$

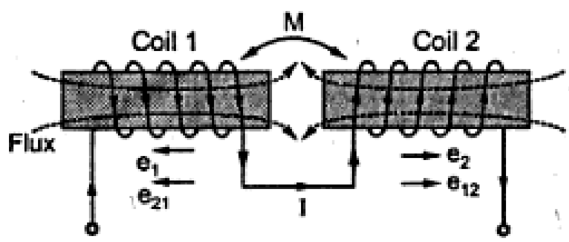
$$= -[L_1 + L_2 + 2M]\frac{di}{dt} = -L_{eq} \frac{di}{dt}$$

Where L_{eq} = Equivalent inductance

$\therefore$ $\boxed{L_{eq} = L_1 + L_2 + 2M}$

4.9.3 Series Opposition or Differentially Coupled Connection

Two coils are said to be differently coupled if their fluxes are always in the opposite direction at any instant.



Winding direction opposite.
Hence current direction opposite

Fig. 4.14 Series opposition

Such a connection is shown in the Fig. 4.14.

Coil 1 has self inductance L_1

Coil 2 has self inductance L_2

and the mutual inductance between the two is M.

4.9.4 Equivalent Inductance of Series Opposition Connection

In series opposition, flux produced by coil 2 is in opposite direction to the flux produced by coil 1.

If current in the circuit is changed at a rate $\frac{di}{dt}$ then their self induced e.m.f.s will oppose the applied voltage but mutually induced e.m.f. will assist the applied voltage.

Similar to the cumulative connection there will exist four e.m.f.s which are,

$$e_1 = -L_1 \frac{di}{dt}, \qquad e_{21} = +M\frac{di}{dt}$$

$$e_{12} = +M\frac{di}{dt} \quad \text{and} \quad e_2 = -L_2 \frac{di}{dt}$$

Hence the total e.m.f. is the addition of these four e.m.f.s,

$$\therefore \quad e = e_1 + e_{21} + e_{12} + e_2$$

$$= -L_1 \frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} - L_2 \frac{di}{dt}$$

$$= -[L_1 + L_2 - 2M]\frac{di}{dt} = -L_{eq} \frac{di}{dt}$$

Where L_{eq} = Equivalent inductance of the differentially coupled connection.

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2 - 2M}$$

Example 4.5 : *Two coils A and B are kept in parallel planes, such that 70 % of the flux produced by coil A links with coil B . Coil A has 10,000 turns. Coil B has 12,000 turns. A current of 4 A in coil A produces a flux of 0.04 mWb while a current of 4A in coil B produces a flux of 0.08 mWb. Calculate,*
i) Self inductances L_A and L_B ii) Mutual inductance M iii) Coupling coefficient.

Solution : The given values are,

$$N_A = 10{,}000, \quad N_B = 12{,}000, \quad \phi_B = 0.7\,\phi_A$$

$$\therefore \quad K_A = \frac{\phi_B}{\phi_A} = 0.7$$

$$\phi_A = 0.04\times10^{-3}\text{ Wb} \quad \text{for } I_A = 4\text{ A.}$$

$$\phi_B = 0.08\times10^{-3}\text{ Wb} \quad \text{for } I_B = 4\text{ A}$$

i) Self Inductance $$L_A = \frac{N_A \phi_A}{I_A} = \frac{10,000 \times 0.04 \times 10^{-3}}{4} = \mathbf{0.1\ H}$$

and $$L_B = \frac{N_B \phi_B}{I_B} = \frac{12000 \times 0.08 \times 10^{-3}}{4} = \mathbf{0.24\ \ H}$$

ii) Mutual Inductance $$M = \frac{N_B \phi_B}{I_A} = \frac{N_B K_A \phi_A}{I_A}$$

$$= \frac{12000 \times 0.7 \times 0.04 \times 10^{-3}}{4} = \mathbf{0.084\ H}$$

iii) Coupling Coefficient $$K = \frac{M}{\sqrt{L_A L_B}} = \frac{0.084}{\sqrt{0.1 \times 0.24}} = \mathbf{0.5422}$$

4.10 Energy Stored in the Magnetic Field

We know that energy is required to establish flux i.e. magnetic field but it is not required to maintain it. This is similar to the fact that the energy is required to raise the water through a certain height (h) which is 'mgh' joules. But energy is not required to maintain the water at height 'h'. This energy 'mgh' gets stored in it as its potential energy and can be utilized for many purposes.

Key Point : *The energy required to establish magnetic field then gets stored into it as a potential energy. This energy can be recovered when magnetic field established, collapses.*

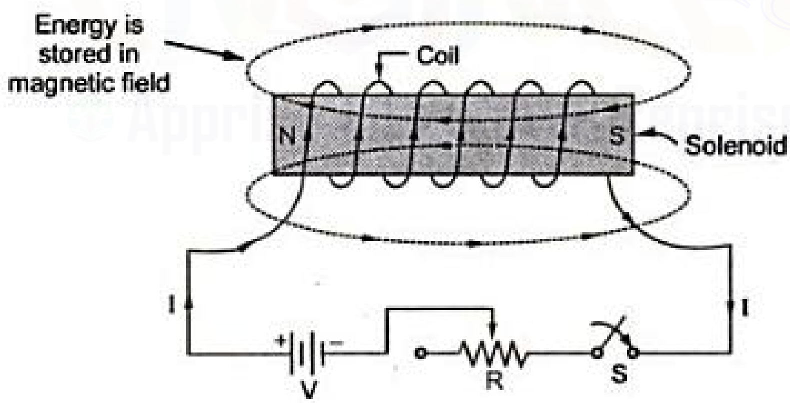


Fig. 4.15 Energy stored in magnetic field

This can be explained as below.

Consider a solenoid, the current through which can be controlled with the help of switch S, resistance R shown in the Fig. 4.15.

Initially switch 'S' is open, so current through coil, I is zero. When switch is closed, current will try to built its value equal to I. Neglect the resistance of coil.

It will take some time to increase the current from 'zero' to 'I' say 'dt' seconds.

In the mean time, flux linkages associated with the coil will change, due to which there will be self induced e.m.f. in the coil whose value is given by,

$$e = -L\frac{dI}{dt}$$

So at every instant, coil will try to oppose the increase in the current. To overcome this opposition, supply has to provide the energy to the circuit. This is nothing but the energy required to establish the current i.e. magnetic field or flux around the coil.

Once current achieves its maximum value 'I' then change in current stops. Hence there can not be any induced e.m.f. in the coil and no energy will be drawn from the supply. So no energy is required to maintain the established flux. This is because, induced e.m.f. lasts as long as there is change in flux lines associated with the coil, according to Faraday's law.

Key Point : *Now the energy which supply has provided, gets stored in the coil which is energy stored in the magnetic field, as its potential energy.*

When current is again reduced to zero by opening the switch then current through the coil starts decreasing and flux starts decreasing. So there is induced e.m.f. in the coil according to Faraday's law. But as per Lenz's law it will try to oppose cause producing it which is decrease in current. So this induced e.m.f. now will try to maintain current to its original value. So instantaneously this induced e.m.f. acts as a source and supplies the energy to the source. This is nothing but the same energy which is stored in the magnetic field which gets recovered while field collapses. So energy stored while increase in the current is returned back to the supply when current decreases i.e. when field collapses.

Key Point : *The energy which is stored in the coil earlier is returned back to the supply. No additional energy can exist as coil can not generate any energy.*

The expression for this energy stored is derived below.

4.10.1 Expression for Energy Stored in the Magnetic Field

Let the induced e.m.f. in a coil be,

$$e = -L\frac{dI}{dt}$$

This opposes a supply voltage. So supply voltage 'V' supplies energy to overcome this, which ultimately gets stored in the magnetic field.

$$\therefore \quad V = -e = -\left[-L\frac{dI}{dt}\right] = L\frac{dI}{dt}$$

$$\therefore \quad \text{Power supplied} = V \times I = L\frac{dI}{dt}\times I$$

$\therefore$ Energy supplied in time dt is,

$$E = \text{Power} \times \text{Time} = L\frac{dI}{dt}\times I \times dt$$

$$= L\,di \times I \quad \text{joules.}$$

This is energy supplied for change in current of dI but actually current changes from zero to I.

$\therefore$ Integrating above total energy stored is,

$$E = \int_0^I L\,dI\,I = L\int_0^I dI\,I$$

$$= L\left[\frac{I^2}{2}\right]_0^I = L\left[\frac{I^2}{2} - 0\right]$$

$$\therefore \quad \boxed{E = \frac{1}{2} L I^2 \quad \text{joules}}$$

4.10.2 Energy Stored Per Unit Volume

The above expression for the energy stored can be expressed in the different form as,

$$E = \frac{1}{2} L I^2 \text{ joules}$$

Now $\quad L = \dfrac{N\phi}{I}$

$$\therefore \quad E = \frac{1}{2}\frac{N\phi}{I} I^2 \text{ joules} = \frac{1}{2} N\phi I \text{ joules}$$

Now $\quad NI = Hl \quad$ ampere-turns

$$\phi = B a$$

$$\therefore \quad E = \frac{1}{2} B a H l$$

But $\quad a \times l = \text{Area} \times \text{Length} = \text{Volume of magnetic circuit}$

$\therefore$ Energy stored per unit volume is,

$$= \frac{1}{2} BH$$

But $\quad B = \mu H$

$\therefore$ Energy per unit volume,

$$= \frac{1}{2}\mu H^2 \quad \text{joules / m}^3$$

$$\boxed{E / \text{unit volume} = \frac{1}{2}\frac{B^2}{\mu} \quad \text{joules / m}^3}$$

Where $\quad \mu = \mu_0 \mu_r$

In case of inductive circuit when circuit is opened with the help of switch then current decays and finally becomes zero. In such case energy stored is recovered and if there is resistance in the circuit, appears in the form of heat across the resistance.

While if the resistance is not present then this energy appears in the form of an arc across the switch, when switch is opened.

If the medium is air, $\mu_r = 1$ hence $\mu = \mu_0$ must be used in the above expressions of energy.

Example 4.6 : *A coil is wound on an iron core to form a solenoid. A certain current is passed through the coil which is producing a flux of 40 µWb. The length of magnetic circuit is 75 cm while its cross-sectional area is 3 cm². Calculate the energy stored in the circuit. Assume relative permeability of iron as 1500.*

Solution :

$$l = 75 \text{ cm} = 0.75 \text{ m}, \quad a = 3 \text{ cm}^2 = 3\times10^{-4} \text{ m}^2$$

$$\phi = 40 \ \mu\text{Wb} = 40\times10^{-6} \text{ Wb}, \quad \mu_r = 1500$$

$$\therefore \quad B = \frac{\phi}{a} = \frac{40\times10^{-6}}{3\times10^{-4}} = 0.133 \text{ Wb/m}^2$$

$\therefore$ Energy stored per unit volume,

$$\frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r} = \frac{1}{2}\frac{(0.133)^2}{4\pi\times10^{-7}\times1500} = 4.7157 \quad \text{J/m}^3$$

$$\therefore \text{Total energy stored} = \text{Energy per unit volume}\times\text{Volume} = E\times(a\times l)$$

$$= 4.7157\times(3\times10^{-4}\times0.75) = \mathbf{0.00106 \ joules}$$

4.11 Lifting Power of Electromagnets

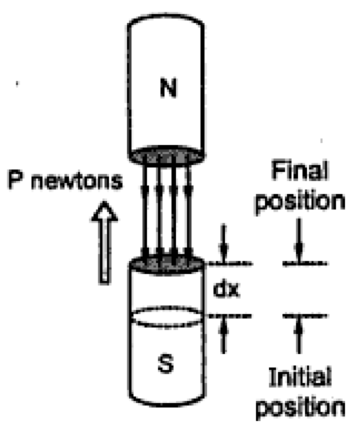


Fig. 4.16

Force of attraction between the two magnetized surfaces forms the basis of operation of devices like lifting magnets, solenoid valves, magnetically operated contactors, clutches etc.

Consider two poles of two magnetized surface N and S having an air gap of length 'l' m between them and a cross-sectional area of 'a' sq.m. Let P newtons be the force of attraction between them. This is shown in the Fig. 4.16.

The energy stored in a magnetic field in air per unit volume is,

$$E = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{J/m}^3 \qquad \ldots \ \mu_r = 1$$

$$\therefore \quad \text{Energy stored} = \frac{1}{2}\frac{B^2}{\mu_0} a \times l \quad J$$

If south pole is moved further by distance dx then energy stored will further increase by,

$$= \frac{B^2}{2\mu_0} a \times dx \quad \text{joules}$$

This increased energy must be equal to the mechanical work done to move pole by distance dx which is,

$$P \times dx = (\text{Force} \times \text{displacement})$$

$$\therefore \quad P\,dx = \frac{B^2}{2\mu_0} a \times dx$$

$$\boxed{P = \frac{B^2 a}{2\mu_0} \quad \text{newtons}}$$

This is the force in newtons existing between two magnetized surfaces.

Examples with Solutions

Example 4.7 : *A square coil of 20 cm side is rotated about its axis at a speed of 200 revolutions per minute in a magnetic field of density 0.8 Wb/m^2. If the number of turns of coil is 25, determine maximum e.m.f. induced in the coil.*

Solution : The arrangement is shown in the Fig. 4.17.

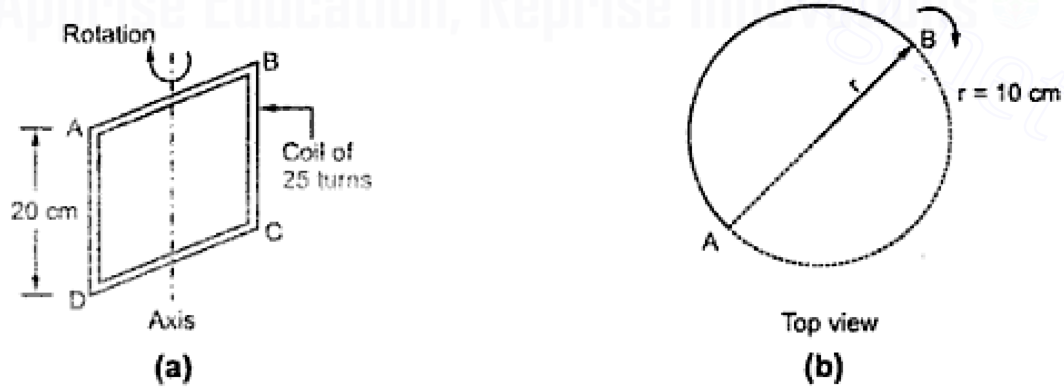


Fig. 4.17

As shown in the Fig. 4.17 (b) the active length responsible for cutting flux lines becomes l = 20 cm = 0.2 m.

Now $\quad N = 200$ r.p.m.

We want the velocity of m/sec

$$v = r\,\omega$$

$$= r \times \frac{2\pi N}{60} \text{ [where N is in r.p.m. and r = 10 cm = 0.1 m]}$$

$$\therefore \quad v = 0.1 \times \frac{2\pi \times 200}{60} = 2.094 \text{ m/sec.}$$

$$B = 0.8 \text{ Wb/m}^2 \quad \text{and} \quad \text{Active length} = 0.2 \text{ m}$$

The maximum e.m.f. induced in conductor AB, shown in Fig. 4.17 (b) will be,

$$e = B\,l\,v \sin\theta$$

For e_{max}, $\theta = 90°$

$$\therefore \quad e = 0.8 \times 0.2 \times 2.094 = 0.335 \text{ volts}$$

The e.m.f. induced in sides BC and AD is almost zero as their plane of rotation becomes parallel to plane of field.

And maximum e.m.f. induced in conductor CD will be same as AB = 0.335 volts.

∴ e.m.f. induced in one turn of the coil [AB + CD]

$$= 2 \times 0.335 = 0.67 \text{ volts}$$

In all, there are 25 turns in that coil,

∴ Total e.m.f. induced in a coil is

$$= 25 \times 0.67 = \mathbf{16.75 \text{ volts}}$$

Example 4.8 : *A conductor has 50 cm length is mounted on the periphery of a rotating part of d.c. machine. The diameter of a rotating drum is 75 cm. The drum is rotated at a speed of 1500 r.p.m. The flux density through which conductor passes at right angles is 1.1 T. Calculate the induced e.m.f. in the conductor.*

Solution : The active length l = 50 cm = 0.5 m., N = 1500 r.p.m., B = 1.1 T, $\theta = 90°$.

The rotating drum on which conductor is mounted is called **armature** of a d.c. machine.

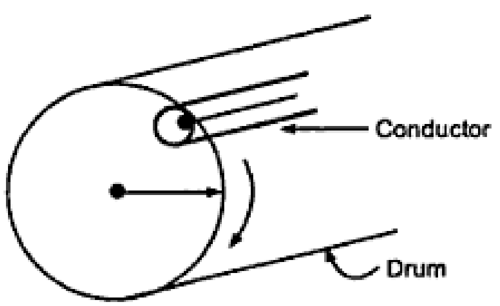


Fig. 4.18

The arrangement is as shown in Fig. 4.18.

The linear velocity

$$v = r\,\omega = r \times \frac{2\pi N}{60}$$

$$= 0.375 \times \frac{2\pi \times 1500}{60} = 58.9 \text{ m/sec.}$$

$$r = \frac{75}{2} = 37.5 \text{ cm}$$

$$\therefore \quad \text{Induced e.m.f. in a conductor} = B\, l\, v = 1.1 \times 0.5 \times 58.9$$

$$= \mathbf{32.397 \text{ volts}}$$

Example 4.9 : *Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius.*

Solution : Given values are, N = 200, l = 25 cm = 0.25 m, r = 5 cm = 0.05 m

$$\therefore \quad \text{c/s area} = \frac{\pi}{4} d^2 \quad \text{where d = Diameter}$$

$$a = \frac{\pi}{4}(2r)^2 = \frac{\pi}{4} \times (2 \times 0.05)^2$$

$$a = 7.853 \times 10^{-3} \text{ m}^2$$

For paper, $\mu_r = 1$

$$\therefore \quad S = \frac{l}{\mu_0 a} = \frac{0.25}{4\pi \times 10^{-7} \times 7.853 \times 10^{-3}} = 2.533 \times 10^7 \text{ AT/Wb}$$

$$L = \frac{N^2}{S} = \frac{(200)^2}{2.533 \times 10^7} = 1.579 \times 10^{-3} \text{H}$$

$$= \mathbf{1.579 \text{ mH}}$$

Example 4.10 : *An electromagnet is wound with 800 turns. Find the value of average e.m.f. induced and current through coil, if it is moved to that magnetic field is changed from 1 mWb to 0.25 mWb in 0.2 sec. The resistance of the coil is 500 Ω.*

Solution : Given values are, N = 800, ϕ_2 = 0.25 mWb, ϕ_1 = 1 mWb, t = 0.2 sec., R = 500Ω

$$\text{Induced e.m.f} \quad e = -N\frac{d\phi}{dt} = -800\left[\frac{\phi_2 - \phi_1}{dt}\right] = -\left[\frac{0.25 \times 10^{-3} - 1 \times 10^{-3}}{0.2}\right]$$

$$= 3 \text{ volts}$$

$$\therefore \quad \text{Current I} = \frac{\text{e.m.f.}}{R} = \frac{3}{500} = 6 \times 10^{-3} \text{ A} = \mathbf{6 \text{ mA}}$$

Example 4.11 : *A solenoid is wound with 1000 turns having area of cross-section 25 cm². When 2.5 A current flows through the coil, the flux density is 0.8 Wb/m² and when current is increased to 5 A, the flux density becomes 1.2 Wb/m². Find the average value of self inductance within given current limits. If this change in current is achieved within 0.04 sec., calculate the self induced e.m.f.*

Solution : Given values are, $N = 1000$, $a = 25\ cm^2 = 25 \times 10^{-4}\ m^2$, $I_1 = 2.5$ A, $B_1 = 0.8\ Wb/m^2$, $I_2 = 5$ A, $B_2 = 1.2\ Wb/m^2$, $t = 0.04$ sec.

$$L = \frac{N\phi}{I} \quad \text{i.e.} \quad L = N\frac{d\phi}{dI}$$

$$L = N\left[\frac{\phi_2 - \phi_1}{I_2 - I_1}\right] \quad \text{as } B = \frac{\phi}{a}$$

$$= Na\left[\frac{\frac{\phi_2}{a} - \frac{\phi_1}{a}}{I_2 - I_1}\right] = Na\left[\frac{B_2 - B_1}{I_2 - I_1}\right]$$

$$\therefore \quad L = 1000 \times 25 \times 10^{-4} \times \left[\frac{1.2 - 0.8}{5 - 2.5}\right] = \mathbf{0.4\ H}$$

Now $$e = -L\frac{dI}{dt} = -0.4\left[\frac{I_2 - I_1}{dt}\right] = -0.4\left[\frac{5 - 2.5}{0.004}\right] = \mathbf{-25\ volts}$$

Negative sign indicates that it opposes change in current.

Example 4.12 : *An iron ring of mean length of 100 cm and cross-sectional area of 10 cm^2 has an air gap of 1 mm cut in it. It is wound with a coil of 100 turns. Assuming relative permeability of iron as 500, calculate the inductance of a coil.*

Solution : Given values are, $N = 100$, $a = 10\ cm^2 = 10 \times 10^{-4}\ m^2$, $\mu_r = 500$

Length of iron is l_i = Mean length – Air gap length

$$= 100\ cm - 1 \times 10^{-1}\ cm = 99.9\ cm = 0.999\ m$$

Length of air gap is $l_g = 1\ mm = 1 \times 10^{-3}$ m

$$\therefore \quad S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.999}{4\pi \times 10^{-7} \times 500 \times 10 \times 10^{-4}}$$

$$= 1.5899 \times 10^6\ AT/Wb$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 7.9577 \times 10^5\ AT/Wb.$$

$$\therefore \quad \text{Total } S = S_i + S_g = 2.38567 \times 10^6\ AT/Wb$$

$$\therefore \quad L = \frac{N^2}{S} = \frac{(100)^2}{2.38597 \times 10^6} = 4.191 \times 10^{-3}\ H$$

$$= \mathbf{4.191\ mH}$$

Example 4.13 : *An iron cored toroid of relative permeability 980 has a mean length of 120 cm and core area of 100 mm². A current of 0.3 A establishes a flux of 40 μWb, calculate*

i) the number of turns of coil ii) self inductance iii) energy stored in magnetic field.

Solution : Given values are $\mu_r = 980$, $l = 120$ cm = 1.2 m, $a = 100$ mm² $= 100 \times 10^{-6}$ m

$$I = 0.3 \text{ A}, \quad \phi = 40\,\mu\text{ Wb} = 40 \times 10^{-6} \text{ Wb}$$

i) Flux density $$B = \frac{\phi}{a} = \frac{40 \times 10^{-6}}{100 \times 10^{-6}} = 0.4 \text{ Wb}$$

Field strength $$H = \frac{B}{\mu_0 \mu_r} = \frac{0.4}{4\pi \times 10^{-7} \times 980} = 324.8 \text{ AT/m}$$

$$H = \frac{NI}{l}$$

$$\therefore \quad 324.8 = \frac{N \times 0.3}{1.2}$$

$$\therefore \quad N = 1299.2 \approx \mathbf{1300 \text{ turns}}$$

ii) $$L = \frac{N\phi}{I} = \frac{1300 \times 40 \times 10^{-6}}{0.3} = \mathbf{0.1733 \text{ H}}$$

iii) Energy stored $$E = \frac{1}{2} L I^2 = \frac{1}{2} \times 0.1733 \times (0.3)^2 = \mathbf{7.8 \times 10^{-3} \text{ joules}}$$

Otherwise alternatively energy stored can be calculated as,

$$E = \frac{1}{2} \frac{B^2}{\mu_0 \mu_r} \times \text{Volume} = \frac{1}{2} \times \frac{(0.4)^2}{4\pi \times 10^{-7} \times 980} \times (a \times l)$$

$$= \frac{1}{2} \times 129.9224 \times \frac{1}{2} \times (100 \times 10^{-6} \times 1.2)$$

$$= \mathbf{7.8 \times 10^{-3} \text{ J}}$$

Example 4.14 : *A coil of 200 turns having a mean diameter of 6 cm is placed coaxially at the centre of a solenoid of 50 cm long with 1500 turns and carrying current of 2.5 A. Calculate the mutual inductance between the two coils.*

Solution : Given values are,

$$N_1 = 1500 \text{ (solenoid)}, \quad N_2 = 200, \quad l_1 = 50 \text{ cm} = 0.5 \text{ m}, \quad I_1 = 2.5 \text{ A}$$

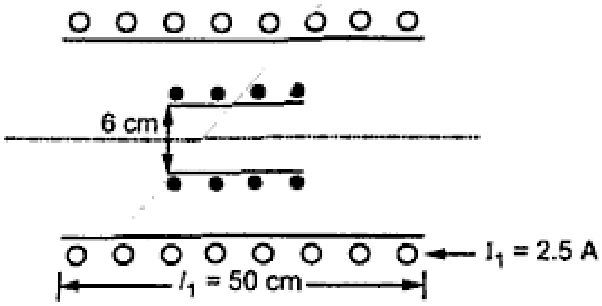


Fig. 4.19

Now magnetic field strength H at the centre of coil due to solenoid current is

$$H = \frac{N_1 I_1}{l_1}$$

$$= \frac{1500 \times 2.5}{0.5} = 7500 \text{ AT/m}$$

$\therefore$ Flux density at centre is $\quad B = \mu_0 H \quad (\mu_r = 1)$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 7500 = 9.424 \times 10^{-3} \text{ Wb/m}^2$$

$\therefore$ Flux linking with second coil is,

$$\phi_2 = B \times a_2 = 9.424 \times 10^{-3} \times \frac{\pi}{4} \times (d_2)^2$$

$$= 9.424 \times 10^{-3} \times \frac{\pi}{4} \times (6 \times 10^{-2})^2 = 2.664 \times 10^{-5} \text{ Wb}$$

$\therefore$ Mutual inductance between the coils is,

$$M = \frac{N_2 \phi_2}{I_1} = \frac{200 \times 2.664 \times 10^{-5}}{2.5} = 2.1318 \times 10^{-3} \text{ H}$$

Example 4.15 : *Two coils with a coefficient of coupling of 0.5 between them are connected in series so as to magnetize a) in the same direction (series aiding), b) in the opposite direction (series opposition). The corresponding values of equivalent inductance for a) is 1.9 H and b) 0.7 H. Find the self inductance of each coil, mutual inductance between the coil.*

(May-2006)

Solution : Given values are, $\quad K = 0.5$

Now for series aiding, $\quad L_{eq} = L_1 + L_2 - 2M = 1.9 \text{ H} \quad$ ...(1)

For series opposition, $\quad L_{eq} = L_1 + L_2 - 2M = 0.7 \text{ H} \quad$ ...(2)

and $\quad M = K\sqrt{L_1 L_2} = 0.5\sqrt{L_1 L_2} \quad$ ... (3)

Subtracting (2) from (1), $\quad 4M = 1.2 \quad$ i.e. $\quad M = 0.3$ H

Substituting in (3), $\quad 0.3 = 0.5\sqrt{L_1 L_2} \quad$ i.e. $\quad L_1 L_2 = 0.36$

$$L_2 = \frac{0.36}{L_1}$$

Substituting in (1), $L_1 + \frac{0.36}{L_1} + 2 \times 0.3 = 1.9$

$$\therefore L_1^2 + 0.36 - 1.3\, L_1 = 0$$

$$\therefore \quad L_1 = \frac{1.3 \pm \sqrt{(1.3)^2 - 4 \times 0.36}}{2}$$

$$L_1 = 0.9 \text{ H or } L_1 = 0.4 \text{ H}$$

$$L_2 = 0.4 \text{ H or } L_2 = 0.9 \text{ H}$$

Example 4.16 : *A coil of 800 turns of copper wire those diameter is of 0.375 mm. The length of the core is 90 cm. The diameter of core is 2.5 cm. Find the resistance and inductance of the coil. Assume specific resistance of copper as 1.73×10^{-6} Ω-cm.*

Solution :

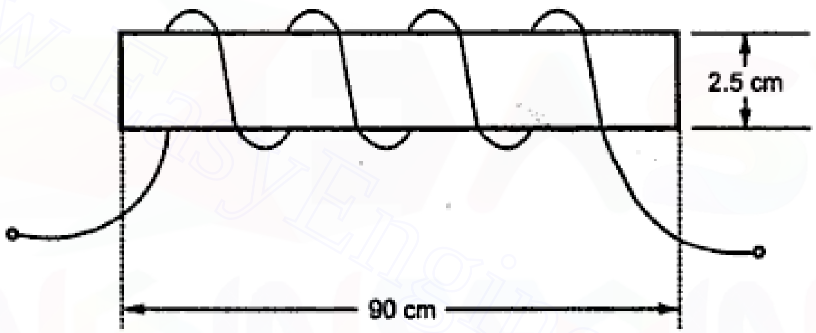


Fig. 4.20

Length of the coil = $(\pi \times d) \times$ Number of turns

As $\pi \times d$ = Circumference of 1 turn

And d = Diameter of the core

$\therefore$ Length of the coil $= (\pi \times 2.5 \times 10^{-2}) \times 800 = 62.83$ m

$$\rho = 1.73 \times 10^{-6} \ \Omega\text{-cm} = 1.73 \times 10^{-8} \ \Omega\text{-m}$$

$$\therefore \quad R = \frac{\rho l}{a}$$

Where $a = \frac{\pi}{4} d^2$ where d = Diameter of coil

$$\therefore \quad d = 0.375 \text{ mm} = 0.375 \times 10^{-3} \text{ m}$$

$$\therefore \quad a = \frac{\pi}{4} \times (0.375 \times 10^{-3})^2 = 1.104 \times 10^{-7} \text{ m}^2$$

$$\therefore \quad R = \frac{\rho l}{a} = \frac{1.73\times10^{-8}\times62.68}{1.1044\times10^{-7}} = 9.84\ \Omega$$

While $$L = \frac{N^2}{S}$$

Reluctance $$S = \frac{l}{\mu_0\mu_r a} \quad \text{where} \quad l = \text{Length of core} = 0.9\text{ m}$$

$$a = \text{c/s area of core} = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(2.5\times10^{-2})^2$$

$$= 4.908\times10^{-4}\text{m}^2$$

Assume $$\mu_r = 1$$

$$\therefore \quad S = \frac{0.9}{4\pi\times10^{-7}\times1\times4.908\times10^{-4}} = 1.45902\times10^9\ \text{AT/Wb}$$

$$L = \frac{N^2}{S} = \frac{(800)^2}{1.45902\times10^9} = 4.386\times10^{-4}\text{ H}$$

$$= \mathbf{0.4386\ mH}$$

Example 4.17 : *A length of an air cored solenoid is 1.7 m and area of cross-section is 12 cm^2. The number of turns of coil is 1000.*
Calculate :
i) The self inductance. ii) The energy stored in magnetic field when a current of 10 A flows through the coil. **(Dec.-97)**

Solution : l = 1.7 m, a = 12 cm^2 = 12×10^{-4}m^2, $\mu_0 = 4\pi\times10^{-7}$, N = 1000, I = 10 A

$$S = \frac{l}{\mu_0 a} = \frac{1.7}{4\pi\times10^{-7}\times12\times10^{-4}}$$

$$= 1.1273\times10^9\ \text{AT/Wb} \qquad \ldots\ \mu_r = 1 \text{ as air cored}$$

$$L = \frac{N^2}{S} = \frac{(1000)^2}{1.1273\times10^9} = 8.87\times10^{-4}\text{ H} = \mathbf{88.7\ mH}$$

Now if $$E = \frac{1}{2}LI^2 = \frac{1}{2}\times(88.7\times10^{-3})\times(10)^2 = \mathbf{0.0443\ J}$$

Example 4.18 : *Two coils having 3000 and 2000 turns are wound on a magnetic ring. 60% of flux produced in first coil links with the second coil. A current of 3 A produces flux of 0.5 mWb in the first coil and 0.3 mWb in the second coil. Determine the mutual inductance and coefficient of coupling.* **(Dec.-98)**

Solution : $N_1 = 3000$, $N_2 = 2000$, $\phi_1 = 0.5$ mWb, $\phi_2 = 0.3$ mWb

$$I_1 = I_2 = 3 \text{ A} \quad \text{and} \quad \phi_2 = 0.6\,\phi_1$$

$$M = \frac{N_2\,\phi_2}{I_1} = \frac{2000 \times 0.3 \times 10^{-3}}{3} = 0.2 \text{ H}$$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{3000 \times 0.5 \times 10^{-3}}{3} = 0.5 \text{ H}$$

$$L_2 = \frac{N_2 \phi_2}{I_2} = \frac{2000 \times 0.3 \times 10^{-3}}{3} = 0.2 \text{ H}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.2}{\sqrt{0.5 \times 0.2}} = 0.6324$$

Example 4.19 : *Two coils having 1000 and 300 turns are wound on a common magnetic path with perfect magnetic coupling. The reluctance of the path is 3×10^6 AT/Wb. Find the mutual inductance between them. If the current in 1000 turns coil changes uniformly from 5 A to zero in 10 milliseconds, find the induced e.m.f. in the other coil.*

(Dec. -99, Dec.-2000)

Solution : $N_1 = 1000$, $N_2 = 300$, $K = 1$, $S = 3 \times 10^6$ AT/Wb

Now,
$$M = \frac{N_1 N_2}{S} = \frac{1000 \times 300}{3 \times 10^6} = 0.1 \text{ H}$$

$$e_2 = -M\frac{dI_1}{dt} = -0.1 \times \left(\frac{0-5}{10 \times 10^{-3}}\right) = 50 \text{ V}$$

This the induced e.m.f. in other coil.

Example 4.20 : *Two coils A and B are placed such that 40 % of flux produced by coil A links with coil B coils A and B have 2000 and 1000 turns respectively. A current of 2.5 A in coil A produces a flux of 0.035 mWb in coil B. For the above coil combination, find out (i) M, the mutual inductance and (ii) the coefficient of coupling K_A, K_B and K (iii) Self inductances L_A and L_B.*

(May-2000)

Solution : $N_A = 2000$, $N_B = 1000$, $K_A = 0.4$, $\phi_B = 0.4\,\phi_A$

$$I_A = 2.5 \text{ A} \quad \text{and} \quad \phi_B = 0.035 \text{ mWb}$$

(i) Mutual Inductance,
$$M = \frac{N_B\,\phi_B}{I_A} = \frac{1000 \times 0.035 \times 10^{-3}}{2.5} = 0.014 \text{ H}$$

(ii)
$$\phi_B = 0.035 \text{ mWb} \quad \text{and} \quad \phi_B = 0.4\,\phi_A$$

$$\therefore \quad \phi_A = \frac{\phi_B}{0.4} = \frac{0.035}{0.4} = 0.0875 \text{ mWb}$$

$$\therefore \quad L_A = \frac{N_A \phi_A}{I_A} = \frac{2000 \times 0.0875 \times 10^{-3}}{2.5}$$

$$\therefore \quad L_A = \mathbf{0.07\ H}$$

Assuming that same current in coil B produces 0.035 mWb in coil B.

$$\therefore \quad L_B = \frac{N_B \phi_B}{I_B} = \frac{1000 \times 0.035 \times 10^{-3}}{2.5} = \mathbf{0.014\ H}$$

(iii) $$M = \frac{N_A \phi_A}{I_B} \qquad M = \frac{N_A K_B \phi_B}{I_B}$$

$$\therefore \quad 0.014 = \frac{2000 \times K_B \times 0.035 \times 10^{-3}}{2.5}$$

$$\therefore \quad K_B = \mathbf{0.5}$$

$\phi_B = K_A \phi_A$ and it is given that 40% of ϕ_A links with coil B,

$$\therefore \quad K_A = \mathbf{0.4}$$

$$K = \sqrt{K_A K_B} = \sqrt{0.4 \times 0.5} = \mathbf{0.4472}$$

Example 4.21 : *Two windings connected in series are wound on a ferromagnetic ring having cross-sectional area of 750 mm^2 and a mean diameter of 175 mm. The two windings have 250 and 750 turns, while the relative permeability of material is 1500. Assuming no leakage of flux, calculate the self inductances of each winding and the mutual inductance as well. Calculate e.m.f. induced in coil 2 if current is coil 1 in increased uniformly from zero to 5 A in 0.01 sec.* **(Dec.-2001)**

Solution : l = Length of magnetic circuit = $\pi \times d_{mean}$

$$l = \pi \times 175 \times 10^{-3} = 0.5497 \text{ m}$$

$$a = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2 = 7.5 \times 10^{-4} \text{ m}^2$$

$$N_1 = 250, \quad N_2 = 750, \ \mu_r = 1500$$

Self inductance, $$L = \frac{N\phi}{I} \text{ but } \phi = \frac{NI}{S}$$

$$\therefore \quad L = \frac{N}{I}\frac{NI}{S} = \frac{N^2}{S}$$

We have, $$S = \frac{l}{\mu a}$$

$$\therefore \quad S = \frac{l}{\mu_0 \mu_r a} = \frac{0.5497}{(4\pi\times 10^{-7})(1500)(7.5\times 10^{-4})}$$

$$= 388833.2 \text{ AT/Wb}$$

$$\therefore \quad L_1 = \frac{N_1^2}{S} = \frac{(250)^2}{388833.2} = \mathbf{0.1607\ H}$$

$$L_2 = \frac{N_2^2}{S} = \frac{(750)^2}{388833.2} = \mathbf{1.4466\ H}$$

The mutual inductance between the two windings is given by,

$$M = \frac{N_1 N_2}{S} = \frac{(250)(750)}{388833.20} = \mathbf{0.4822\ H}$$

$$\therefore \quad M = \mathbf{0.4822\ H}$$

E.M.F. induced in coil 2 is,

$$e_2 = -M\frac{dI_1}{dt} = -0.4822\times\frac{(5-0)}{0.01} = \mathbf{-241.1\ V}$$

Example 4.22 : *If a current of 5 A flowing in coil with 1000 turns wound on a ring of ferromagnetic material produces a flux of 0.5 mWb in the ring. Calculate (i) self inductance of coil (ii) e.m.f. induced in the coil when current is switched off and reaches zero value in 2 millisec. (iii) mutual inductance between the coils, if a second coil with 750 turns is wound uniformly over the first one.* **(May-2003)**

Solution : $\phi = 0.5$ mWb, N = 1000, I = 5 A

i) $$L = \frac{N\phi}{I} = \frac{1000\times 0.5\times 10^{-3}}{5} = \mathbf{0.1\ H}$$

ii) $$e = -L\frac{dI}{dt} = -0.1\left[\frac{0-5}{2\times 10^{-3}}\right] = \mathbf{250\ V}$$

iii) Let N_2 = 750 of other coil

As other coil is wound on first, all the flux produced by coil 1 links with the second coil.

$$\therefore \quad \phi_2 = K_1\,\phi_1 = \phi_1 \quad \text{as} \quad K_1 = 1$$

$$\therefore \quad M = \frac{N_2\phi_2}{I_1} = \frac{N_2[K_1\phi_1]}{I_1}$$

$$= \frac{750\times 05\times 10^{-3}}{5} = 0.075 \text{ H}$$

Example 4.23 : *An electric conductor of effective length of 0.3 metre is made to move with a constant velocity of 5 metre per second perpendicular to a magnetic field of uniform flux density 0.5 tesla. Find the e.m.f. induced in it. If this e.m.f. is used to supply a current of 25 A, find the force on the conductor, and state its direction w.r.t. motion of conductor, ignoring friction. Find the power required to keep the conductor moving across the field.* **(Dec.-2003)**

Solution : $l = 0.3$ m, $v = 5$ m/s, $B = 0.5$ T

$$\therefore \quad e = Blv = 0.3\times 5\times 0.5 = \mathbf{0.75\ V}$$

Now $I = 25$ A

$$\therefore \quad F = B\,I\,l = 0.5\times 25\times 0.3 = \mathbf{3.75\ N}$$

The direction of this force is so as to oppose the motion of conductor, as per Lenz's law.

The power required to keep the conductor moving is,

$$P = e\times I = 0.75\times 3.75 = \mathbf{2.8125\ W}$$

Example 4.24 : *Two identical coils P and Q, each with 1500 turns, are placed in parallel planes near to each other, so that 70% of the flux produced by current in coil P links with coil Q. If a current of 4 A is passed through any one coil, it produces a flux of 0.04 mWb linking with itself. Find the self inductances of the two coils, the mutual inductance and coefficient of coupling between them.* **(Dec.-2003)**

Solution : $N_P = N_Q = 1500$, $\phi_Q = 0.7\ \phi_P$ ... 70% linking

Let $I_P = 4$ A and $\phi_P = 0.04$ mWb

$$\therefore \quad L_P = \frac{N_P\,\phi_P}{I_P} = \frac{1500\times 0.04\times 10^{-3}}{4} = \mathbf{15\ mH}$$

Let $I_Q = 4$ A then $\phi_Q = 0.04$ mWb

$$\therefore \quad L_Q = \frac{N_Q\,\phi_Q}{I_Q} = \frac{1500\times 0.04\times 10^{-3}}{4} = \mathbf{15\ mH}$$

$$M = \frac{N_Q\,\phi_Q}{I_P} = \frac{N_Q\ 0.7\,\phi_P}{I_P} = \frac{1500\times 0.7\times 0.04\times 10^{-3}}{4} = \mathbf{10.5\ mH}$$

And

$$K = \frac{M}{\sqrt{L_P\,L_Q}} = \frac{10.5\times 10^{-3}}{\sqrt{(15\times 10^{-3})^2}} = \mathbf{0.7}$$

➠ **Example 4.25 :** *A coil of 450 turns is uniformly wound around a ring of an iron alloy of mean circumference of 100 cm and cross-sectional area 1.125 sq. cm. When a current of 0.5 ampere is linearly reduced to zero in 0.01 second, the e.m.f. induced in the coil is 2 V. Find the relative permeability of the iron alloy.* **(May-2004)**

Solution : N = 450, l_i = 100 cm, a = 1.125 cm^2

$$e = -L\frac{dI}{dt}$$

$$dI = +0.5 \text{ to zero i.e. } 0 - 0.5 = -0.5$$

$$dt = 0.01 \text{ sec}, \quad e = 2 \text{ V}$$

$$\therefore \quad 2 = -L\frac{(-0.5)}{0.01} \quad \text{i.e.} \quad \mathbf{L = 0.04\ H}$$

Now $$L = \frac{N^2}{S} \quad \text{and} \quad S = \frac{l_i}{\mu_0 \mu_r a}$$

$$\therefore \quad 0.04 = \frac{(450)^2}{\dfrac{l_i}{\mu_0 \mu_r a}}$$

$$\therefore \quad \frac{1}{\mu_r} = \frac{(450)^2 \times (1.125\times10^{-4}) \times (4\pi\times10^{-7})}{100\times10^{-2}\times0.04}$$

$$\therefore \quad \mu_r = 1397.245 \approx \mathbf{1398}$$

➠ **Example 4.26 :** *A straight conductor 1.5 m long lies in a plane perpendicular to a uniform magnetic field of flux density 1.2 tesla. When a current of 'I' ampere is passed through it, it makes the conductor move across the magnetic field with a velocity of 1 m/s. Ignoring resistance of the conductor and friction, find the current 'I', if the power of the moving conductor is 90 watt. Find the e.m.f. induced in the conductor and the force on it. State the sense of the force w.r.t. the velocity, and sense of the e.m.f. induced w.r.t. current.* **(May-2004)**

Solution : l = 1.5 m, B = 1.2 T, v = 1 m/s, P = 90 W

$$e = B\,l\,v = 1.2\times1.5\times1 = 1.8 \text{ V}$$

$$P = e\times I$$

$$\therefore \quad 90 = 1.8\times I$$

$$\therefore \quad I = 50 \text{ A}$$

$$\therefore \quad F = B\,I\,l = 1.2\times50\times1.5 = \mathbf{90\ N}$$

The force is so as to oppose the velocity while the sense of e.m.f. is so as to oppose the current.

Example 4.27 : *Two coils A and B, have self inductances of 120 μH and 300 μH respectively. A current of 1 A through coil 'A' produces flux linkage of 100 μWb turns in coil 'B'. Calculate*

i) mutual inductance between the coil.

ii) average e.m.f. induced in coil 'B' if current of 1 A in coil 'A' is reversed at a uniform rate in 0.1 sec. Also find coefficient of coupling. **(Dec.-2004)**

Solution : $L_A = 120\ \mu H,\ L_B = 300\ \mu H$

$$I_A = 1\text{ A produces } N_B\phi_B = 100\ \mu\text{Wb}$$

i) $$M = \frac{N_B\phi_B}{I_A} = \frac{100\times10^{-6}}{1} = \mathbf{100\ \mu H} \qquad \text{...Mutual inductance}$$

ii) $$e_B = -M\frac{dI_A}{dt}$$

The current in coil A is reversed i.e. it is –1 A in 0.1 sec.

$$\therefore \quad \Delta I = (\text{New value} - \text{Original value}) = (-1-1) = -2\text{ A}$$

and $$\Delta t = 0.1\text{ sec}$$

$$\therefore \quad \frac{dI_A}{dt} = \frac{\Delta I}{\Delta t} = \frac{-2}{0.1} = -20\text{ A/sec}$$

$$\therefore \quad e_B = -100\times10^{6}\times(-20) = \mathbf{2\ mV} \qquad \text{... Induced e.m.f. in B}$$

$$K = \frac{M}{\sqrt{L_A L_B}} = \frac{100\times10^{-6}}{\sqrt{120\times10^{-6}\times300\times10^{-6}}}$$

$$= \mathbf{0.527} \qquad \text{... Coefficient of coupling}$$

Example 4.28 : *A magnetic core is in the form of a closed ring of mean length 20 cm and cross-sectional area 1 cms². Its relative permeability is 2400. A coil of 2000 turns is uniformly wound around it. Find the flux density set up in the core if a current of 66 mA is passed through the coil. Find the energy stored in the magnetic field set up.*

Find the inductance of the coil, if an air gap of 1 mm is cut in the ring perpendicular to the direction of the flux. **(May-2005)**

Solution : Given $l = 20$ cm, $a = 1\text{ cm}^2$, $\mu_r = 2400$, $N = 2000$, $I = 66$ mA

Case 1 : $$S = \frac{l}{\mu_0\mu_r a} = \frac{20\times10^{-2}}{4\pi\times10^{-7}\times2400\times1\times10^{-4}} = 663.1455\times10^{3}\text{ AT/Wb}$$

$$\text{m.m.f} = NI = 2000\times66\times10^{-3} = 132\text{ AT}$$

$$\therefore \quad \phi = \frac{NI}{S} = \frac{132}{663.1455\times10^{3}} = 1.9905\times10^{-4}\text{ Wb}$$

$\therefore \quad B = \frac{\phi}{a} = \frac{1.9905\times10^{-4}}{1\times10^{-4}} = \mathbf{1.9905\ Wb/m^2}$ i.e. T ...Flux density

$$L = \frac{N^2}{S} = \frac{(2000)^2}{663.1455\times10^3} = 6.03185 \text{ H or } L = \frac{N\phi}{I}$$

$\therefore \quad E = \frac{1}{2}LI^2 = \frac{1}{2}\times 6.03185\times(66\times10^{-3})^2 = \mathbf{13.1373\ mJ}$ ...Energy stored

Case 2 : New air gap is cut of length $l_g = 1$ mm in the ring.

$\therefore \quad l_i = \text{Iron length} = l - l_g = 20\times10^{-2} - 1\times10^{-3} = 0.199$ m

$\therefore \quad S = S_i + S_g = \frac{l_i}{\mu_0\mu_r a} + \frac{l_g}{\mu_0 a}$ ... $\mu_r = 1$ for air gap

$$= \frac{1}{\mu_0 a}\left[\frac{li}{\mu_r} + l_g\right] = \frac{1}{4\pi\times10^{-7}\times1\times10^{-4}}\left[\frac{0.199}{2400} + 1\times10^{-3}\right]$$

$= 8.6175\times10^6$ AT/Wb ...Total reluctance

$\therefore \quad L = \frac{N^2}{S} = \frac{(2000)^2}{8.6175\times10^6} = \mathbf{0.4641\ H}$ ...New inductance.

Example 4.29 : *Two long, single-layered solenoids 'x' and 'y' have the same length and the same number of turns. The cross-sectional areas of the two are 'a_x' and 'a_y' respectively, with 'a_y' < 'a_x'. They are placed coaxially, with solenoid 'y' placed within the solenoid 'x'. Show that the coefficient of coupling between them is equal to $\sqrt{a_y / a_x}$.* **(May-2005)**

Solution : The arrangement is shown in the Fig. 4.21.

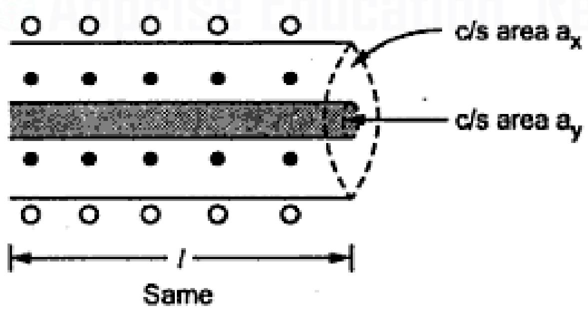


Fig. 4.21

It is known that

$$L = \frac{N^2\mu_0\mu_r a}{l}$$

For coil x,

$$L_x = \frac{N^2\mu_0\mu_r a_x}{l}$$

and $$L_y = \frac{N^2\mu_0\mu_r a_y}{l}$$

The number of turns N and μ_r is same for both.

Considering coil y,

$$M = \frac{N_1 N_2 a_y \mu_0 \mu_r}{l} \quad \text{where } N_1 = N_2 = N$$

$$\therefore \quad M = \frac{N^2 a_y \mu_0 \mu_r}{l}$$

The coefficient of coupling is given by,

$$K = \frac{M}{\sqrt{L_x L_y}} = \frac{\dfrac{N^2 a_y \mu_0 \mu_r}{l}}{\sqrt{\dfrac{N^2 \mu_0 \mu_r a_x}{l} \times \dfrac{N^2 \mu_0 \mu_r a_y}{l}}}$$

$$= \frac{\left(\dfrac{N^2 \mu_0 \mu_r}{l}\right) a_y}{\left(\dfrac{N^2 \mu_0 \mu_r}{l}\right)\sqrt{a_x a_y}} = \sqrt{\frac{a_y}{a_x}}$$

$$\therefore \quad K = \sqrt{\frac{a_y}{a_x}} \qquad \text{...Proved}$$

Example 4.30 : *An iron ring wound with 500 turns solenoid produces a flux density of 0.94 tesla in the ring carrying a current of 2.4 Amp. The mean length of iron path is 80 cm and that of air gap is 1 mm. Determine i) the relative permeability of iron, ii) the self inductance and iii) energy stored in the above arrangement, if the area of cross-section of ring is 20 cm².* **(Dec.-2005)**

Solution : $N = 500$, $I = 2.4$ A, $B = 0.94$ T, $a = 20$ cm^2

$$l_i = \text{Length of iron path} = 80 \text{ cm}$$

$$l_g = \text{Length of air gap} = 1 \text{ mm}$$

i) $$\phi = B \times a = 0.94 \times 20 \times 10^{-4} = 1.88 \times 10^{-3} \text{ Wb}$$

$$\phi = \frac{\text{m.m.f}}{\text{reluctance}} = \frac{NI}{S}$$

$$\therefore \quad S = \frac{500 \times 2.4}{1.88 \times 10^{-3}} = 638297.8723 \text{ AT/Wb}$$

But $$S = S_i + S_g = \frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a} \qquad \text{... } \mu_r = 1 \text{ for air gap}$$

$$\therefore \quad 638297.8723 = \left[\frac{80 \times 10^{-2}}{\mu_r} + \frac{1 \times 10^{-3}}{1}\right] \frac{1}{4\pi \times 10^{-7} \times 20 \times 10^{-4}}$$

$$\therefore \quad \mu_r = 1324.02$$

ii) $$L = \frac{N^2}{S} = \frac{(500)^2}{638297.8723} = \mathbf{0.3916\ H}$$

iii) $$E = \frac{1}{2}LI^2 = \frac{1}{2}\times 0.3916\times(2.4)^2 = \mathbf{1.1278\ J}$$

Example 4.31 : *An air cored solenoid 1 m in length and 10 cm in diameter has 5,000 turns. Calculate : (i) the self inductance and (ii) the energy stored in the magnetic field when current of 2 A flows in solenoid.* **(Dec.-2006)**

Solution : $l = 1$ m, $d = 10$ cm, $N = 5000$, $\mu = \mu_0$ as air cored

$$a = \frac{\pi}{4}d^2 = \frac{100\pi}{4}\text{cm}^2 = 7.854\times 10^{-3}\text{ m}^2$$

$$\therefore \quad S = \frac{l}{\mu_0 a} = \frac{1}{4\pi\times 10^{-7}\times 7.854\times 10^{-3}} = 101.3209\times 10^6 \text{ AT/Wb}$$

i) $$L = \frac{N^2}{S} = \frac{(5000)^2}{101.3209\times 10^6} = \mathbf{0.2467\ H}$$

ii) $$I = 2 \text{ A}$$

$$\therefore \quad E = \frac{1}{2}LI^2 = \frac{1}{2}\times 0.2467\times 2^2 = \mathbf{0.4934\ J}$$

Example 4.32 : *An iron ring of 10 cm in diameter and 8 cm^2 in cross-section is wound with 300 turns of wire. For a flux density of 1.2 Wb/m^2 and relative permeability of 500, find the exciting current, the inductance and the energy stored.* **(May-2007)**

Solution : $d = 10$ cm, $a = 8$ cm^2, $N = 300$, $B = 1.2$ Wb/m^2, $\mu_r = 500$

$$l = \pi\times d = \pi\times 10 \text{ cm} = 0.3141 \text{ m}$$

$$S = \frac{l}{\mu_0\mu_r a} = \frac{0.3141}{4\pi\times 10^{-7}\times 500\times 8\times 10^{-4}} = 624.882\times 10^3 \text{ AT/Wb}$$

$$\phi = B\times a = 1.2\times 8\times 10^{-4} = 9.6\times 10^{-4} \text{ Wb}$$

$$\therefore \quad \phi = \frac{NI}{S}$$

$$\therefore \quad 9.6\times 10^{-4} = \frac{300\times I}{624.882\times 10^3}$$

$$\therefore \quad I = 2 \text{ A}$$

$$L = \frac{N^2}{S} = \frac{(300)^2}{624.882\times 10^3} = \mathbf{0.14402\ H}$$

$$\therefore \quad E = \frac{1}{2}LI^2 = \frac{1}{2}\times 0.14402\times(2)^2 = \mathbf{0.288\ J}$$

Review Questions

1. *State the Faraday's laws of electromagnetism.*
2. *What is the difference between dynamically induced e.m.f. and statically induced e.m.f. ?*
3. *Derive the expression for the magnitude of the dynamically induced e.m.f.*
4. *Explain clearly the difference between self inductance and mutual inductance. State their units.*
5. *Derive the various expressions for the self inductance.*
6. *Explain the factors on which self inductances depends.*
7. *Derive the various expressions for the mutual inductance.*
8. *Derive the expression for coefficient of coupling.*
9. *Derive the expression for the equivalent inductance when two inductances are connected in i) Series aiding (cumulatively coupled) ii) Series opposition (differentially coupled).*
10. *How energy gets stored in the magnetic field ?*
11. *Derive the expression for energy stored in the magnetic field.*
12. *Write a note on lifting power of an electromagnet.*
13. *Two identical 1000 turn coils X and Y lie in parallel planes such that 60% of the flux produced by one coil links with the other. A current of 5 A in X produces a flux of 5×10^{-6} Wb in itself. If the current in X changes from + 6 A to – 6 A in 0.01 sec, what will be the magnitude of the e.m.f. induced in Y ? Calculate the self inductance of each coil.* **(Ans. : 0.72 V, 0.001 H)**
14. *Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius. Also calculate energy stored in it if current rises from zero to 5 A.* **(Ans. : 1.579 mH, 0.01973 J)**
15. *Two 200 turns, air cored solenoids, 25 cm long have a cross-sectional area of 3 cm^2 each. The mutual inductance between them is 0.5 μH. Find the self inductance of the coils and the coefficient of coupling.* **(Ans. : 60.31 μH, 0.00828)**
16. *Two coils A and B having 5000 and 2500 turns respectively are wound on a magnetic ring. 60 % of the flux produced by coil A links with coil B. A current of 1 A produces a flux of 0.25 mWb in coil A while same current produces a flux of 0.15 mWb in coil B. Find the mutual inductance and coefficient of coupling.* **(Ans. : 1.25 H, 0.375 H, 0.5477)**
17. *A conductor has 1.9 m length. It moves at right angles to a uniform magnetic field. The flux density of the magnetic field is 0.9 tesla. The velocity of the conductor is 65 m/sec. Calculate the e.m.f. induced in the conductor.* **(Ans. : 111.15 volts)**
18. *An air cored coil has 800 turns. Length of the coil is 6 cm while its diameter is 4 cm. Find the current required to establish flux density of 0.01 T in core and self inductance of the coil.* **(Ans. : 0.5968 A, 16.844 mH)**

19. *A flux of 0.25 mWb is produced when a current of 2.5 a passes through a coil of 1000 turns. Calculate*

    i) Self inductance

    ii) E.M.F. induced in the coil if the current of 2.5 A is reduced to zero in 1 milliseconds.

    iii) If second coil of 100 turns is placed near to the first on the same iron ring, calculate the mutual inductance between the coils. **(Ans. : 0.1 H, 250 V, 0.01 H)**

20. *Two coils A and B having 180 and 275 number of turns respectively are closely wound on an iron magnetic circuit, which has a mean length of 1.5 m and cross-sectional area of 150 cm. The relative permeability of iron is 1500. Determine mutual inductance between the coils. When will be the e.m.f. induced in a coil B if the current in coil A changes uniformly from 0 to 2.5 a in 0.03 seconds?* **(Ans. : 0.933 H, – 77.75 Volts)**

❑❑❑

5 Electrostatics

5.1 Introduction

The branch of electrical engineering which deals with electricity at rest is called **electrostatics.** All the electric phenomena are produced due to the various types of charges. The moving charges produce current and magnetic effects. The accelerated charges produce radiation. Apart from moving and accelerated charges, there exists one more type of charges called **stationary charges** or **static charges.** Static charges are responsible for the generation of the forces on other charges which are called **electrostatic forces.** Electrostatics means the study of the static charges and the associated effects.

Such static charges may be situated at a point when they are called **point charges.** When the static charges are distributed along the telephone lines or power lines, they are called **line charges.** When distributed over the surfaces such as surfaces of plates of capacitor, they are called **surface charges.** Static charges may exist in the entire volume in the form of a charge cloud then they are called **volume charges.** In this chapter, we will discuss the behaviour of electricity due to the static charges, the laws governing such behaviour and concept of a capacitor.

5.2 Concept of an Electric Charge

The matter on the earth which occupies the space may be solid, liquid or gaseous. The matter is made up of one or more elements. Each element is made up of many atoms which are of similar nature. Now a days, scientists are successful in breaking atoms and studying the resulting products.

According to modern electron theory, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron**, the **proton** and the **electron**. The proton is positively charged while the electron is negatively charged. The neutron is electrically neutral i.e. possessing no charge. The mass of neutron and proton is same which is 1.675×10^{-27} kg while the mass of electron is 9.107×10^{-31} kg. The magnitude of positive charge on proton is same as the magnitude of negative charge on electron. Under normal conditions, number of protons is equal to number of electrons hence, the atom as a whole is electrically neutral. All the protons and neutrons are bound together into a compact nucleus. Nucleus may be thought as a central sun, about which, the electrons revolve in a particular fashion. The electrons are arranged in different **orbits** i.e. levels. The orbits are also called **shells.**

The orbits are more or less elliptical. The electrons revolving in various orbits are held by force of attraction exerted by nucleus. The orbit which is closest to the nucleus is under tremendous force of attraction while orbit which is farthest is under very weak force of attraction. Hence, electrons revolving in farthest orbit are loosely held to the nucleus. Such a shell is called **valence shell** and the electrons in this shell are **valence electrons.** In some atoms, at room temperature only, these valence electrons gain an additional energy and they escape from the shell. Such electrons exist in an atom as **free electrons.** Now, if such electrons are removed from an atom, it will loose negative charge and will become positively charged. Such positively charged atom is called **anion.** As against this, if excess electrons are added to an atom, it will become negatively charged. Such negatively charged atom is called **cation.**

Key Point : *This total deficiency or addition of excess electrons in an atom is called as its charge and the atom is said to be* ***charged.*** *The unit of charge is* ***coulomb.***

The deficiency or excess of electrons can be achieved by different methods. One of such methods is to rub two dissimilar materials against each other. When an ebonite rod is rubbed on a fur cloth, then the rod extracts electrons from fur cloth and behaves as negatively charged while fur cloth behaves as positively charged. This charged condition of rod cannot be sensed by eyes or by any sense organs. But, we can observe the effect of it by simple experiment. Such charged ebonite rod, when brought near light pieces of paper, attracts these pieces. This attraction is nothing but the effect of static charge present on the rod. This is the basic principle of the **static electricity.**

Such phenomena due to static charges are governed by some laws called **laws of electrostatics.** Let us study these laws.

5.3 Laws of Electrostatics

The two fundamental laws of electrostatics are as below :-

1) Like charges repel each other and unlike charges attract each other.

The law can be demonstrated by another simple experiment. The ebonite rod becomes negatively charged when rubbed against fur cloth. Now, if glass rod is rubbed against fur cloth, it gets positively charged. And if they are brought near each other, they try to attract each other. While two ebonite rods after rubbing against fur cloth, brought nearby, try to repel each other. This shows that like charges repel while unlike charges attract each other.

2) Coulomb's Inverse Square Law.

The law states that the mechanical force, attraction or repulsion, between the two small charged bodies is

i) directly proportional to the product of the charges present on the bodies.

ii) inversely proportional to the square of the distance between the bodies and

iii) depends upon the nature of the medium surrounding the bodies.

(a) Unlike charges attract (b) Like charges repel

Fig. 5.1 Force between charges

The Fig. 5.1 shows two point charges, separated by distance 'd' metres. The charges are Q_1 and Q_2 coulombs and K is the constant of proportionality.

According to Coulomb's law, force between the charges can be mathematically expressed as,

$$F \propto \frac{Q_1 Q_2}{d^2}$$

So, $$\boxed{F = \frac{K Q_1 Q_2}{d^2} \quad \text{Newtons}}$$

The constant of proportionality, K depends on the surrounding medium and is given by,

$$\boxed{K = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_r \varepsilon_0}}$$

where ε = Absolute permittivity of the medium = $\varepsilon_0 \varepsilon_r$

ε_0 = Permittivity of free space and ε_r = Relative permittivity of the medium

And $$\boxed{\varepsilon_0 = \frac{1}{36\pi\times 10^9} = 8.854 \times 10^{-12} \quad \text{F/m}}$$

For air, $\varepsilon_r = 1$

The concept of permittivity is discussed later in this chapter.

If $Q_1 = Q_2 = 1$ C and $d = 1$ m,

then, $$F = \frac{1}{4\pi\times 8.854\times 10^{-12}} = 9 \times 10^9 \text{ N}$$

Key Point : *Thus, one coulomb of charge may be defined as that charge, which, when placed in the air or vacuum at a distance of one metre away from an equal and similar charge, is repelled by a force of 9 × 10⁹ N.*

Example 5.1 : *The two equal charges $Q_1 = 5\ \mu C$ and $Q_2 = 1\ \mu C$ are seperated by 50 cm, are kept in a vacuum. Find the force of repulsion.*

To have same force of repulsion, what should be the distance between them, if they are kept in a material having $\varepsilon_r = 5$?

Solution : Case 1 : $Q_1 = 5\ \mu C$, $Q_2 = 1\ \mu C$, $d = 50 \times 10^{-2}$ m

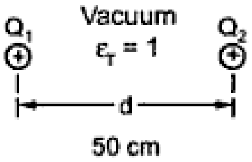


Fig. 5.2 (a)

$$\varepsilon_r = 1,\ \varepsilon = \varepsilon_0 = 8.854 \times 10^{-12}$$

$$\therefore\ F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2} = \frac{5\times10^{-6}\times1\times10^{-6}}{4\pi\times8.854\times10^{-12}\times(50\times10^{-2})^2}$$

$$= 0.1797\ \text{N}$$

Case 2 : The force must be same, 0.1797 N, but $\varepsilon_r = 5$

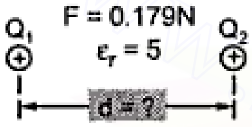


Fig. 5.2 (b)

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}$$

$$\therefore\ 0.1797 = \frac{5\times10^{-6}\times1\times10^{-6}}{4\pi\times8.854\times10^{-12}\times5\times(d)^2}$$

$$\therefore\ d^2 = 0.05$$

$\therefore$ d = 0.2236 m = 22.36 cm ... New distance

5.4 Electrostatic Field

As we have seen in the previous section that unlike charges attract and like charges repel each other. Positively charged particle exerts a force of attraction on negatively charged while exerts a force of repulsion on positively charged particle. It must be kept in mind that the second charged particle also produces the electrostatic force on the first particle. So, it can be concluded that the space around the charge is always under the stress and exerts a force on another charge which is placed around it. The region or space around a charge or charged body in which the influence of electrostatic force or stress exists is called **electric field or dielectric field or electrostatic field.**

5.4.1 Electric Lines of Force

The electric field around a charge is imagined in terms of presence of lines of force around it. The imaginary lines, distributed around a charge, representing the stress of the charge around it are called as **electric or electrostatic lines of force.** The pattern of lines of force around isolated positive charge is shown in Fig. 5.3 (a) while the pattern of lines of force around isolated negative charge is shown in Fig. 5.3 (b). Such lines of force originate from the positive charge and terminate on the negative charge, when these charges are placed near each other. They exert the force of attraction on each other. This is shown in

Fig. 5.3 (c). While when two like charges are near each other, such lines will be in opposite direction as shown in Fig. 5.3 (d). There exists a force of repulsion between them.

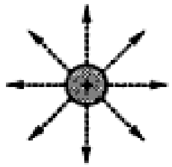


Fig. 5.3 (a) Isolated positive charge

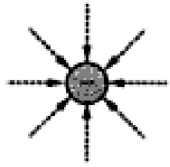


Fig. 5.3 (b) Isolated negative charge

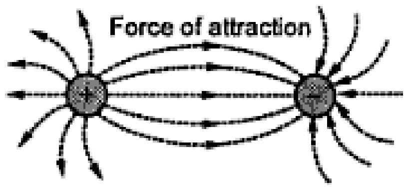


Fig. 5.3 (c) Two equal unlike charges

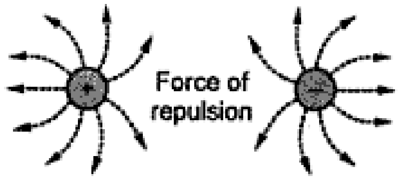


Fig. 5.3 (d) Two equal like charges

Key Point : *In general, the directions of the lines of force at any point is the direction of movement of a unit positive charge placed at that point, if free to do so.*

5.4.2 Properties of Electric Lines of Force

The properties of electric lines of force are,

1) The lines of force always originate from a positive charge and terminate at negative charge.
2) They always enter or leave a conducting surface, normally.
3) They are always parallel and never cross each other.
4) The lines travelling in the same direction repel each other, while traveling in the opposite directions attract one another.
5) They behave like a stretched rubber band and always try to contract.
6) They pass only through the insulating medium between the charges and do not enter the charged bodies.

Key Point : *Hence, they cannot form a closed loop as in case of the magnetic lines of force.*

5.5 Electric Flux

Theoretically, the lines of force emanating from a charge are infinite. Faraday suggested that the electric field should be assumed to be composed of very small bunches containing a fixed number of electric lines of force. Such a bunch or a closed area is called **a tube of flux.**

Key Point : *The total number of lines of force or tubes of flux in any particular electric field is called as the electric flux.*

This is represented by the symbol ψ. Similar to charge, unit of electric flux is also coulomb C.

One coulomb of electric flux is defined as that flux which emanates from a positive charge of one Coulomb.

In general, if the charge on a body is $\pm$ Q Coulombs, then the number of tubes of flux or total electric flux, starting or terminating on it is also Q.

So, for a charge of $\pm$ Q coulombs,

$$\boxed{\text{Electric Flux, } \psi = \text{Q coulombs (numerically)}}$$

5.6 Electric Flux Density

This is defined as the flux passing at right angles through unit area of surface. It is represented by symbol D and measured in Coulomb per square metre.

If a flux of ψ Coulombs passes normally (at right angles) through an area of A m^2, then

$$D = \frac{\psi}{A} = \frac{Q}{A} \quad C/m^2 \qquad \text{... as } \psi = Q$$

Let a point charge of Q coulombs placed at the centre of an imaginary sphere of radius 'r' metres.

$$\text{Total flux, } \psi = Q$$

This flux falls normally on a surface area of $4\pi r^2$ (metre)^2 of sphere. So, electric flux density,

$$\boxed{D = \frac{\psi}{A} = \frac{Q}{4\pi r^2} \; C/m^2}$$

The flux density is also called **displacement density.**

5.6.1 Surface Charge Density

If the charge is distributed over the surface, then the surface charge density is defined as the charge per unit area of the surface over which the charge is distributed. It is denoted as δ.

$$\therefore \qquad \boxed{\delta = \frac{Q}{A} \quad C/m^2}$$

5.7 Electric Field Strength or Field Intensity

It is defined as the force experienced by a unit positive charge placed at any point in the electric field. It is represented by symbol E and measured in newton per coulomb.

Suppose a charge of Q coulombs, placed at a point within an electric field, experiences a force of F newtons, then the intensity of the electric field at that point is given by,

$$E = \frac{F}{Q} \quad \text{N/C}$$

Higher the value of E, stronger is the electric field.

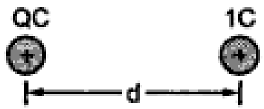


Fig. 5.4

Consider a positive charge of Q coulombs placed in a medium as shown in the Fig. 5.4.

Let a unit positive charge is placed at a distance of d metres from the charge Q.

The field intensity at the point where unit positive charge is placed can be obtained from force experienced by unit positive charge.

Now, $E = \frac{F}{Q}$ but here, $Q = 1C$ unit charge

$\therefore$ $E = F$

But, $F = \frac{Q \times 1}{4\pi\varepsilon d^2}$ by Coulomb's law

$\therefore$ $E = \frac{Q}{4\pi\varepsilon d^2}$ N/C

But, $\varepsilon = \varepsilon_0 \, \varepsilon_r$

$$\therefore \quad \boxed{E = \frac{Q}{4\pi\varepsilon_0 \, \varepsilon_r \, d^2}}$$

The similar concept can be used to obtain the relation between electric field intensity and electric flux density.

5.7.1 Relation between D and E

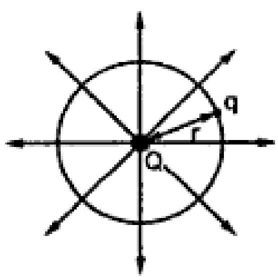


Fig. 5.5 Relation between D and E

Let there be a point charge of 'Q' coulombs placed at the centre of the sphere of radius 'r' metres. The small positive charge 'q' coulombs is placed at a distance 'r' from 'Q' on the surface of the sphere as shown in Fig. 5.5.

The force experienced on the charge 'q' due to 'Q' is given by

$$F = \frac{Q \cdot q}{4\pi\varepsilon_0 \, \varepsilon_r \, r^2} \quad \text{... by Coulomb's law}$$

Electric field strength is given by force per charge.

$$\therefore \quad \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 \, \varepsilon_r \, r^2}$$

$$\therefore \qquad E = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2} \quad N/C$$

The flux density is, $D = \frac{\psi}{a}$

Now, $\psi = Q$

while a = surface area of the sphere = $4 \pi r^2 \ m^2$

$$\therefore \qquad D = \frac{Q}{4\pi r^2} \ C/m^2$$

Substituting in the equation for 'E' we get,

$$\therefore \qquad E = \frac{D}{\varepsilon_0 \varepsilon_r}$$

and

$$\boxed{D = E \varepsilon_0 \varepsilon_r \quad C/m^2}$$

5.8 Permittivity

From the relation derived above, we can say that electric flux density depends on electric field strength. Now the value of electric flux density depends on the value of electric field strength E along with the dielectric property of the medium which is known as **permittivity.**

Key point : *Permittivity can be defined as the ease with which a dielectric medium permits an electric flux to be established in it.*

5.8.1 Absolute Permittivity

The ratio of the electric flux density D to electric field strength E at any point is defined as the **absolute permittivity.**

It is denoted by ε and measured in units farads/metre, (F/m).

$$\boxed{\varepsilon = \frac{D}{E} \qquad F/m}$$

5.8.2 Permittivity and Free Space

It is also called as **electric space constant.**

Key Point : *The ratio of the electric flux density in a vacuum (or free space) to the corresponding electric field is defined as permittivity of the free space.*

It is denoted by ε_0 and measured in unit farads/m (F/m).

$$\boxed{\varepsilon_0 = \frac{D}{E} \qquad F/m \qquad \text{in vacuum}}$$

The value of ε_0 is less than the value of permittivity of any medium. Experimentally, its value has been derived as,

$$\boxed{\varepsilon_0 = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12} \quad \text{F/m}}$$

5.8.3 Relative Permittivity

To define the permittivity of the dielectric medium, the vacuum or free space is considered to be a reference medium. So, **relative permittivity** of vacuum with respect to itself is unity.

The ratio of electric flux density in a dielectric medium to that produced in a vacuum by the same electric field strength under identical conditions is called **relative permittivity.**

It is denoted by ε_r and has no units.

$$\varepsilon_r = \frac{D}{D_0}$$

Now $\quad D = \varepsilon E \quad$ and $\quad D_0 = \varepsilon_0 E$

$$\therefore \quad \varepsilon_r = \frac{\varepsilon E}{\varepsilon_0 E}$$

$$\therefore \quad \boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

$$\therefore \quad \varepsilon = \varepsilon_r \varepsilon_0$$

It can also be defined as the ratio of the absolute permittivity of the dielectric medium to the permittivity of the free space. The relative permittivity of the air is also taken as unity, though its actual value is 1.0006. Most of the other materials have value of relative permittivity between 1 to 10.

No.	Material	Relative Permittivity, ε_r
1	Free Space	1
2	Air	1.0006 ≈ 1
3	Rubber	2 to 3.5
4	Paper	2 to 2.5
5	Mica	3 to 7
6	Porcelain	6 to 7
7	Bakelite	4.5 to 5.5
8	Glass	5 to 10

Table 5.1 Material and ε_r

The relative permittivity of air is assumed to be one for all practical purposes.

Key Point : *Higher the value of ε_r, easier is the flow of electric flux through the materials.*

In practice, paper and mica are extensively used for manufacturing of capacitors.

Key Point : *The relative permittivity is nothing but the dielectric constant of the material.*

5.9 Electric Potential and Potential Difference

When a mass is raised above the ground level, work is done against the force of gravity. This work done is stored in the mass as a potential energy (mgh). Hence, due to such potential energy, it is said that the mass, when raised above the ground level has a gravitational potential. Such potential of mass depends upon the position of the mass with respect to the ground.

An electric charge gives rise to an electric field around it, analogous to gravitational field around the earth. If any charge is introduced in this field, it gets attracted or repelled, depending on the nature of the charge. At the time of movement of this charge, work is done against or by the force acting on the charge due to the electric field. This depends on the position of the charge in the electric field and is analogous to the potential of mass due to gravitation field, when lifted upwards.

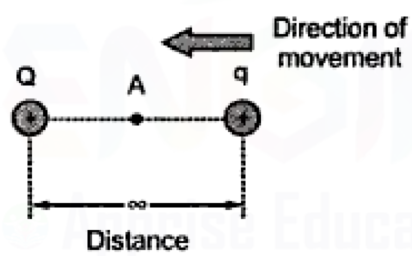


Fig. 5.6 Electric potential

Now, consider a small isolated positive charge 'q' placed at infinity with respect to another isolated positive charge 'Q' as shown in the Fig. 5.6. Theoretically, the electric field of charge 'Q' extends upto infinity but has a zero influence at infinity, where 'q' is placed. When charge 'q' is moved towards 'Q', work is done against the force of repulsion between these two like charges.

Due to this work done, when charge 'q' reaches position A, it acquires a potential energy. If charge 'q' is released, due to force of repulsion, it will go back to infinity i.e. position of zero potential. So, at point A, charge 'q' has some potential exactly equal to work done in bringing it from infinity to the point A, called **electric potential.**

It can be defined as the work done in joules, in moving a unit positive charge from infinity (position of zero potential) to the point against the electric field.

It is denoted by symbol V and is measured in joule per coulomb or volt.

Thus, $$\boxed{\text{Electric Potential V} = \frac{\text{Workdone (W)}}{\text{Charge (Q)}}} \quad \text{... volts}$$

Definition of 1 volt :

The electric potential at a point in an electric field is said to be one volt when the work done in bringing a unit positive charge from infinity to that point from infinity against the electric field is one joule.

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

5.9.1 Potential Difference

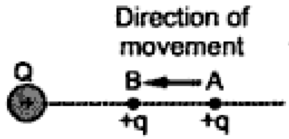


Fig. 5.7 Potential difference

Consider two points A and B in an electric field as shown in Fig. 5.7. The positive charge '+q' is moved from point A to B in an electric field. At point A, charge acquires certain electric potential say V_A. Some additional work is done in bringing it to point B. At point B, it has an electric potential say V_B.

Key Point : *The difference between these two potentials per unit positive charge is called potential difference.*

So, the potential difference between the two points in an electric field is defined as the work done in moving a unit positive charge from the point of lower potential to the higher potential.

i.e. $$\boxed{V_{AB} = V_A - V_B = \frac{W_A - W_B}{q} \text{ volts}}$$

5.9.2 Expressions for Potential and Potential Difference

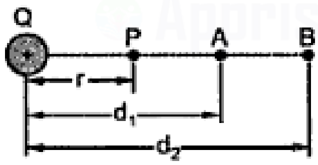


Fig. 5.8

Consider a positive charge Q placed in a medium of relative permittivity ε_r. Consider a point P at a distance r from the charge Q. Now, a unit positive charge of 1 C is placed at point P, there will exist a force of repulsion between the two charges. This is shown in the Fig. 5.8.

The force of repulsion is given by,

$$F = \frac{Q \times 1}{4 \pi \varepsilon r^2} = \frac{Q}{4 \pi \varepsilon_0 \varepsilon_r r^2} \quad \text{N}$$

Now, electric field intensity at point P is the ratio of force to charge at point P. But charge at P is unit charge,

$$\therefore \quad E = \frac{F}{1 \text{C}} = \frac{Q}{4 \pi \varepsilon_0 \varepsilon_r r^2} \quad \text{N/m}$$

Now, move the unit charge at point P towards charge Q against the force of repulsion.

Work done :

Let the distance moved by charge at P towards Q be dr and for this work is done against force of repulsion, given by,

$$dW = -E\,dr = -\frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r^2}\,.\,dr$$

The negative sign indicates that the work is done against the force of repulsion.

Now, to find electrical potential at point P, consider that the unit positive charge is moved from infinity to the point P. Hence, total work done in moving unit positive charge from infinity to point P can be obtained by integrating dW as,

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} -\frac{Q}{4\,\pi\varepsilon_0\,\varepsilon_r\,r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\int_{\infty}^{r}\frac{1}{r^2}\,dr$$

Now, $\int \frac{1}{r^2}\,dr = -\frac{1}{r}$

$$\therefore \quad W = -\frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$W = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r}$$

Key Point: *But this total work done is nothing but potential at point P.*

$$\therefore \quad \boxed{V_P = W = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r} \quad \text{volts}}$$

Thus, as r increases, potential decreases till it becomes zero at infinity.

Potential difference :

Consider point A at a distance d_1 from charge Q. Hence, potential of point A is given by,

$$V_A = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,d_1}$$

While the potential of point B which is at a distance d_2 from the charge Q is,

$$V_B = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,d_2}$$

Hence, the potential difference between the points A and B is given by,

$$V_{AB} = V_A - V_B$$

$$\therefore \quad \boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r}\left[\frac{1}{d_1}-\frac{1}{d_2}\right]}$$

We know that field intensity at a distance 'd' due to charge Q is given by,

$$E = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d^2}$$

While the potential of the same point is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d}$$

Substituting V in expression for E, we can write,

$$\boxed{E = \frac{V}{d}}$$

5.10 Potential Gradient

In practice, the electric field intensity is not uniform but varies from point to point in an electric field. Let the electric field strength at any point A in an electric field be E (N/C). Now, the unit positive charge at point A is displaced by distance dx metres in the direction of the field so that the electric field strength remains constant. Work done in moving this charge can be determined by force × displacement.

$$\text{Work done} = +E \times dx \text{ joules}$$

Let dV be the potential drop over this distance in the direction of the electric field. It is moved from point of **higher potential** to **lower potential.** So, by the definition of a potential difference,

$$dV = +E\,dx$$

i.e

$$\boxed{E = +\frac{dV}{dx}}$$

The term $\frac{dV}{dx}$ in the above expression is called the **potential gradient.**

> **Key Point**: *Potential gradient is defined as the drop in potential per metre in the direction of the electric field.*

It is measured in units volts/metre (V/m).

If the change in potential is from lower potential to higher potential, i.e. against the direction of the electric field then potential gradient is said to be negative i.e.

$$\boxed{E = -\frac{dV}{dx}}$$

> **Key Point**: *From the above expression, it follows that numerically,*
>
> *Electric Field Strength = Potential Gradient*

5.11 Capacitor

A capacitor is nothing but the two conducting surfaces, separated by an insulating medium called **dielectric.** These conducting surfaces could be in the form of rectangular, circular, spherical or cylindrical in shape.

A capacitor is also called **condenser**. The commonly used dielectrics in capacitors are paper, mica, air etc.

5.12 Capacitance

Capacitance is defined as the amount of charge required to create a unit potential difference between the plates.

> **Key Point :** *The property of a capacitor to store an electric energy in the form of static charges is called its* ***capacitance.***

5.13 Action of a Capacitor

Consider a capacitor formed by two flat metal plates X and Y, facing each other and separated by an air gap or other insulating material used as a dielectric medium. There is no electrical contact or connection between them. Such a capacitor is called **parallel plate capacitor.**

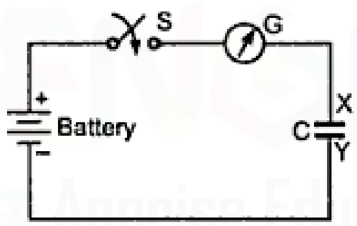


Fig. 5.9 A capacitor

Consider a circuit in which such a capacitor across a battery with the help of a switch 'S' and a galvanometer 'G' in series. The arrangement is shown in the Fig. 5.9.

Let us see what happens when the switch 'S' is closed. As soon as the switch 'S' is closed, the positive terminal of the battery attracts some of the free electrons from the plate 'X' of the capacitor. The electrons are then pumped from positive terminal of the battery to the negative terminal of the battery due to e.m.f. of the battery. Now, negative terminal and electrons are like charges and hence, electrons are repelled by the negative terminal to the plate 'Y' of the capacitor.

The action is shown in Fig. 5.10.

So, plate 'X' becomes positively charged while plate 'Y' becomes negatively charged. The flow of electrons constitutes a current, in the direction opposite to the flow of electrons. This is the conventional current called

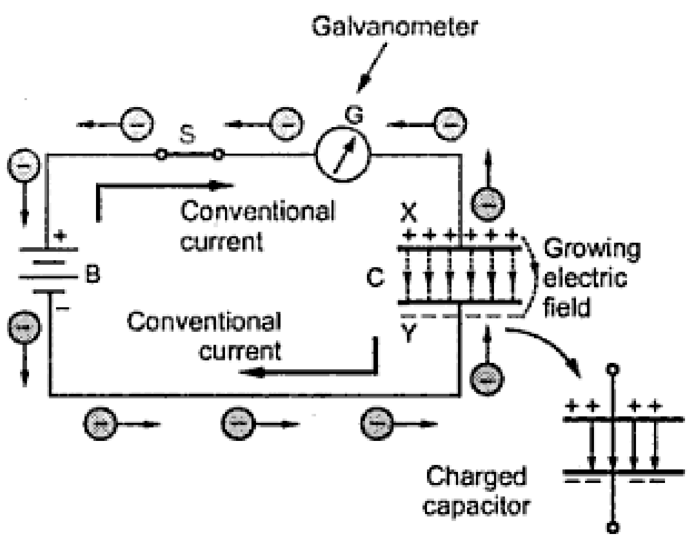


Fig. 5.10 Action of a capacitor

charging current of the capacitor as shown in the Fig. 5.10. This can be experienced from the momentary deflection of the galvanometer 'G'. Because of this, there builds a potential difference across the plates 'X' and 'Y'. There builds an electric field between the two fields.

But this potential difference across the plates, acts as a counter e.m.f. and starts opposing the movement of the electrons. The magnitude of this potential difference is proportional to the charge that accumulates on the plates. When this potential difference becomes equal to the battery e.m.f., the flow of electrons ceases.

If under such condition, the battery is disconnected then the capacitor remains in the charged condition, for a long time. It stores an electrical energy and can be regarded as a reservoir of electricity. Now, if a conducting wire is connected across the two plates of capacitor, with the galvanometer in series, then galvanometer shows a momentary deflection again but in the opposite direction.

This is due to the fact that electrons rush back to plate X from plate Y through the wire. So, there is a rush of current through the wire. This is called **discharging current of a capacitor.** Thus, the energy stored in the capacitor is released and is dissipated in the form of the heat energy in the resistance of the wire connected.

The direction of the conventional current is always opposite to the flow of electrons. If the voltage of the battery is increased, the deflection of the galvanometer also increases at the time of charging and discharging.

Key Point: *So, charge on the capacitor is proportional to the voltage applied to it.*

5.14 Relation between Charge and Applied Voltage

As seen earlier, the charge on capacitor plates depends on the applied voltage. Let 'V' be the voltage applied to the capacitor and 'Q' be the charge accumulated on the capacitor plates, then mathematically, it can be written as,

$$Q \propto V$$

i.e. $$Q = C\,V$$

The constant of proportionality 'C' is called **capacitance** of the capacitor, defined earlier.

$$\therefore \quad C = \frac{Q}{V}$$

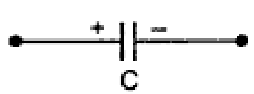


Fig. 5.11 Symbol of capacitance

From the above expression, the **capacitance** is defined as the ratio of charge acquired to attain the potential difference between the plates. It is the charge required per unit potential difference. It is measured in unit **farads**.

One farad capacitance is defined as the capacitance of a capacitor which requires a charge of one coulomb to establish a potential difference of one volt between its plates.

The capacitance is symbolically denoted as shown in the Fig. 5.11.

For practical use, the farad is too large unit and hence, micro farad (μF), nano farad (nF) and pico farad (pF) are commonly used.

$$\boxed{\begin{aligned} 1\ \mu F &= 10^{-6}\ F \\ 1\ nF &= 10^{-9}\ F \\ 1\ pF &= 10^{-12}\ F \end{aligned}}$$

5.15 Capacitance of a Parallel Plate Capacitor

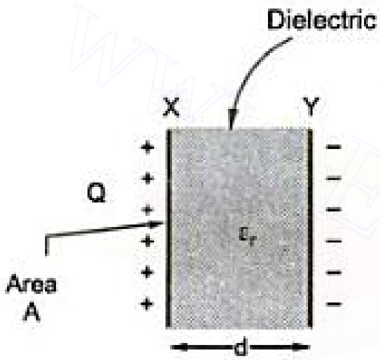


Fig. 5.12 Charged capacitor

Consider a parallel plate capacitor, fully charged, as shown in the Fig. 5.12.

The area of each plate X and Y is say A m^2 and plates are separated by distance 'd'.

The relative permittivity of the dielectric used in between is say ε_r.

Let Q be the charge accumulated on plate X, then the flux passing through the medium is $\psi = Q$.

The flux density, $D = \dfrac{\psi}{A} = \dfrac{Q}{A}$

The electric field intensity,

$$E = \frac{V}{d}$$

We know that $D = \varepsilon E$

$$\therefore \quad \frac{Q}{A} = \varepsilon \frac{V}{d}$$

$$\therefore \quad \frac{Q}{V} = \frac{\varepsilon A}{d}$$

But, $\dfrac{Q}{V} = C$

$$\therefore \quad \boxed{C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad F}$$

Key Point: *When the capacitor is fully charged, the potential difference across it is equal to the voltage applied to it.*

Example 5.2 : *A parallel plate capacitor has an area of 10 cm^2 and distance between the plates is 2mm. The dielectric used between the plates has relative permittivity of 3. Determine the capacitance of the parallel plate capacitor.*

Solution : The capacitance can be calculated as,

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{3 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 1.328 \times 10^{-11} = \mathbf{13.28\ pF}$$

Example 5.3 : *The potential gradient between the plates in the above capacitor is 12 kV/cm, determine the voltage across the plates, charge, electric flux density and electricity flux between the plates.*

Solution : Electric intensity = Potential gradient

$$E = 12 \text{ k V/cm} = \frac{12 \times 10^3}{1 \times 10^{-2}} \text{ V/m} = 1200 \times 10^3 \text{ V/m}$$

And $C = 13.28$ pF ... Calculated in Ex. 5.1.

Now, $E = \dfrac{V}{d}$

$\therefore \quad 1200 \times 10^3 = \dfrac{V}{2 \times 10^{-3}}$ i.e. $V = 2400$ V

This is the voltage across capacitor plates.

$$C = \frac{Q}{V}$$

$\therefore \quad Q = C V = 13.28 \times 10^{-12} \times 2400 = 31.87 \times 10^{-9}$ C

$\therefore$ Charge $=$ **31.87 nC**

Electric flux, $\psi = Q = 31.87$ nC

Electric flux density, $D = \dfrac{Q}{A} = \dfrac{31.87 \times 10^{-9}}{10 \times 10^{-4}} = 3.187 \times 10^{-5}$ C/m^2

$= \mathbf{31.87\ \mu C / m^2}$

5.16 Dielectric Strength

We know that, $E = \dfrac{V}{d}$

So, as the voltage on the capacitor is increased with a given thickness (d) or the thickness (d) is reduced with a given voltage (V), the electric intensity E increases.

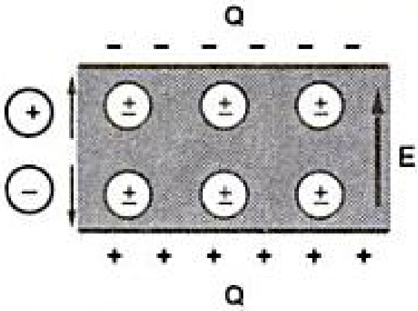


Fig. 5.13 Dielectric strength

This intensity represents the force exerted on the charges on the molecules or the dielectric material.

As E is increased, the centre of the positive charges is pushed in the direction of E and centre of the negative charges in the opposite direction.

Now, every dielectric medium has its capacity to withstand the increasing E. If the applied voltage and hence E is increased beyond a certain limit, then forces on the molecules become sufficiently large. The electrons break away from the molecules causing ionization and free charges.

The material then conducts due to ionization and the charge recombine, thereby vanish from the capacitor plates. The capacitor can no more hold the charge and is said to be breakdown. The dielectric medium is said to be punctured and becomes useless from using it as a dielectric.

The ability of an insulating medium to resist its breakdown when a voltage is increased across it, is called its **dielectric strength.**

This depends upon the temperature of the material and presence of air pockets and imperfections in the molecular arrangement of that material. It is generally expressed in kV/cm of kV/mm.

Key Point : *The voltage at which the dielectric medium of the capacitor breaksdown is known as breakdown voltage of the capacitor.*

The factors affecting the dielectric strength are,

1. Temperature
2. Type of material
3. Size, thickness and shape of the plates.
4. Presence of air pockets in the material.
5. Moisture content of the material.
6. Molecular arrangement of the material.

Dielectric strength and dielectric constants of some materials are quoted below from published literature.

Sr.No.	Material	Dielectric constant	Dielectric strength in kV/mm
1	Air	1	3
2	Bakelite	5	15 to 25
3	Mica	6	46 to 200
4	Dry paper	2.2	5 to 10
5	Glass	6	6 to 26

Table 5.2 Dielectric strengths

The dielectric strength varies as thickness of dielectric material hence the range of values are given in the Table 5.2. The value indicates that if material is subjected to electric field more than specified dielectric strength then it will breakdown.

5.16.1 Dielectric Leakage and Losses

If there is no leakage of current in the dielectric and the insulation is perfect, then the charge on the capacitor plates can be held on for hours.

The fact however remains that the insulation resistance of most of the dielectric materials is only of the order of megaohms and hence charge on the capacitor leaks away through the insulating material in a few minutes.

Key Point : *In any case, it is dangerous to touch a charged capacitor even after it is disconnected from the supply.*

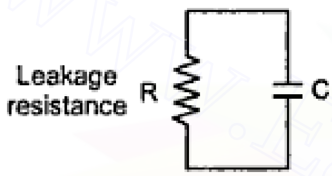


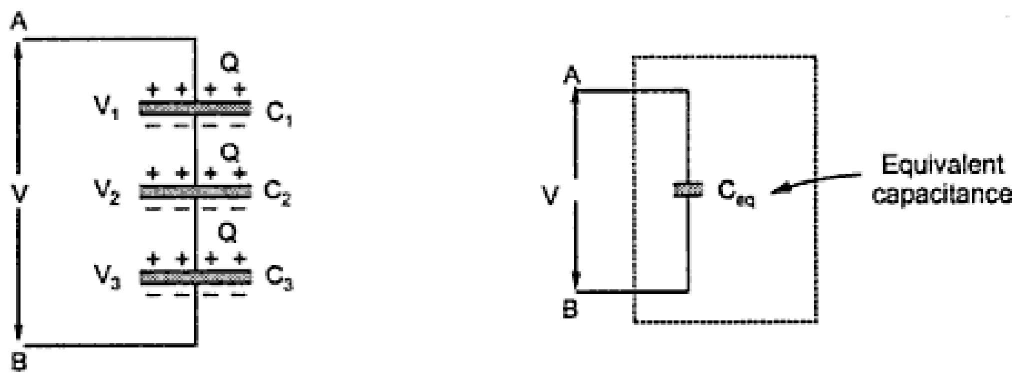
Fig. 5.14 Practical capacitor

In case of d.c. a practical capacitor is considered to be a charge storing device in parallel with a leakage resistance (R) as shown in the Fig. 5.14.

Further, when the voltage applied to the capacitor is alternating, due to molecular friction of dipoles created in the material, the value of R becomes frequency dependent. The loss due to such molecular friction is called **dielectric loss.**

5.17 Capacitors in Series

Consider the three capacitors in series connected across the applied voltage V as shown in the Fig. 5.15. Suppose this pushes charge Q on C_1 then the opposite plate of C_1 must have the same charge. This charge which is negative must have been obtained from the connecting leads by the charge separation which means that the charge on the upper plate of C_2 is also Q. In short, **all the three capacitors have the same charge Q.**



(a) **Fig. 5.15 Capacitors in series** (b)

$$\boxed{Q = C_1 V_1 = C_2 V_2 = C_3 V_3}$$

Giving, $V_1 = \dfrac{Q}{C_1}; \quad V_2 = \dfrac{Q}{C_2}; \quad V_3 = \dfrac{Q}{C_3}$

If an equivalent capacitor also stores the same charge, when applied with the same voltage, then it is obvious that,

$$C_{eq} = \frac{Q}{V} \quad \text{or} \quad V = \frac{Q}{C_{eq}}$$

But, $V = V_1 + V_2 + V_3$

$$\therefore \quad \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

It is easy to find V_1, V_2 and V_3 if Q is known.

$$\boxed{\text{For 'n' capacitors in series, } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

Key Point: *For all the capacitors in series, the charge on all of them is always same, but the voltage across them is different.*

5.17.1 Voltage Distribution in Two Capacitors in Series

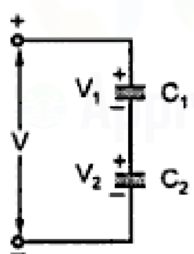


Fig. 5.16 Capacitors in series

Consider two capacitors C_1 and C_2 connected in series. The voltage across them is say, V volts. This is shown in Fig. 5.16.

As capacitors are in series, the charge on them is same, say Q.

$$\therefore \quad Q = C_1 V_1 = C_2 V_2$$

where V_1 is voltage across C_1 and V_2 is voltage across C_2

Now, $V = V_1 + V_2$

From the expression of Q, we can write,

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

Adding 1 to both sides, $\dfrac{V_1}{V_2} + 1 = \dfrac{C_2}{C_1} + 1$

$$\therefore \quad \frac{V_1 + V_2}{V_2} = \frac{C_2 + C_1}{C_1}$$

Now, if the n plates are used, we can write that

$$\boxed{C_{eq} = \frac{(n-1)\ \varepsilon_0\ \varepsilon_r\ A}{d}}$$

Example 5.6 : *A multiple plate capacitor has 4 plates in all and the distance between the plates is 1 mm. If the area of the plates is effectively 5 cm^2 and the dielectric constant of material between the plates is 10, determine the capacitance.*

Solution :

$$C = \frac{(n-1)\ A\varepsilon_0\ \varepsilon_r}{d} = \frac{(4-1)\times(5\times10^{-4})}{1\times10^{-3}} \times \frac{1}{36\,\pi\times10^{9}} \times 10$$

$$= \frac{3\times5\times10\times10^{-4}}{36\,\pi\times10^{6}} = 1.32 \times 10^{-10}\ \text{F} \quad = \mathbf{132\ pF}$$

5.20 Composite Dielectric Capacitors

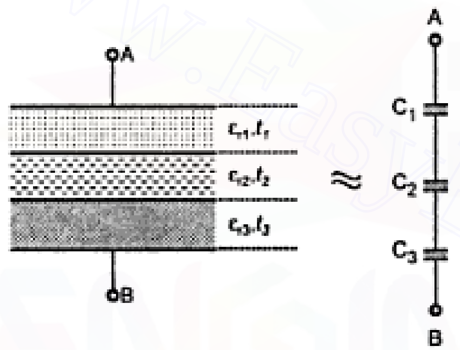


Fig. 5.19 Composite capacitor

When a parallel plate capacitor has two dielectrics or more between the plates, it is said to be **composite capacitor.** The various types of such composite capacitor exists in practice. Let us study few types of such composite capacitors.

Type 1 : In this type, number of dielectrics having different thicknesses and relative permittivities are filled in between the two parallel plates. The composite capacitor with three different dielectrics with permittivities ε_{r1}, ε_{r2} and ε_{r3} and thicknesses t_1, t_2 and t_3 is shown in Fig.5.19.

Let V be the voltage applied across the capacitor.

It can be seen that there exists three capacitors in series. The values of three capacitors are different. Hence, the equivalent capacitance across the terminals A-B is,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

And $$C_1 = \frac{\varepsilon_0\ \varepsilon_{r1}\ A}{t_1},\quad C_2 = \frac{\varepsilon_0\ \varepsilon_{r1}\ A}{t_2},\quad C_3 = \frac{\varepsilon_0\ \varepsilon_{r1}\ A}{t_3}$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{\varepsilon_0\ A}\left[\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right]$$

$$\therefore \quad C_{eq} = \frac{\varepsilon_0\ A}{\left[\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right]}$$

In general, for a composite capacitor with 'n' dielectrics,

$$\boxed{C_{eq} = \frac{\varepsilon_0 A}{\sum_{k=1}^{n} \frac{t_k}{\varepsilon_{rk}}} = \frac{\varepsilon_0 A}{\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \ldots + \frac{t_n}{\varepsilon_{rn}}}}$$

The voltage across each dielectric will be different,

$$\therefore \quad V = V_1 + V_2 + V_3$$

But, $$E = \frac{V}{t} \quad \text{i.e.} \quad V = E\,t$$

$$\therefore \quad V = E_1 t_1 + E_2 t_2 + E_3 t_3$$

where E_1, E_2 and E_3 are the values of electric intensities in the different dielectrics.

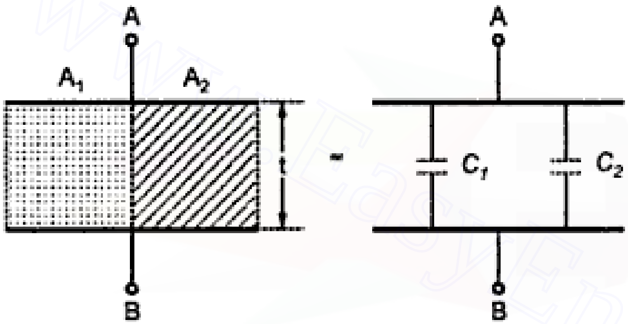


Fig. 5.20 Composite capacitor

Type 2 : In this type, in the same thickness, 't', the two dielectrics are arranged as shown in the Fig. 5.20.

Let the relative permittivity values for the two dielectrics be ε_{r1} and ε_{r2}. The thickness for both is same but the areas are different. It can be seen from the equivalent circuit that there exists two capacitors in parallel due to two different dielectrics.

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{t} + \frac{\varepsilon_0 \varepsilon_{r2} A_2}{t} = \frac{\varepsilon_0}{t}(A_1 \varepsilon_{r1} + A_2 \varepsilon_{r2})$$

If one of the two dielectrics is air, then the corresponding relative permittivity is one, to be used in the above expression.

For 'n' dielectrics arranged in same thickness 't',

$$\boxed{C_{eq} = \frac{\varepsilon_0}{t}[A_1\varepsilon_{r1} + A_2\varepsilon_{r2} + \ldots\ldots + A_n\varepsilon_{rn}]}$$

Type 3 : In practice we can have the capacitor which is combination of above two types. One such capacitor is shown in the Fig. 5.21.

Basically it is Type 2 capacitor, consisting of Type 1 capacitor. So there are two capacitors in parallel. The C_1 is having thickness t_1, relative permitivity ε_{r1} and area A_1.

$$\therefore \quad C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{t_1}$$

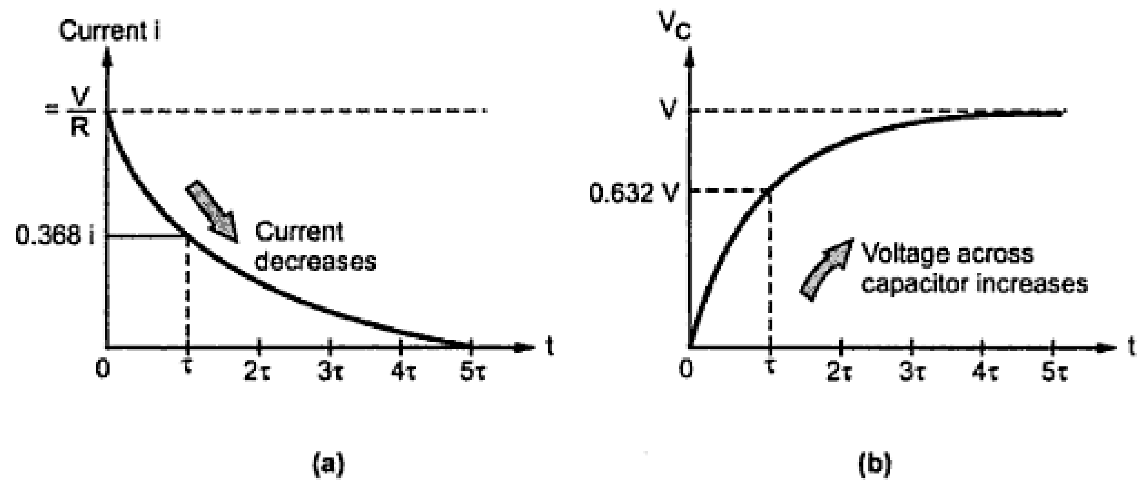


Fig. 5.24 Variation of charging current and voltage V_C

5.24.1 Mathematical Analysis

Let V_C = Voltage across capacitor at any instant.

q = Charge on capacitor in coulombs at any instant.

i = Charging current at any instant in amperes.

By Kirchhoff's voltage law,

$$V = V_R + V_C$$

$$= i\,R + V_C$$

but
$$i = C\,\frac{dV_C}{dt}$$

$$\therefore \quad V = CR\,\frac{dV_C}{dt} + V_C$$

$$\therefore \quad V - V_C = RC\,\frac{dV_C}{dt}$$

$$\therefore \quad \frac{dt}{RC} = \frac{dV_C}{V - V_C}$$

Integrating both sides of the above equation,

$$\frac{t}{CR} = -\ln(V - V_C) + K$$

where K = Constant of integration.

At t = 0, $V_C = 0$, substituting in above,

$$0 = -\ln(V) + K$$

$$\therefore \quad K = \ln(V)$$

$$\therefore \qquad \frac{t}{CR} = -ln\,(V - V_C) + ln\,(V)$$

$$\therefore \qquad \frac{t}{CR} = ln\frac{V}{V - V_C}$$

$$\therefore \qquad \frac{V}{V - V_C} = e^{t/CR}$$

$$\therefore \qquad V - V_C = V\,e^{-t/CR}$$

$$\therefore \qquad \boxed{V_C = V\,(1 - e^{-t/CR})}$$

When the steady state is achieved, the total charge on the capacitor is Q coulombs.

$$\therefore \qquad V = \frac{Q}{C}$$

Similarly at any instant $\qquad V_C = \frac{q}{C}$

$$\therefore \qquad \frac{q}{C} = \frac{Q}{C}(1 - e^{-t/CR})$$

$$\therefore \qquad q = Q\,(1 - e^{-t/CR})$$

Now $\qquad V - V_C = i\,R$

$$\therefore \qquad i = \frac{V - V_C}{R}$$

$$\therefore \qquad i = \frac{V - e^{-t/CR}}{R}$$

$$\therefore \qquad \boxed{i = \frac{V}{R}e^{-t/CR}}$$

So at t = 0, $i = \frac{V}{R}$ is maximum and in steady state it becomes zero.

Thus capacitor acts as short circuit at start and acts as open circuit in steady state.

5.24.2 Time Constant

The term CR in all the above equation is called the **Time Constant of the R-C** charging circuit and denoted by τ, measured in seconds.

When $\qquad t = CR = \tau \quad$ then ,

$$V_C = V\,(1 - e^{-1})$$

$$V_C = 0.632\ V$$

So time constant of the R-C series circuit is defined as **time required by the capacitor voltage to rise from zero to 0.632 of its final steady state value during charging.**

Incidentally after t = 2τ, 3τ, 4τ the capacitor voltage attains the values as 0.863 V, 0.95 V, 0.982 V respectively and practically capacitor requires the time 4 to 5 times the time constant to charge fully.

When $\quad t = CR$ then

$$\boxed{i = \frac{V}{R} e^{-1} = 0.368\left(\frac{V}{R}\right)}$$

Now $\left(\frac{V}{R}\right)$ is starting charging current. From this time constant can be defined as below.

> **Key Point :** *Time constant is the time required for the charging current of the capacitor to fall to 0.368 of its initial maximum value, starting from its maximum value.*

5.24.3 Initial Rate of Rise of Capacitor Voltage

The initial rate of rise of capacitor voltage is fast however when the capacitor charges this rate is reduced.

Let us find initial $\frac{dV_C}{dt}$, by differentiating the equation of V_C.

Since $\quad \frac{dV_C}{dt} = V\left(+\frac{1}{CR}\right)e^{-t/CR}$

$$\boxed{\text{At } t = 0, \quad \frac{dV_C}{dt} = \frac{V}{CR} = \frac{V}{\tau}}$$

If the same rate is maintained through out after

$$t = \tau, V_C = V$$

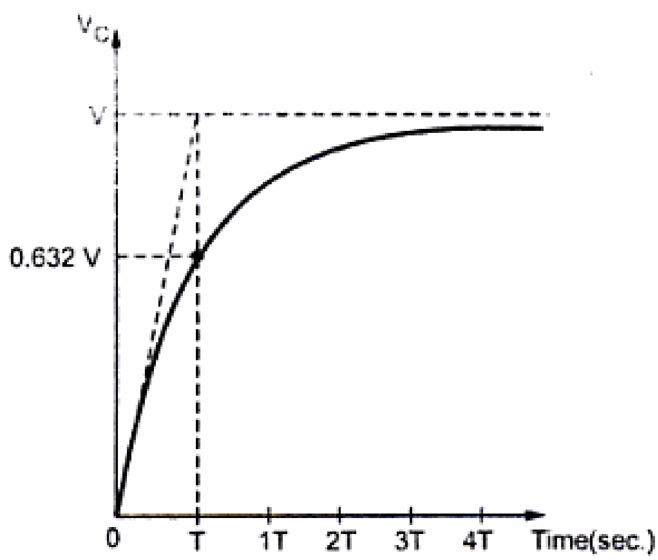


Fig. 5.25

Thus the tangent to the initial part of V_C joins V_C = V at t = τ as shown in the graph.

From the above discussion, the time constant of R-C series circuit can be defined as the time in seconds during which the voltage across the capacitor, starting from zero, would reach its final steady value if its rate of change was maintained constant at its initial value throughout the charging period.

5.25 Discharging a Capacitor through a Resistance

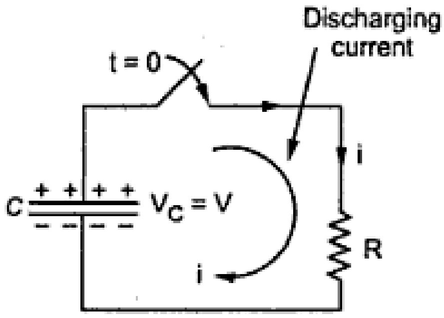


Fig. 5.26 Discharging of a capacitor

Now consider that a capacitor 'C' is being discharged through a resistor R by closing the switch at t = 0. At the time of closing the switch the capacitor 'C' is fully charged to V volts and it discharges through resistance 'R' and current through resistance flows in opposite direction to that at the time of charging.

As time passes, charge and hence the capacitor voltage V_C decreases gradually and hence discharge current also gradually decreases exponentially from maximum to zero.

The variation of capacitor voltage and discharging current as a function of time is shown in the Fig. 5.27.

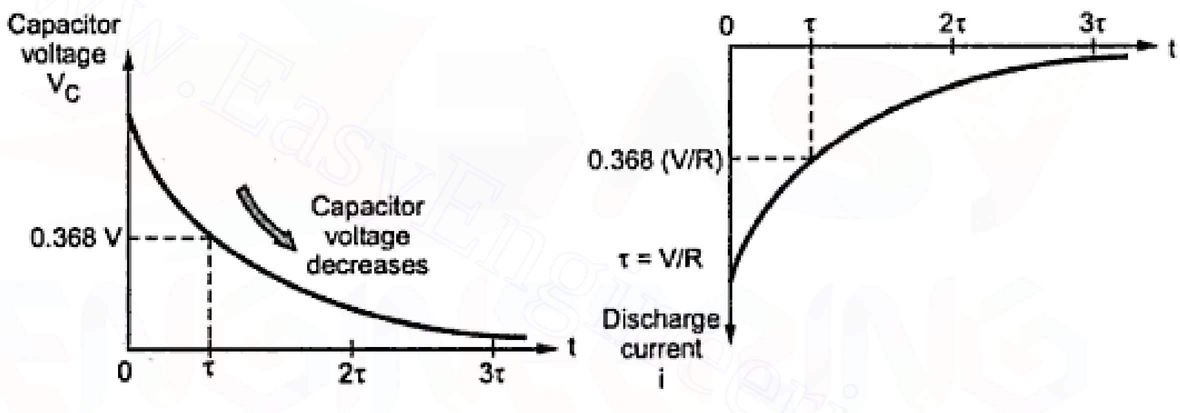


Fig. 5.27 Variation of discharge current and voltage

As direction of current is opposite to that of charging current, it is mathematically considered as negative. Hence graph of current against time is in fourth quadrant.

5.25.1 Mathematical Analysis

Applying Kirchhoff's voltage law, we had

$$V = V_C + i R$$

But V = 0,

$$\therefore \quad 0 = V_C + i R$$

$$\therefore \quad V_C = -i R$$

But

$$i = +C \frac{d V_C}{d t}$$

5.25.2 Time Constant

Similar to the previous definition, at the time of discharging also the term CR is called as **time constant** denoted by τ.

$$\tau = t = CR \text{ seconds}$$

When $$t = CR$$

$$V_C = V e^{-1} = 0.368 \text{ V}$$

and $$i = -\frac{V}{R} e^{-1}$$

$\therefore$ $$\boxed{i = -0.368 \left(\frac{V}{R}\right) \text{amps.}}$$

So **time constant** can be defined as

i) The time required for capacitor voltage to fall to 0.368 of its initial maximum value on discharge from its initial maximum value.

ii) The time required during which the capacitor discharge current falls to 0.368 of its initial maximum value.

5.25.3 Significance of Time Constant

The charging and discharging of a capacitor under the conditions discussed is said to be exponential. The 'time constant' (τ) of the circuit has following significance.

i) The whole charging or discharging process can be considered to be completed in a time equal to 4 times the time constant and the current falls to insignificant value (Theoretically the process takes infinite time).

ii) The charging current falls to 36.8% of its initial value in a time equal to time constant (τ) and to nearly 1.8% of initial value in $t = 4\,\tau$.

iii) The capacitor charges to nearly 63.2% of its final value in $t = \tau$ and nearly 98.2% of the final value in $t = 4\,\tau$ provided it is initially uncharged.

iv) The capacitor discharges to nearly 36.8% of its initial value in $t = \tau$ and nearly 1.8% of its initial value in $t = 4\,\tau$.

v) If the initial rate of rise of voltage is maintained then the capacitor charges to its final value in a time = τ.

5.26 Electrostatic and Electromagnetic Terms - A Comparison

Electrostatics				Electromagnetism		
Sr. No	**Term**	**Symbol**	**Unit**	**Term**	**Symbol**	**Unit**
1	Electric flux	ψ	C	Magnetic flux	ϕ	Wb
2	Electric flux density	D	C/m^2	Magnetic flux density	B	Wb/m^2
3	Electric field strength	E	N/C	Magnetic field strength	H	A/m
4	Electromotive force	E	V	Magnetomotive force	F	A
5	Permittivity of free space	ε_0	F/m	Permeability of free space	μ_0	H/m
6	Absolute permittivity	ε	F/m	Absolute permeability	μ	H/m
7	Relative permittivity	ε_r	–	Relative permeability	μ_r	–
	$\varepsilon = \frac{D}{E} = \varepsilon_0 \varepsilon_r$			$\mu = \frac{B}{H} = \mu_0 \mu_r$		

Table 5.3 Comparison of electrostatic and electromagnetic terms

Examples with Solutions

Example 5.8 : *A parallel plate capacitor has plates 0.1 cm apart, a plate area of 100 cm^2 and a dielectric with relative permittivity of 4. Determine the electric flux, electric flux density, electric field intensity, voltage between the plates, value of capacitance and energy stored if the capacitor has a charge of 0.05 μC.*

Solution : Now electric flux ψ = charge on C i.e. Q

$$\therefore \quad \psi = 0.05\ \mu C$$

Electric flux density

$$D = \frac{Q}{A} = \frac{0.05\times10^{-6}}{100\times10^{-4}} = 5\ \mu C/m^2$$

Now

$$D = \varepsilon E$$

$$\therefore \quad E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{5\times10^{-6}}{8.854\times10^{-12}\times4} = 141.179\ kV/m$$

$$E = \frac{V}{d}$$

$$V = 141.179\times10^3\times0.1\times10^{-2} = 141.179\ V$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$= \frac{8.854\times10^{-12}\times4\times100\times10^{-4}}{0.1\times10^{-2}} = 354.16\ pF$$

$$W = \frac{1}{2} C V^2 = \frac{1}{2} 354.16\times10^{-12}\times(141.179)^2$$

$$= 3.53\ \mu C$$

Example 5.9 : *Three capacitors are connected in series across a 120 V supply, the voltage across them are 30, 40 and 50 and the charge on each is 4500 μC. What is the value of each capacitor and equivalent capacitor of the series combination ?*

Solution : $Q = 4500\times10^{-6}$ C

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3$$

$$\therefore \quad C_1 = \frac{Q}{V_1} = \frac{4500\times10^{-6}}{30} = 150\ \mu F$$

$$\therefore \quad C_2 = \frac{Q}{V_2} = \frac{4500\times10^{-6}}{40} = 112.5\ \mu F$$

$$\therefore \quad C_3 = \frac{Q}{V_3} = \frac{4500\times10^{-6}}{50} = 90\ \mu F$$

Now $$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{150\times10^{-6}}+\frac{1}{112.5\times10^{-6}}+\frac{1}{90\times10^{-6}}$$

$$\therefore \quad C = 37.5\ \mu F$$

Example 5.10 : *Four capacitors are connected in parallel across a 250 V supply. The charges taken by them are 750, 1000, 1500 and 2000 μC. What is the equivalent capacitance ?*

Solution: $Q_1 = 750\ \mu C$, $Q_2 = 1000\ \mu C$, $Q_3 = 1500\ \mu C$, $Q_4 = 2000\ \mu C$.

$$V = 250\ V$$

$$Q_1 = C_1 V$$

$$\therefore \quad C_1 = \frac{Q_1}{V} = \frac{750\times10^{-6}}{250} = 3\ \mu F, \quad C_2 = \frac{Q_2}{V} = \frac{1000\times10^{-6}}{250} = 4\ \mu F$$

$$\therefore \quad C_3 = \frac{Q_3}{V} = \frac{1500\times10^{-6}}{250} = 6\ \mu F, \quad C_4 = \frac{Q_4}{V} = \frac{2000\times10^{-6}}{250} = 8\ \mu F$$

The equivalent $$C = C_1 + C_2 + C_3 + C_4 = 21\ \mu F$$

Example 5.11 : *A capacitor is charged with 10 mC. If the energy stored in it is 1 joule, calculate the voltage across it and its capacitance.*

Solution : The energy stored $= \frac{1}{2} C V^2$

but $$Q = C\,V$$

$\therefore$ $$W = \frac{1}{2} Q\,V$$

$\therefore$ $$1 = \frac{1}{2} \times 10 \times 10^{-3} \times V$$

$\therefore$ $$V = \mathbf{200\ V}$$

Now $$C = \frac{Q}{V} = \frac{10 \times 10^{-3}}{200} = \mathbf{50\ \mu F}$$

Example 5.12 : *80 μF capacitor in series with a 1000 Ω resistor is connected suddenly across a 110 V supply. Find :*

i) Initial value of current. ii) Time constant of the circuit.

iii) Equation of the current. iv) Value of current at t = 0.08 sec.

v) Rate at which current begins to decrease.

Solution : Given : $C = 80\ \mu F$, $R = 1000\ \Omega$, $V = 110\ V$

i) Initially current is maximum

$$i = \frac{V}{R} = \frac{110}{1000} = \mathbf{0.11\ A}$$

ii) Time constant τ

$$\tau = R\,C = 1000 \times 80 \times 10^{-6} = \mathbf{0.08\ sec}$$

iii) Equation of the current

$$i = \frac{V}{R}\left(e^{-t/CR}\right) \quad \text{i.e. } \mathbf{i = 0.11\ e^{-12.5t}\ A}$$

iv) At $$t = 0.08\ \text{sec}$$

$$i = 0.11\ e^{-12.5 \times 0.08} = \mathbf{0.0405\ A}$$

v) $$\frac{di}{dt} = 0.11\,(-12.5)\,e^{-12.5t}$$

At start $$t = 0$$

$\therefore$ Rate of decrease in current = **– 1.375 A/sec.**

Negative sign indicates decrease.

Example 5.13 : *A 80 μF capacitor is in series with, 10 kΩ resistance. The combination is connected suddenly across a 100 V supply. Find after 0.04 sec,*

i) Voltage across resistance. ii) Voltage across capacitor.

iii) Find the time at which voltage across resistance becomes 40 V.

iv) Find current at this time. v) What is charge on capacitor after 0.2 sec?

Solution : Given : $C = 80\ \mu F$ and $R = 10\ k\Omega = 10\times10^{3}\,\Omega$

$$\text{Initial current} = \frac{V}{R} = \frac{100}{10\times10^{3}} = 0.01\ \text{A}$$

$$\text{Time constant} \quad \tau = RC = 80\times10^{-6}\times10\times10^{3} = 0.8\ \text{sec}$$

$$\therefore \quad i = \frac{V}{R}\left(e^{-t/RC}\right)$$

$$\therefore \quad i = 0.01\ e^{-1.25t}$$

$$\text{After} \quad t = 0.4$$

$$i = 6.0653\times10^{-3}\ \text{A}$$

i) Voltage across resistance $= i R = 6.0653\times10\times10^{-3}\times10\times10^{3}$

$= \mathbf{60.653\ V}$

ii) Voltage across capacitor $= V - V_R = 100 - 60.653 = \mathbf{39.34\ V}$

iii) Voltage across resistance $= 40$ V

$$\therefore \quad iR = 40\ \text{V}$$

$$\therefore \quad i = \frac{40}{10\times10^{3}} = 4\times10^{-3}\ \text{A}$$

$$\therefore \quad 4\times10^{-3} = 0.01\ e^{-1.25t}$$

$$\therefore \quad t = \mathbf{0.733\ sec}$$

iv) Current at this time $\quad i = \mathbf{4\times10^{-3}\ A}$

v) After $t = 0.2$ sec, $\quad V_C = V(1 - e^{-1.25t}) = 100\,(1 - e^{-1.25\times0.2})$

$= \mathbf{22.119\ V}$

$$\therefore \quad Q = C\times V_C = 80\times10^{-6}\times22.119$$

$$= \mathbf{1.7695\times10^{-3}\ C}$$

Example 5.14 : *A capacitor of 0.5 μF is charged to 500 V and then disconnected from the supply. It is then allowed to be discharged through its own insulation resistance. If the voltage is reduced to 300 volts in 20 sec, determine the insulation resistance of the capacitor.*

Solution : Let R be the insulation resistance of capacitor. The equation for the capacitor voltage at the time of discharging is,

$$V_C = V\,e^{-t/RC}$$

$$\therefore \quad 300 = 500\,e^{-20/R\times 0.5\times 10^{-6}}$$

$$\therefore \quad (0.6) = e^{-40\times 10^{6}/R}$$

$$\therefore \quad \ln(0.6) = -0.5108 = -\frac{40\times 10^{6}}{R}$$

$$\therefore \quad R = 78.30\ \text{M}\Omega$$

Example 5.15 : *A capacitor is composed of two plates separated by a sheet of insulating material 3 mm thick and of relative permittivity = 4. The distance between plates is increased to allow the insertion of a second sheet 5 mm thick and relative permittivity 'ε_r'. If the capacitance of the capacitor so formed is one half of the original capacitance, find the value of ε_r.* **(Dec. - 2003)**

Solution : Initially, $t_1 = 3$ mm and $\varepsilon_{r1} = 4$ thus $\varepsilon = \varepsilon_0\varepsilon_{r1} = 4\varepsilon_0$

$$\therefore \quad C_1 = \frac{\varepsilon A}{t_1} = \frac{\varepsilon_0\,4\,A}{3\times 10^{-3}} = 1333.333\,\varepsilon_0\,A \qquad \text{... (1)}$$

With $t_2 = 5$ mm and $\varepsilon_r = \varepsilon_{r2}$, $\quad C_2 = 0.5\,C_1$

$$\text{Now} \quad C_2 = \frac{A\varepsilon_0}{\dfrac{t_1}{\varepsilon_{r1}} + \dfrac{t_2}{\varepsilon_{r2}}} = \frac{A\varepsilon_0}{\dfrac{3\times 10^{-3}}{4} + \dfrac{5\times 10^{-3}}{\varepsilon_{r2}}} \qquad \text{... (2)}$$

Take ratio of (1) and (2),

$$\therefore \quad \frac{C_1}{C_2} = \frac{1333.333\,\varepsilon_0\,A}{\left[\dfrac{A\varepsilon_0}{\dfrac{3\times 10^{-3}}{4} + \dfrac{5\times 10^{-3}}{\varepsilon_{r2}}}\right]}$$

$$\therefore \quad \frac{1}{0.5} = 1333.333\left[7.5\times 10^{-4} + \frac{5\times 10^{-3}}{\varepsilon_{r2}}\right]$$

$$\therefore \quad \frac{5\times 10^{-3}}{\varepsilon_{r2}} = 7.5\times 10^{-4}$$

$$\therefore \quad \varepsilon_{r2} = 6.667$$

Example 5.16 : *A capacitor having capacitance of 4 μF is connected in series with a resistance of 1 MΩ across 200 volt d.c. supply.*
Calculate
i) The time constant.
ii) The initial charging current.
iii) The time taken by capacitor to raise upto 160 volt. **(Dec. - 2003)**

Solution : C = 4 μF, R = 1 MΩ, V = 200 V

The equation of charging current

$$i = \frac{V}{R} e^{-t/RC}$$

i) Time constant = $\tau = RC = 1\times10^6 \times 4\times10^{-6}$ = **4 sec**

ii) Initial charging current at t = 0 is,

$$i = \frac{V}{R} = \frac{200}{1\times10^6} = \textbf{200 μA}$$

iii) $$V_C = V\left(1-e^{-t/RC}\right) \text{ and } V_C = 160 \text{ V}$$

$$\therefore \quad 160 = 200\left(1-e^{-t/4}\right) \qquad \ldots RC = 4$$

$$\therefore \quad e^{-t/4} = 0.2$$

$$\therefore \quad -t/4 = ln\,(0.2) = -1.6094$$

$$\therefore \quad t = \textbf{6.4377 sec}$$

Example 5.17 : *The plate area of a parallel plate capacitor is 0.01-sq.m. The distance between the plates is 2.5 cm. The insulating medium is air. Find its capacitance. What would be its capacitance, if the space between the plates is filled with an insulating material of relative permittivity 5 ?* **(May - 2004)**

Solution : A = 0.01 m^2, d = 2.5 cm

Case 1 : Dielectric is air

$$\therefore \quad C = \frac{\varepsilon_0 A}{d} = \frac{8.854\times10^{-12}\times0.01}{2.5\times10^{-2}} \qquad \ldots \varepsilon_r = 1$$

$$= 3.5416\times10^{-12} \text{ F} = \textbf{3.5416 pF}$$

Case 2 : Dielectric ε_r = 5

$$\therefore \quad C = \frac{\varepsilon_0\,\varepsilon_r\,A}{d} = \frac{8.854\times10^{-12}\times5\times0.01}{2.5\times10^{-2}} = \textbf{17.708 pF}$$

It is 5 times more than previous value.

$$\therefore \qquad C_{eq} = \frac{\frac{\varepsilon_0 \varepsilon_{r1} A}{t_1} \times \frac{\varepsilon_0 A}{t_2}}{\frac{\varepsilon_0 \varepsilon_{r1} A}{t_1} + \frac{\varepsilon_0 A}{t_2}} = \frac{\frac{\varepsilon_0^2 A^2}{t_1 t_2}[\varepsilon_{r1}]}{\frac{\varepsilon_0 A}{t_1 t_2}[t_2 \varepsilon_{r1} + t_1]}$$

$$= \frac{\varepsilon_0 A\, \varepsilon_{r1}}{\left[0.5\times 10^{-3} \times 6 + 2\times 10^{-3}\right]} = \frac{6\times 8.854\times 10^{-12} \times 11\times 10^{-4}}{5\times 10^{-3}}$$

$$= \mathbf{11.6872\ pF}$$

Example 5.20 : *A capacitor of 2 μF capacitance charged to p.d. of 200 V is discharged through a resistor of 2 MΩ.*

Calculate :

1) The initial value of discharged current

2) Its value 4 seconds later, and

3) Initial rate of decay of the capacitor voltage. **(May-2006)**

Solution : C = 2 μF, V = 200 V, R = 2 MΩ

i) The discharging current is given by

$$i = -\frac{V}{R}\, e^{-t/RC}$$

Initially t = 0,
$$i = -\frac{V}{R} = -\frac{200}{2\times 10^6} = -100\ \mu A$$

ii) At t = 4 sec,

$$i = -\frac{200}{2\times 10^6}\, e^{-4/2\times 10^6 \times 2\times 10^{-6}} = -\mathbf{36.7879\ \mu A}$$

iii)
$$V_C = V\, e^{-t/RC}$$

$$\therefore \qquad \frac{d V_C}{dt} = V\left(-\frac{1}{RC}\right) e^{-t/RC}$$

Initial rate of decay is at t = 0,

$$\left.\frac{d V_C}{dt}\right|_{t=0} = -\frac{V}{RC} = -\frac{200}{2\times 10^6 \times 2\times 10^{-6}} = -\mathbf{50\ V/sec}$$

Negative sign indicates decay of voltage and current.

Example 5.21 : *A capacitor consist of two parallel rectangular plates each 120 mm square separated by 1 mm in air. When a voltage of 1000 V is applied between the plates, calculate : i) The charge on the capacitor, ii) The electric flux density and iii) The electric field strength in the dielectric.* **(Dec.-2006)**

$$E_2 = \frac{1}{2}C_2V^2 = \frac{1}{2}\times4\times10^{-6}\times(100)^2 = \mathbf{0.02\ J}$$

The equivalent capacitance is,

$$C_{eq} = C_1 + C_2 = 2 + 4 = 6\ \mu F$$

ii) **Series :** The charge across them remains same.

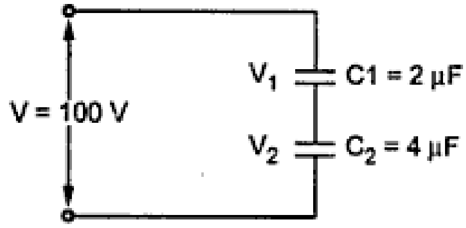


Fig. 5.31

$\therefore \quad Q = C_1 V_1 = C_2 V_2$

and $\quad V_1 + V_2 = V$

$\therefore \quad V_2 = V - V_1 = 100 - V_1$

$\therefore \quad C_1 V_1 = C_2 (100 - V_1)$

$\therefore \quad 2\times10^{-6} V_1 = 4\times10^{-6}(100 - V_1)$ i.e. $1.5\ V_1 = 100$

$\therefore \quad V_1 = \mathbf{66.667\ V}, \quad \mathbf{V_2 = 33.333\ V}$

Thus the energy stored in each is,

$$E_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times2\times10^{-6}\times(66.667)^2 = \mathbf{4.4444\ mJ}$$

$$E_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times4\times10^{-6}\times(33.333)^2 = \mathbf{2.2222\ mJ}$$

And $\quad C_{eq} = \dfrac{C_1C_2}{C_1+C_2} = \dfrac{2\times10^{-6}\times4\times10^{-6}}{2\times10^{-6}+4\times10^{-6}} = \mathbf{1.3333\ \mu F}$

Review Questions

1. *Explain the concept of charge. What is its unit ?*
2. *Explain the laws of electrostatics.*
3. *What is electric field and electric lines of force ? State the properties of electric lines of force.*
4. *State and explain Coulomb's law.*
5. *Define the following terms stating their units.*

   i) Electric flux   *ii) Electric field intensity*

   iii) Electric flux density   *iv) Surface charge density*

   v) Absolute permittivity   *vi) Relative permittivity*
6. *Derive the relation between electric field intensity and electric flux density.*
7. *What is electric potential ?What is its unit ? Define the unit.*
8. *Explain the concept of potential difference.*
9. *Derive the expressions for an electric potential and potential difference.*
10. *Explain what is potential gradient in an electric field.*

11. *What is capacitor question? Define its unit.*
12. *Obtain expressions for equivalent capacitance when the capacitors are connected in*
    i) Series and ii) Parallel
13. *With usual notation derive an expression for capacitance of a parallel plate capacitor.*
14. *Derive the equation for the capacitance of a multiple plate capacitor.*
15. *Derive the equation of the capacitance of a composite capacitor consisting of three different dielectric media with different thicknesses and relative permittivities.*
16. *Derive the expression for the energy stored in a capacitor.*
17. *Explain the action of a capacitor, when connected to a battery of V volts.*
18. *A parallel plate capacitor has plates, each of area 100 cm^2, separated by a distance of 3 cms. The dielectric between the plates has relative permittivity of 2.2. The potential difference between the plates is 10 kV.*
    Find (i) Capacitance of the capacitor ; (ii) Surface charge density; (iii) Field intensity; (iv) Energy stored. **(Ans. : 6.493 pF, 6.493 $\times 10^{-6}$ C/m^2, 3.3 $\times 10^5$ V/m, 3.246 $\times 10^{-4}$ J)**
19. *Calculate the energy stored in a parallel plate capacitor which consists of two metal plates 60 cm^2, separated by a dielectric of 1.5 mm thick and of relative permittivity 3.5, if a potential difference of 1000 V is applied across it.* **(Ans. : 6.1978 $\times 10^{-5}$ J)**
20. *A potential difference of 400 V is maintained across a capacitor of 25 μF. Calculate (i) charge; (ii) electric field strength ; (iii) electric flux density. The distance between the plates of a capacitor is 0.5 mm and area of cross-section of plates is 1.2 cm^2. Find also the energy stored in the capacitor.* **(Ans. : .01 C, 8 $\times 10^5$ V/m, 83.33 C/m^2, 2 J)**
21. *The capacity of a parallel plate capacitor is 0.0005 μF. The capacitor is made up of plates having area 200 cm^2, separated by a dielectric of 5 mm thickness. A p.d. of 10,000 V is applied across the condenser. Find (i) charge; (ii) potential gradient in the dielectric; (iii) relative permittivity and (i) electric flux density.*
    (Ans. : 5 $\times 10^{-6}$ C, 2$\times 10^6$ V/m, 14.12, 2.5 $\times 10^{-4}$ C/m^2)
22. *Three capacitors of 5, 10 and 15 μF are connected in series across a 100 V supply. Find the equivalent capacitance and the voltage across each.*
    If the capacitor, after being charged in series are disconnected and then connected in parallel, with plates of like polarity together, find the total charge of the parallel combination.
    (Ans. : 2.727 μF, 54.54 V, 27.27 V, 18.18 V, 8.181 $\times 10^{-4}$ C)
23. *Three capacitors A, B and C are charged as follows : A = 10 μF, 100 V; B = 15 μF, 150 V and C = 25 μF, 200 V. They are now connected in parallel with terminals of like polarity together. Find the voltage across the combination.* **(Ans. : 50 μF, 165 V)**
24. *A capacitor consists of two metallic plates, each 40 cm × 40 cm and placed 6 mm apart. The space between plates is filled with a glass plate 5 mm thick and a layer of paper 1 mm thick. The relative permittivities are 8 and 2 respectively. Calculate its capacitance.* **(Ans. : 1.26 nF)**

25. *A capacitor of 10 μF is charged to a p.d. of 200 V and then connected in parallel with an uncharged capacitor of 5 μF. Find the p.d. across the parallel combination and the energy stored in each capacitor.* **(Ans. : 133.33 V, 0.089 J, 0.044 J)**

26. *Determine the equivalent capacitance of the combination shown in Fig. 5.32.* **(Ans. : 2.85 F)**

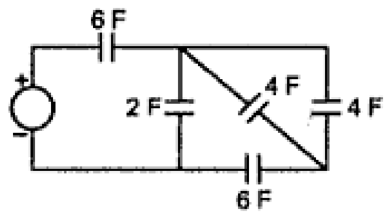


Fig. 5.32

27. *Three capacitors A, B and C are connected in series across 100 V d.c. supply. The p.d. across the capacitors are 20 V, 30 V and 50 V respectively. If the capacitance of A is 10 μF, calculate the capacitances of B and C.* **(Ans. : 6.67 μF, 4 μF)**

6

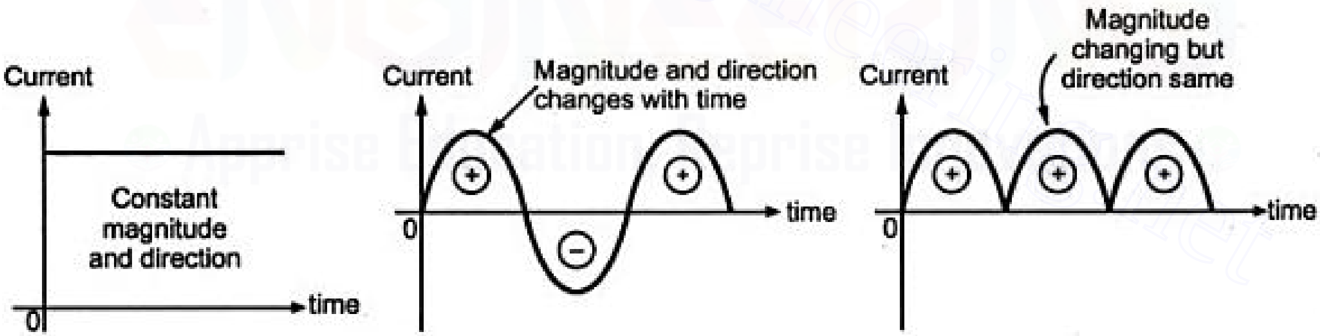
A.C. Fundamentals

6.1 Introduction

Uptill now, we have discussed about D.C. supply and D.C. circuits. But 90 % of electrical energy used now a days is a.c. in nature. Electrical supply used for commercial purposes is alternating. The d.c. supply has constant magnitude with respect to time. The Fig. 6.1(a) shows a graph of such current with respect to time.

> **Key Point**: *An alternating current (a.c.) is the current which changes periodically both in magnitude and direction.*

Such change in magnitude and direction is measured in terms of cycles. Each cycle of a.c. consists of two half cycles namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly. The Fig. 6.1 (b) shows a graph of alternating current against time.



(a) Direct current **(b) Alternating current** **(c) Pulsating d.c.**

Fig. 6.1

In practice some waveforms are available in which magnitude changes but its direction remains same as positive or negative. This is shown in the Fig. 6.1(c). Such waveform is called **pulsating d.c.** The waveform obtained as output of full wave rectifier is an example of pulsating d.c.

Let us see, why in practice, there is generation of a.c. Use of a.c. definitely offers certain advantages.

6.2 Advantages of A.C.

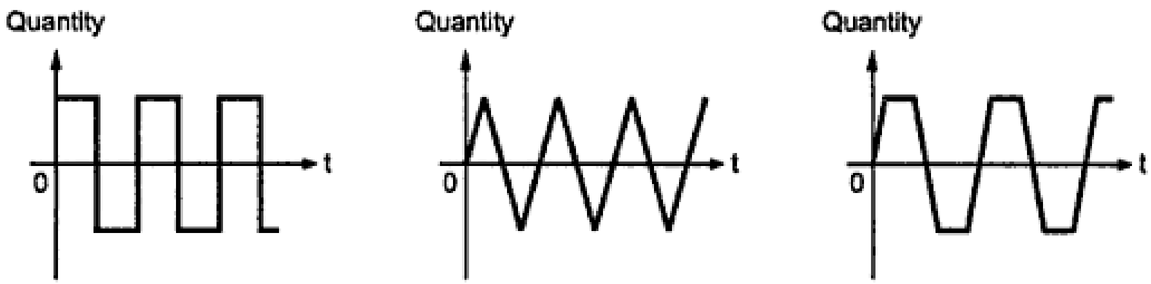
The various advantages of a.c. are,

1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible. Now, higher the voltage, lesser is the current flowing through transmission line. Less the current, lesser are the copper losses and lesser is the conducting material required. This makes a.c. transmission always economical and efficient.
3. It is possible to build up high a.c. voltage; high speed a.c. generators of large capacities. The construction and cost of such generators are very low. High a.c. voltages of about 11 kV can be generated and can be raised upto 220 kV for transmission purpose at sending end, while can be lowered down at 400 V at receiving end. This is not possible in case of d.c.
4. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
5. Whenever it is necessary, a.c. supply can be easily converted to obtain d.c. supply. This is required as d.c. is very much essential for the applications like cranes, printing process, battery charging, telephone system, etc. But, such requirement of d.c. is very small compared to a.c.

Due to these advantages, a.c. is used extensively in practice and hence, it is necessary to study a.c. fundamentals.

6.3 Types of A.C. Waveforms

The waveform of alternating voltage or current is shown purely sinusoidal in the Fig. 6.1 (b). But, in practice, a quantity which undergoes variations in its instantaneous values, in magnitude as well as direction with respect to some zero reference is called an **alternating quantity.** The graph of such quantity against time is called its **waveform.** Various types of alternating waveforms other than sinusoidal are shown in the Fig. 6.2 (a), (b) and (c).



(a) Square (b) Triangular (c) Trapezoidal

Fig. 6.2

Out of all these types of alternating waveforms, purely sinusoidal waveform is preferred for a.c. system. There are few advantages of selecting purely sinusoidal as the standard waveform.

6.3.1 Advantages of Purely Sinusoidal Waveform

1) Mathematically, it is very easy to write the equations for purely sinusoidal waveform.

2) Any other type of waveform can be resolved into a series of sine or cosine waves of fundamental and higher frequencies, sum of all these waves gives the original wave form. Hence, it is always better to have sinusoidal waveform as the standard waveform.

3) The sine and cosine waves are the only waves which can pass through linear circuits containing resistance, inductance and capacitance without distortion. In case of other waveforms, there is a possibility of distortion when it passes through a linear circuit.

4) The integration and derivative of a sinusoidal function is again a sinusoidal function. This makes the analysis of linear electrical networks with sinusoidal inputs, very easy.

6.4 Generation of A.C. Voltage

The machines which are used to generate electrical voltages are called **generators.** The generators which generate purely sinusoidal a.c. voltages are called **alternators.**

The basic principle of an alternator is the principle of electromagnetic induction. The sine wave is generated according to **Faraday's law of electromagnetic induction.** It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Such an induced e.m.f. then can be used to supply the electrical load.

Let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called **single turn or single loop alternator.**

6.4.1 Single Turn Alternator

Construction : It consists of a permanent magnet of two poles. A single turn rectangular coil is kept in the vicinity of the permanent magnet. The coil is made up of same conducting material like copper or aluminium. The coil is made up of two conductors namely a-b and c-d. Such two conductors are connected at one end to form a coil. This is shown in the Fig. 6.3.

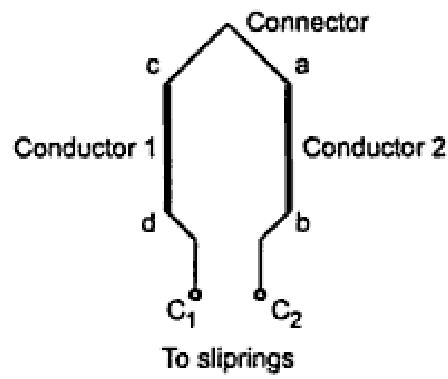


Fig. 6.3 Single turn coil

The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. The remaining two ends C1 and C2 of the coil are connected to the rings mounted on the shaft called **slip rings.** Slip rings are also rotating members of the alternator. The two brushes P and Q are resting on the slip rings. The **brushes are stationary** and just making contact with the slip rings. The slip rings and brush assembly is necessary to collect the current induced in the rotating coil and make it available to the stationary external resistance. The overall construction is shown in the Fig. 6.4.

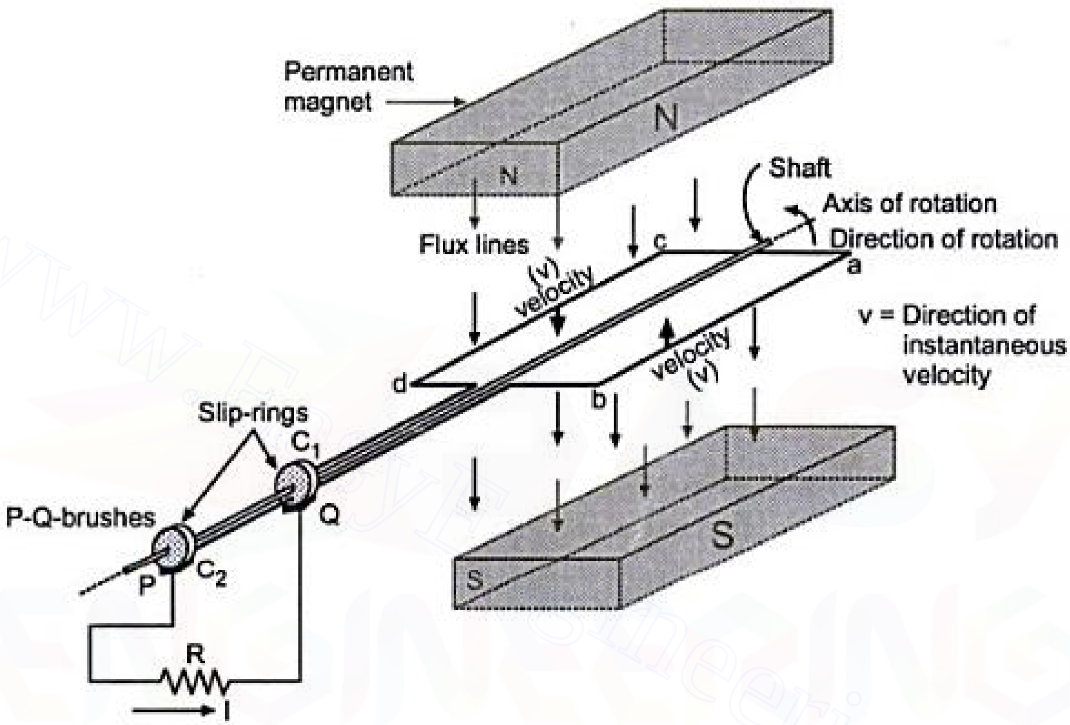


Fig. 6.4 Single turn alternator

Working : The coil is rotated in anticlockwise direction. While rotating, the conductors ab and cd cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction, an e.m.f. gets induced in the conductors. This e.m.f. drives a current through resistance R connected across the brushes P and Q. The magnitude of the induced e.m.f. depends on the position of the coil in the magnetic field. Let us see the relation between magnitude of the induced e.m.f. and the positions of the coil. Consider different instants and the different positions of the coil.

Instant 1 : Let the initial position of the coil be as shown in the Fig. 6.4. The plane of the coil is perpendicular to the direction of the magnetic field. The instantaneous component of velocity of conductors ab and cd, is parallel to the magnetic field as shown and there cannot be the cutting of the flux lines by the conductors. Hence, no e.m.f. will be generated in the conductors ab and cd and no current will flow through the external resistance R. This position can be represented by considering the front view of the Fig. 6.4 as shown in the Fig. 6.5 (a).

Instant 6 : As the coil rotates beyond θ = 180°, the conductor ab uptil now cutting flux lines in one particular direction reverses the direction of cutting the flux lines. Similar is the behaviour of conductor cd. So, direction of induced e.m.f. in conductor ab is opposite to the direction of induced e.m.f. in it for the rotation of θ = 0° to 180°. Similarly, the direction of induced e.m.f. in conductor cd also reverses. This change in direction of induced e.m.f. occurs because the direction of rotation of conductors ab and cd reverses with respect to the field as θ varies from 180° to 360°. This process continues as coil rotates further. At θ = 270° again, the induced e.m.f. achieves its maximum value but the direction of this e.m.f. in both the conductors is opposite to the previous maximum position i.e. at θ = 90°. From θ = 270° to 360°, induced e.m.f. decreases without change in direction and at θ = 360°, coil achieves the starting position with zero induced e.m.f.

So, as θ varies from 0° to 360°, the e.m.f. in a conductor ab or cd varies in an alternating manner i.e. zero, increasing to achieve maximum in one direction, decreasing to zero, increasing to achieve maximum in other direction and again decreasing to zero. This set of variation repeats for every revolution as the conductors rotate in a circular motion with a certain speed.

This variation of e.m.f. in a conductor can be graphically represented.

6.4.2 Graphical Representation of the Induced E.M.F.

The instantaneous values of the induced e.m.f. in any conductor, as it is rotated from θ = 0° to 360°, i.e. through one complete revolution can be represented as shown in the Fig. 6.6.

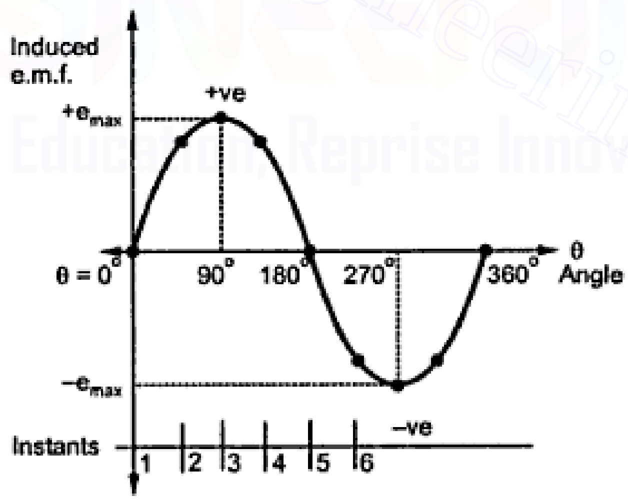


Fig. 6.6 Graphical representation of the induced e.m.f.

From the Fig. 6.6, it is clear that the waveform generated by the instantaneous values of the induced e.m.f. in any conductor (ab or cd) is purely sinusoidal in nature.

6.6 Equation of an Alternating Quantity

For the derivation of the equation of an alternating quantity, consider single turn, 2 pole alternator discussed earlier. The coil is rotated with constant angular velocity in the magnetic field. An alternating e.m.f. induced is purely sinusoidal in nature.

Let B = Flux density of the magnetic field in Wb/m^2

l = Active length of each conductor in metres

r = Radius of circular path traced by conductors in metres.

ω = Angular velocity of coil in radians / second

v = linear velocity of each conductor in m / sec.

Now, $v = r\,\omega$

Consider an instant where coil has rotated through angle θ from the position corresponding to $\theta = 0°$ i.e. from the instant when induced e.m.f. is zero. It requires time 't' to rotate through θ. So, θ in radians can be expressed as,

$$\boxed{\theta = \omega t \quad \text{radians}}$$

The position of the coil is shown in the Fig. 6.9 (a). The instantaneous peripheral velocity of any conductor can be resolved into two components as shown in the Fig. 6.9 (b).

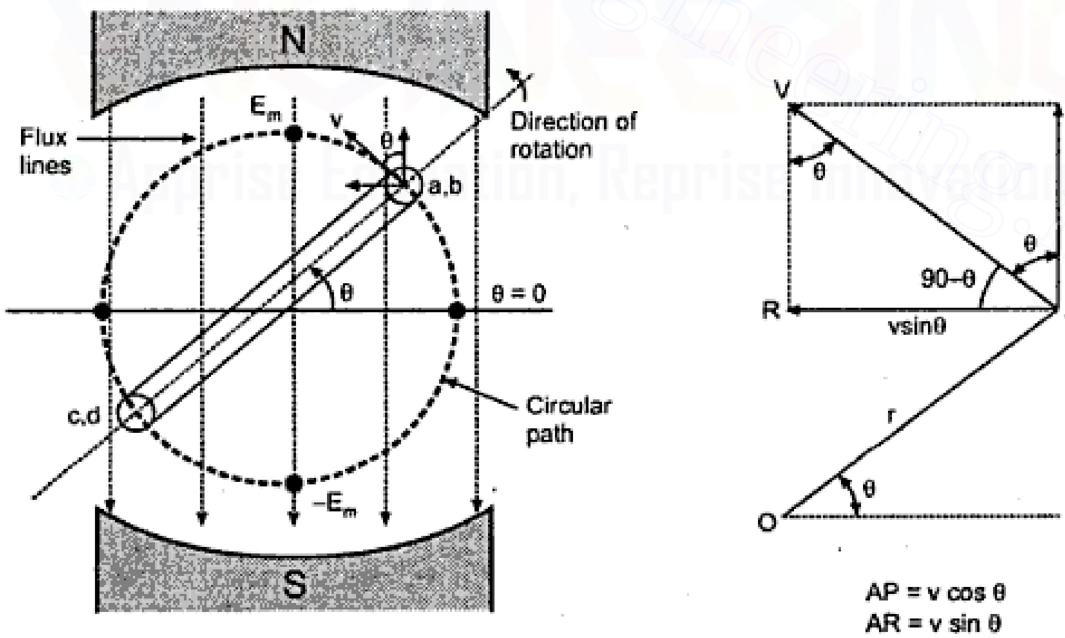


Fig. 6.9 Instantaneous value of an induced e.m.f.

The components of velocity, v are,

1) Parallel to the magnetic flux lines, $(AP) = v \cos\theta$
2) Perpendicular to the magnetic flux lines, $(AR) = v \sin\theta$

Out of the two, due to the component parallel to the flux, there cannot be the generation of the e.m.f. as there cannot be the cutting of the flux lines. Hence, the component which is acting perpendicular to the magnetic flux lines i.e. v sin θ is solely responsible for the generation of the e.m.f.

According to the Faraday's law of electromagnetic induction, the expression for the generated e.m.f. in each conductor is,

$$e = B\,l\,v \sin\theta \quad \text{volts}$$

The active length 'l' means the length of the conductor which is under the influence of the magnetic field.

Now, $E_m = B\,l\,v$

$= $ maximum value of induced e.m.f. in conductor

This is achieved at $\theta = 90°$ and is the peak value or amplitude of the sinusoidal induced e.m.f.

Hence, equation giving instantaneous value of the generated e.m.f. can be expressed as,

$$\boxed{e = E_m \sin\theta \quad \text{volts}}$$

6.6.1 Different Forms of E.M.F. Equation

Now, $\theta = \omega t$ radians

$$\therefore \quad \boxed{e = E_m \sin(\omega t)} \qquad \ldots (1)$$

But, $\omega = 2\pi f$ rad / sec.

$$\therefore \quad \boxed{e = E_m \sin(2\pi f t)} \qquad \ldots (2)$$

But, $f = \frac{1}{T}$ seconds

$$\therefore \quad \boxed{e = E_m \sin\left(\frac{2\pi}{T} t\right)} \qquad \ldots (3)$$

Important Note : *In all the above equations, the angle θ is expressed in radians. Hence, while calculating the instantaneous value of the e.m.f., it is necessary to calculate the sine of the angle expressed in radians.*

> **Key Point** : *Mode of the calculator should be converted to radians, to calculate the sine of the angle expressed in radians, before substituting in any of the above equations.*

This alternating e.m.f. drives a current through the electrical load which also varies in similar manner.

Its frequency is the same as the frequency of the generated e.m.f. Hence, it can be expressed as,

$$\boxed{i = I_m \sin \theta}$$

where I_m = maximum or peak value of the current. This maximum value depends on the resistance of the electrical circuit to which an e.m.f. is applied. The instantaneous value of this sinusoidal current set up by the e.m.f. can be expressed as,

$$i = I_m \sin \omega t$$

or

$$i = I_m \sin 2 \pi f t$$

or

$$i = I_m \sin\left(\frac{2 \pi t}{T}\right)$$

Example 6.1 : *Write the 4 ways of representing an a.c. voltage given by a magnitude of 5 V and frequency of 50 Hz.*

Solution : Given values are, $E_m = 5$ V and $f = 50$ Hz

So $\omega = 2 \pi f = 100 \pi$ rad/sec and $T = 1/f = 1/50$ sec

The voltage can be represented as,

1) $e = E_m \sin(\omega t) = 5 \sin(100 \pi t)$ V

2) $e = E_m \sin \theta = 5 \sin \theta$ V

3) $e = E_m \sin(2 \pi ft) = 5 \sin(100 \pi t)$ V

4) $e = E_m \sin\left(\frac{2 \pi t}{T}\right) = 5 \sin\left(\frac{2\pi t}{1/50}\right)$

Note that, after substituting the values of E_m, f, ω and T the resultant equation obtained remains same by all four ways.

Example 6.2 : *An alternating current of frequency 60 Hz has a maximum value of 12 A :*
i) Write down the equation for instantaneous values. ii) Find the value of the current after 1/360 second. iii) Time taken to reach 9.6 A for the first time.
In the above cases assume that time is reckoned as zero when current wave is passing through zero and increasing in the positive direction. (May - 98)

Solution : $f = 60$ Hz and $I_m = 12$ A

$\omega = 2\pi f = 2\pi \times 60 = 377$ rad/sec

i) Equation of instantaneous value is

$$i = I_m \sin \omega t$$

$\therefore$ $$i = 12 \sin 377\, t$$

ii) $$t = \frac{1}{360} \text{ sec}$$

$$i = 12 \sin 377 \frac{1}{360} = 12 \sin 1.0472 = \mathbf{10.3924\ A}$$

Note : sin of 1.0472 must be calculated in **radian** mode.

iii) $$i = 9.6 \text{ A}$$

$\therefore$ $$9.6 = 12 \sin 377\, t$$

$\therefore$ $$\sin 377\, t = 0.8$$

$\therefore$ $$377\, t = 0.9272$$

Note : find inverse of sin in **radian** mode.

$$t = \mathbf{2.459 \times 10^{-3}\ sec.}$$

Example 6.3 : *A sinusoidal voltage of 50 Hz has a maximum value of $200\sqrt{2}$ volts. At what time measured from a positive maximum value will the instantaneous voltage be equal to 141.4 volts ?* **(Dec. - 2001)**

Solution : f = 50 Hz, $V_m = 200\sqrt{2}$ V, $v_1 = 141.4$ V

The equation of the voltage is,

$$v = V_m \sin(2\pi f t) = 200\sqrt{2} \sin(2\pi \times 50\, t)\ \text{V}$$

For $v = v_1$

$$141.4 = 200\sqrt{2} \sin(2\pi \times 50 \times t_1)$$

$\therefore$ $$t_1 = 1.666 \times 10^{-3} \text{ sec}$$ ... **Use radian mode for sin**

But this time is measured from t = 0. At positive maximum, time is $\frac{T}{4} = \frac{1}{4f} = 5 \times 10^{-3}$ sec

so $t = t_1 = 1.666 \times 10^{-3}$ sec is before positive maximum.

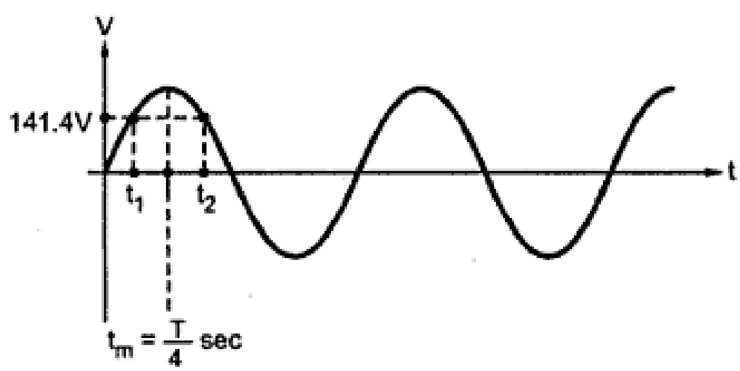


Fig. 6.10

From Fig. 6.10.

$$t_m - t_1 = 5 \times 10^{-3} - 1.666 \times 10^{-3}$$
$$= \mathbf{3.314 \times 10^{-3}\ sec}$$

As the waveform is symmetrical, at the time of **3.314×10^{-3} sec measured after positive maximum value, the instantaneous voltage will be again 141.4 V.**

6.7 Effective Value or R.M.S. Value

An alternating current varies from instant to instant, while the direct current is constant, with respect to time. So, for the comparison of the two, there must be some common platform. Such platform can be the effect produced by the two currents. One of the such effects is heating of the resistance, due to current passing through it. The heating effect can be used to compare the alternating and direct current. From this, r.m.s. value of an alternating current can be defined as,

> **Key Point:** *The effective or r.m.s. value of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

The following simple experiment gives the clear understanding of the r.m.s. value of an alternating current. The arrangement is shown in the Fig. 6.11.

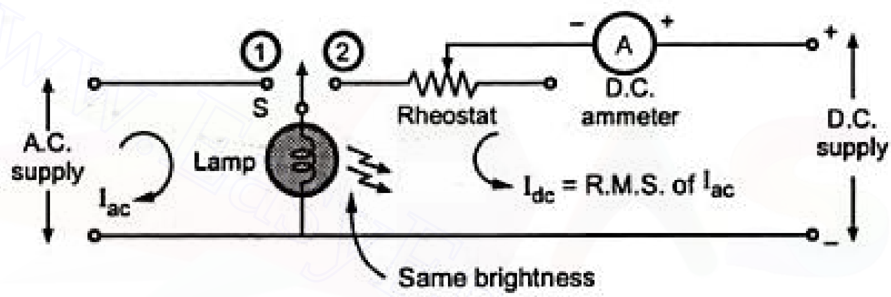


Fig. 6.11 Experiment to demonstrate r.m.s. value

A lamp is provided with double throw switch S. On position 1, it gets connected to an a.c. supply. The brightness of filament is observed.

Then, switch is thrown in position 2 and using the rheostats, the d.c. current is adjusted so as to achieve the same brightness of the filament.

The reading on the ammeter on d.c. side gives the value of the direct current that produces the same heating effect as that produced by the alternating current. This ammeter reading is nothing but the r.m.s. value of the alternating current.

R.M.S. value can be determined by two methods :

1) **Graphical Method :** This can be used for an alternating current having any wave form i.e. sinusoidal, triangular, square, etc.

2) **Analytical Method :** This is to be used for purely sinusoidally varying alternating current.

6.7.1 Graphical Method

Consider sinusoidally varying current.The r.m.s. value is to be obtained by comparing heat produced. Heat produced is proportional to square of current (i^2 R) so heat produced in both positive and negative half cycles will be the same. Hence, consider only positive half cycle, which is divided into 'n' intervals as shown in the Fig. 6.12. The width of each interval is 't/n' seconds and average height of each interval is assumed to be the average instantaneous values of current i.e. $i_1, i_2, ..., i_n$.

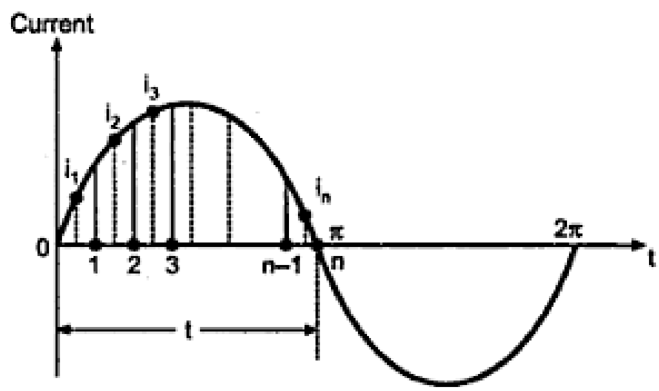


Fig. 6.12 Determining r.m.s. value

Let this current be passing through resistance 'R' ohms. Hence, heat produced can be calculated as,

$$\text{Heat Produced} = i^2 R t \quad \text{joules}$$

$$\therefore \quad \text{Heat produced due to } 1^{st} \text{ interval} = i_1^2 R \frac{t}{n} \quad \text{joules}$$

$$\therefore \quad \text{Heat produced due to } 2^{nd} \text{ interval} = i_2^2 R \frac{t}{n} \quad \text{joules}$$

$$\therefore \quad \text{Heat produced due to } n^{th} \text{ interval} = i_n^2 R \frac{t}{n} \quad \text{joules}$$

$$\therefore \quad \text{Total heat produced in 't' seconds} = R \times t \times \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

Now, heat produced by direct current I amperes passing through same resistance 'R' for the same time 't' is $= I^2 R t$ joules

For I to be the r.m.s. value of an alternating current, these two heats must be equal.

$$\therefore \quad I^2 R t = R \times t \times \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

$$\therefore \quad I^2 = \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

$$\therefore \quad \boxed{I = \sqrt{\frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}} = I_{r.m.s.}}$$

$I_{r.m.s.}$ = square root of the mean of the squares of ordinates of the current.

This is called **Effective** value of an alternating current or **Virtual value** of an alternating current. This expression is equally applicable to sinusoidally varying alternating voltage as,

$$V_{r.m.s.} = \sqrt{\frac{[V_1^2 + V_2^2 + \ldots + V_n^2]}{n}}$$

6.7.2 Analytical Method

Consider sinusoidally varying alternating current and square of this current as shown in the Fig. 6.13.

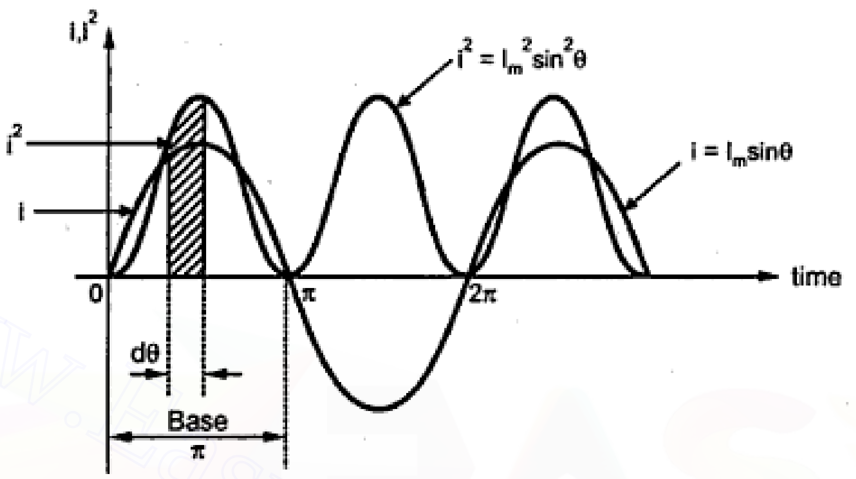


Fig. 6.13 Waveform of current and square of the current

The current $i = I_m \sin\theta$ while

square of current $i^2 = I_m^2 \sin^2\theta$

Area of curve over half a cycle can be calculated by considering an interval dθ as shown.

Area of square curve over half cycle = $\int_0^{\pi} i^2 \, d\theta$ and Length of the base is π .

∴ Average value of square of the current over half cycle

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2 \, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta = \frac{I_m^2}{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$= \frac{I_m^2}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{I_m^2}{2\pi}[\pi]$$

$$= \frac{I_m^2}{2}$$

Hence, root mean square value i.e. r.m.s. value can be calculated as,

$$I_{r.m.s.} = \sqrt{\text{mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{I_{r.m.s.} = 0.707\ I_m}$$

The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.

Key Point : *The **instantaneous** values are denoted by **small** letters like i, e etc. while **r.m.s.** values are represented by **capital** letters like I, E etc.*

The above result is also applicable to **sinusoidal** alternating voltages.

So, we can write,

$$\therefore \quad \boxed{V_{r.m.s.} = 0.707\ V_m}$$

6.7.3 Importance of R.M.S. Value

1. In case of alternating quantities, the r.m.s. values are used for specifying magnitudes of alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quatntities unless and otherwise specicied to beother than r.m.s.

Key Point : *In practice, everywhere, r.m.s. values are used to analyze alternating quantities.*

2. The ammeters and voltmeters record the r.m.s. values of current and voltage respectively.
3. The heat produced due to a.c. is proportional to square of the r.m.s. value of the current.

Steps to find r.m.s. value of an a.c. quantity :

1. Write the equation of an a.c. quantity. Observe its behaviour during various time intervals.
2. Find square of the a.c. quantity from its equation.
3. Find average value of square of an alternating quantity as,

$$\text{Average} = \frac{\text{Area of curve over one cycle of squared waveform}}{\text{Length of the cycle}}$$

4. Find square root of average value which gives r.m.s. value of an alternating quantity.

$$\text{Time period T} = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

$$\text{Time at A} = \frac{T}{4} = 0.005 \text{ sec}$$

i) $t_1 = 0.005 + 0.0025 = 0.0075$ sec

$\therefore$ $i = 14.1421 \sin(314.159 \times 0.0075) = \mathbf{10\ A}$

ii) $$\text{Time at C} = \frac{T}{2} = \frac{0.02}{2} = 0.01 \text{ sec}$$

$\therefore$ $t_2 = 0.01 + 0.0075 = 0.0175$ sec

$\therefore$ $i = 14.1421 \sin(314.159 \times 0.0175) = \mathbf{-10\ A}$

* Calculate sin in radian mode.

Example 6.5 : *Calculate the r.m.s. value, average value, form factor, peak factor of a periodic current having following values for equal time intervals changing suddenly from one value to next as 0, 2, 4, 6, 8, 10, 8, 6, 4, 2, 0, –2, –4, –6, –8, –10, –8, ...* **(May - 2001)**

Solution : The waveform can be represented as shown in the Fig. 6.16.

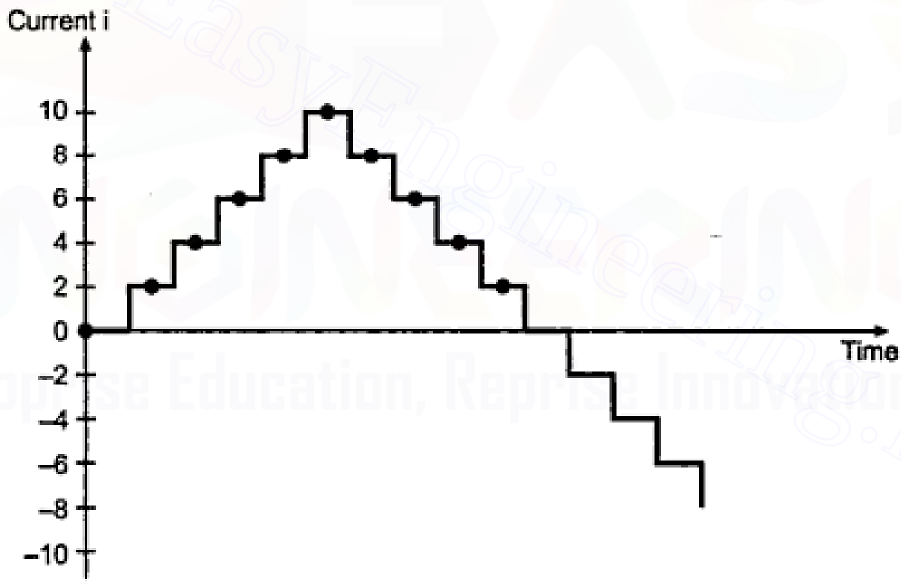


Fig. 6.16

The average value of the current is given by,

$$\text{Average value} = \frac{0+2+4+6+8+10+8+6+4+2}{10} = \mathbf{5\ A}$$

$$\text{The r.m.s. value of the current} = \sqrt{\frac{0^2+2^2+4^2+6^2+8^2+10^2+8^2+6^2+4^2+2^2}{10}}$$

$$= \mathbf{5.8309\ A}$$

$$\text{Form factor} \quad K_f = \frac{\text{r.m.s.}}{\text{average}} = \frac{5.8309}{5} = \mathbf{1.1661}$$

Peak factor $K_p = \frac{\text{maximum}}{\text{r.m.s.}} = \frac{10}{5.8309} = 1.715$

Example 6.6 : *A sinusoidally varying alternating current has r.m.s. value of 20 A and periodic time of 20 milliseconds. If the waveform of this current enters into its positive half cycle at t = 0, find the instantaneous values of the current at $t_1 = 6$ milliseconds and $t_2 = 12$ milliseconds.* **(May - 2002)**

Solution : $I_{rms} = 20$ A, T = 20 ms

$$\therefore \quad I_m = \sqrt{2}\, I_{rms} = 28.2842 \text{ A}$$

$$f = \frac{1}{T} = 50 \text{ Hz}$$

So $\omega = 2\pi f = 314.1592$ rad/sec

Hence equation for the current is,

$$i = I_m \sin \omega t = 28.2842 \sin [314.1592\, t] \text{ A}$$

At $t_1 = 6$ ms, $i = 26.899$ A ... Use radian mode

At $t_2 = 12$ ms, $i = -16.625$ A ... Use radian mode

6.11 R.M.S. Value of Combined Waveform

Consider a wire carrying simultaneously more than one alternating current of different magnitudes and frequencies alongwith certain d.c. current. It is required to calculate resultant **r.m.s. value i.e. effective value** of the current.

Let the wire carries three different currents as shown in the Fig. 6.17 It is required to obtain resultant I_{rms} through the wire.

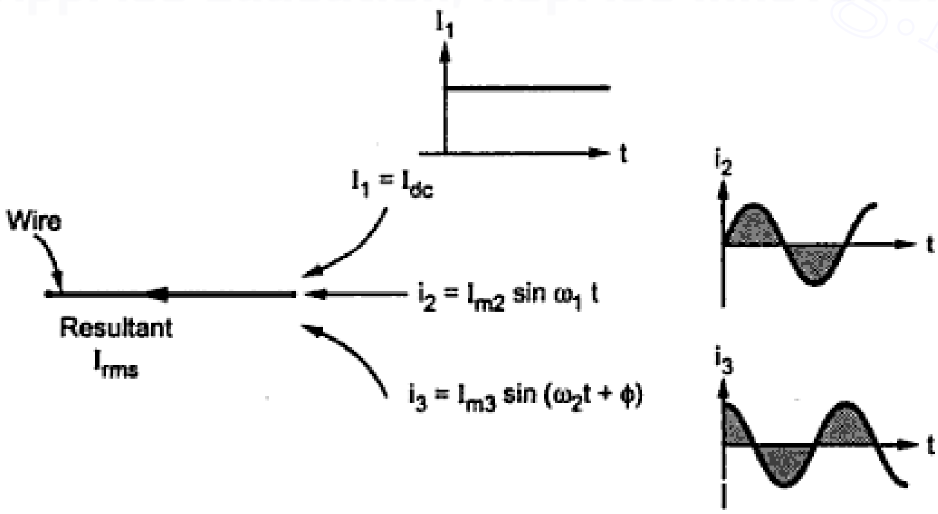


Fig. 6.17 Wire carrying 3 different currents simultaneously

Method : It is based on heating effect of various currents.

Let I_{rms} = Resultant r.m.s. value of current

$$\therefore \quad I_{rms}^2 \times R \times t = I_{dc}^2 \times R \times t + \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 \times R \times t + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t$$

> Note that heat produced = (rms value)$^2 \times R \times t$
>
> and r.m.s. of a.c. = $\dfrac{I_m}{\sqrt{2}}$

$$\therefore \quad I_{rms}^2 = 10^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 = 198$$

$\therefore \quad I_{rms} = 14.0712$ A ...Effective value of the resultant

6.12 Phasor Representation of an Alternating Quantity

In the analysis of a.c. circuits, it is very difficult to deal with alternating quantities in terms of their waveforms and mathematical equations. The job of adding, subtracting, etc. of the two alternating quantities is tedious and time consuming in terms of their mathematical equations. Hence, it is necessary to study a method which gives an easier way of representing an alternating quantity. Such a representation is called **phasor representation** of an alternating quantity.

The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in this method. The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a **phasor.**

Key Point: *The phasors are assumed to be rotated in anticlockwise direction.*

One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.

Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. 6.18. If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as $\theta = \omega t$), we get a sine waveform.

Consider the various positions shown in the Fig. 6.18.

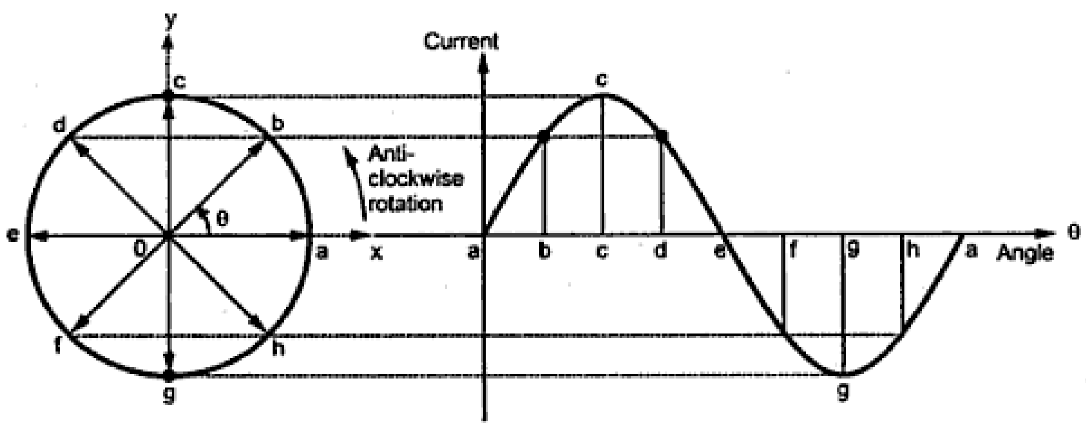


Fig. 6.18 Phasor representation of an alternating quantity

Phase : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

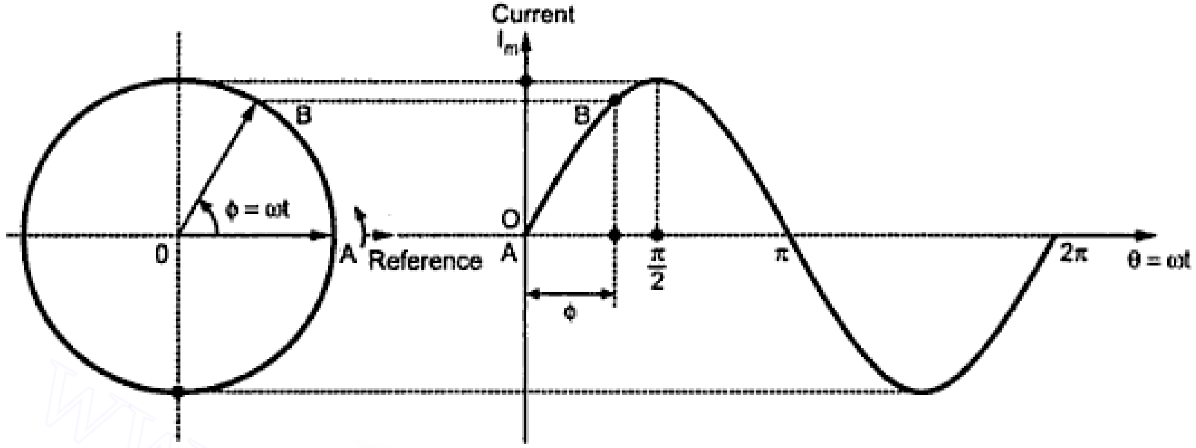


Fig. 6.19 Concept of phase

Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. 6.20 at the instant A is $\phi = 0°$. While the phase of the current at the instant B is the angle ϕ through which the phasor has travelled, measured from the reference axis i.e. X-axis.

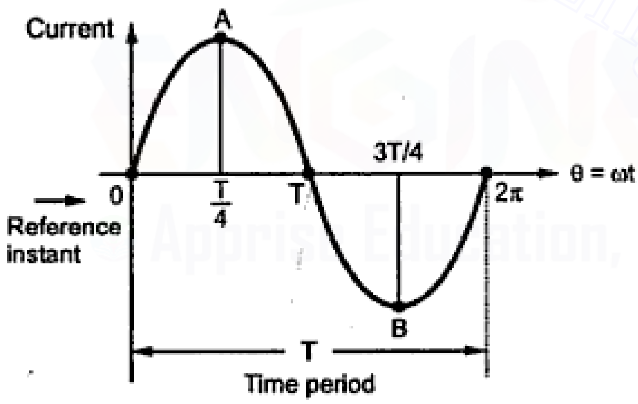


Fig. 6.20

In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0°$ to $360°$.

Another way of defining the phase is in terms of a time period T. The phase of an alternating quantity at any particular instant is the fraction of the time period (T) through which the quantity is advanced from the reference instant.

Consider alternating quantity represented in the Fig. 6.20. As per above definition, the phase of quantity at instant A is $\frac{T}{4}$, while phase at instant B is $\frac{3T}{4}$. Generally, the phase is expressed in terms of angle ϕ which varies from 0 to 2π radians and measured with respect to positive x-axis direction.

In terms of phase the equation of alternating quantity can be modified as,

$$\boxed{e = E_m \sin(\omega t \pm \phi)}$$

where ϕ = Phase of the alternating quantity.

Let us consider three cases;

Case 1 : $\phi = 0°$

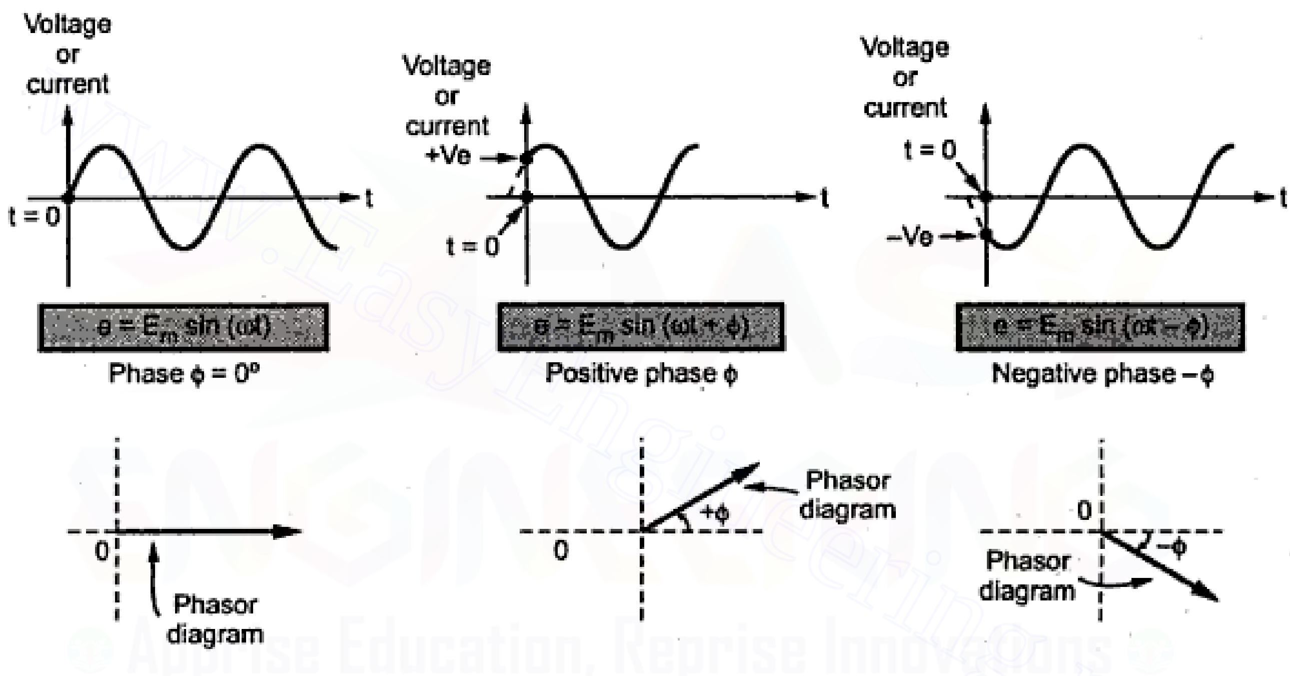
When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig. 6.21 (a).

Case 2 : Positive phase ϕ

When phase of an alternating quantity is **positive** it means that quantity has some **positive instantaneous value at t = 0.** This is shown in the Fig. 6.21 (b).

Case 3 : Negative phase ϕ

When phase of an alternating quanting is **negative** it means that quantity has some **negative instantaneous value at t = 0.** This is shown in the Fig. 6.21 (c).



(a) Zero phase (b) Positive phase (c) Negative phase

Fig. 6.21 Concept of phase

1. The **phase** is measured with respect to **reference direction** i.e. positive x axis direction.
2. The **phase** measured in **anticlockwise** direction is **positive** while the **phase** measured in **clockwise** direction is **negative.**

6.13.1 Phase Difference

Consider the two alternating quantities having same frequency f Hz having different maximum values.

$$e = E_m \sin(\omega t)$$

and $$i = I_m \sin(\omega t)$$

where $$E_m > I_m$$

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin(\omega t + \phi)$$

'i' is said to lead 'e' by angle ϕ

Key Point : *Thus, related to the phase difference, it can be remembered that a plus (+) sign of angle indicates lead where as a minus (–) sign of angle indicates lag with respect to the reference.*

6.13.2 Phasor Diagram

The diagram in which different alternating quantities of the same frequency, sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as **phasor diagram.**

The phasors are rotating in anticlockwise direction with an angular velocity of $\omega = 2\pi f$ rad/sec. Hence, all phasors have a particular fixed position with respect to each other.

Key Point : *Hence, phasor diagram can be considered as a still picture of these phasors at a particular instant.*

To clear this point, consider two alternating quantities in phase with each other.

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin \omega t$$

At any instant, phase difference between them is zero i.e. angle difference between the two phasors is zero. Hence, the phasor diagram for such case drawn at different instants will be alike giving us the same information that two quantities are in phase. The phasor diagram drawn at different instants are shown in the Fig. 6.25.

Fig. 6.25 Same phasor diagram at different instants

Consider another example where current i is lagging voltage e by angle ϕ So, difference between the angles of the phasors representing the two quantities is angle ϕ

$$e = E_m \sin \omega t$$

and $$i = I_m \sin(\omega t - \phi)$$

The phasor diagram for such case, at various instants will be same, as shown in the Fig. 6.26 (a), (b) and (c).

The phasor diagram drawn at any instant gives the same information.

Key Point : *Remember that the lagging and leading word is relative to the reference. In the above case, if we take current as reference, we have to say that the voltage leads current by angle ϕ . The direction of rotation of phasors is always anticlockwise.*

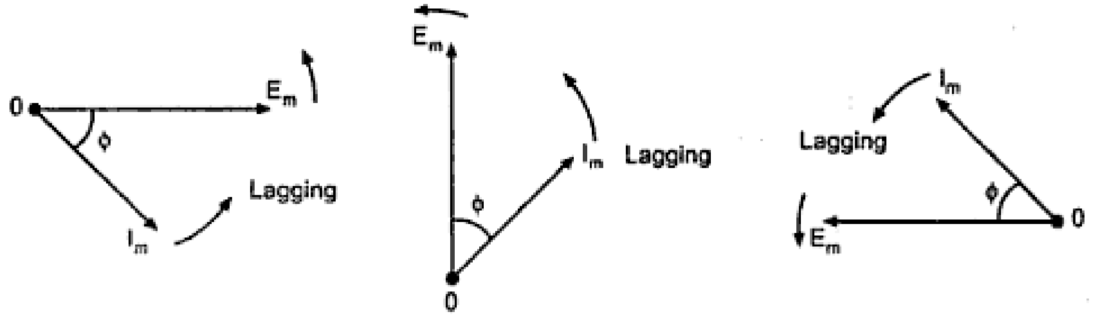


Fig. 6.26

Important Points Regarding Phasor Diagram :

1) As phasor diagram can be drawn at any instant, X and Y axis are not included in it. But, generally, the reference phasor chosen is shown along the positive X axis direction and at that instant other phasors are shown. This is just from convenience point of view. **The individual phase of an alternating quantity is always referred with respect to the positive x-axis direction.**

2) There may be more than two quantities represented in phasor diagram. Some of them may be current and some may be voltages or any other alternating quantities like flux, etc. The frequency of all of them must be the same.

3) Generally, length of phasor is drawn equal to r.m.s. value of an alternating quantity, rather than maximum value.

4) The phasors which are ahead, in anticlockwise direction, with respect to reference phasor are said to be leading with respect to reference and phasors behind are said to be lagging.

5) Different arrow heads may be used to differentiate phasors drawn for different alternating quantities like current, voltage, flux, etc.

Example 6.8 : *Two sinusoidal currents are given by,*
$i_1 = 10 \sin (\omega t + \pi/3)$ *and*

$i_2 = 15 \sin (\omega t - \pi/4)$
Calculate the phase difference between them in degrees.

Solution : The phase of current i_1 is $\pi/3$ radians i.e. 60° while the phase of the current i_2 is $-\pi/4$ radians i.e. $-45°$. This is shown in the Fig. 6.27.

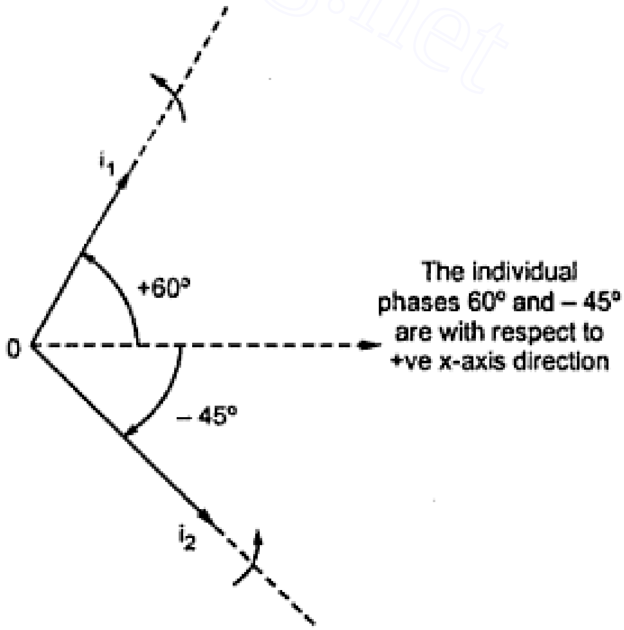


Fig. 6.27

Hence the phase difference between the two is,

$$\phi = \theta_1 - \theta_2 = 60° - (-45°) = 105°$$

And i_2 lags i_1.

Polar system,	$r \angle \pm \phi$ while	Rectangular system, $x \pm j\,y$	
and	$x = r\cos\phi$	$y = r\sin\phi$	...(1)
while	$r = \sqrt{x^2+y^2}$,	$\phi = \tan^{-1}\left(\frac{y}{x}\right)$	...(2)

The equations (1) and (2) can be used to convert rectangular form to polar or vice versa.

Such rectangular to polar and polar to rectangular conversion is often required in phasor mathematical operations like addition, subtraction, multiplication and division.

Key Point : *Instead of using above relations, students can use the polar to rectangular $(P \to R)$ and rectangular to polar $(R \to P)$ functions available on calculator for the required conversions.*

As the graphical method is time consuming which includes plotting the phasors to the scale, generally, analytical method is used. Also, graphical method may give certain error which will vary from person to person depending upon the skills of plotting the phasors. The answer by analytical method is always accurate.

Very Important :

The polar form of an alternating quantity can be easily obtained from its equation or phase as,

If $\quad e = E_m \sin(\omega t \pm \phi)$ then

E *in polar* $= E \angle \pm \phi$ *where* E = *r.m.s. value*

Example 6.9 : *Write the polar form of the voltage given by,*
$V = 100 \sin(100\,\pi\,t + \pi/6)$ V

Obtain its rectangular form.

Solution : $V_m = 100$ V and $\phi = +\frac{\pi}{6}$ rad $= +30°$, $\quad V_{rms} = \frac{V_m}{\sqrt{2}} = 70.7106$ V

$\therefore$ In polar form $= 70.7106 \angle +30°$ V

$\therefore$ Rectangular form $= 61.2371 + j\,35.3553$ V

Key Point: *The r.m.s. vlaue of an alternating quantity exists in its polar form and not in rectangular form. Thus to find r.m.s. value of an alternating quantity express it in polar form.*

Example 6.10 : *Find r.m.s. value and phase of the current $I = 25 + j\,40$ A.*

Solution : The r.m.s. value is not 25 or 40 as it exists in polar form.

Converting it to polar form,

$$I = 47.1699\angle 57.99° \text{A} = I_{rms} \angle \phi \text{ A}$$

$\therefore$ r.m.s. value of current = **47.1699 A**

$$\text{Phase} = 57.99°$$

Key Point: *To obtain phase, express the equation in sine form if given in cosine as,*

If $\quad e = E_m \cos(\omega t)$

then $\quad e = E_m \sin(\omega t + 90)$ as $\sin(90 + \theta) = \cos\theta$

Thus the phase is 90° and not zero.

In general, $\quad e = E_m \cos(\omega t \pm \phi)$

then $\quad e = E_m \sin(\omega t + 90 \pm \phi)$

$\therefore \quad$ The phase $= 90 \pm \phi$

Example 6.11 : *A voltage is defined as $-E_m \cos \omega t$. Express it in polar form.*

Solution : To express a voltage in polar form express it in the form, $e = E_m \sin \omega t$

Now $\quad e = -E_m \cos \omega t = -E_m \sin\left(\omega t + \frac{\pi}{2}\right)$ as $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$

$\quad = E_m \sin\left(\omega t + \frac{3\pi}{2}\right)$ as $\sin(\pi + \theta) = -\sin\theta$

Now it can be expressed in polar form as,

$$e = E_m \angle + \frac{3\pi}{2} \text{ rad} = E_m \angle + 270° \text{ V}$$

But + 270° phase is nothing but − 90°

$$\therefore \quad e = E_m \angle - 90° \text{ V}$$

Example 6.12 : *Find the resultant of the three voltages e_1, e_2 and e_3 where,*

$$e_1 = 20 \sin(\omega t), \quad e_2 = 30 \sin\left(\omega t - \frac{\pi}{4}\right) \text{ and } e_3 = \cos\left(\omega t + \frac{\pi}{6}\right)$$

Solution : Express all the voltages interms of $\sin(\omega t \pm \phi)$.

$$e_1 = 20 \sin(\omega t + 0°)$$

$$e_2 = 30 \sin\left(\omega t - \frac{\pi}{4}\right) = 30 \sin(\omega t - 45°)$$

$$\therefore \quad i = I_m \sin(\omega t - \phi) = 14.142 \sin\left(100\pi t - \frac{\pi}{3}\right) A$$

ii) $$I_m = 8 \text{ A, zero at } \omega t = -\frac{\pi}{6} \text{ rad} = -30^\circ$$

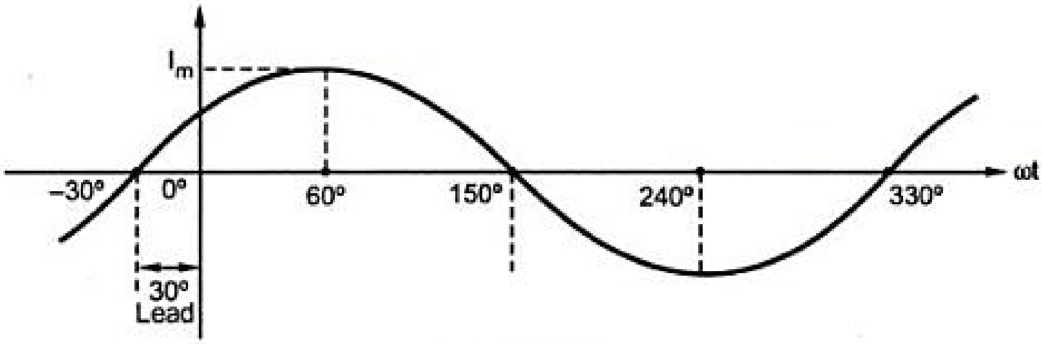


Fig. 6.31 (b)

$$\therefore \quad i = I_m \sin(\omega t + \phi)$$

$$= 8 \sin\left(100\pi t + \frac{\pi}{6}\right) A$$

6.15 Multiplication and Division of Phasors

In the last section, the addition and subtraction of phasors is discussed, which is to be carried out using rectangular form of phasors. But the rectangular form is not suitable to perform multiplication and division of phasors. Hence multiplication and division **must be performed using polar form** of the phasors.

Let P and Q be the two phasors such that,

$$P = x_1 + jy_1 \quad \text{and} \quad Q = x_2 + jy_2$$

To obtain the multiplication $P \times Q$ both must be expressed in polar form

$$\therefore \quad P = r_1 \angle \phi_1 \quad \text{and} \quad Q = r_2 \angle \phi_2$$

Then $$\boxed{P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2] = [r_1 \times r_2] \angle \phi_1 + \phi_2}$$

Key Point : *Thus in multiplication of complex numbers in polar form, the magnitudes get multiplied while their angles get added.*

The result then can be expressed back to rectangular form, if required. Now consider the division of the phasors P and Q.

$$\boxed{\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2}$$

Key Point: *Thus in division of complex numbers in polar form, the magnitudes get divided while their angles get subtracted.*

Remember :

While addition and subtraction, use rectangular form.

While multiplication and division, use polar form.

Examples with Solutions

Example 6.15 : *The mathematical expression for the instantaneous value of an alternating current is* $i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$ *A. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the wave-form from* $t = 0$ *over one complete cycle.* **(Dec.-99, 2006)**

Solution : The given current is, $i = 7.071 \sin\left(157.08\,t - \frac{\pi}{4}\right)$ ... Amp

Comparing this with, $i = I_m \sin(\omega t - \phi)$ ... Amp

We get, $I_m = 7.071$ A

$$\text{Effective value} = \frac{I_m}{\sqrt{2}} = I_{rms}$$

$$\therefore \quad I_{rms} = \frac{7.071}{\sqrt{2}} = \mathbf{5\ A}$$

$$\omega = 157.08 \quad \text{i.e.} \quad \frac{2\pi}{T} = 157.08$$

$$\therefore \quad T = \frac{2\pi}{157.08} = 0.04 \text{ sec} = \mathbf{40\ msec}$$

Positive maximum is, $i = I_m = +7.071$ A

$$\therefore \quad 7.071 = 7.071 \sin\left(157.08\,t - \frac{\pi}{t}\right)$$

$$\therefore \quad \text{Sin}\left(157.08\,t - \frac{\pi}{t}\right) = 1$$

$$\therefore \quad 157.08\,t - \frac{\pi}{t} = 1.5707 \qquad \text{Use radian mode}$$

$$\therefore \quad 157.08\ t = 2.3561$$

$$\therefore \quad t = 0.015 \text{ sec} = \mathbf{15\ msec}$$

Example 6.17 : *An alternating current is represented by the expression,*

$i = 10 \sin\left(2\pi \times 60 \times t - \frac{\pi}{6}\right)$ ampere. Find its periodic time. Also find (i) its instantaneous value at t = 0, (ii) time 't' at which it first reaches zero value after t = 0, and (iii) time at which it first reaches its negative maximum value after t = 0. Draw a neat sketch of its wave form for one cycle from time t = 0, and indicate in it the coordinates of the above three points.

(Dec. - 2002)

Solution : $i = 10 \sin\left(2\pi \times 60 \times t - \frac{\pi}{6}\right) A$

Comparing with, $i = I_m \sin(2\pi f \times t - \phi) A$

$\therefore \quad f = 60 \text{ Hz} \quad \text{hence} \quad T = \frac{1}{f} = 0.01666 \text{ sec}$

i) At $t = 0, \quad i = -5$ A ... Use radian mode for sin

ii) It first reaches zero value after t = 0 when,

$$0 = 10 \sin\left(2\pi \times 60t - \frac{\pi}{6}\right)$$

$$\therefore \quad 0 = 2\pi \times 60t - \frac{\pi}{6}$$

$$\therefore \quad t = \frac{\pi}{6 \times 2\pi \times 60} = 1.388 \text{ m sec}$$

iii) It reaches negative maximum after $\frac{3}{4}$ time period from the time at which it has achieved zero value.

$\therefore \quad t$ = time at which it will achieve negative maximum

= t at which it is zero + $\frac{3}{4}$ T

$= 1.3888 \text{ msec} + \frac{3}{4} \times 0.01666$

= **0.01388 sec**

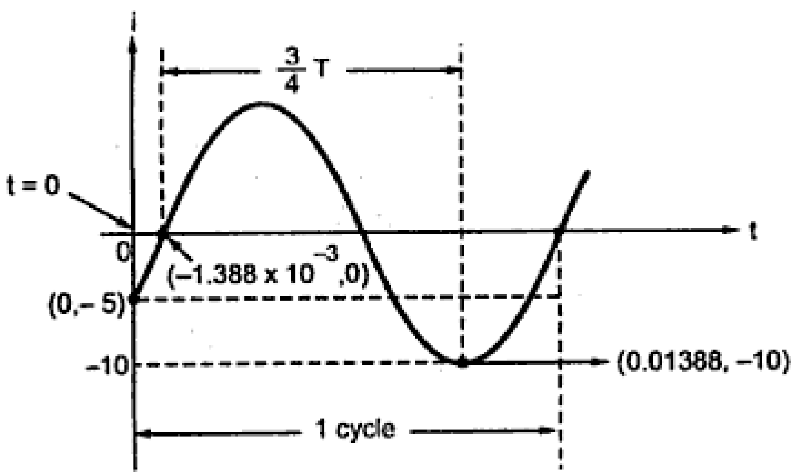


Fig. 6.35

Solution : i) The resultant is shown in the Fig. 6.37.

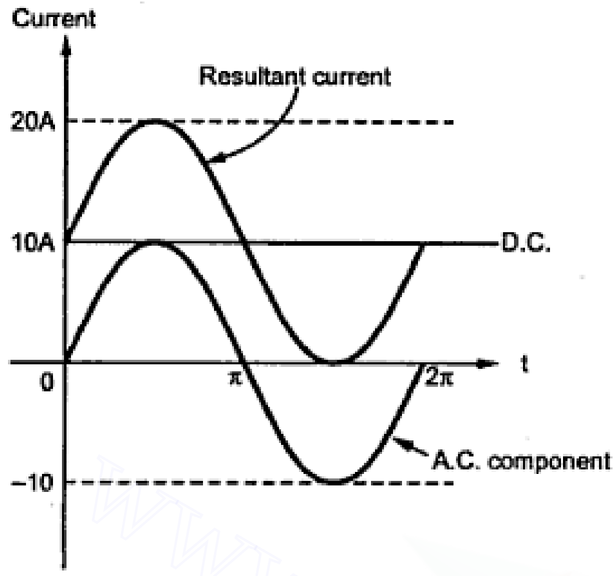


Fig. 6.37

ii) For d.c., $I_{dc} = 10$ A

For a.c. $i = I_m \sin\theta = 10 \sin\theta$

So the resultant is,

$$i_R = I_{dc} + i = 10 + 10\sin\theta$$

This is the expression for the resultant wave.

iii) Now $i_R = 10 + 10 \sin\theta$

The average value can be obtained as,

$$i_R(\text{average}) = \frac{1}{2\pi}\int_0^{2\pi} i_R \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} [10 + 10\sin\theta]\, d\theta$$

$$= \frac{1}{2\pi}[10\theta - 10\cos\theta]_0^{2\pi}$$

$$= \frac{1}{2\pi}[10(2\pi - 0) - 10(\cos 2\pi - \cos 0)]$$

$$= \textbf{10 A}$$

iv) The r.m.s. value is given by,

$$i_R\,(\text{r.m.s.}) = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_R^2 \, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\theta)^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [100 + 200\sin\theta + 100\sin^2\theta]\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \left[100 + 200\sin\theta + 100\left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[100\,\theta - 200\cos\theta + 100\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_0^{2\pi}}$$

$$= \sqrt{\frac{1}{2\pi}\left[100(2\pi - 0) - 200(\cos 2\pi - \cos 0) + \frac{100}{2}(2\pi - 0) - \frac{100}{4}(\sin 4\pi - \sin 0)\right]}$$

$$= \sqrt{\frac{1}{2\pi}[300 \times 0\,\pi]} = \sqrt{150} = \textbf{12.2474 A}$$

v) Form factor $= \dfrac{\text{r.m.s.}}{\text{average}} = \dfrac{12.2474}{10} = \mathbf{1.2247}$

Peak factor $= \dfrac{\text{maximum}}{\text{r.m.s.}} = \dfrac{20}{12.2474} = \mathbf{1.633}$

Example 6.20 : *Calculate the average and effective values of the saw tooth waveform shown in Fig. 6.38.*

The voltage completes the cycle by falling back to zero instantaneously after regular interval of time.

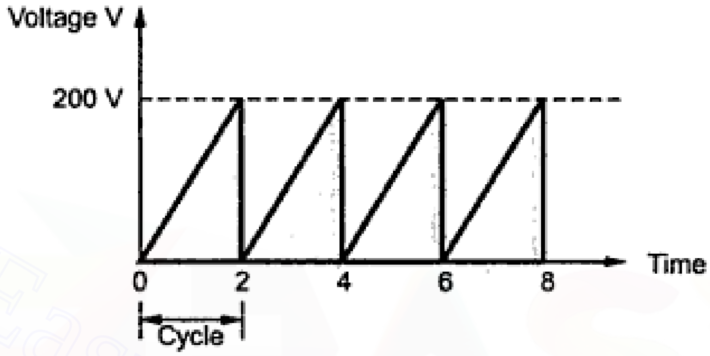


Fig. 6.38

Solution : Let us calculate equation for the instantaneous value of the voltage. The voltage increases linearly from 0 to 200 V in two seconds. So slope between 0 to 2 seconds is,

$$= \frac{200-0}{2} = 100$$

$\therefore$ Equation for the instantaneous value is,

$$V = 100\ t$$

The average value $= \dfrac{\text{Area under curve}}{\text{base}} = \int_0^2 \dfrac{(100\ t)\ dt}{2}$

$$= \frac{1}{2}\left[100\frac{t^2}{2}\right]_0^2 = 50 \times 2 = \mathbf{100\ volts.}$$

The r.m.s. value = root of the mean of square

$$= \sqrt{\frac{\int_0^2 (100\ t)^2\ dt}{2}} = \sqrt{\frac{\frac{1}{2}\times(100)^2\times\left[\frac{t^3}{3}\right]_0^2}{2}}$$

$$= \sqrt{5000\times\frac{8}{3}} = \mathbf{115.47\ volts}$$

Example 6.21 : *Calculate the average value, r.m.s. value and form factor of the output of a half wave rectifier when input to rectifier is purely sinusoidal alternating current.*

(May-2008)

Solution : Input to rectifier $i = I_m \sin\theta$

Half-wave rectifier is one which rectifies the half cycle of the input applied and its output is as shown in the Fig. 6.39.

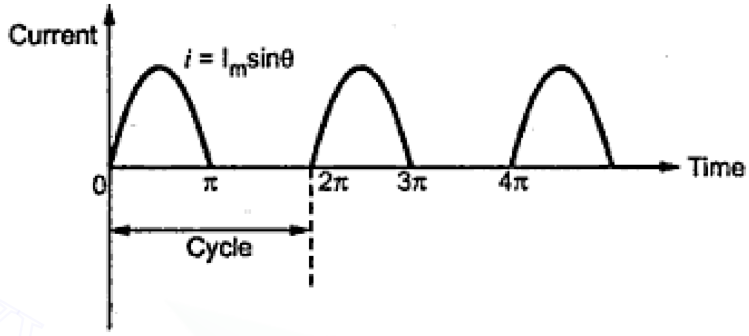


Fig. 6.39

Since the waveform is unsymmetrical, the average and r.m.s. values must be calculated for one complete cycle.

$$\text{The average value} = \frac{\text{Area of curve over a cycle}}{\text{Length of base over a cycle}}$$

$$= \frac{\int_0^{\pi} i\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{\pi} i\,d\theta$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\,d\theta = \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi} = \frac{I_m}{2\pi}[2] = \frac{I_m}{\pi}$$

$$I_{av} = 0.318\ I_m$$

R.M.S. value = root of mean of square

$$= \sqrt{\frac{\text{Area of curve over a square wave cycle}}{\text{Length of base over a cycle}}}$$

$$= \sqrt{\frac{\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} (0)^2 d\theta}{2\pi}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta} = \sqrt{\frac{I_m^2}{4\pi}\left[\frac{\theta - \sin 2\theta}{2}\right]_0^{\pi}}$$

Now $I_m = 9.2$ A and $f = 50$ Hz

$$\therefore \quad i = I_m \sin 2\pi ft = 9.2 \sin 100\pi t \text{ A}$$

a) At $t = 0.002$ sec,

$$i = 9.2 \sin(100\pi \times 0.002) = \mathbf{5.4076\ A} \quad \text{... Use sin in radians}$$

b) At $t = 0.0045$ sec after positive maximum as shown, time period $T = \frac{1}{f} = 0.02$ sec so positive maximum occurs at $t = \frac{T}{4} = \frac{0.02}{4} = 5\times10^{-3}$ sec. After this 0.0045 sec means value of i at $t = 5\times10^{-3} + 0.0045 = 9.5\times10^{-3}$ sec from t = 0.

$$\therefore \quad i = 9.2 \sin(100\pi \times 9.5\times10^{-3}) = \mathbf{1.4391\ A}$$

Example 6.25 : *At t = 0, the instantaneous value of a 60-Hz sinusoidal current is + 5 ampere and increases in magnitude further. Its r.m.s. value is 10-A.*
i) Write the expression for its instantaneous value.
ii) Find the current at t = 0.01 and t = 0.015 second.
iii) Sketch the wave form indicating these values. **(May-2004)**

Solution : t = 0, i = 5 A, f = 60 Hz, $I_{rms} = 10$ A

i) $I_m = \sqrt{2}\, I_{rms} = 10\sqrt{2}$ A = 14.1421 A

Let the equation for instantaneous value is,

$$i = I_m \sin(2\pi f t + \phi)$$

$$\text{Now} \quad 5 = 14.1421 \sin(2\pi \times 60 \times 0 + \phi)$$

$$\therefore \quad \phi = \sin^{-1}\frac{5}{14.1421} = 20.704° = 0.3613 \text{ rad}$$

$$\therefore \quad i = 14.1421 \sin(120\pi t + 20.704°) \text{ A}$$

$$\text{i.e.} \quad i = \mathbf{14.1421 \sin(120\pi t + 0.3613)\ A}$$

ii) To find i, calculate sin in radian mode.

At $t = 0.01$, $i = 14.1421 \sin(120\pi \times 0.01 + 0.3613) = \mathbf{-11.8202\ A}$

At $t = 0.015$, $i = 14.1421 \sin(120\pi \times 0.015 + 0.3613) = \mathbf{-3.7314\ A}$

iii) The waveform is as shown, (See Fig. 6.43 on next page)

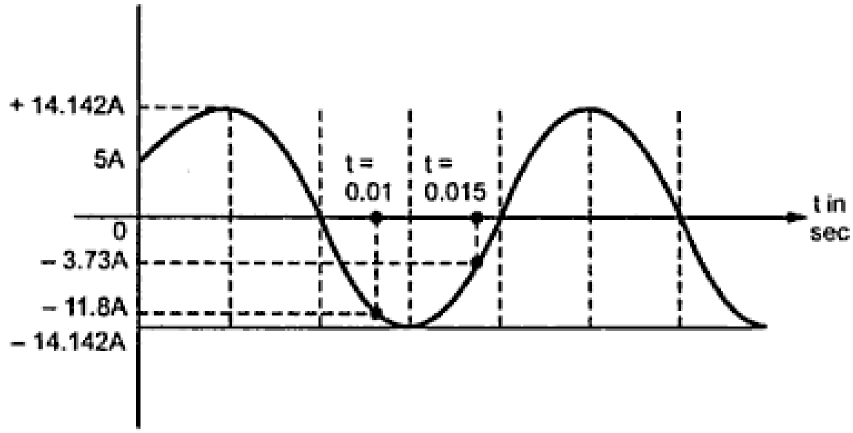


Fig. 6.43

Example 6.26 : *A 50-Hz sinusoidal voltage applied to a single phase circuit has its r.m.s. value of 200 V. Its value at t = 0 is $(\sqrt{2}\times 200)$ V positive. The current drawn by the circuit is 5 ampere (rms) and lags behind the voltage by one-sixth of a cycle. Write the expressions for the instantaneous values of voltage and current. Sketch their wave-forms, and find their values at t = 0.0125 second.* **(May-2004)**

Solution : $f = 50$ Hz, $V_{rms} = 200$ V

$$V_m = \sqrt{2}\ V_{rms} = 282.842 \text{ V}$$

The equation for voltage is $V = V_m \sin(2\pi f t + \phi)$

At t = 0, $V = 200\times\sqrt{2}$ V

$\therefore \quad 200\times\sqrt{2} = 282.842 \sin(0+\phi)$

$\therefore \quad \phi = \dfrac{\pi}{2}$ rad $= 1.5707$ rad $= 90°$

$\therefore \quad \mathbf{V = 282.842 \sin(100\pi t + 1.5707)\ V}$

Now $I_{rms} = 5$A hence $I_m = \sqrt{2}\ I_{rms} = 7.071$ A

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ sec}$$

I lags by $\dfrac{1}{6}^{th}$ of cycle i.e. by $\dfrac{T}{6}$ i.e. $\dfrac{0.02}{6} = 3.333\times 10^{-3}$ sec

$\therefore$ I lags by angle $= \omega t = 2\pi f t = 100\pi \times 3.3333\times 10^{-3} = 1.04709$ rad

$\therefore \quad \theta = 1.04709$ rad. where θ = angle by which I lags V

$\therefore \quad i = I_m \sin(2\pi f t + \phi - \theta)$

$\therefore \quad i = 7.071 \sin(100\pi t + 1.5707 - 1.04709)$

$\therefore \quad \mathbf{i = 7.071 \sin(100\pi t + 0.5236)\ A}$

At t = 0.0125 sec, find v and i. **Use sin in radians.**

$\therefore \quad V = 282.842 \sin(100\pi \times 0.0125 + 1.5707) = \mathbf{-200\ V}$

$$i = 7.071 \sin(100\pi \times 0.0125 + 0.5236) = -6.83 \text{ A}$$

The waveforms are shown in the Fig. 6.44.

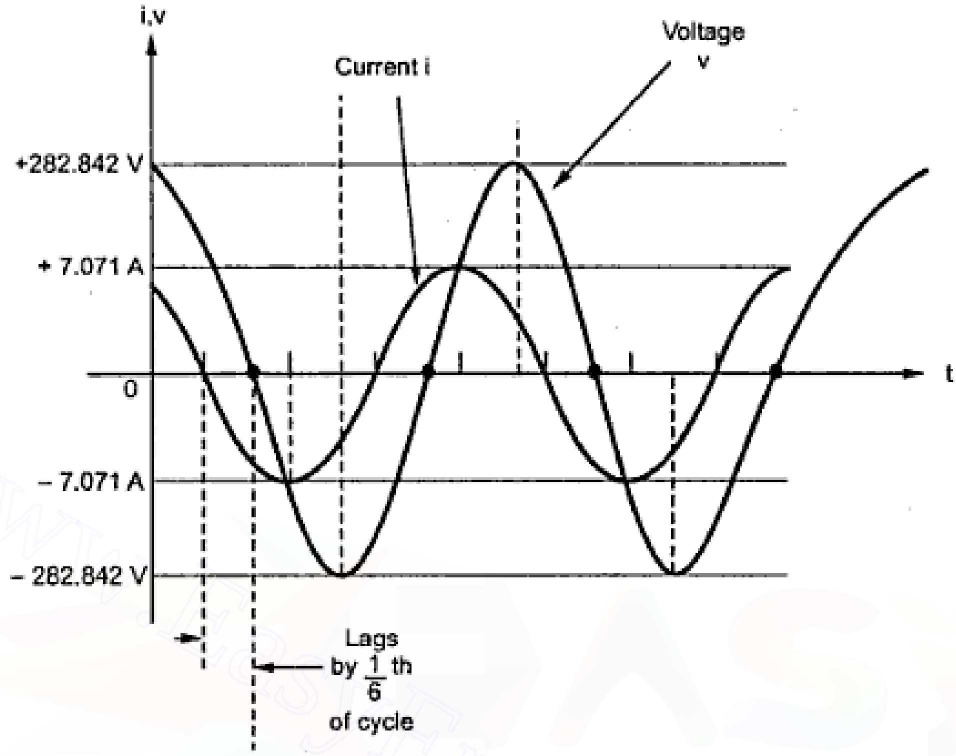


Fig. 6.44

Example 6.27 : *A sinusoidal current of frequency 25 Hz has a maximum value of 100 A. How long will it take for the current to attain value of 20 A and 50 A, starting from zero. Sketch the waveform and show the times and currents.* **(Dec.-2004)**

Solution : $f = 25$ Hz, $I_m = 100$ A

Thus equation for instantaneous value is,

$$i = I_m \sin(2\pi f t) \quad \text{...Starting from zero at } t = 0$$

$$\therefore \quad i = 100 \sin(50\pi t) \text{ A}$$

for i = 20 A, $$20 = 100 \sin(50\pi t_1)$$

$$\therefore \quad t_1 = \frac{\sin^{-1}\left(\frac{20}{100}\right)}{50\pi} \quad \text{... Calculate } \sin^{-1} \text{ in radians}$$

$$= \frac{0.20135}{50\pi} = 1.2818 \text{ ms}$$

for i = 50 A, $$50 = 100 \sin(50\pi t_2)$$

$$\therefore \quad t_2 = \frac{\sin^{-1}\left(\frac{50}{100}\right)}{50\pi} = \frac{0.52359}{50\pi} = 3.333 \text{ ms}$$

So resultant has peak value 14.9575 V and phase 52.3239°, so its equation is,

$$V_R = \mathbf{14.9575\ sin(\omega t + 52.3239°)V}$$

Example 6.29 : *In a certain circuit supplied from 50 Hz mains, the potential difference has a maximum value of 500 volt and the current has a maximum value of 10 Amp. At the instant t = 0, the instantaneous vlaues of potential difference and current are 400 volt and 4 Amp respectively both increasing in positive direction. State expressions for instantaneous values of potential difference and current at time 't'. Calculate the instataneous values at time t = 0.015 second. Find phase angle between potential difference and current.*

(May-2005)

Solution : $f = 50$ Hz, $V_m = 500$ V, $I_m = 10$ A

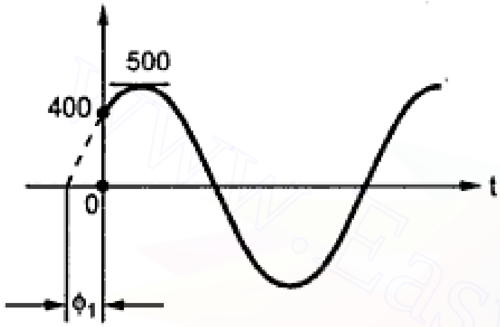


Fig. 6.46

It is given that t = 0, v = 400 V and not zero hence it has phase ϕ_1. So its equation is,

$$v = V_m \sin(\omega t + \phi_1)$$

So at $t = 0$,

$$400 = 500 \sin(0 + \phi_1)$$

$$\therefore \quad \phi_1 = \sin^{-1}\left(\frac{4}{5}\right) = 53.13° = \mathbf{0.9272\ rad}$$

$$\therefore \quad v = 500 \sin(2\pi f t + \phi_1) = \mathbf{500\ sin(100\ \pi\ t + 0.9272)V}$$

While at t = 0, i = 4 A hence it has phase ϕ_2. So its equation is,

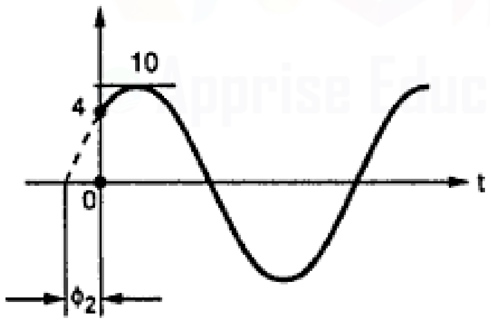


Fig. 6.47

$$i = I_m \sin(\omega t + \phi_2)$$

So at $t = 0$,

$$4 = 10 \sin(0 + \phi_2)$$

$$\therefore \quad \phi_2 = \sin^{-1}\left(\frac{4}{10}\right) = 23.5781° = \mathbf{0.4115\ rad}$$

$$\therefore \quad i = 10 \sin(2\pi f t + \phi_2)$$

$$\therefore \quad i = \mathbf{10\ sin(100\ \pi t + 0.4115)\ A}$$

At t = 0.015 sec, $v = 500 \sin(100\pi \times 0.015 + 0.9272)$ **...Use radian mode**

$$= \mathbf{-300.038\ V}$$

$$i = 10 \sin(100\pi \times 0.015 + 0.4115) = \mathbf{-9.1652\ A}$$

Example 6.31 : *An alternating current varying sinusoidally with a frequency of 50 Hz has a r.m.s. value of current of 20 Amp. At what time, measured from negative maximum value, instantaneous current will be $10\sqrt{2}$ Amp. ?* **(Dec.-2005)**

Solution : $I = 20$ A, $f = 50$ Hz

$\therefore \quad I_m = \sqrt{2}I = 20\sqrt{2} = 28.2842$ A

$\therefore \quad i = I_m \sin(2\pi ft) = 20\sqrt{2}\sin(100\pi t)$ A

The waveform is shown in the Fig. 6.50.

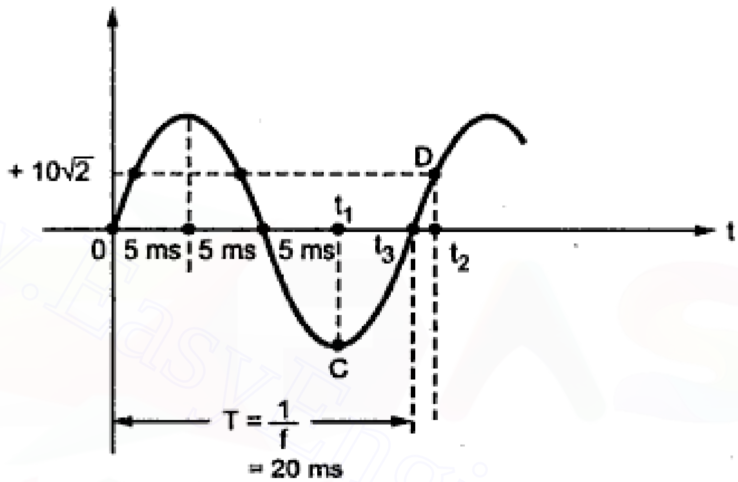


Fig. 6.50

To find time corresponding to point D after point C which is negative maximum.

At point C, $t_1 = 15$ ms

For $\quad i = 10\sqrt{2}$

$$10\sqrt{2} = 20\sqrt{2}\sin(100\pi t)$$

$\therefore \quad t = 1.66$ ms

Thus from t_3 and t_2 time is 1.66 ms and t_1 to t_3 is quarter half cycle i.e. 5 ms.

$\therefore$ Total time for current to achieve $10\sqrt{2}$ A from negative maximum value is,

$$t_1 \text{ to } t_2 = 5 + 1.66 = 6.666 \text{ ms}$$

Example 6.32 : *Draw a neat sketch in each case, of the waveform and write expression of Instantaneous Value for the following :* **(May-2006)**

1) Sinusoidal current of amplitude 10 A, 50 Hz passing through its zero value at $\omega t = \pi/3$ and rising positively.

2) Sinusoidal current of amplitude 8 A, 50 Hz passing through its zero value at $\omega t = -\pi/6$ and rising positively.

Solution : 1) $I_m = 10$ A, f = 50 Hz, $\omega = 2\pi f = 100\,\pi$ rad/sec

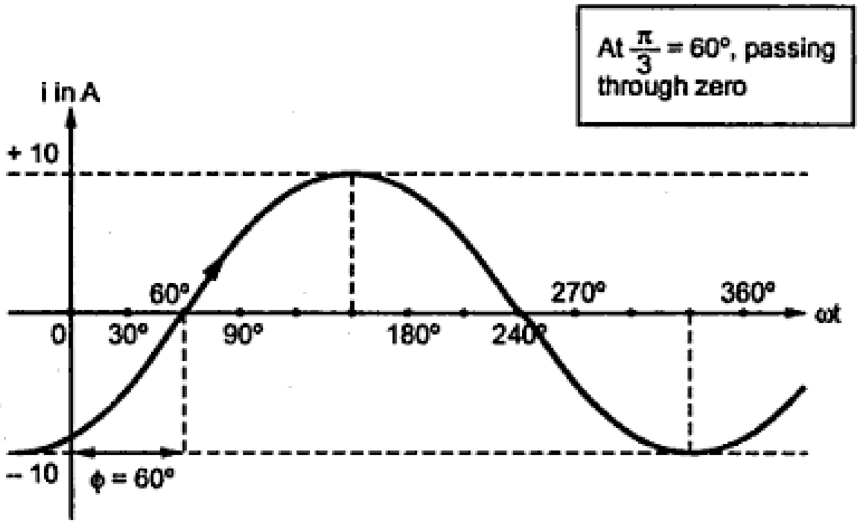


Fig. 6.51

$$\therefore \qquad i = I_m \sin(\omega t - \phi) = 10 \sin(100\pi t - 60°)$$

$$\therefore \qquad i = 10 \sin\left(100\pi t - \frac{\pi}{3}\right) \text{ A}$$

2) $I_m = 8$ A, $\omega = 100\,\pi$ rad/sec

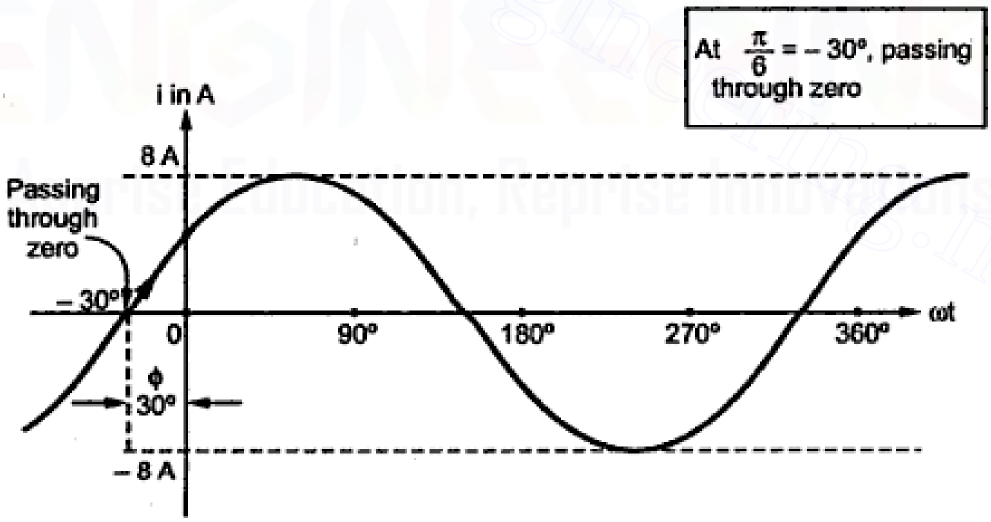


Fig. 6.52

$$\therefore \qquad i = I_m \sin(\omega t + 30°) = 8 \sin(100\pi t + 30°)$$

$$\therefore \qquad i = 8 \sin\left(100\pi t + \frac{\pi}{6}\right) \text{ A}$$

11. *Show that alternating quantity can be represented by a rotating phasor.*
12. *Sketch the waveform of a current $i = 10 \sin 314.159\, t$ amp. and find its maximum, r.m.s. and average values. Find also the frequency.*
13. *Estimate the average value, r.m.s. value and form factor if the instantaneous values of a voltage are 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, –5, – 10 volts etc at equal intervals, changing suddenly from one value to another.* **(Hint : Refer Ex. 6.5, Ans. : 23 V, 31.06 V, 1.3506) (6)**
14. *A sinusoidal voltage of 50 Hz has a maximum value of $200\sqrt{2}$ volts. At what time measured from a positive maximum value will the instantaneous voltage be equal to 141.4 volts ?* **(Ans. : 1.666 msec)**

❑❑❑

7 Single Phase A.C. Circuits

7.1 Introduction

The resistance, inductance and capacitance are three basic elements of any electrical network. In order to analyze any electric circuit, it is necessary to understand the following three cases,

1) A.C. through pure resistive circuit.
2) A.C. through pure inductive circuit.
3) A.C. through pure capacitive circuit.

In each case, it is assumed that a purely sinusoidal alternating voltage given by the equation $v = V_m \sin(\omega t)$ is applied to the circuit. The equation for the current, power and phase shift are developed in each case. The voltage applied having zero phase angle is assumed reference while plotting the phasor diagram in each case.

Once the behaviour of pure R, L and C is discussed, then the various series and parallel combinations of R, L and C are discussed in this chapter. The concept of impedance, admittance, susceptance, phasor diagrams of series and parallel circuits and resonance in series and parallel circuits are also included in this chapter.

7.2 A.C. through Pure Resistance

Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$. The circuit is shown in the Fig. 7.1.

According to Ohm's law, we can find the equation for the current i as,

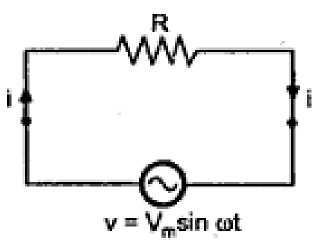


Fig. 7.1 Pure resistive circuit

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \quad \text{i.e.} \quad i = \left(\frac{V_m}{R}\right) \sin(\omega t)$$

This is the equation giving instantaneous value of the current.

Comparing this with standard equation,

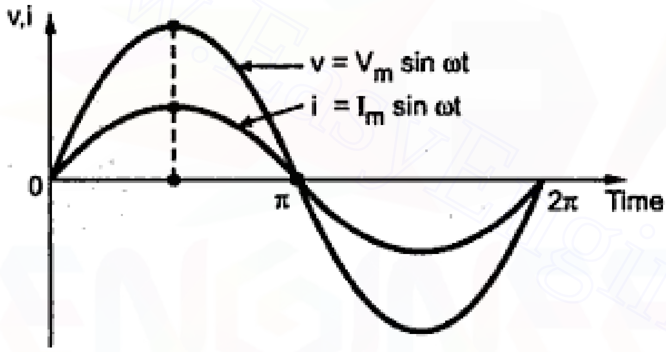
$$i = I_m \sin(\omega t + \phi)$$

$$\boxed{I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0}$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while, as $\phi = 0$, it indicates that it is in phase with the voltage applied. There is no phase difference between the two. The current is going to achieve its maximum (positive and negative) and zero whenever voltage is going to achieve its maximum (positive and negative) and zero values.

Key Point : *In purely resistive circuit, the current and the voltage applied are in phase with each other.*

The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. 7.2 (a) and (b).



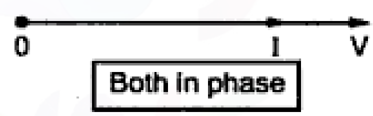


Fig. 7.2 A.C. through purely resistive circuit

In the phasor diagram, the phasors are drawn in phase and there is no phase difference in between them. Phasors represent the **r.m.s. values** of alternating quantities.

7.2.1 Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin \omega t = V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos(2\omega t)}$$

From the above equation, it is clear that the instantaneous power consists of two components,

1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

2) Fluctuating component $\left[\frac{V_m I_m}{2}\cos(2\,\omega t)\right]$ having frequency, double the frequency of the applied voltage.

Now, the average value of the fluctuating cosine component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$.

$$\therefore \qquad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\therefore \qquad \boxed{P_{av} = V_{rms} \times I_{rms} \quad \text{watts}}$$

Generally, r.m.s. values are indicated by capital letters

$$\therefore \qquad \boxed{P_{av} = V \times I \quad \text{watts} = I^2 R \quad \text{watts}}$$

The Fig. 7.3 shows the waveforms of voltage, current and power.

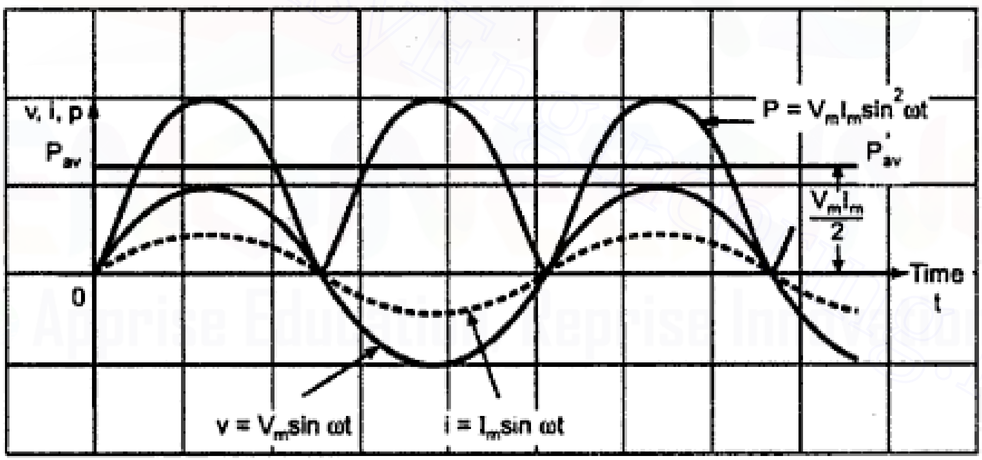


Fig. 7.3 v, i and p for purely resistive circuit

7.3 A.C. through Pure Inductance

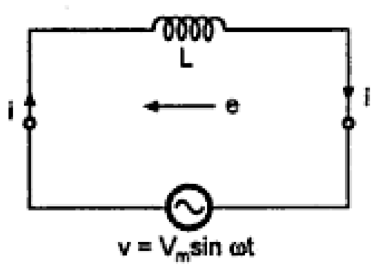


Fig. 7.4 Purely inductive circuit

Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 7.4.

Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries (H).

When alternating current 'i' flows through inductance 'L', it sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance, e.m.f. gets induced in the coil. This e.m.f. opposes the applied voltage.

The self induced e.m.f. in the coil is given by,

Self induced e.m.f., $$e = -L\frac{di}{dt}$$

At all instants, applied voltage, V is equal and opposite to the self induced e.m.f., e

$$\therefore \quad v = -e = -\left(-L\frac{di}{dt}\right)$$

$$\therefore \quad v = L\frac{di}{dt} \quad \text{i.e. } V_m \sin \omega t = L\frac{di}{dt}$$

$$\therefore \quad di = \frac{V_m}{L}\sin \omega t\, dt$$

$$\therefore \quad i = \int di = \int \frac{V_m}{L}\sin \omega t\, dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right) \quad \text{as } \cos \omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$\therefore \quad i = \frac{V_m}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \quad \text{as } \sin\left(\frac{\pi}{2}-\omega t\right) = -\sin\left(\omega t-\frac{\pi}{2}\right)$$

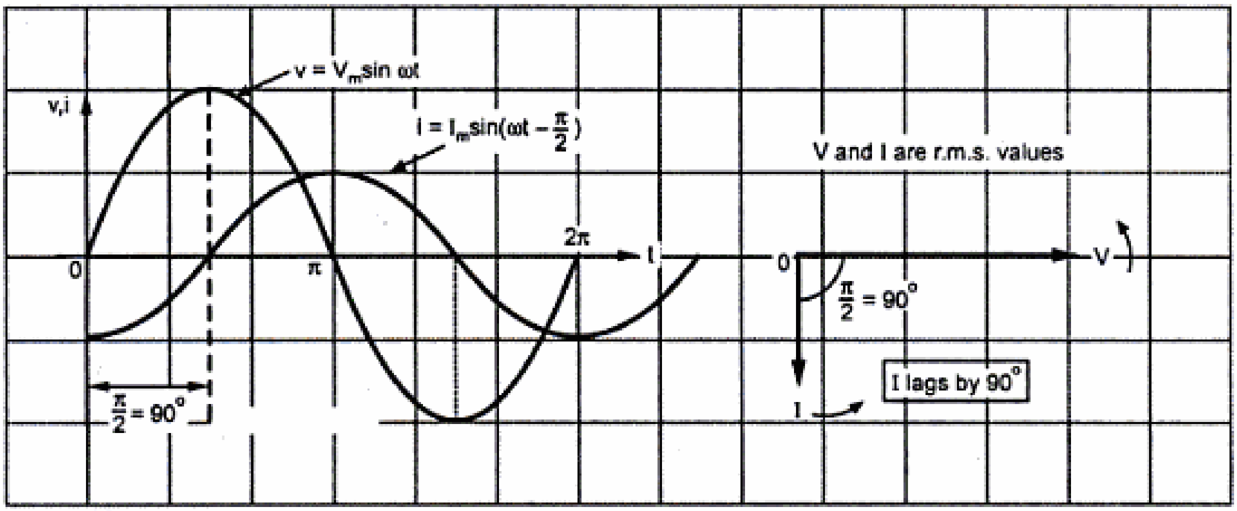
$$\therefore \quad \boxed{i = I_m \sin\left(\omega t-\frac{\pi}{2}\right)}$$

Where $$I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$$

Where $$\boxed{X_L = \omega L = 2\pi f L \ \Omega}$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. $-90°$. **This means that the current lags voltage applied by 90°.** The negative sign indicates lagging nature of the current. If current is assumed as a reference, we can say that the voltage across inductance leads the current passing through the inductance by 90°.

The Fig. 7.5 shows the waveforms and the corresponding phasor diagram.



(a) Waveforms (b) Phasor diagram

Fig. 7.5 A.C. through purely inductive circuit

Key Point: *In purely inductive circuit, current lags voltage by 90°.*

7.3.1 Concept of Inductive Reactance

We have seen that in purely inductive circuit,

$$I_m = \frac{V_m}{X_L}$$

where $\boxed{X_L = \omega L = 2\pi f L\ \Omega}$

The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**

So, **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.

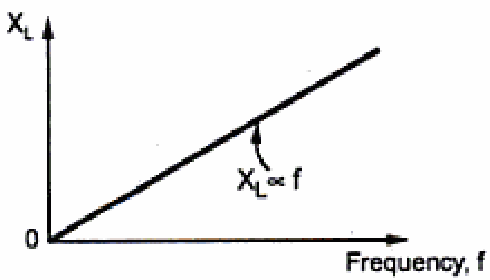


Fig. 7.6 X_L Vs f

It is measured in ohms and it depends on the frequency of the applied voltage.

The inductive reactance is directly proportional to the frequency for constant L.

$X_L \propto f$, for constant L

So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. 7.6.

Key Point : *If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.*

7.3.2 Power

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t)\cos(\omega t) \quad \text{as } \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos \omega t$$

$$\therefore \quad \boxed{P = -\frac{V_m I_m}{2}\sin(2\omega t)} \quad \text{as } 2\sin \omega t \cos \omega t = \sin 2\omega t$$

Key Point : *This power curve is a sine curve of frequency double than that of applied voltage.*

The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2}\sin(2\omega t)\, d(\omega t) = 0$$

The Fig. 7.7 shows voltage, current and power waveforms.

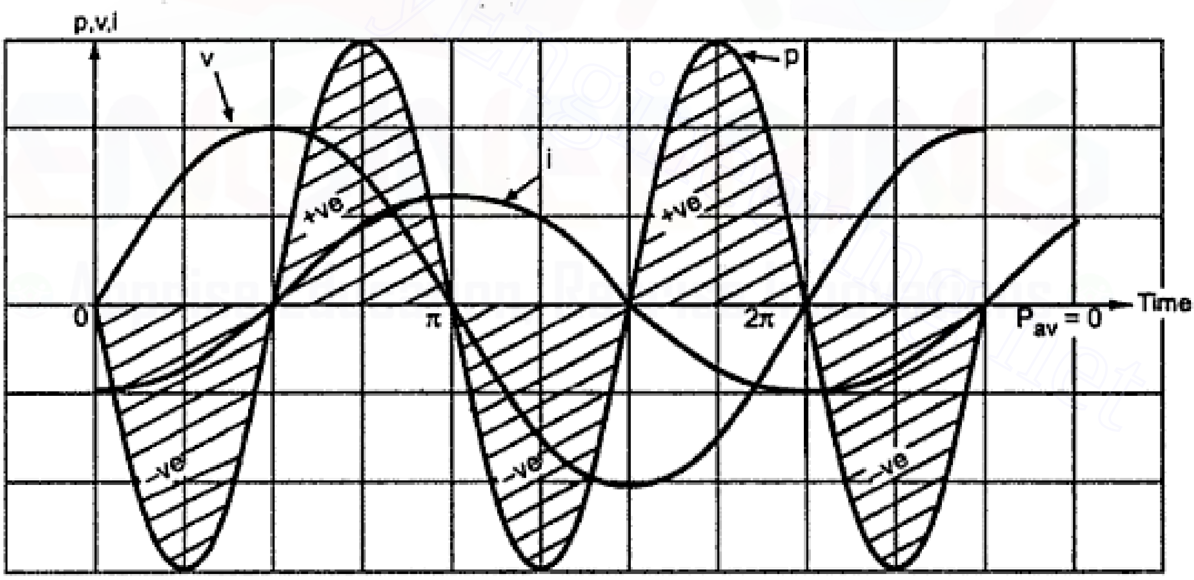


Fig. 7.7 Waveforms of voltage, current and power

It can be observed from it that when power curve is positive, energy gets stored in the magnetic field established due to the increasing current while during negative power curve, this power is returned back to the supply.

The areas of positive loop and negative loop are exactly same and hence, average power consumption is zero.

Key Point : *Pure inductance never consumes power.*

7.4 A.C. through Pure Capacitance

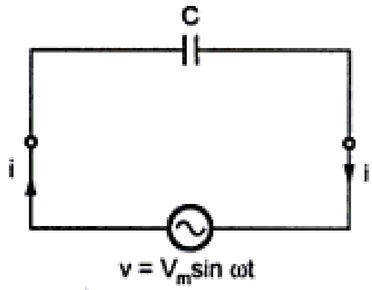


Fig. 7.8 Purely capacitive circuit

Consider a simple circuit consisting of a pure capacitor of C– farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 7.8.

The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C v$$

$$\therefore \quad q = C V_m \sin \omega t$$

Now, current is rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin \omega t)$$

$$\therefore \quad i = C V_m \frac{d}{dt}(\sin \omega t) = C V_m \omega \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \quad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $$\boxed{I_m = \frac{V_m}{X_C}}$$

where $$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \Omega}$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.

This means **current leads voltage applied by 90°**. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor lags the current passing through the capacitor by 90°.

The Fig. 7.9 shows waveforms of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

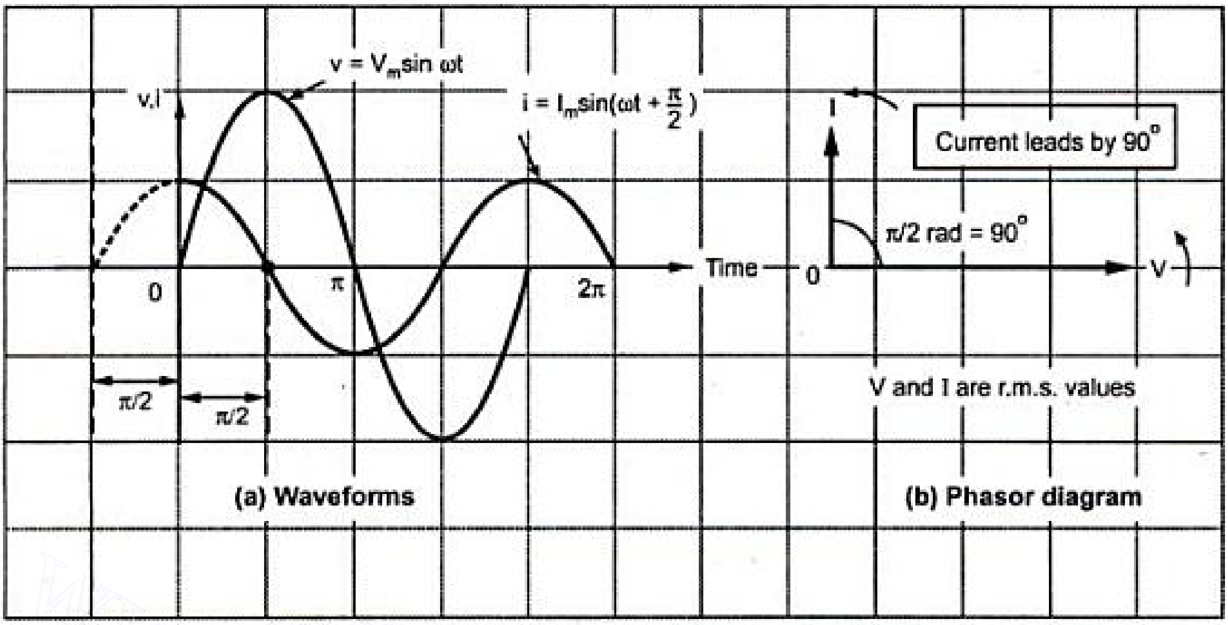


Fig. 7.9 A.C. through purely capacitive circuit

Key Point : *In purely capacitive circuit, current leads voltage by 90°.*

7.4.1 Concept of Capacitive Reactance

We have seen while expressing current equation in the standard form that,

$$I_m = \frac{V_m}{X_C} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\,\Omega$$

The term X_C is called **Capacitive Reactance** and is measured in ohms.

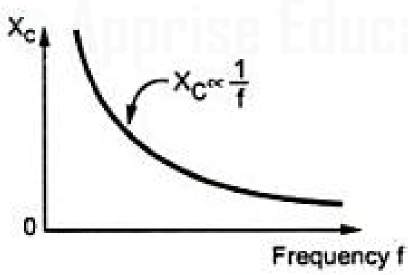


Fig. 7.10 X_C Vs f

So, **capacitive reactance** is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.

X_C is measured in ohms and it depends on the frequency of the applied voltage.

The capacitive reactance is inversely proportional to the frequency for constant C.

$$X_C \propto \frac{1}{f} \quad \text{for constant C}$$

The graph of X_C Vs f is a rectangular hyperbola as shown in Fig. 7.10.

Key Point : *If the frequency is zero, which is so for d.c. voltage, the capacitive reactance is infinite. Therefore, it is said that the capacitance offers open circuit to the d.c. or it blocks d.c.*

Example 7.1 : *A 50 Hz, alternating voltage of 150 V (r.m.s.) is applied independently to (i) Resistance of 10 Ω (2) Inductance of 0.2 H (3) Capacitance of 50 μF*

Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.

Solution : Case 1 : R = 10 Ω

$$V = V_m \sin \omega t$$

$$V_m = \sqrt{2}\, V_{r.m.s.} = \sqrt{2} \times 150 = 212.13 \text{ V}$$

$$I_m = \frac{V_m}{R} = \frac{212.13}{10}$$

$$= 21.213 \text{ A}$$

In pure resistive circuit, current is in phase with the voltage.

$$\therefore \quad \phi = \text{Phase Difference} = 0°$$

$$\therefore \quad i = I_m \sin \omega t = I_m \sin (2 \pi f t)$$

$$\therefore \quad i = 21.213 \sin (100 \pi t) \text{ A}$$

The phasor diagram is shown in the Fig. 7.12 (a).

Fig. 7.12 (a)

Case 2 : $L = 0.2 \ \Omega$

Inductive reactance, $X_L = \omega L = 2 \pi f L$

$$\therefore \quad X_L = 2 \pi \times 50 \times 0.2 = 62.83 \ \Omega$$

$$\therefore \quad I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.37 \text{ A}$$

In pure inductive circuit, current lags voltage by 90°.

$$\therefore \quad \phi = \text{Phase difference} = -90° = \frac{\pi}{2} \text{ rad}$$

$$\therefore \quad i = I_m \sin (\omega t - \phi)$$

$$\therefore \quad i = 3.37 \sin \left(100 \pi t - \frac{\pi}{2}\right) \text{A}$$

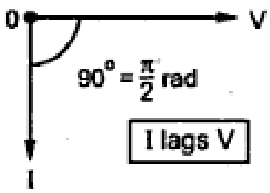


Fig. 7.12 (b)

The phasor diagram is shown in the Fig. 7.12 (b).

Case 3 : $C = 50\ \mu F$

Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi\times 50\times 50\times 10^{-6}} = 63.66\ \Omega$

$\therefore \quad I_m = \dfrac{V_m}{X_C} = \dfrac{212.13}{63.66} = 3.33\ A$

In pure capacitive circuit, current leads voltage by 90°.

$\therefore \quad \phi = \text{Phase Difference} = 90° = \dfrac{\pi}{2}\ \text{rad}$

$\therefore \quad i = I_m \sin(\omega t + \phi)$

$\therefore \quad i = 3.33 \sin\left(100\pi t + \dfrac{\pi}{2}\right) A$

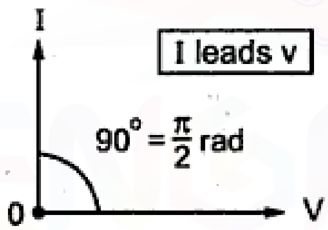


Fig. 7.12 (c)

The phasor diagram is shown in the Fig. 7.12 (c).

All the phasor diagrams represent r.m.s. values of voltage and current.

Example 7.2 : *A voltage $v = 141 \sin\{314t + \pi/3\}$ is applied to*

i) Resistor of 20 ohms ii) Inductance of 0.1 Henry iii) Capacitance of 100 μF

Find in each case rms value of current and power dissipated.

Draw the phasor diagram in each case. **(May-2003)**

Solution : Comparing given voltage with $v = V_m \sin(\omega t + \theta)$ we get,

$$V_m = 141\ V \text{ and hence } V = V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 99.702\ V$$

$$\omega = 314 \text{ and hence } f = \frac{\omega}{2\pi} = 50\ Hz,\ \theta = \frac{\pi}{3} = 60°$$

Hence the polar form of applied voltage becomes,

$$V = 99.702 \angle 60°\ V$$

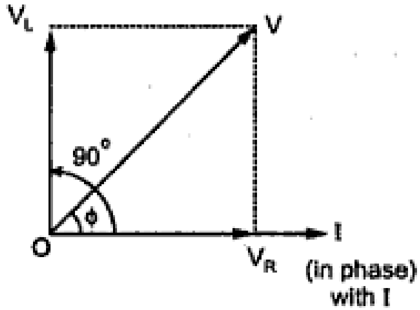


Fig. 7.14 (b) Phasor diagram

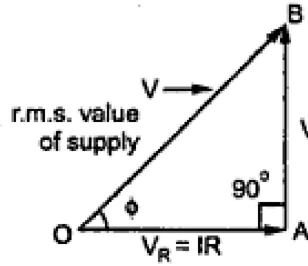


Fig. 7.14 (c) Voltage triangle

7.5.1 Impedance

Impedance is defined as the opposition of circuit to flow of alternating current. It is denoted by Z and its unit is ohms.

For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle φ From the voltage triangle, we can write,

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}$$

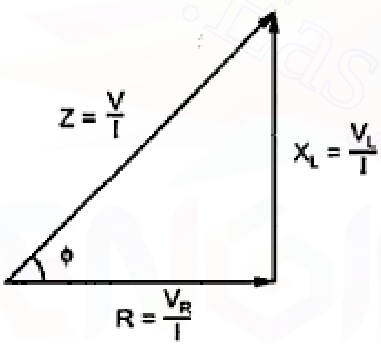


Fig. 7.15 Impedance triangle

If all the sides of the voltage triangle are divided by current, we get a triangle called impedance triangle as shown in the Fig. 7.15.

Sides of this triangle are resistance R, inductive reactance X_L and an impedance Z.

From this impedance triangle, we can see that the X component of impedance is R and is given by,

$$R = Z\cos\phi$$

and Y component of impedance is X_L and is given by,

$$X_L = Z\sin\phi$$

In rectangular form the impedance is denoted as,

$$\boxed{Z = R + j X_L \quad \Omega}$$

While in polar form, it is denoted as,

where

$$\boxed{\begin{gathered} Z = |Z| \angle\phi \quad \Omega \\ |Z| = \sqrt{R^2 + X_L^2}, \qquad \phi = \tan^{-1}\left[\frac{X_L}{R}\right] \end{gathered}}$$

Key Point: *Thus φ is positive for inductive impedance.*

7.5.2 Power and Power Triangle

The expression for the current in the series R-L circuit is,

$$i = I_m \sin(\omega t - \phi) \text{ as current lags voltage.}$$

The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - \phi) = V_m I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right] = \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\therefore \quad \boxed{P = V I \cos\phi \quad \text{watts}}$$ where V and I are r.m.s. values

If we multiply voltage equation by current I, we get the power equation.

$$\overline{VI} = \overline{V_R I} + \overline{V_L I}$$

$$\therefore \quad \overline{VI} = \overline{V\cos\phi I} + \overline{V \sin\phi I}$$

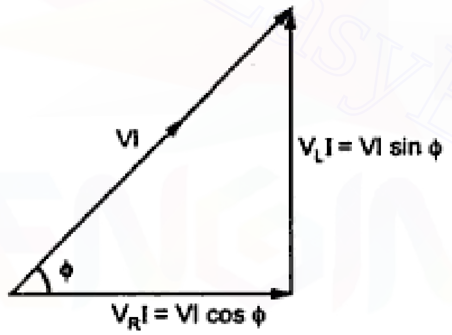


Fig. 7.16 Power triangle

From this equation, power triangle can be obtained as shown in the Fig. 7.16.

So, three sides of this triangle are,

1) VI, 2) VI $\cos\phi$, 3) VI $\sin\phi$

These three terms can be defined as below.

7.5.3 Apparent Power (S)

It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$$\therefore \quad \boxed{S = V I \quad \text{VA}}$$

It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

7.5.4 Real or True Power (P)

It is defined as the product of the applied voltage and the active component of the current.

It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

$$\boxed{P = V I \cos\phi \quad \text{watts}}$$

7.5.5 Reactive Power (Q)

It is defined as product of the applied voltage and the reactive component of the current.

It is also defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

$$\boxed{Q = V I \sin\phi \quad \text{VAR}}$$

Apparent power, $S = V I$ VA

True power $P = V I \cos\phi$ W (Average Power)

Reactive power $Q = V I \sin\phi$ VAR

7.5.6 Power Factor (cos ϕ)

It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

It is the ratio of true power to apparent power.

$$\boxed{\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{V I \cos\phi}{V I} = \cos\phi}$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than 1.

It is also defined as the ratio of resistance to the impedance.

$$\boxed{\cos\phi = \frac{R}{Z}}$$

Key Point: *The nature of power factor is always determined by position of current with respect to the voltage.*

If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. $\phi = 0$. Therefore, power factor is cos (0°) = 1. Such circuit is called unity power factor circuit.

$$\boxed{\text{Power factor} = \cos\phi}$$

ϕ is the angle between supply voltage and current.

Key Point: *Nature of power factor always tells position of current with respect to voltage.*

Example 7.3 : *An alternating current, i = 414 Sin (2 π × 50 × t) - A, is passed through a series circuit consisting of a resistance of 100-ohm and an inductance of 0.31831 henry. Find the expressions for the instantaneous values of the voltages across (i) the resistance, (ii) the inductance and (iii) the combination.* **(Dec.-2000)**

Solution : The circuit is shown in the Fig. 7.17.

$$i = 1.414 \sin(2\pi \times 50\, t) \text{ A}$$

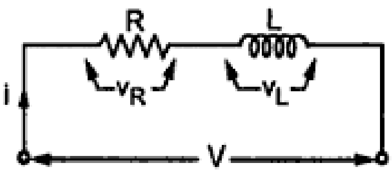


Fig. 7.17

$$\therefore \quad \omega = 2\pi \times 50 = 2\pi f$$

$$\therefore \quad f = 50 \text{ Hz}, \quad R = 100\ \Omega, \quad L = 0.31831 \text{ H}$$

$$\therefore \quad X_L = 2\pi f L = 2\pi \times 50 \times 0.31831 = 100\ \Omega$$

i) The voltage across the resistance is,

$$v_R = i R = 1.414 \sin(2\pi \times 50\, t) \times 100 = \mathbf{141.4 \sin(2\pi \times 50\, t) \text{ V}}$$

ii) The voltage across L leads current by 90° as current lags by 90° with respect to voltage.

$$\therefore \quad v_L = i X_L \text{ but leading current by } 90° = \mathbf{141.4 \sin(2\pi \times 50\, t + 90°) \text{ V}}$$

iii) From the expression of V_R we can write,

$$\text{r.m.s. value of } V_R = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 0°$$

$$\therefore \quad V_R = 100 \angle 0° = 100 + j0 \text{ V}$$

$$\text{r.m.s. value of } V_L = \frac{141.4}{\sqrt{2}} = 100 \text{ V}, \qquad \phi = 90°$$

$$\therefore \quad V_L = 100 \angle 90° = 0 + j\,100 \text{ V}$$

$$\therefore \quad V = \overline{V}_R + \overline{V}_L = 100 + j0 + 0 + j100$$

$$= 100 + j\,100 = 141.42 \angle 45° \text{ V}$$

$$\therefore \quad V_m = \sqrt{2} \times 141.42 = 200 \text{ V}$$

Hence expression of instantaneous value of resultant voltage is,

$$v = \mathbf{200 \sin(2\pi \times 50\, t + 45°) \text{ V}}$$

Example. 7.4 : *A voltage $e = 200 \sin 100 \pi t$ is applied to a load having $R = 200\ \Omega$ in series with $L = 638$ mH.*
Estimate :-

i) Expression for current in $i = I_m \sin(\omega t \pm \phi)$ form ii) Power consumed by the load iii) Reactive power of the load iv) voltage across R and L. **(May-2001)**

Solution : The circuit is shown in the Fig. 7.18.

$$e_m = 200 \text{ V} \quad \therefore \ V = \frac{200}{\sqrt{2}} = 141.421 \text{ V (r.m.s.)}$$

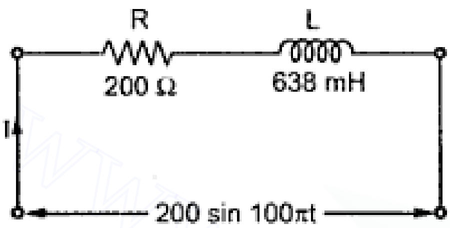


Fig. 7.18

$$\omega = 100\,\pi \quad \therefore \quad f = 50 \text{ Hz}$$

$$\therefore \quad X_L = \omega L = 100\,\pi \times 638 \times 10^{-3}$$

$$= 200.433\ \Omega$$

$$Z = R + j\,X_L = 200 + j\,200.433\ \Omega$$

$$= 283.149 \angle 45.06°\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{141.421 \angle 0°}{283.149 \angle 45.06°}$$

$$= 0.5 \angle -45.06° \text{ A,}$$ Current lags voltage by 45.06°.

$$\therefore \quad I_m = \sqrt{2} \times 0.5 = 0.7071 \text{ A}, \ \phi = -45.06°$$

i) $i = I_m \sin(\omega t - \phi) = \mathbf{0.7071 \sin(100\,\pi\, t - 45.06°)\ A}$

ii) $P = VI \cos\phi = 141.421 \times 0.5 \times \cos(45.06°) = 49.9474 \approx \mathbf{50\ W}$

iii) $Q = VI \sin\phi = 141.421 \times 0.5 \times \sin(45.06°) = \mathbf{50\ VAR}$

iv) $V_R = IR = 0.5 \times 200 = \mathbf{100\ V}$

$V_L = I\,X_L = 0.5 \times 200.433 = \mathbf{100.21\ V}$

7.6 A.C. through Series R-C Circuit

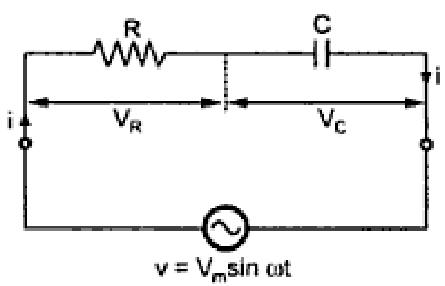


Fig. 7.19 Series R-C circuit

Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. 7.19.

The series combination is connected across a.c. supply given by

$$v = V_m \sin \omega t$$

Circuit draws a current I, then there are two voltage drops,

a) Drop across pure resistance $V_R = I \times R$

b) Drop across pure capacitance $V_C = I \times X_C$

Where $X_C = \frac{1}{2\pi f\ C}$ and I, V_R, V_C are the r.m.s. values

The Kirchhoff's voltage law can be applied to get,

$$V = \overline{V_R} + \overline{V_C} \qquad \text{... (Phasor Addition)}$$

$$\therefore \quad \overline{V} = \overline{IR} + \overline{IX_C}$$

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.

Following are the steps to draw the phasor diagram :

1) Take current as reference phasor.
2) In case of resistance, voltage and current are in phase. So, V_R will be along current phasor.
3) In case of pure capacitance, current leads voltage by 90° i.e. voltage lags current by 90° so V_C is shown downwards i.e. lagging current by 90°.
4) The supply voltage being vector sum of these two voltages V_C and V_R obtained by completing parallelogram.

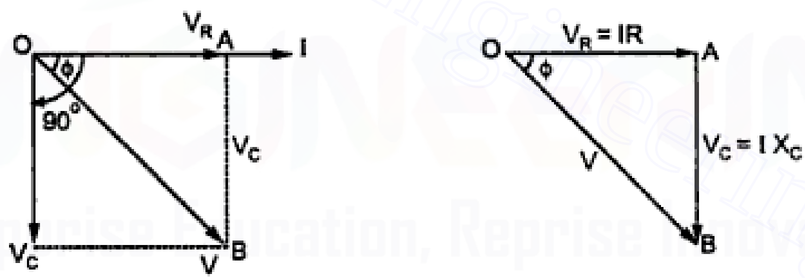


Fig. 7.20 Phasor diagram and voltage triangle

From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (I\,X_C)^2} = I\sqrt{(R)^2 + (X_C)^2}$$

$$\therefore \quad V = I\,Z$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_C)^2}}$ is the impedance of the circuit.

7.6.1 Impedance

Similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current. It is measured in ohms given by $Z = \sqrt{(R)^2 + (X_C)^2}$ where $X_C = \frac{1}{2\pi f C}$ Ω called **capacitive reactance.**

In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 7.20.

From voltage triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \qquad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \qquad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.**

Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.

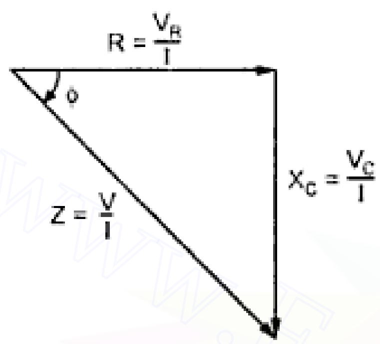


Fig. 7.21 Impedance triangle

The X component of impedance is R and is given by

$$R = Z\cos\phi$$

and Y component of impedance is X_C and is given by

$$X_C = Z\sin\phi$$

But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$\boxed{Z = R - j\,X_C\ \Omega}$$

While in polar form, it is denoted as,

$$\boxed{Z = |\,Z\,| \angle -\phi\ \Omega}$$

$$Z = R - j\,X_C = |\,Z\,| \angle -\phi$$

where $\quad |\,Z\,| = \sqrt{R^2 + X_C^2},\ \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$

Key Point: *Thus ϕ is negative for capacitive impedance.*

7.6.2 Power and Power Triangle

The current leads voltage by angle ϕ, hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) \cdot \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi) \qquad \text{as } \cos(-\phi) = \cos\phi$$

Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\therefore \quad \boxed{P = V I \cos\phi \quad \text{watts}} \quad \text{where V and I are r.m.s. values}$$

If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I}$$

$$\therefore \quad \overline{VI} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

Hence, the power triangle can be shown as in the Fig. 7.22.

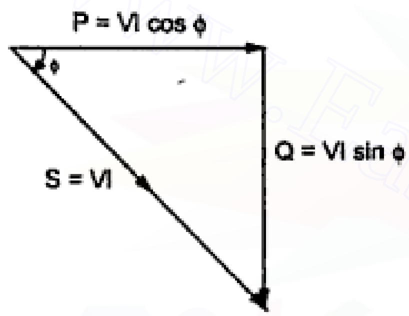


Fig. 7.22

Thus, the various powers are,

Apparent power,	$S = V I$	VA
True or average power,	$P = V I \cos\phi$	W
Reactive power,	$Q = V I \sin\phi$	VAR

Remember that, X_L term appears positive in Z.

$$Z = R + j X_L = |Z| \angle \phi \qquad \phi \text{ is positive for inductive Z}$$

While X_C term appears negative in Z.

$$Z = R - j X_C = |Z| \angle -\phi \qquad \phi \text{ is negative for capacitive Z}$$

For any single phase a.c. circuit, the average power is given by,

$$\mathbf{P = V I \cos\phi \quad watts}$$

Where V, I are r.m.s. values

$\cos\phi$ = Power factor of circuit

$\cos\phi$ **is lagging for inductive circuit and** $\cos\phi$ **is leading for capacitive circuit.**

Example 7.5 : *Calculate the resistance and inductance or capacitance in series for each of the following impedances. Assume the frequency to be 60 Hz.*

i) (12 + j 30) ohms ii) – j 60 ohms iii) 20 ∠ 60° ohms. **(May-99)**

Solution : i) $12 + j\,30\ \Omega$

Comparing the value of impedance with,

$$Z = R + j X_L, \quad R = 12\ \Omega \quad \text{and } X_L = 30\ \Omega = 2\pi f L$$

$$\therefore \quad L = \frac{30}{2\pi f} = \frac{30}{2\pi\times 60} = \mathbf{79.58\ mH}$$

ii) $0 - j\,60\ \Omega$

Comparing with, $Z = R - j X_C$

$$R = 0\ \Omega$$

$$X_C = 60\ \Omega = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi\times 60\times 60} = \mathbf{44.209\ \mu F}$$

iii) $20\angle 60°\Omega$

Converting to rectangular form, $Z = 10 + j\,17.32$

Comparing with, $Z = R + j X_L$

$$R = \mathbf{10\ \Omega}$$

$$X_L = 17.32\ \Omega = 2\pi f L$$

$$\therefore \quad L = \frac{1732}{2\pi\times 60} = \mathbf{45.94\ mH}$$

Example 7.6 : *The waveforms of the voltage and current of a circuit are given by,*

$$e = 120 \sin(314\, t) \text{ and } i = 10 \sin(314\, t + \pi/6)$$

Calculate the values of the resistance, capacitance which are connected in series to form the circuit.

Also draw waveforms for current, voltage and phasor diagram. Calculate power consumed by the circuit. **(Dec.-97)**

Solution : $e = 120 \sin(314\, t)$

Comparing with, $v = V_m \sin(\omega t)$

$$\therefore \quad V_m = 120 \quad \text{and} \quad \omega = 314$$

Now $\omega = 2\pi f$ i.e. $314 = 2\pi f$

$$\therefore \quad f = 50\ \text{Hz}$$

$$V = \frac{V_m}{\sqrt{2}} = \frac{120}{\sqrt{2}} = 84.85\ \text{V}$$

Similarly $i = 10 \sin(314\, t + \pi/6)$

Comparing with, $i = I_m \sin(\omega t + \phi)$

$$I_m = 10 \quad \text{and} \quad \phi = \frac{\pi}{6} = 30^\circ$$

$$\therefore \quad I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

$$|Z| = \frac{V}{I} = \frac{84.85}{7.07} = 12 \, \Omega$$

As current leads voltage by 30°, the circuit is R-C series circuit, capacitive in nature. As impedance is capacitive, ϕ must be negative.

$$\therefore \quad Z = 12 \angle -30^\circ \Omega = 10.393 - j\,6 \, \Omega \ldots \text{ use } P \rightarrow R$$

Comparing with $\quad Z = R - j\,X_C$

$$R = \mathbf{10.393 \, \Omega} \qquad \text{and } X_C = 6 \, \Omega$$

Now $\quad X_C = \dfrac{1}{2\pi f C}$

$$\therefore \quad 6 = \frac{1}{2\pi \times 50 \times C}$$

$$\therefore \quad C = \mathbf{530.45 \, \mu F}$$

$$P = VI \cos\phi = 84.85 \times 7.07 \times \cos(30^\circ) = \mathbf{519.52 \ W}$$

The waveforms are shown in the Fig. 7.23 (a), while phasor diagram is shown in the Fig. 7.23 (b).

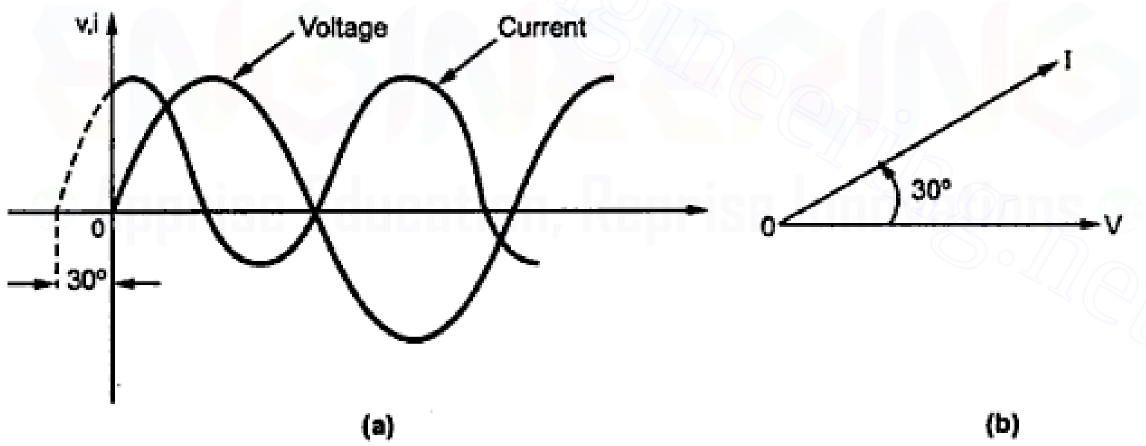


Fig. 7.23

➠ Example 7.7 : *A resistance of 120 ohms and a capacitive reactance of 250 ohms are connected in series across a A.C. voltage source. If a current of 0.9 A is flowing in the circuit find out (i) Power factor, (ii) Supply voltage (iii) Voltages across resistance and capacitance (iv) Active power and reactive power.* **[May-2003]**

Solution : The circuit is shown in the Fig. 7.24.

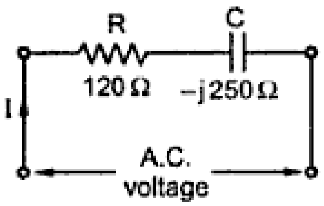


Fig. 7.24

$R = 120\ \Omega,\ X_C = 250\ \Omega,\ I = 0.9\ A$

$Z = R - j\ X_C = 120 - j250\ \Omega = 277.308\ \angle -64.358°$

Take current as reference.

$\therefore\ I = 0.9\ \angle 0°\ A$

i) Power factor $\cos\phi = \cos(-64.358°) = \mathbf{0.4327\ leading}$

ii) Supply voltage $V = I \times Z = [0.9\ \angle 0°] \times [277.308\ \angle -64.358°]$

$\therefore\quad V = \mathbf{249.5772\ \angle -64.358°\ V}$

iii) $V_R = I \times R = 0.9 \times 120 = \mathbf{108\ V}$ (magnitude)

$V_C = I \times X_C = 0.9 \times 250 = \mathbf{225\ V}$ (magnitude)

iv) P = active power = $V\ I\cos\phi = 249.5772 \times 0.9 \times 0.4327$

$= \mathbf{97.1928\ W}$

Q = reactive power = $VI \sin\phi$

$= 249.5772 \times 0.9 \times \sin(-64.358°)$

$= \mathbf{-202.498\ VAR}$

The negative sign indicates leading nature of reactive volt-amperes.

7.7 A.C. through Series R-L-C Circuit

Consider a circuit consisting of resistance R ohms pure inductance L henries and capacitance C farads connected in series with each other across a.c. supply. The circuit is shown in the Fig. 7.25.

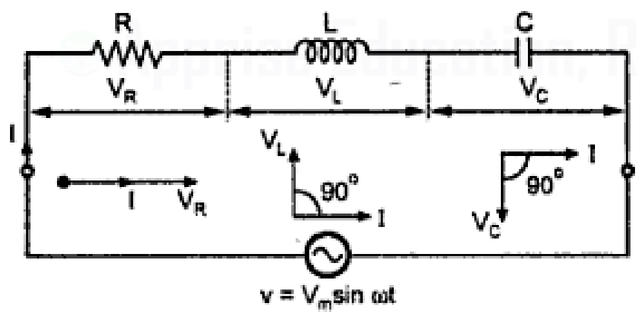


Fig. 7.25 R-L-C series circuit

The a.c. supply is given by,

$v = V_m \sin \omega t$. The circuit draws a current I. Due to current I, there are different voltage drops across R, L and C which are given by,

a) Drop across resistance R is $V_R = I\ R$

b) Drop across inductance L is $V_L = I\ X_L$

c) Drop across capacitance C is $V_C = I\ X_C$

The values of I, V_R, V_L and V_C are r.m.s. values

The characteristics of three drops are,

a) V_R is in phase with current I.

b) V_L leads current I by 90°.

c) V_C lags current I by 90°.

According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

Let us see the phasor diagram. Current I is taken as reference as it is common to all the elements.

Following are the steps to draw the phasor diagram :

1) Take current as reference. 2) V_R is in phase with I.

3) V_L leads current I by 90°. 4) V_C lags current I by 90°.

5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition (180° out of phase).

6) Add that with V_R by law of parallelogram to get the supply voltage.

The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

7.7.1 $X_L > X_C$

When $X_L > X_C$, obviously, I X_L i.e. V_L is greater than I X_C i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I. Current I will lag the resultant of V_L and V_C i.e. $(V_L - V_C)$.

The circuit is said to be **inductive** in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig. 7.26.

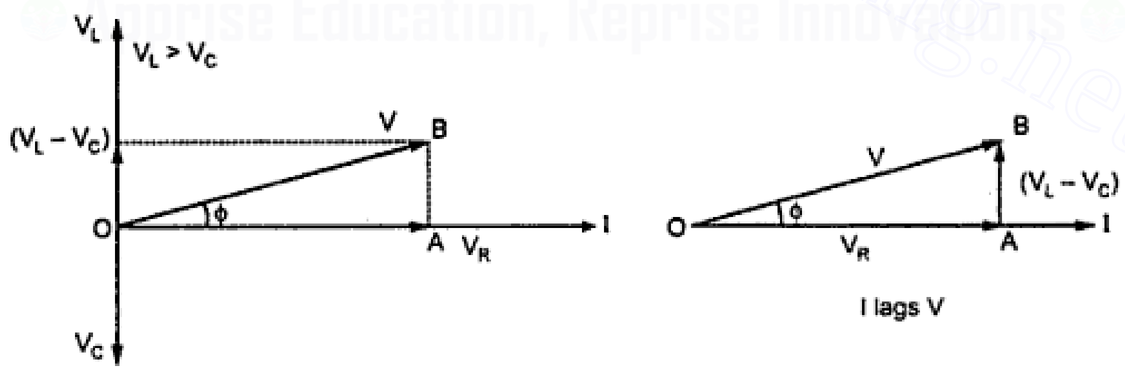


Fig. 7.26 Phasor diagram and voltage triangle for $X_L > X_C$

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(I R)^2 + (I X_L - I X_C)^2}$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\therefore \quad V = I Z$$

$\therefore$ $V = I Z$

Where $Z = R$

7.7.4 Impedance

In general, for RLC series circuit impedance is given by,

$$Z = R + j X$$

Where $X = X_L - X_C$ = total reactance of circuit

If $X_L > X_C$, X is positive and circuit is inductive.

If $X_L < X_C$, X is negative and circuit is capacitive.

If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan \phi = \left[\frac{X_L - X_C}{R}\right], \quad \cos \phi = \frac{R}{Z} \text{ and } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

7.7.5 Impedance Triangle

The impedance is expressed as,

$$Z = R + j X \quad \text{where} \quad X = X_L - X_C$$

For $X_L > X_C$, ϕ is positive and the impedance triangle is as shown in the Fig. 7.29 (a).

For $X_L < X_C$, $X_L - X_C$ is negative, so ϕ is negative and the impedance triangle is as shown in Fig. 7.29 (b).

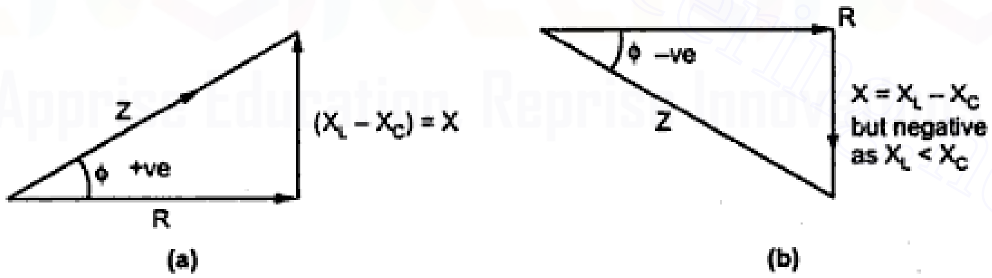


Fig. 7.29 Impedance triangles

In both the cases, $R = Z \cos \phi$ and $X = Z \sin \phi$

7.7.6 Power and Power Triangle

The average power consumed by the circuit is,

P_{av} = Average power consumed by R+ Average power consumed by L + Average power consumed by C

But, pure L and C never consume any power.

$\therefore$ P_{av} = Power taken by R $= I^2 R = I (I R) = I V_R$

But, $V_R = V \cos \phi$ in both the cases

Example 7.9 : *A series circuit having pure resistance of 40 ohms, pure inductance of 50.07 mH and a capacitor is connected across a 400 V, 50 Hz, A.C. supply. This R, L, C combination draws a current of 10 A. Calculate (i) Power factor of the circuit and (ii) Capacitor value.* **(May-2000)**

Solution : The arrangement is shown in the Fig. 7.33.

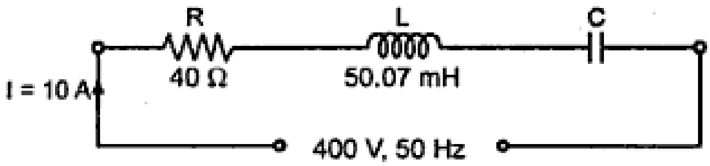


Fig. 7.33

$$X_L = 2\pi f L$$

$$= 2\pi \times 50 \times 50.07 \times 10^{-3}$$

$$= 15.73\ \Omega,\quad X_C = \frac{1}{2\pi f C}$$

$$Z = R + j(X_L - X_C)$$

$$\therefore \quad |Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (15.73 - X_C)^2} \qquad \dots (1)$$

$$|Z| = \frac{|V|}{|I|} = \frac{400}{10} = 40\ \Omega \qquad \dots (2)$$

$$\therefore \quad 40 = \sqrt{R^2 + (15.73 - X_C)^2} \quad \text{i.e.} \quad 1600 = R^2 + (15.73 - X_C)^2 \qquad \dots (3)$$

Substitute R = 40 Ω in (3), $1600 = 1600 + (15.73 - X_C)^2$

$$\therefore \quad (15.73 - X_C)^2 = 0$$

$$\therefore \quad X_C = 15.73\ \Omega \quad \text{i.e.} \quad 15.73 = \frac{1}{2\pi f C}$$

$$\therefore \quad C = 2.023 \times 10^{-4}\ \text{F}$$

$$\therefore \quad Z = 40 + j(15.73 - 15.73) = 40 + j\,0\ \Omega$$

$$= 40 \angle 0^\circ\ \Omega$$

$$\therefore \quad \text{p.f.} = \cos(0^\circ) = 1$$

7.8 Complex Power

As seen earlier in a.c. circuits there are three types of powers exist. These are apparent power (S), active power (P) and reactive power (Q). The P and Q are the components of apparent power (S) such that,

$$\boxed{|S| = \sqrt{P^2 + Q^2}}$$

While
$$\phi = \tan^{-1}\left[\frac{\sin\phi}{\cos\phi}\right] = \tan^{-1}\left[\frac{VI\sin\phi}{VI\cos\phi}\right] = \tan^{-1}\left[\frac{Q}{P}\right]$$

Where
$$P = VI\cos\phi \quad \text{and} \quad Q = VI\sin\phi$$

Thus the apparent power can be expressed in the rectangular form as,

$$\boxed{S = P \pm jQ}$$

This is called **complex power** where,

Real part = Active, true or real power in watts (W)

Imaginary part = Reactive power in reactive volt-amp (VAR)

> **Key Point** : *The reactive power Q may be positive or negative, depending upon nature of the circuit.*

The postive sign indicates lagging nature of reactive power while negative sign indicates leading nature of reactive power.

The complex power is generally indicated in a **power triangle** as shown in the Fig. 7.34.

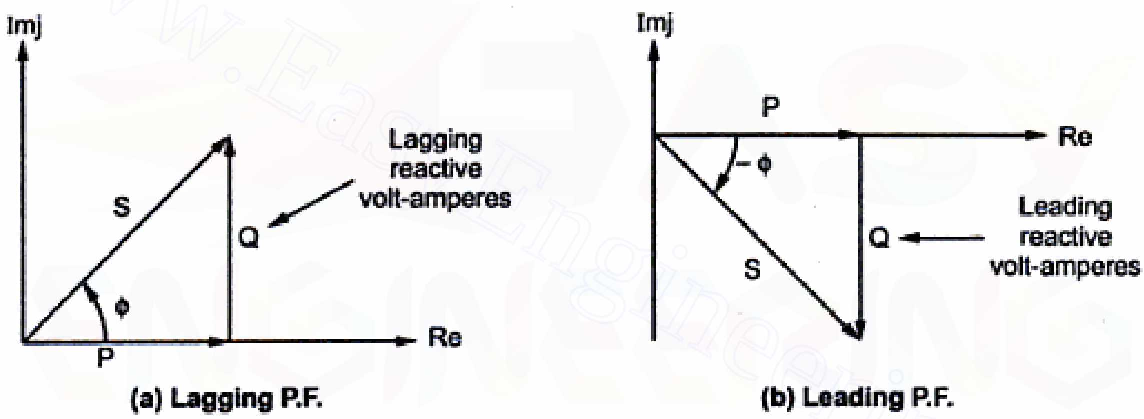


Fig. 7.34 Power triangle

The angle ϕ is p.f. angle i.e. angle between V and I.

In general if	$V = V_1 \angle \theta_1$ and $I = I_1 \angle \theta_2$	
Then	$\phi = \theta_1 - \theta_2$	
And	$P = VI \cos(\theta_1 - \theta_2)$ W	... Active power
	$Q = VI \sin(\theta_1 - \theta_2)$ VAR	... Reactive power
	$S = P + jQ$ VA	... Complex power

If $\theta_1 - \theta_2 > 0$, Q is positive indicating lagging p.f. while

If $\theta_1 - \theta_2 < 0$, Q is negative indicating leading p.f.

Sign of reactive power	Nature of power factor	Nature of load
Q is positive	Lagging	Inductive
Q is negative	Leading	Capacitive

Table 7.2

Physical significance of reactive power :

The reative power is that component of power which is supplied to the reactive components of the load from the source during positive half cycle while it is returned back to supply from the components to the source during negative half cycle. It is rate of change of energy with time which keeps on flowing from the source to reative components and back from the components to the source, alternately. The reactive power charges and discharges the reactive components alternately.

> **Key Point :** *Thus reactive power never gets consumed by the circuit but flows alternately back and forth from the source to the reactive components and vice-versa.*

➠ **Example 7.10 :** *Find the complex power delivered by the source.*

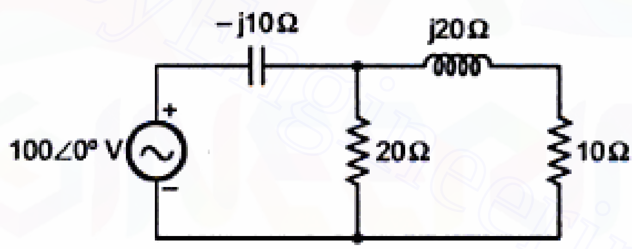


Fig. 7.35

Solution : The circuit can be analysed as,

Applying KVL to the two loops,

$$-(I)(-j10) - 20I_1 + 100\angle 0° = 0$$

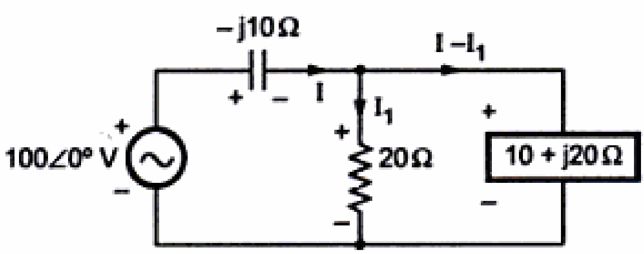


Fig. 7.35 (a)

i.e. $(-j10)\,I + 20\,I_1 = 100\angle 0°$ ... (1)

$$-(10+j20)(I - I_1) + 20\,I_1 = 0$$

$\therefore$ $(10+j20)\,I + (-30-j20)\,I_1 = 0$ ... (2)

$$D = \begin{vmatrix} -j10 & 20 \\ 10+j20 & -30-j20 \end{vmatrix} = +j300 - 200 - 200 - j400 = -400 - j100$$

ii) $I_m = \frac{V}{R} = \frac{100}{10} = 10\ A$ ... Current is maximum at resonance

iii) $P_m = I_m^2\ R = (10)^2 \times 10 = 1000\ W$

iv) Power factor is unity, as impedance is purely resistive at resonance

v) $V_R = I_m\ R = 10 \times 10 = 100\ V$

$X_L = 2\ \pi\ f_r\ L = 2\pi \times 56.2697 \times 0.2 = 70.7105\ \Omega$

$\therefore$ $V_L = I_m\ X_L = 10 \times 70.7105 = 707.105\ V$

And $X_C = \frac{1}{2\pi f_r C} = \frac{1}{2\pi \times 56.2697 \times 40 \times 10^{-6}} = 70.7105\ \Omega$

$\therefore$ $\boxed{V_C = I_m\ X_C = 707.105\ V}$

Thus $V_L = V_C$ at resonance

vi) $Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = 7.071$

vii) $\Delta f = \frac{R}{4\pi L} = \frac{10}{4\pi \times 0.2} = 3.9788$

$\therefore$ $f_1 = f_r - \Delta f = 56.2697 - 3.9788 = 52.2909$ Hz

and $f_2 = f_r + \Delta f = 56.2697 + 3.9788 = 60.2485$ Hz

viii) B.W. $= f_2 - f_1 = 60.2485 - 52.2909 = 7.9576$ Hz

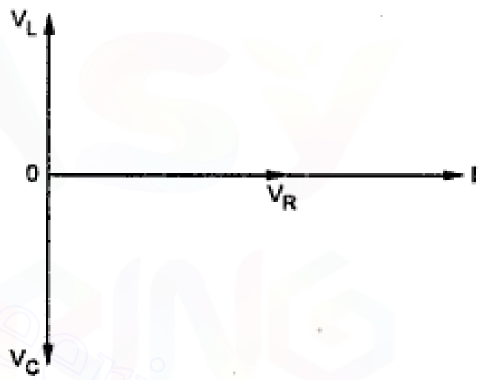


Fig. 7.39

The phasor diagram is shown in the Fig. 7.39.

Example 7.12 : *A series R-L-C circuit is connected to 230 V a.c. supply. The current drawn the circuit at the resonance is 25 A. The voltage drop across the capacitor is 4000 V, at the series resonance.*

Calculate the resistance, inductance if capacitance is 5 μF, also calculate the resonant frequency.

Solution : At resonance,

$$R = \frac{V}{I} = \frac{230}{25} = 9.2\ \Omega$$

Voltage drop across capacitor

$V_C = I\ X_C$ i.e. $4000 = 25 \times X_C$

$X_C = 160\ \Omega$

At resonant frequency, $f = f_r$ And $X_C = \frac{1}{2\pi f_r C}$

$$\therefore \quad 160 = \frac{1}{2\pi f_r \times 10^{-6}}$$

$$\therefore \quad f_r = 198.943 \text{ Hz}$$

At resonance $\quad X_L = X_C = 160\ \Omega$

$$2\pi f_r L = 160$$

$$\therefore \quad L = \frac{160}{2\pi f_r} = \frac{160}{2\pi \times 198.943}$$

$$\therefore \quad L = 0.128 \text{ H}$$

7.10 A.C. Parallel Circuit

A parallel circuit is one in which two or more impedances are connected in parallel across the supply voltage. Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.

The Fig. 7.40 shows a parallel circuit consisting of three impedances connected in parallel across an a.c. supply of V volts.

Key Point : *The voltage across all the impedances is same as supply voltage of V volts.*

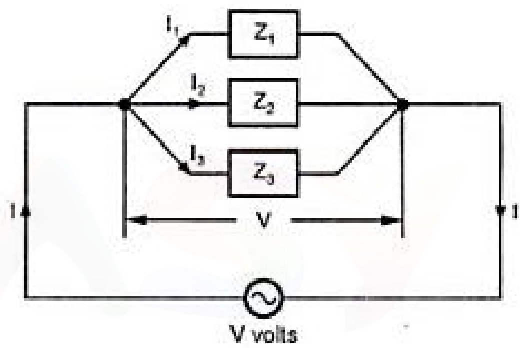


Fig. 7.40 A.C. parallel circuit

The current taken by each impedance is different.

Applying Kirchhoff's law, $\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$ ... (Phasor addition)

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z_1}} + \frac{\bar{V}}{\bar{Z_2}} + \frac{\bar{V}}{\bar{Z_3}}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z_1}} + \frac{1}{\bar{Z_2}} + \frac{1}{\bar{Z_3}}$$

Where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

7.10.1 Two Impedances in Parallel

If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents.

$$\boxed{\begin{aligned} \bar{I}_1 &= \bar{I}_T \times \frac{\bar{Z_2}}{\bar{Z_1}+\bar{Z_2}} \\ \bar{I}_2 &= \bar{I}_T \times \frac{\bar{Z_1}}{\bar{Z_1}+\bar{Z_2}} \end{aligned}}$$

Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1}, \quad \bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2}, ..., \quad \bar{I}_n = \frac{\bar{V}}{\bar{Z}_n}$$

While the individual phase angles can be calculated by the relation,

$$\tan \phi_1 = \frac{X_1}{R_1}, \tan \phi_2 = \frac{X_2}{R_2}, ..., \tan \phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.
3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in rectangular form. This is the resultant current drawn from the supply.
4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit..

7.10.2 Concept of Admittance

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit siemens or mho.

Now, current equation for the circuit shown in the Fig. 7.41 is,

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

$$\bar{I} = \bar{V} \times \left(\frac{1}{\bar{Z}_1}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_2}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

where Y is the admittance of the total circuit. The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series, as shown in the Fig. 7.41.

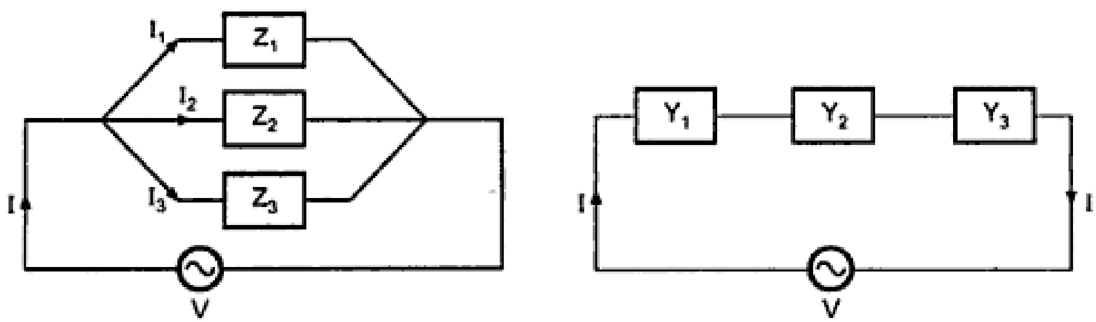


Fig. 7.41 Equivalent parallel circuit using admittances

7.10.3 Components of Admittance

Consider an impedance given as,

$$Z = R \pm jX$$

Positive sign for inductive and negative for capacitive circuit.

Admittance $$Y = \frac{1}{Z} = \frac{1}{R \pm jX}$$

Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2+X^2}\right) \mp j\left(\frac{X}{R^2+X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$\therefore$ $$\boxed{Y = G \mp jB}$$

In the above expression, G = Conductance = $\frac{R}{Z^2}$

and B = Susceptance = $\frac{X}{Z^2}$

7.10.4 Conductance (G)

It is defined as the ratio of the resistance to the square of the impedance. It is measured in the unit siemens.

7.10.5 Susceptance (B)

It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit siemens.

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

Note : The sign convention for the reactance and the susceptance are opposite to each other.

Remember,

$$Y = G + jB = |Y| \angle \phi \text{ siemens or mho}$$

$$|Y| = \sqrt{G^2 + B^2}, \ \phi = \tan^{-1}\frac{B}{G}$$

B is negative if inductive and B is positive if capacitive.

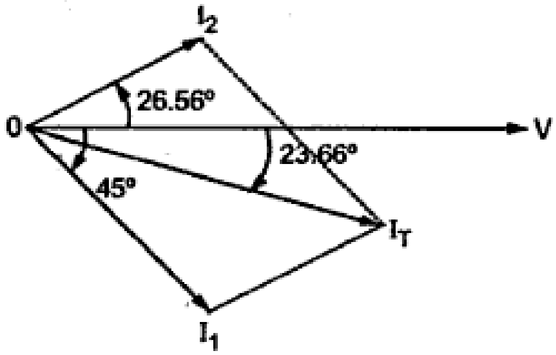


Fig. 7.46

$= 11.18 \angle +26.56° \quad A = 10 + j\,5\ A$

$\therefore \quad I_T = \bar{I}_1 + \bar{I}_2 = 16.699 - j\,16.699 + 10 + j\,5$

$= 26.699 - j\,11.699\ A$

$= 29.15 \angle -23.66°\ A$

p.f. $= \cos(-23.66°) = 0.9159$ lagging

The phasor diagram is shown in the Fig. 7.46.

Example 7.15 : *Two impedances Z_1 and Z_2 are connected in parallel across applied voltage of $(100+j200)$ volts. The total power supplied to the circuit is 5 kW. The first branch takes a leading current of 16 A and has a resistance of 5 ohms while the second branch takes a lagging current at 0.8 power factor. Calculate*
i) Current in second branch ii) Total current iii) Circuit constants. **(Dec.-2001)**

Solution : The circuit is shown in the Fig. 7.47.

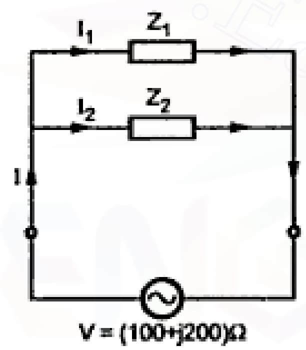


Fig. 7.47

$V = 100 + j\,200 = 223.60 \angle 63.43°$ volts

$I_1 = 16$ A, $R_1 = 5\ \Omega$, $\cos\phi_2 = 0.8$

Now, $|Z_1| = \dfrac{V}{I_1} = \dfrac{223.60}{16} = 13.975\ \Omega$

$R_1 = Z_1 \cos\phi_1$

$\therefore \cos\phi_1 = \dfrac{R_1}{Z_1} = \dfrac{5}{13.975} = 0.3577$

$\therefore \quad \phi_1 = -69.03°$ (... negative, as leading in nature)

$\therefore \quad \sin\phi_1 = 0.9338$

$X_1 = Z_1 \sin\phi_1 = (13.975)(0.9338) = 13.04\ \Omega$

Power consumed in Z_1, $P_1 = I_1^2\ R_1 = (16)^2(5) = 1280$ watt

Power consumed in Z_2 = Total power supplied – Power consumed in Z_1

$\therefore \quad P_2 = 5000 - 1280 = 3720$ watt

Power consumed in Z_2, $P_2 = V\ I_2\ \cos\phi_2$

$\therefore \quad I_2 = \dfrac{P_2}{V\cos\phi_2}$

$\therefore \quad I_2 = \dfrac{3720}{223.60 \times 0.8}$

$\therefore \quad I_2 = 20.79$ A

Now we have, $P_2 = I_2^2 \; R_2$

$\therefore \quad R_2 = \frac{P_2}{I_2^2} = \frac{3720}{(20.79)^2} = 8.60 \; \Omega$

Now, $\cos\phi_2 = 0.8$

$\therefore \quad \sin\phi_2 = 0.6$

$\therefore \quad X_2 = |Z_2| \sin\phi_2 = \frac{V}{I_2} \sin\phi_2 = \frac{223.60}{20.79} \times 0.6$

$X_2 = 6.4531 \; \Omega$

$\therefore \quad Z_2 = R_2 + jX_2 = 8.60 + j6.4531 = 10.75 \angle 36.88° \; \Omega$

$\therefore \quad \bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{223.60 \angle 63.43°}{10.75 \angle 36.88°} = 20.79 \angle 26.55° \; A$

Similarly, $Z_1 = R_1 - jX_1 = (5 - j\,13.04) = 13.965 \angle -69.02° \; \Omega$

$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{223.60 \angle 63.43°}{13.965 \angle -69.02°} = 16 \angle 132.45° \; A$

$\therefore$ Total current, $\bar{I} = \bar{I}_1 + \bar{I}_2 = (16 \angle 132.45°) + (20.79 \angle 26.55°)$

$= (-10.799 + j\,11.80) + (18.59 + j\,9.29) = 7.8 + j\,21.09$

$\therefore \quad \bar{I} = 22.48 \angle 69.70° \; A$

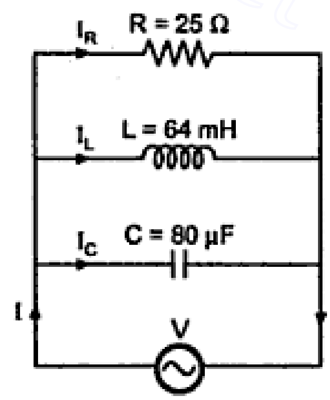
$\therefore$ Total current = **22.48** A

Current in branch 2 = **20.79** A

Circuit constants are $R_1 = 5\;\Omega$, $R_2 = 8.60\;\Omega$, $X_1 = 13.04\;\Omega$, $X_2 = 6.4531\;\Omega$

Example 7.16 : *A parallel circuit of 25 Ω resistor, 64 mH inductor and 80 μF capacitor connected across a 110 V, 50 Hz, single phase supply, is shown in Fig. 7.46. Calculate the current in individual element, the total current drawn from the supply and the overall p.f. of the circuit. Draw a neat phasor diagram showing $\bar{V}$, $\bar{I}_R$, $\bar{I}_L$, $\bar{I}_C$ and $\bar{I}$.*

(May-2002, Dec.-2007)



110 volt, 50 Hz supply

Fig. 7.48

Solution : From Fig. 7.48,

$R = 25\;\Omega, \quad X_L = 2\pi fL = 2\pi \times 50 \times 64 \times 10^{-3}$

$\therefore \quad X_L = 20.10\;\Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.78\;\Omega$

Let $V = 110 \angle 0°$ volts

$$I_R = \frac{V}{R} = \frac{110 \angle 0^\circ}{25} = 4.4 \angle 0^\circ \text{ A}$$

$$I_L = \frac{V}{jX_L} = \frac{110 \angle 0^\circ}{20.10 \angle 90^\circ} = 5.47 \angle -90^\circ \text{ A}$$

$$I_C = \frac{V}{-j X_C} = \frac{110 \angle 0^\circ}{39.78 \angle -90^\circ} = 2.76 \angle 90^\circ \text{ A}$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$= (4.4 \angle 0^\circ) + (5.47 \angle -90^\circ) + (2.76 \angle 90^\circ)$$

$$= (4.4 + j0) + (0 - j5.47) + (0 + 2.76)$$

$$= (4.4 - j2.71) \text{ A}$$

$$\therefore \quad \bar{I} = 5.1676 \angle -31.62^\circ \text{ A}$$

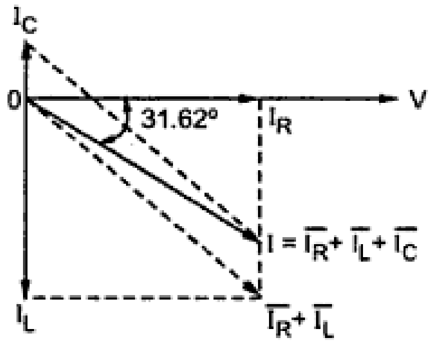


Fig. 7.49

Overall p.f. $= \cos \phi = \cos(31.62) =$ **0.851 lagging.**

Phasor diagram : The phasor diagram is shown in the Fig. 7.49.

7.12 Resonance in Parallel Circuit

Similar to a series a.c. circuit, there can be a resonance in parallel a.c. circuit. When the power factor of a parallel a.c. circuit is unity i.e. the voltage and total current are in phase at a particular frequency then the parallel circuit is said to be at **resonance.** The frequency at which the parallel resonance occurs is called **resonant frequency** denoted as f_r Hz.

7.12.1 Characteristics of Parallel Resonance

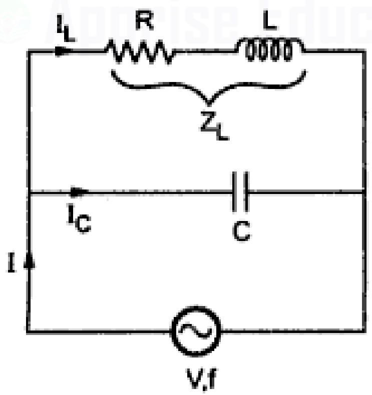


Fig. 7.50 Practical parallel circuit

Consider a practical parallel circuit used for the parallel resonance as shown in the Fig. 7.50.

The one branch consists of resistance R in series with inductor L. So it is series R-L circuit with impedance Z_L. The other branch is pure capacitive with a capacitor C. Both the branches are connected in parallel across a variable frequency constant voltage source.

The current drawn by inductive branch is I_L while drawn by capacitive branch is I_C.

$$I_L = \frac{V}{Z_L} \quad \text{where } Z_L = R + j X_L$$

And

$$I_C = \frac{V}{X_C} \quad \text{where } X_C = \frac{1}{2\pi f C}$$

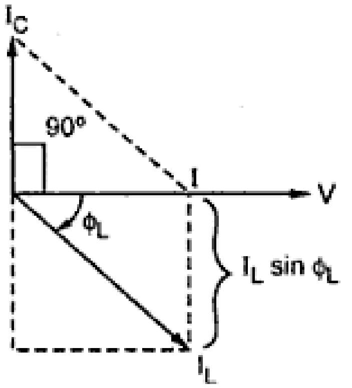


Fig. 7.51

The current I_L lags voltage V by angle ϕ_L which is decided by R and X_L while the current I_C leads voltage V by 90°. The total current I is phasor addition of I_L and I_C. The phasor diagram is shown in the Fig. 7.51.

For the parallel resonance V and I must be in phase. To achieve this unity p.f. condition,

$$\boxed{\begin{aligned} I &= I_L \cos \phi_L \\ \text{and} \quad I_C &= I_L \sin \phi_L \end{aligned}}$$

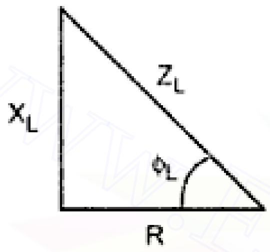


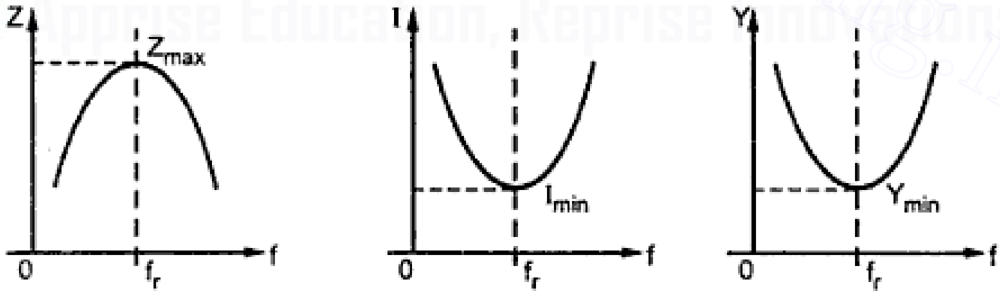
Fig. 7.52 Impedance triangle

From the impedance triangle of R-L series circuit we can write,

$$\tan \phi_L = \frac{X_L}{R}, \cos \phi_L = \frac{R}{Z_L}, \sin \phi_L = \frac{X_L}{Z_L}$$

As frequency is increased, $X_L = 2\pi f L$ increases due to which $Z_L = \sqrt{R^2 + X_L^2}$ also increases. Hence $\cos \phi_L$ decreases and $\sin \phi_L$ increases. As Z_L increases, the current I_L also decreases.

At resonance $f = f_r$ and $I_L \cos \phi_L$ is at its minimum. Thus at resonance **current is minimum** while the total **impedance** of the circuit is **maximum**. As admittance is reciprocal of impedance, as frequency is changed, **admittance** decreases and is **minimum** at resonance. The three curves are shown in the Fig. 7.53 (a), (b) and (c).



(a) Impedance (b) Current (c) Admittance

Fig. 7.53 Characteristics of parallel resonance

7.12.2 Expression for Resonant Frequency

At resonance $I_C = I_L \sin \phi_L$

$$\therefore \quad \frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L} = \frac{V X_L}{Z_L^2}$$

$$\therefore \quad Z_L^2 = X_L X_C$$

$$\therefore \quad R^2 + (2\pi f_r L)^2 = (2\pi f_r L) \times \frac{1}{2\pi f_r C} \quad \text{as } f = f_r$$

$$\therefore \quad R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$\therefore \quad (2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$\therefore \quad (2\pi f_r)^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \quad \boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

Thus if R is very small compared to L and C, $\frac{R^2}{L^2} << \frac{1}{LC}$

$$\therefore \quad \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

This is same as that for series resonance.

Key Point : *The net susceptance of the whole circuit is zero at resonance.*

7.12.3 Dynamic Impedance at Resonance

The impedance offered by the parallel circuit at resonance is called **dynamic impedance** denoted as Z_D. This is maximum at resonance. As current drawn at resonance is minimum, the parallel circuit at resonance is called **rejector circuit**. This indicates that it rejects the unwanted frequencies and hence it is used as filter in radio receiver.

From $I_C = I_L \sin \phi_L$ we have seen that,

$$Z_L^2 = \frac{L}{C}$$

while

$$I = I_L \cos \phi_L = \frac{V}{Z_L} \cdot \frac{R}{Z_L} = \frac{VR}{Z_L^2}$$

$$\therefore \quad I = \frac{VR}{\frac{L}{C}} = \frac{V}{(L/RC)}$$

$$\therefore \quad I = \frac{V}{Z_D}$$

Where

$$\boxed{Z_D = \frac{L}{RC} = \text{Dynamic impedance}}$$

7.12.4 Quality Factor of Parallel Circuit

The parallel circuit is used to magnify the current and hence known as current resonance circuit.

The quality factor of the parallel circuit is defined as the current magnification in the circuit at resonance.

The current magnification is defined as,

$$\text{Current magnification} = \frac{\text{Current in the inductive branch}}{\text{Current in supply at resonance}} = \frac{I_L}{I}$$

$$= \frac{\frac{V}{Z_L}}{\frac{V}{Z_D}} = \frac{Z_D}{Z_L} = \frac{\frac{L}{RC}}{\sqrt{\frac{L}{C}}} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{as} \quad Z_L = \sqrt{X_L X_C} = \sqrt{\frac{L}{C}}$$

This is nothing but the quality factor at resonance.

$$\therefore \qquad \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Example 7.17 : *An inductive coil of resistance 10 Ω and inductance 0.1 henries is connected in parallel with a 150 μF capacitor to a variable frequency, 200 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with the supply voltage. Also find the value of this current. Draw the phasor diagram.*

Solution : The circuit is shown in the Fig. 7.54.

The resonant frequency is,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 150 \times 10^{-6}} - \frac{(10)^2}{(0.1)^2}}$$

$$= 37.8865 \quad \text{Hz}$$

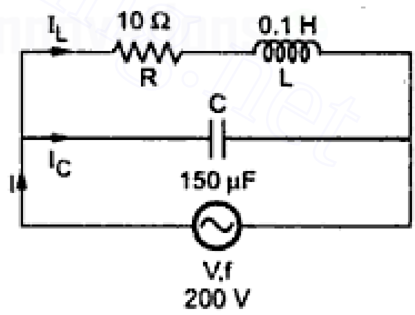


Fig. 7.54

Now $\quad Z_L = R + j\,X_L = 10 + j\,(2\pi f_r\,L)$

$$= 10 + j\,23.805 = 25.82 \angle 67.21° \,\Omega$$

$$\therefore \quad I_L = \frac{V}{Z_L} = \frac{200\angle 0°}{25.82\angle 67.21°} = 7.7459 \angle -67.21° \text{ A}$$

and $\quad I_C = \frac{V}{X_C} = \frac{200\angle 0°}{\frac{1}{2\pi f_r C}\angle -90°} = \frac{200\angle 0°}{28\angle -90°} = 7.143 \angle +90° \text{ A}$

where $Z_C = 0 - j\,X_C = 0 - j\,28 = 28 \angle -90° \,\Omega$

$$\therefore \quad Z_T = \frac{Z_C\,Z_L}{Z_C + Z_L} = \frac{28\angle -90° \times 25.82 \angle 67.21°}{0 - j\,28 + 10 + j\,23.805} = \frac{722.96\angle -22.79°}{10 - j\,4.195}$$

$$= \frac{722.96\angle -22.79}{10.844\angle -22.79} = 66.67\ \Omega \text{ pure resistive}$$

$$\therefore \quad Z_T = Z_D = \frac{L}{CR} = \frac{0.1}{150 \times 10^{-6} \times 10} = 66.67\ \Omega$$

$$\therefore \quad I = \frac{V}{Z_D} = \frac{200}{66.67} = 3\ \text{A}$$

The phasor diagram is shown in the Fig. 7.55.

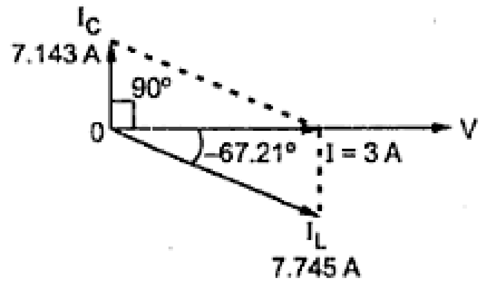
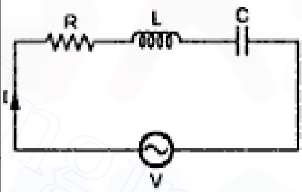
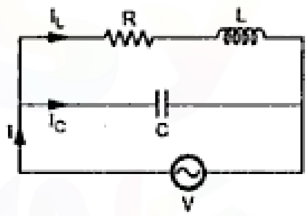


Fig. 7.55

7.13 Comparison of Resonant Circuits

No.	Parameter	Series resonant	Parallel resonant
1.	Circuit		
2.	Type of circuit	Purely resistive	Purely resistive
3.	Power factor	Unity	Unity
4.	Impedance	Minimum Z = R	Dynamic but maximum $Z_D = \frac{L}{RC}$
5.	Frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi\sqrt{LC}}$
6.	Current	Maximum $I = \frac{V}{R}$	Minimum $I = \frac{V}{Z_D}$
7.	Magnification	Voltage magnification	Current magnification
8.	Quality factor	$Q = \frac{\omega_r L}{R} = \frac{\omega_r}{B.W.}$	$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
9.	Nature	Acceptor	Rejector
10.	Practical use	Radio circuits sharpness of tunning circuit	Impedance for matching, tuning, as a filter

Examples with Solutions

Example 7.18 : *A room heater of 2 kW, 125 V rating is to be operated on 230 V, 50 Hz, a.c., supply. Calculate the value of inductance, that must be connected in series with the heater so that heater will not get damaged due to over voltage.*

Solution : Heater is to be assumed as purely resistive load.

$$P = 2\text{ kW} = 2000\text{ W},\ V_R = 125\text{ V}$$

$$I_R = \frac{P}{V_R} = \frac{2000}{125} = 16\text{ A} = I$$

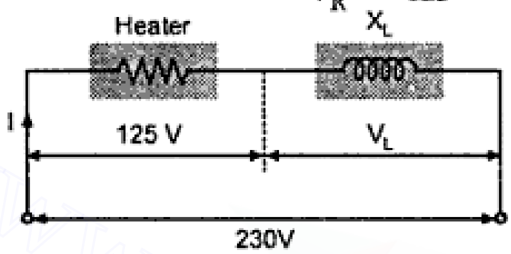


Fig. 7.56 (a)

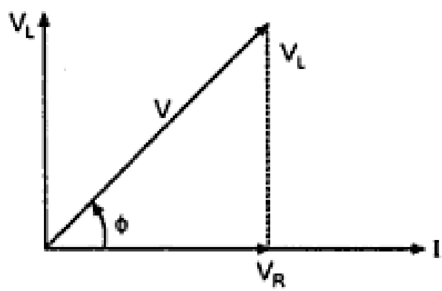


Fig. 7.56 (b)

$$V = \sqrt{V_R^2 + V_L^2}$$

$\therefore$ $$230 = \sqrt{(125)^2 + (V_L)^2}$$

$\therefore$ $$V_L = 193.067\text{ V}$$

Now $$V_L = I\,X_L$$

$\therefore$ $$X_L = \frac{V_L}{I} = \frac{193.067}{16} = 12.066\ \Omega$$

And $$X_L = 2\pi f L$$

$\therefore$ $$L = \frac{X_L}{2\pi f} = \frac{12.066}{2\pi\times 50} = 0.0384\text{ H}$$

An inductor of 0.0384 H must be connected in series with the heater.

Example 7.19 : *A choke coil and pure resistance are connected in series across 230 V, 50 Hz, a.c. supply. If the voltage drop across coil is 190 V and across resistance is 80 V while current drawn by the circuit is 5 A. Calculate, i) Internal resistance of coil ii) Inductance of coil iii) Resistance R, iv) Power factor of the circuit v) Power consumed by the circuit.*

Solution :

$$V = 230\text{ V},\quad I = 5\text{ A}$$

$$Z_T = \frac{V}{I} = \frac{230}{5} = 46\ \Omega$$

$$Z_T = \sqrt{(R+r)^2 + (X_L)^2}$$

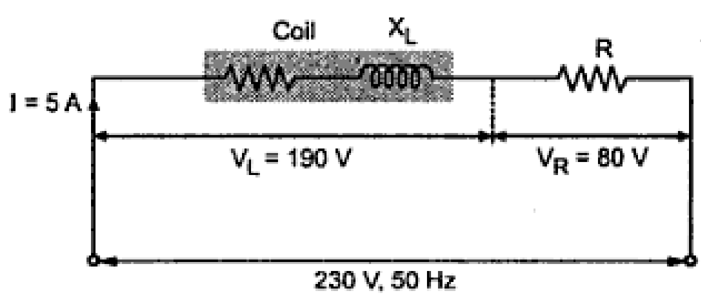


Fig. 7.57

Impedance of coil

$$Z_L = \frac{V_L}{I} = \frac{190}{5} = 38\ \Omega \quad \text{... (1)}$$

$$Z_L = r + j X_L$$

$$\therefore \quad 38 = \sqrt{r^2 + (X_L)^2}$$

From (1) $$(46)^2 = (R + r)^2 + (X_L)^2 \quad \text{... (2)}$$

From (2) $$(38)^2 = r^2 + (X_L)^2 \quad \text{... (3)}$$

Now $$V_R = 80\ V = I\ R \quad \text{... (4)}$$

$$\therefore \quad R = \frac{V_R}{I} = \frac{80}{5} = 16\ \Omega$$

$\therefore$ From (3) $$2116 = (R)^2 + 2\ R\ r + r^2 + (X_L)^2$$

Substituting (4) in (3) $$2116 = (16)^2 + 2 \times 16 \times r + (38)^2$$

$$\therefore \quad r = 13\ \Omega$$

Now $$38 = \sqrt{r^2 + (X_L)^2}$$

$$(38)^2 = (13)^2 + (X_L)^2$$

$$X_L = 35.707\ \Omega$$

Now $$X_L = 2\ \pi\ f\ L \quad \text{hence } L = \frac{X_L}{2\ \pi\ f} = \frac{35.707}{2\pi \times 50}$$

$$\therefore \quad L = 0.1136\ H$$

$$Z_T = (R + r) + j\ (X_L) = (16 + 13) + j\ (35.707) = 29 + j\ 35.707\ \Omega$$

$$\therefore \quad \cos\phi = \frac{(R + r)}{Z_T} = \frac{29}{46} = 0.6304 \text{ lagging}$$

Power consumed $P = V\ I \cos\phi = 230 \times 5 \times 0.6304 = $ **724.96 W**

Example 7.20 : *Two coils A and B are connected in series across 200 V, 50 Hz a.c. supply. The power input to the circuit is 2.2. kW and 1.5 kVAR. If the resistance of coil A is 4 Ω and the reactance is 8 Ω. Calculate resistance and reactance of coil B. Also calculate active power consumed by coil A and B, total impedance of the circuit.*

Solution :

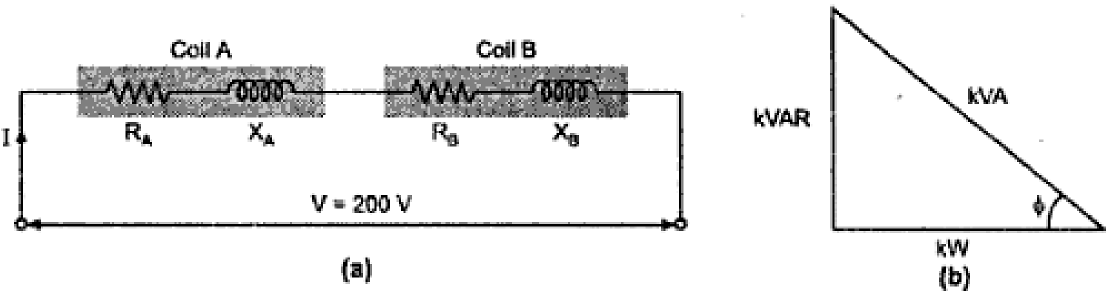


Fig. 7.58

$$R_A = 4\ \Omega \text{ and } X_A = 8\ \Omega,\ V = 200 \text{ volt}$$

Total power $\quad P = 2.2 \text{ kW}, \quad Q = 1.5 \text{ kVAR}$

Consider the power triangle shown in Fig. 7.58 (b)

$$\text{kVA} = VI \times 10^{-3} \quad (10^{-3} \text{ to convert to kVA from VA})$$

And $\quad \text{kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2} = \sqrt{(2.2)^2 + (1.5)^2} = 2.66 \text{ kVA}$

$$2.66 = VI \times 10^{-3}$$

$\therefore \quad 2.66 = 200 \times I \times 10^{-3}$

$\therefore \quad I = 13.3 \text{ A}$

$$P_T = P_A + P_B$$

$$P_A = \text{Power consumed in coil A}$$

$$= I^2 R_A = (13.3)^2 \times 4 = 707.56 \text{ W}$$

$$P_B = \text{Power consumed in coil B} = I^2 R_B$$

$$2.2 \times 10^3 = 707.56 + I^2 R_B$$

$$R_B = 8.4\ \Omega$$

$\therefore \quad P_B = I^2 R_B = (13.3)^2 \times 8.4 = 1.4858 \text{ kW}$

Now, $\quad Z_T = \dfrac{V}{I} = \dfrac{200}{13.3} = 15.037\ \Omega$

And $\quad R_T = R_A + R_B = 4 + 8.4 = 12.4\ \Omega$

$$Z_T = \sqrt{(R_T)^2 + (X_T)^2} \quad \text{i.e.} \quad 15.037 = \sqrt{(12.4)^2 + (X_T)^2}$$

$\therefore \quad X_T = 8.5\ \Omega$

$$X_T = X_A - X_B \quad \text{i.e. } X_B = X_T - X_A = 8.5 - 8$$

$\therefore \quad X_B = 0.5\ \Omega$

Example 7.21 : *A series R-C circuit is connected across 200 V, 50 Hz a.c. supply draws a current of 20 A. When the frequency of the supply is increased to 100 Hz, the current increases to 23.4082 A. Calculate the value of resistance and capacitance of the circuit.*

Solution : When frequency is 50 Hz i.e. $f_1 = 50$ Hz

$$Z_1 = \frac{V}{I_1} = \frac{200}{20} = 10\ \Omega$$

When frequency is 100 Hz i.e. $f_2 = 100$ Hz

$$Z_2 = \frac{V}{I_2} = \frac{200}{23.4082} = 8.544\ \Omega$$

$$Z_1 = \sqrt{R^2 + (X_{C_1})^2} \quad \text{... (1) and} \quad Z_2 = \sqrt{R^2 + (X_{C_2})^2} \quad \text{... (2)}$$

$$\therefore \quad (10)^2 = R^2 + (X_{C_1})^2 \quad \text{...(3) and} \quad (8.544)^2 = R^2 + (X_{C_2})^2 \quad \text{... (4)}$$

Subtracting (3) and (4),

$$\therefore \quad (X_{C_1})^2 - (X_{C_2})^2 = 27 \quad \text{i.e.} \left(\frac{1}{2\pi f_1 C}\right)^2 - \left(\frac{1}{2\pi f_2 C}\right)^2 = 27$$

$$\therefore \quad \frac{1}{4\pi^2 \times 50^2 \times C^2} - \frac{1}{4\pi^2 \times 100^2 \times C^2} = 27$$

$$\frac{1}{C^2}(7.6 \times 10^{-6}) = 27 \quad \text{i.e.} \quad \therefore C^2 = 2.814 \times 10^{-7}$$

$$\therefore \quad C = 5.305 \times 10^{-4}\ \text{F}$$

Substituting in (3), $R = 8\ \Omega$

Example 7.22 : *Two impedances Z_1 and Z_2 having same numerical value are connected in series. If Z_1 is having p.f. of 0.866 lagging and Z_2 is having p.f. of 0.6 leading. Calculate the p.f. of the series combination.*

Solution : Z_1 has p.f. $\cos\phi_1 = 0.866$ lagging

Z_2 has p.f. $\cos\phi_2 = 0.6$ leading

$$|Z_1| = |Z_2| = Z$$

$$\cos\phi_1 = 0.866 \text{ and } \sin\phi_1 = 0.5$$

$$\cos\phi_2 = 0.6 \text{ and } \sin\phi_2 = 0.8$$

$$Z_1 = [Z\cos\phi_1 + j\sin\phi_1] = Z\,[0.866 + j\,0.5]$$

$$Z_2 = Z\cos\phi_2 + j\,Z\sin\phi_2 = Z\,[0.6 + j\,0.8]$$

$$\therefore \quad Z_T = Z\,[1.466 - j\,0.3] = Z\,1.4963 \angle -11.565°$$

$\therefore$ Power factor of the combination is cos (– 11.565) i.e.

$$\cos\phi_T = \mathbf{0.9796\ leading.}$$

Example 7.23 : *A circuit is shown in Fig. 7.59. Draw its equivalent admittance circuit. Also calculate admittance, conductance and susceptance.*

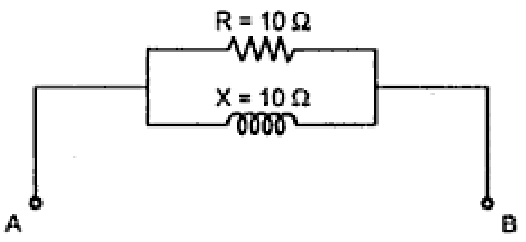


Fig. 7.59

Solution : The impedance of branch 1, $Z_1 = R + j\,0$ where $R = 10\ \Omega$

$$\therefore \quad Z_1 = 10 + j\,0 = 10\angle 0^\circ\ \Omega$$

The impedance of branch 2, $Z_2 = 0 + j\,X$

Where $X = 20\ \Omega$

$$\therefore \quad Z_2 = 0 + j\,20 = 20\angle 90^\circ\ \Omega$$

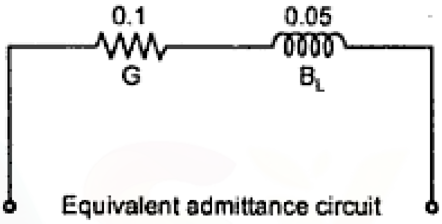


Fig. 7.60

$$\text{Admittance } Y_1 = \frac{1}{Z_1} = \frac{1}{10\angle 0^\circ}$$

$$= 0.1\angle 0^\circ \text{ siemens}$$

$$\text{Admittance } Y_2 = \frac{1}{Z_2} = \frac{1}{20\angle 90^\circ} = 0.05\angle -90^\circ \text{ siemens}$$

$$Y_1 = 0.1\angle 0^\circ = 0.1 + j\,0 \text{ siemens}$$

$$Y_2 = 0.05\angle -90^\circ = 0 - j\,0.05 \text{ siemens}$$

$$\overline{Y} = \overline{Y}_1 + \overline{Y}_2 = 0.1 + j\,0 + 0 - j\,0.05$$

$$= 0.1118\angle -26.56^\circ$$

$\therefore$ Conductance $G = \mathbf{0.1}$

Susceptance $B = \mathbf{0.05}$

and admittance $Y = \mathbf{0.1118\ siemens}$

Example 7.24 : *A heater operates at 100 V, 50 Hz and takes current of 8 A and consumes 1200W power. A choke coil is having ratio of reactance to resistance as 10, is connected in series with the heater. The series combination is connected across 230 V, 50 Hz a.c. supply. Calculate the*

i) Resistance of choke coil *ii) Reactance of choke coil*

iii) Power consumed by choke coil *iv) Total power consumed.*

Solution : For heater $V_{rated} = 100$ V and $I = 8$ A

Heater is purely resistive load.

$$\therefore \quad R_{heater} = \frac{V}{I} = \frac{100}{8} = 12.5\ \Omega$$

$$\text{For choke coil } \frac{X_L}{r} = 10$$

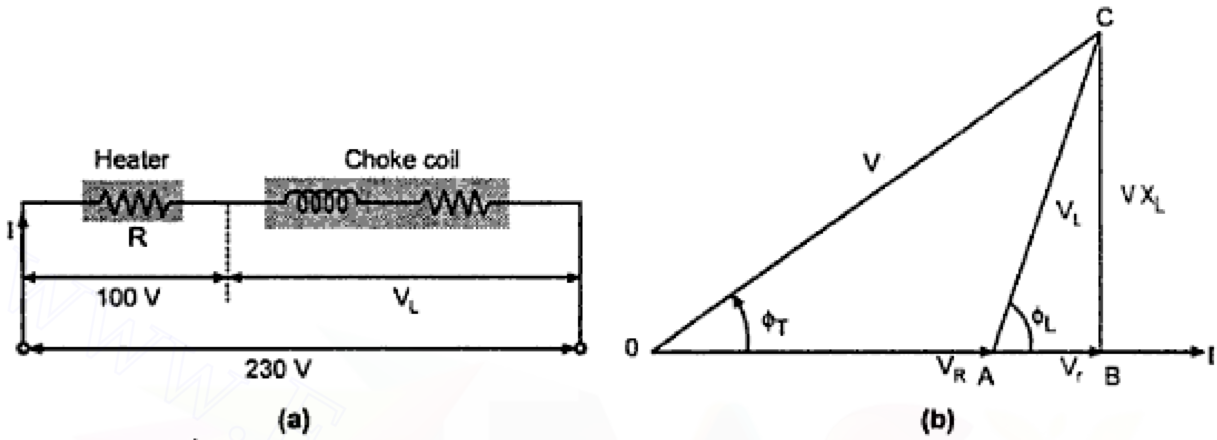


Fig. 7.61

$$Z_{coil} = r + j\,X_L = (r) + j\,(10r)$$

$$|Z| = \sqrt{(r)^2 + (10r)^2} = (10.049\ r) \angle 84.289°$$

And $$\phi_L = \tan^{-1}\left(\frac{10\,r}{r}\right)$$

$$\therefore \quad \phi_L = 84.289°$$

This is shown in the Fig. 7.61 (b)

Resolving V_L into its two components,

i) V_r = drop across resistance of coil = $V_L \cos \phi_L$

ii) V_{X_L} = drop across reactance of coil = $V_L \sin \phi_L$

Consider triangle OBC as shown in Fig. 7.61 (b)

$$\therefore \quad (OC)^2 = (OB)^2 + (BC)^2$$

$$\therefore \quad (OC)^2 = (OA + AB)^2 + (BC)^2$$

$$\therefore \quad V^2 = (V_R + V_r)^2 + (V \times L)^2$$

$$\therefore \quad (230)^2 = (100)^2 + 2 \times 100 \times V_r + (V_r)^2 + (V \times L)^2$$

$$\therefore \quad (230)^2 = (100)^2 + 200 \times (V_L \cos \phi_L) + (V_L \cos \phi_L)^2 + (V_L \sin \phi_L)^2$$

$$\therefore \quad (230)^2 = (100)^2 + 200 \times (V_L)\,(\cos 84.28) + (V_L)^2\,(\cos \phi_L^2 + \sin \phi_L^2)$$

$$\therefore \quad (230)^2 = (100)^2 + 19.93\ V_L + (V_L)^2$$

$$(V_L)^2 + 10.93\ V_L - 42900 = 0$$

Solving for V_L we get, $V_L = 197.397$ V

$$V_r = V_L \cos\phi_L = 19.643 \text{ V}$$

$$V \times L = V_L \sin\phi_L = 196.417 \text{ V}$$

i) $V_r = I \times r$

$$\therefore \quad r = \frac{V_r}{I} = \frac{196.417}{8} = 2.4552\ \Omega$$

ii) $V_{X_L} = I \times X_L$

$$\therefore \quad X_L = \frac{V \times L}{I} = \frac{196.417}{8} = 24.552\ \Omega$$

iii) Power consumed by coil is $= I^2 \times r = (8)^2 \times 2.4532 = 157.13$ **W**

Or $P_{coil} = V_L \times I \times \cos\phi_L = 197.387 \times 8 \times \cos(84.28) = 157.13$ **W**

iv) Total power consumed $= I^2 \times R_{heater} + I^2 r = I^2 (R_{heater} + r)$

$$= 64 \times 14.9552 = 957.1328 \text{ W.}$$

Example 7.25 : *Two impedances Z_1 and Z_2 are connected in series across 200 V, 50 Hz a.c. supply. The total current drawn by the series combination is 2.3 A. The p.f. of Z_1 is 0.8 lagging. The voltage drop across Z_1 is twice the voltage across Z_2 and it is in 90° out of phase with it. Determine the value of Z_2 and power consumed by Z_2 and total power consumed by the circuit.*

Solution : Let V_1 = Drop across Z_1 and V_2 = Drop across Z_2

$$V_1 = 2\ V_2 \quad \text{i.e.} \quad I Z_1 = 2\ (I Z_2)$$

$$V_2 = (0.5)\ V_1 \text{ and } Z_2 = (0.5)\ Z_2$$

Now V_1 is 90° out of phase with V_1.

$$(V)^2 = (V_1)^2 + (V_2)^2$$

$$\therefore \quad (200)^2 = (V_1)^2 + 0.5\ (0.5\ (V_1)^2$$

$$\therefore \quad V_1 = 178.885 \text{ V}$$

$$\therefore \quad V_2 = 89.442 \text{ V}$$

Now $I = 2.3$ A

$$I Z_1 = 178.885$$

$$\therefore \quad |Z_1| = 77.78\ \Omega$$

and $\qquad I\,Z_2 = 89.442$

$\therefore \qquad |Z_2| = 38.38\ \Omega$

Now $\qquad Z_1 = |Z_1| \angle \phi_1,$

$\qquad \phi_1 = \cos^{-1} 0.8 = 36.86°$ lagging

and $\qquad Z_2 = |Z_2| \angle \phi_2$

But $\qquad \phi_2 + \phi_1 = 90°$ $\qquad$ ... out of phase by 90°

Refer to the Fig. 7.62 (b) shown below.

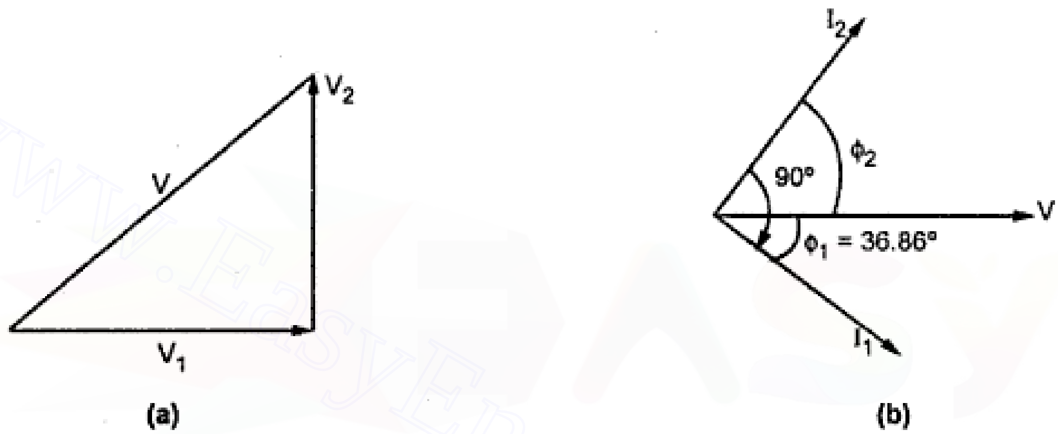


Fig. 7.62

$\therefore \qquad \phi_2 \phi_1 = 90 - 36.86 = 53.13°$ leading

$\therefore \qquad$ Power consumed in $Z_2 = V_2 \times I \times \cos \phi_2 = 89.442 \times 2.3 \times \cos (53.13)$

$= \mathbf{123.43\ W}$

Total power consumed in the circuit

$=$ Power in Z_1 + Power in $Z_2 = V_1\, I \cos \phi_1 + 123.43 = 329.148 + 123.43$

$= \mathbf{452.578\ W}$

Example 7.26 : *An alternating voltage is applied to a circuit, which is given by*

$$v = 141.42 \sin\left(157.08t + \frac{\pi}{12}\right) \text{volts.}$$

An a.c. ammeter, wattmeter and power factor meter are connected in the circuit. The ammeter reads 5 A and power factor meter reads 0.5 lagging. Calculate,

i) Expression for the instantaneous current. *ii) The wattmeter reading.*

iii) Impedance of the circuit in the reactangular form. **(Dec. - 99)**

$$Z_2 = \frac{V\angle 0°}{|I_2|\angle 14.28°} = |Z_2| \angle -14.28°$$

Now $\quad |I_1| = |I_2|$

$\therefore \quad \dfrac{V}{|Z_1|} = \dfrac{V}{|Z_2|}$

$\therefore \quad |Z_1| = |Z_2|$

$|Z_2| = 16.21\ \Omega$

$\therefore \quad Z_2 = |Z_2| \angle -4.28° = 16.21 \angle -14.28° = 15.7 - j\,4\ \Omega$

$\therefore \quad R = 15.7 - 4 = \mathbf{11.7\ \Omega}$

While, external capacitance required with coil is,

$X_C = 15.707 - (-4) = 19.707\ \Omega$

$\therefore \quad C = \dfrac{1}{2\pi f X_C} = \mathbf{1.615 \times 10^{-4}\ F}$

Now $\quad V = 200\ V$

$Z_1 = 16.21 \angle 75.72° \quad$ and $\quad Z_2 = 16.21 \angle -14.28°$

$I_1 = \dfrac{200\angle 0°}{16.21\angle 75.72°} = 12.338 \angle -75.72°\ A = 3.0433 - j\,11.956\ A$

$I_2 = \dfrac{200\angle 0°}{16.21\angle 14.28°} = 12.338 \angle 14.28° = 11.956 + j\,3.0433\ A$

$\therefore \quad \overline{I_T} = \overline{I_1} + \overline{I_2} = 3.0433 - j\,11.956 + 11.956 + j\,3.0433$

$= 15 - j\,8.9127\ A = \mathbf{17.45 \angle -30.71°\ A}$

The resultant current of 17.45 A, lags voltage by 30.71°.

$\therefore$ Total power factor = cos (30.71) = **0.8596 lagging.**

Example 7.28 : *Three impedances $Z_1 = (8 + j6)$ ohm, $Z_2 = (4 + j3)$ ohm and $Z_3 = (18 - j\,9)$ ohm are connected in series across the a.c. supply. If the voltage drop across Z_1 is $(40 + j\,30)$ volts, calculate :*

i) The current in the circuit ii) The voltage drops across Z_2 and Z_3 iii) Total supply voltage iv) Total power consumed by the series circuit v) Power factor of the circuit. Draw phasor diagram for the circuit. **(Dec.-97)**

Solution : The circuit diagram is as shown in the Fig. 7.65.

$$Z_1 = 8 + j6 = 10 \angle 36.86° \, \Omega$$

$$Z_2 = 4 + j3 = 5 \angle 36.86° \, \Omega$$

$$Z_3 = 18 - j\,9 = 20.124 \angle -26.56° \, \Omega$$

$$V_1 = 40 + j\,30 = 50 \angle 36.86° \, \Omega$$

$$I = \frac{V_1}{Z_1} = \frac{50 \angle 36.86°}{10 \angle 36.86°} = 5 \angle 0° \text{ A}$$

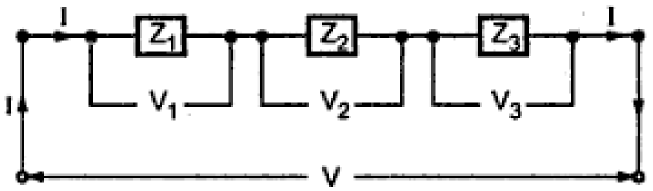


Fig. 7.65

i) Current in the circuit = **5 A**

ii) Drop across $Z_2 = I \times Z_2$

$\therefore \quad V_2 = 5 \angle 0° \times 5 \angle 36.86° = 25 \angle 36.86°$ V = **20 + j 15 V**

Drop across $Z_3 = I \times Z_3$

$\therefore \quad V_3 = 5 \angle 0° \times 20.124 \angle -26.56° = 100.62 \angle -26.56°$ V = **90 − j 45 V**

iii) Total supply voltage

$$V = \overline{V_1} + \overline{V_2} + \overline{V_3} = 40 + j\,30 + 20 + j\,15 + 90 - j\,45$$

$$= 150 + j\,0 \text{ V} = \mathbf{150 \angle 0° \text{ V}}$$

iv) Total power consumed

$$P = V\,I\,\cos\phi$$

$$= 150 \times 5 \times \cos(0)$$

$$= \mathbf{750 \text{ W}}$$

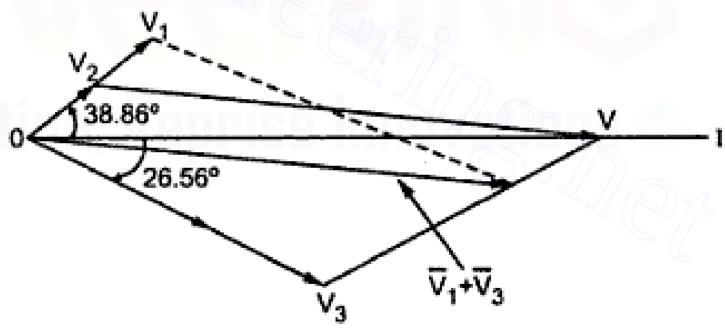


Fig. 7.66

v) Angle between V and I is 0°

Hence $\phi = 0°$

$\therefore \quad \cos\phi = 1$

So power factor of the circuit is one.

The phasor diagram is shown in the Fig. 7.66.

Example 7.29 : *When connected to a 230 V, 50 Hz single phase supply, coil takes 10 kVA and 8 kVAR. For this coil, calculate :*

i) Resistance ii) Inductance of coil and iii) Power consumed. **(Dec.-98)**

Solution : $V_{ph} = 230$ V, kVAR = 8, kVA = 10

The power triangle is shown in the Fig. 7.67.

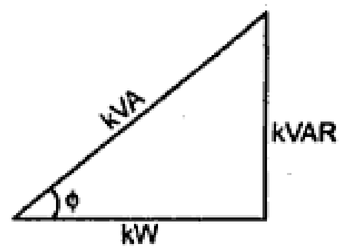


Fig. 7.67

$$\therefore \quad \sin\phi = \frac{\text{k VAR}}{\text{kVA}} = \frac{8}{10} = 0.8$$

$$\therefore \quad \phi = 53.13°$$

$$VA = VI$$

$$\therefore \quad 10\times10^3 = 230\ I$$

$$\therefore \quad I = \frac{10\times10^3}{230} = 43.4782\ A$$

$$\therefore \quad |Z| = \frac{V}{I} = \frac{230}{43.4782} = 5.29\ \Omega$$

$$\therefore \quad Z = |Z|\ \angle\phi$$

$$= 5.29\ \angle+53.13° = 3.17 + j\ 4.323\ \Omega$$

$$\therefore \quad R = 3.17\ \Omega \text{ and } X_L = \mathbf{4.232\ \Omega}$$

$$P = VI\cos\phi = 230\times43.4782\times0.6 = \mathbf{6\ kW}$$

As $X_L = 2\pi fL$ i.e. $4.232 = 2\pi\times50\times L$

$$\therefore \quad L = \mathbf{13.4708\ mH}$$

Example 7.30 : *Two impedances Z_1 and Z_2 when connected separately across 230 volts, 50 Hz, ac supply consumed 100 watts and 60 watts at power factors of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply find.*

i) Total power absorbed and overall power factor.

ii) Value of the impedance to be added in series to raise the overall power factor to unit.

(May-99)

Solution : When Z_1 alone is connected across the supply.

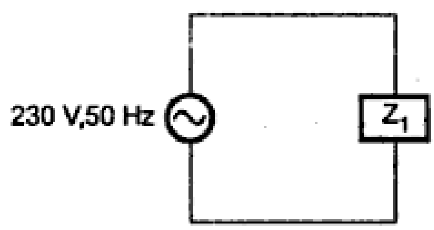


Fig. 7.68

$$\cos\phi = 0.5 \text{ lagging}$$

$$P = 100\ W$$

$$P = VI\cos\phi$$

$$\therefore \quad 100 = 230\times I\times0.5$$

$$\therefore \quad I = 0.8695\ A$$

$$\therefore \quad |Z_1| = \frac{V}{I} = \frac{230}{0.8695} = 264.5198\ \Omega$$

$$\phi = \cos^{-1}0.5 = 60°$$

$$Z_1 = |Z_1| \angle +\phi = 264.5198 \angle +60°\Omega = 132.29 + j\,229.1328\ \Omega$$

Similarly when Z_2 alone is connected.

$$\cos\phi = 0.6 \text{ leading}$$

$$P = 60\ W$$

$$P = VI\cos\phi$$

$$\therefore \quad 60 = 230 \times I \times 0.6$$

$$\therefore \quad I = 0.4347\ A$$

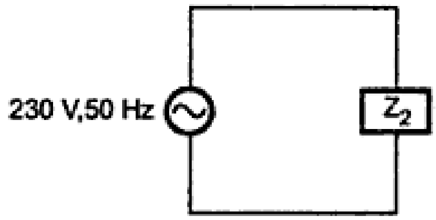


Fig. 7.69

$$|Z_2| = \frac{V}{I} = \frac{230}{0.4347} = 529$$

$$\phi = -53.13° \qquad \text{negative as leading}$$

$$Z_2 = |Z_2| \angle\phi = 529 \angle -53.13°\Omega = 317.4 - j\,423.2\ \Omega$$

i) When connected in series, $Z_T = Z_1 + Z_2 = 449.69 - j\,194.06\ \Omega$

$$= 498.778 \angle -23.3439°\ \Omega$$

$$\cos\phi = \cos(-23.3429)$$

$$\therefore \quad \text{Power factor} = \mathbf{0.9181\ leading}$$

$$I = \frac{V}{Z_T} = \frac{230}{489.778} = 0.4696\ A$$

$$\therefore \quad P = VI\cos\phi = 230 \times 0.4696 \times 0.9181 = \mathbf{99.16\ W}$$

ii) Let Z_X be the impedance to be added to get overall power factor as unity. Hence net impedance is,

$$Z_{total} = Z_1 + Z_2 + Z_x = 449.96$$

$$\therefore \quad 449.69 = 449.69 - j\,194.06 + Z_x$$

For unity power factor,

$$Z_{total} = \text{Purely resistive} = 449.69\ \Omega$$

$$\therefore \quad 449.69 = 449.69 - j\,194.06 + Z_x$$

$$\therefore \quad Z_x = \mathbf{+ j\,194.06\ \Omega}$$

The value of impedance required to get unity power factor is inductive reactance of 194.06 Ω.

Example 7.31 : *A coil draws 5 amps when connected to 100 volts 50 Hz supply. The resistance of the coil is 5 Ω determine*

i) Inductance of the coil ii) Real power, reactive power, apparent power for the coil.

(May-99)

Solution : I = 5 A, V = 100 V, R = 5 Ω

$$|Z| = \frac{V}{I} = \frac{100}{5} = 20 \text{ A}$$

$$Z = R + j X_L$$

$$|Z| = \sqrt{R^2 + X_L^2} \quad \text{i.e. } 20 = \sqrt{25 + X_L^2}$$

$$\therefore \quad 400 = 25 + X_L^2 \quad \text{i.e. } X_L^2 = 375$$

$$\therefore \quad X_L = 19.3649\ \Omega$$

i) $$X_L = 2\pi f L \quad \text{i.e. } 19.3649 = 2\pi \times 50 \times L$$

$$\therefore \quad L = \mathbf{61.64\ mH}$$

ii) $$\text{Real power} = VI \cos\phi$$

$$\cos\phi = \frac{R}{Z} = \frac{5}{20} = 0.25$$

$$\therefore \quad \text{Real power} = 100 \times 5 \times 0.25 = \mathbf{125\ W}$$

$$\text{Reactive power} = VI \sin\phi = 100 \times 5 \times 0.9682 = \mathbf{484.1229\ VAR}$$

$$\text{Apparent power} = VI = 100 \times 5 = \mathbf{500\ VA}$$

Example 7.32 : *Two impedances, $Z_1 = (100 + j\ 0)$ ohm and $Z_2 = (R + jX)$ ohm are in series. An a.c. Voltage of 400 V, 50 Hz is applied across the series combination. The voltage drops across the two impedances are 200 V and 300 V respectively. Sketch a neat connection diagram and phasor diagram and find the current and power consumed by Z_2.* **(Dec.-99)**

Solution : The arrangement is shown in the Fig. 7.70.

Now, $$Z_1 = 100 + j\ 0\ \Omega$$

$$= 100 \angle 0^\circ\ \Omega$$

$$\therefore \quad I = \frac{V_1}{Z_1} = \frac{200}{100} = 2 \text{ A}$$

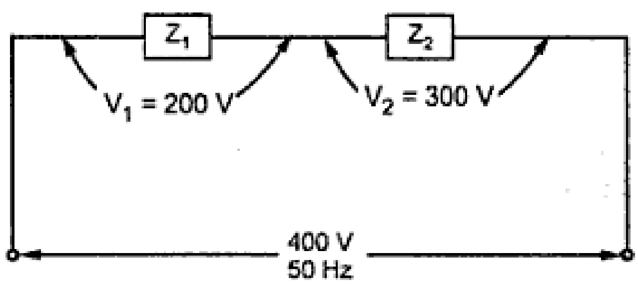


Fig. 7.70

Example 7.34 : *A coil connected to a single phase 230 V, 50 Hz, ac supply dissipates 1.5 kW of power, when it takes 10 A from the supply. Find out the resistance and reactance of the coil. Also, determine phase angle and draw the phase diagram.* **(May-2000)**

Solution : The arrangement is shown in the Fig. 7.73.

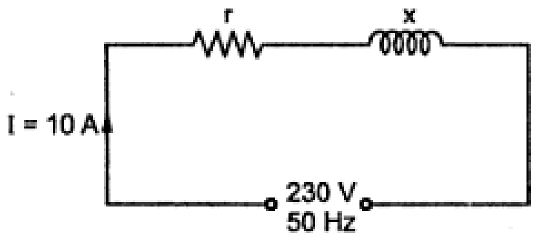


Fig. 7.73

$$P = V I \cos \phi_T$$

$$\therefore \quad 1.5 \times 10^3 = 230 \times 10 \times \cos \phi_T$$

$$\therefore \quad \cos \phi_T = \frac{1.5 \times 10^3}{230 \times 10} = 0.6521 \text{ lagging}$$

$$Z = r + j X$$

$$\cos \phi_T = \frac{r}{Z} \quad \text{i.e.} \quad 0.6521 = \frac{r}{Z} \quad \text{... (1)}$$

$$|Z| = \frac{|V|}{|I|} = \frac{230}{10} = 23 \ \Omega$$

Now, $\quad |Z| = \sqrt{r^2 + X^2} \quad$ i.e. $\quad 23 = \sqrt{r^2 + X^2}$

$$\therefore \quad r^2 + X^2 = 529 \quad \text{... (2)}$$

Now, $\quad 0.6521 = \dfrac{r}{Z} = \dfrac{r}{\sqrt{r^2 + Z^2}}$

Squaring both sides, $\quad (0.6521)^2 = \dfrac{r^2}{r^2 + X^2}$

Using result of (2),

$$(0.6521)^2 = \frac{r^2}{529}$$

$$\therefore \quad r^2 = 224.949$$

$$\therefore \quad r = 14.999 \approx 15 \ \Omega$$

$$\therefore \quad X = 17.4356 \ \Omega$$

$$\therefore \quad \phi_T = \cos^{-1} (0.6521)$$

$$= 49.299^\circ$$

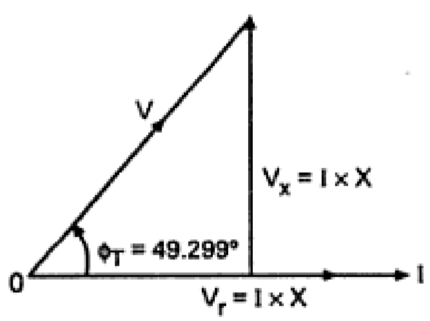


Fig. 7.74

The phasor diagram is shown in Fig. 7.74.

7) The phasor diagram is shown in the Fig. 7.75.

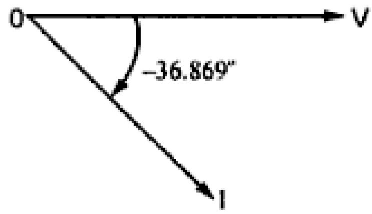


Fig. 7.75

Example 7.37 : *Two circuits having same numerical value of ohmic impedance are connected in parallel. The power factor of one circuit having impedance Z_1 is 0.8 and other impedance Z_2 is 0.6. What is the power factor of the combination, when,*

i) both the impedances are inductive?

ii) Z_1 is inductive and Z_2 is capacitive ? **(Dec.-2003, Dec.-2006)**

Solution : $|Z_1| = |Z_2| = Z\ \Omega$

$$\cos\phi_1 = 0.8 \quad \text{and} \quad \sin\phi_1 = 0.6$$

$$\cos\phi_2 = 0.6 \quad \text{and} \quad \sin\phi_2 = 0.8$$

i) Both are inductive

$$Z_1 = |Z_1|\,[\cos\phi_1 + j\sin\phi_1] = Z(0.8+j0.6)$$

$$Z_2 = |Z_2|\,[\cos\phi_2 + j\sin\phi_2] = Z(0.6+j\,0.8)$$

They are in parallel,

$$\therefore \quad Z_T = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{Z^2(0.8+j\,0.6)(0.6+j\,0.8)}{Z[0.8+j\,0.6+0.6+j\,0.8]}$$

$$= Z\left[\frac{1\angle 36.869°\times 1\angle 53.13°}{1.4+j\,1.4}\right]$$

$$= Z\left[\frac{1\angle 90°}{1.9798\angle 45°}\right] = \frac{Z}{1.9748}\angle 45°$$

$\therefore$ p.f. of combination $= \cos 45° =$ **0.7071 lagging**

ii) Z_1 is inductive and Z_2 is capacitive

$$\therefore \quad Z_1 = Z(0.8+j\,0.6) = Z[1\angle 36.869°]$$

While $$Z_2 = Z(0.6-j\,0.8) = Z[1\angle -53.13°]$$

j term is negative due to capacitive.

$$\therefore \quad Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{Z^2 [1\angle 36.869° \times 1\angle -53.13°]}{Z[0.8 + j0.6 + 0.6 - j0.8]}$$

$$= Z\left\{\frac{1\angle -16.261°}{1.4 - j0.2}\right\} = Z\left\{\frac{1\angle -16.261°}{1.4142\angle -8.13°}\right\} = \frac{Z}{1.4142}\angle -8.1308°$$

$\therefore$ p.f. combination = $\cos(-8.1308°)$ = **0.9899 leading.**

Example 7.38 : *A pure resistance R, a choke coil and a pure capacitor of 15.91 μF are connected in series across a supply of V volts and carry current of 0.25 Amp. The voltage across choke is 40 volts, the voltage across capacitor is 50 volt and voltage across resistance is 20 volt. The voltage across combination of R and choke coil is 45 volts. Calculate,*

i) Supply voltage ii) Frequency iii) Power loss in choke coil. **(Dec.-2003)**

Solution : The circuit is shown in the Fig. 7.76.

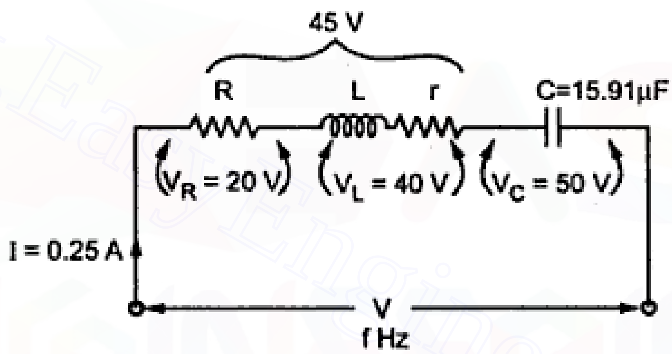


Fig. 7.76

The choke coil resistance is r Ω

$$Z_T = (R+r) + j(X_L - X_C) \quad \text{i.e.} \quad |Z_T| = \sqrt{(R+r)^2 + (X_L - X_C)^2} \quad \text{... (1)}$$

$$|Z_T| = \frac{V}{I} = \frac{V}{0.25} = 4\Omega \quad \text{... (2)}$$

Now $\quad V_R = IR \quad$ i.e. $\quad 20 = 0.25\ R$

$\therefore \quad R = 80\ \Omega$ ... (3)

Then $\quad V_C = I X_C \quad$ i.e. $\quad 50 = 0.25\ X_C$

$\therefore \quad X_C = 200\ \Omega$ ... (4)

and $\quad V_L = I Z_L \quad$ where $\quad Z_L = r + jX_L$

$$\therefore \quad 40 = 0.25\sqrt{r^2 + X_L{}^2}$$

$$\therefore \quad r^2 + X_L^2 = (160)^2 \quad \text{... (5)}$$

Also $\quad \bar{V}_R + \bar{V}_L = 45\ V$

i.e. $\quad I \times |(R+r) + jX_L| = 45$

$$\therefore \quad 0.25\sqrt{(R+r)^2 + X_L^2} = 45$$

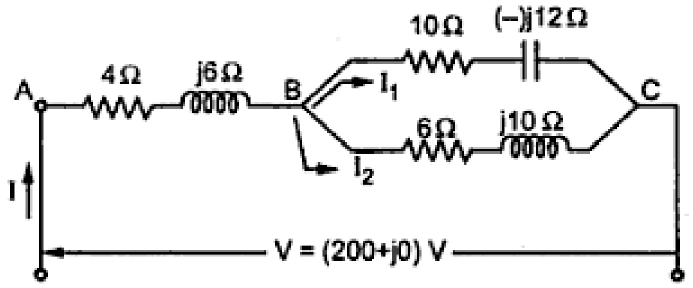


Fig. 7.78

Solution : The two impedances Z_1 and Z_2 are in parallel.

$$Z_1 = 10 - j\,12 = 15.62 \angle -50.194° \Omega$$

$$Z_2 = 6 + j\,10 = 11.662 \angle 59.036° \ \Omega$$

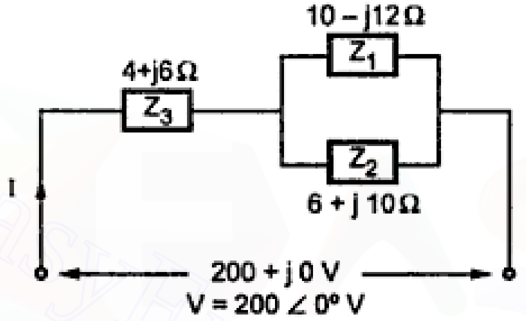


Fig. 7.79 (a)

$$\therefore \quad Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{182.1604 \angle 8.842°}{10 - j\,12 + 6 + j\,10} = \frac{182.1604 \angle 8.842°}{16.124 \angle -7.125°}$$

$$\therefore \quad Z_T = 11.2974 \angle 15.967° \Omega = 10.861 + j\,3.1077 \ \Omega$$

$$\therefore \quad Z_{total} = Z_3 + Z_T = 14.861 + j\,9.1077 = 17.429 \angle 31.502° \ \Omega$$

$$\therefore \quad I = \frac{V_{total}}{Z_{total}} = \frac{200 \angle 0°}{17.429 \angle 31.502°} = 11.475 \angle -31.502° \text{ A}$$

Using current distribution in parallel impedances,

$$I_1 = I \times \frac{Z_2}{Z_1 + Z_2} \quad \text{and} \quad I_2 = I \times \frac{Z_1}{Z_1 + Z_2}$$

$$Z_1 + Z_2 = 10 - j\,12 + 6 + j\,10 = 16 - j\,2 = 16.124 \angle -7.125° \ \Omega$$

$$\therefore \quad I_1 = 11.475 \angle -31.502° \times \frac{11.662 \angle 59.036°}{16.124 \angle -7.125°} = \mathbf{8.3 \angle 34.659° \text{ A}}$$

And
$$I_2 = 11.475 \angle -31.502° \times \frac{15.62 \angle -50.194°}{16.124 \angle -7.125°} = \mathbf{11.1163 \angle -74.571° \text{ A}}$$

The phasor diagram is shown in the Fig. 7.79 (b).

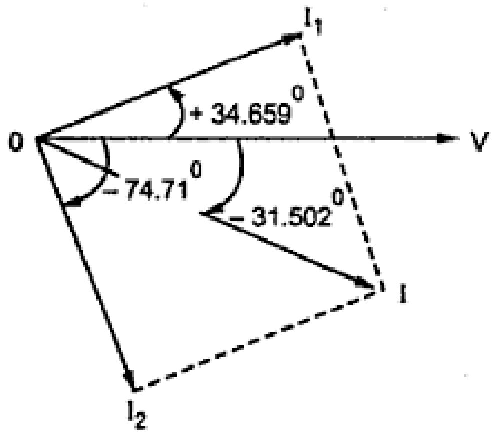


Fig. 7.79 (b)

Example 7.42 : *A series circuit consisting of a 12 Ω resistance, 0.3 henry inductance and a variable capacitor is connected across 100 V, 50 Hz a.c. supply. The capacitance value is adjusted to obtain maximum current. Find this capacitance value and the power drawn by the circuit under this condition.*

Now, the supply frequency is raised to 60 Hz, the voltage remaining same at 100 V. Find the value of capacitor C_1 to be connected across the above series circuit, so that current drawn from supply is the minimum. **(May-2004)**

Solution : Case 1 : The circuit diagram is shown in the Fig. 7.80.

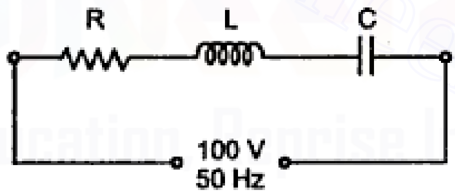


Fig. 7.80

For I_{max}, there must be resonance.

$$\therefore \quad X_L = X_C$$

$$\text{Now} \quad X_L = 2\pi f L = 2\pi \times 50 \times 0.3 = 94.2477\ \Omega = X_C$$

$$\therefore \quad \frac{1}{2\pi \times 50 \times C} = 94.2477$$

$$\therefore \quad C = \mathbf{33.7737\ \mu F}$$

$$\text{And} \quad P = VI = \frac{V^2}{R} \quad \text{... as } I_{max} = \frac{V}{R} \text{ under resonance}$$

$$= \frac{(100)^2}{12} = \mathbf{833.333\ W}$$

Case 2 : f = 60 Hz, V = 100 V

The circuit is shown in the Fig. 7.81 (a).

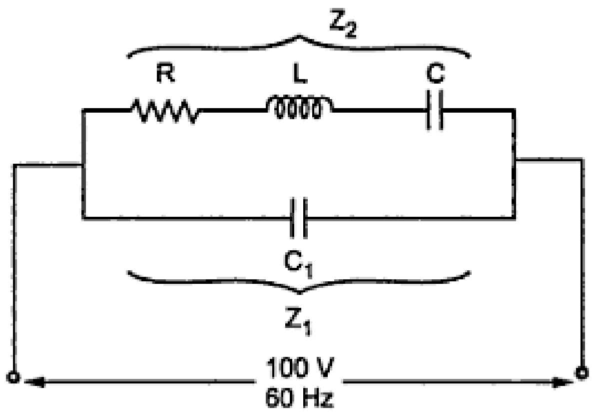


Fig. 7.81 (a)

At 60 Hz,

$$X_L = 2\pi f L = 113.0973\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = 78.5398\ \Omega$$

$$\therefore \quad Z_2 = R - jX_C + jX_L$$

$$= 12 + j\,34.5575\ \Omega$$

$$= 36.5817\ \angle 70.8505°\ \Omega$$

$$\therefore \quad Y_2 = \frac{1}{Z_2} = \frac{1}{36.5817\angle 70.8505°} = 0.02733\ \angle -70.8505°$$

$$= 8.9671\times10^{-3} - j\,0.02582 \ \text{ mho}$$

and $\quad Z_1 = -jX_{C1} = X_{C1}\angle -90°$

$$\therefore \quad Y_1 = \frac{1}{X_{C1}\angle -90°} = \frac{1}{X_{C1}}\ \angle +90° = +j\left[\frac{1}{X_{C1}}\right]$$

$$Y_T = Y_1 + Y_2 = 8.9671\times10^{-3} + j\left(\frac{1}{X_{C1}} - 0.02582\right)$$

For current minimum, there must be parallel resonance and circuit must be having unity p.f. so imaginary part of Y_T must be zero.

$$\therefore \quad \left(\frac{1}{X_{C1}}\right) - 0.02582 = 0$$

$$\therefore \quad \left(\frac{1}{X_{C1}}\right) = 0.02582\ \Omega$$

$$\therefore \quad X_{C1} = 38.7296 = \frac{1}{2\pi f C_1}$$

$$\therefore \quad C_1 = 68.4896 \ \mu F$$

Example 7.43 : *An e.m.f. given by v = 100 sin π t is impressed across a circuit consists of resistance of 40 Ω in series with 100 μF capacitor and 0.25 H inductor.*

Determine -

i) R.M.S. value of current ii) Power consumed

iii) Power factor **(Dec-2004)**

Solution : The circuit diagram is shown in the Fig. 7.82.

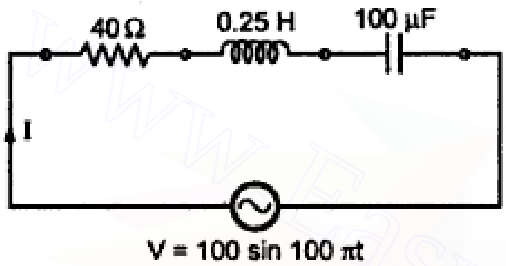


Fig. 7.82

Comparing voltage with,

$$v = V_m \sin \omega t$$

$$V_m = 100 \text{ V and } \omega = 100\,\pi \text{ rad/sec}$$

$$\therefore \quad V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$$= 70.7106 \text{ V} \quad \text{... (r.m.s.)}$$

$$X_L = 2\pi f L = \omega L = 100\pi \times 0.25 = 78.5398 \ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C} = \frac{1}{100\pi \times 100 \times 10^{-6}} = 31.8309 \ \Omega$$

$$\therefore \quad Z_T = R + j X_L - jX_C = 40 + j\,78.5398 - j\,31.8309$$

$$= 40 + j\,46.7089 = 61.4957 \angle 49.4243^\circ \ \Omega$$

In polar form, $V = 70.7106 \angle 0^\circ$ V ... As its phase is zero

$$\therefore \quad I = \frac{V}{Z_T} = \frac{70.7016 \angle 0^\circ}{61.4957 \angle 49.4243^\circ} = 1.1498 \angle -49.4243^\circ \text{ A}$$

i) R.M.S. value of current = **1.1498 A**

ii) Power consumed = VI cos ϕ or I^2R = 70.7106 × 1.1498 × cos (– 49.4243°)

= **52.8837 W**

iii) Power factor = cos (– 49.4243°) = **0.6504 lagging** ... $X_L > X_C$

Example 7.44 : *Two impedances (8 + j 6) Ω and (3 – 4 j) Ω are connected in parallel. If the total current drawn by the combination is 25 Amp, find the current and power taken by each impedance.* **(May-2008)**

Solution : The circuit diagram is shown in the Fig. 7.83.

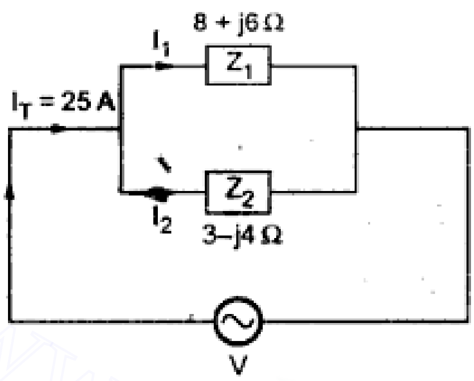


Fig. 7.83

$$Z_1 = 8 + j6\ \Omega$$

$$= 10 \angle 36.869^\circ\ \Omega$$

$$Z_2 = 3 - j4\ \Omega = 5 \angle -53.1301^\circ\ \Omega$$

i) Branch currents

Using current distribution in parallel circuit,

Assume I_T as reference

$$I_1 = I_T \times \frac{Z_2}{Z_1 + Z_2}$$

$$\text{and } I_2 = I_T \times \frac{Z_1}{Z_1 + Z_2}$$

$$Z_1 + Z_2 = 8 + j6 + 3 - j4 = 11 + j2 = 11.1803 \angle 10.3048^\circ\ \Omega$$

$$\therefore \quad I_1 = 25 \angle 0^\circ \times \frac{5 \angle -53.1301^\circ}{11.1803 \angle 10.3048^\circ} = \mathbf{11.1803 \angle -63.4349^\circ\ A}$$

$$\text{And} \quad I_2 = 25 \angle 0^\circ \times \frac{10 \angle 36.869^\circ}{11.1803 \angle 10.3048^\circ}$$

$$= \mathbf{22.3607 \angle 26.5642^\circ\ A}$$

ii) Power consumed

The power gets consumed only in the resistive part given by I^2R

$$\therefore \quad P_1 = I_1^2 R_1 = (11.1803)^2 \times 8 = \mathbf{1000\ W}$$

$$\text{and} \quad P_2 = I_2^2 R_2 = (22.3607)^2 \times 3 = \mathbf{1500\ W}$$

The phaser diagram is shown in the Fig. 7.84

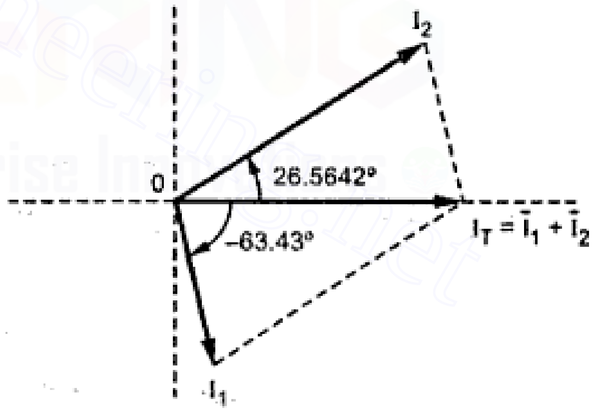


Fig.7.84

Example 7.45 : *A series circuit consisting of a coil and a variable capacitance having reatance X_C. The coil has resistance of 10 Ω, inductive reactance of 20 Ω. It is observed that at certain value of capacitance current in the circuit is maximum, find (1) This value of capacitance (2) Impedance of the circuit (3) Power factor (4) current, if applied voltage is 100 V, 50 Hz.* **(May-2005)**

Solution : The circuit diagram is shown in the Fig. 7.85.

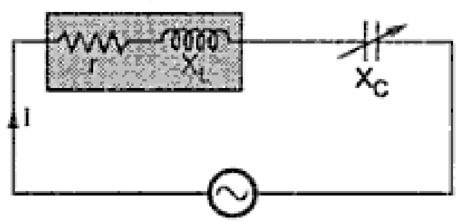


Fig. 7.85

$$r = 10\ \Omega,\ X_L = 20\ \Omega,\ I = I_{max}$$

This is the condition of series resonance for which $I = I_{max}$,

$$Z = r \text{ and } X_L = X_C$$

1) $\therefore\ X_C = 20\ \Omega = \dfrac{1}{2\pi f C}$

$$\therefore\ C = \frac{1}{2\pi\times 50\times 20} = 159.1549\ \mu F$$

2) $Z = R + j X_L - jX_C = 10 + j\,20 - j\,20 = 10\ \Omega$

3) Power factor $= \cos\phi = \cos 0° = 1$

For $X_L = X_C$, the V and I are inphase hence $\phi = 0°$

4) $I = \dfrac{V}{Z} = \dfrac{100}{10} = 10$ A

➡ **Example 7.46 :** *Two impedances ($R_1 - jX_{C1}$) and ($R_2 + jX_{L2}$) are connected in parallel across supply voltage $v = (100\sqrt{2}) \sin (314\ t)$. The current flowing through the two impedances are given by $i_1 = 10\sqrt{2} \sin (314\ t + \pi/4)$ and $i_2 = 10\sqrt{2} \sin (314\ t - \pi/4)$ respectively.*

Find equation for instantaneous value of total current drawn from supply. Also find values of R_1, R_2, X_{C1} and X_{L2}. **(May-2005)**

Solution : The circuit diagram is shown in the Fig. 7.86.

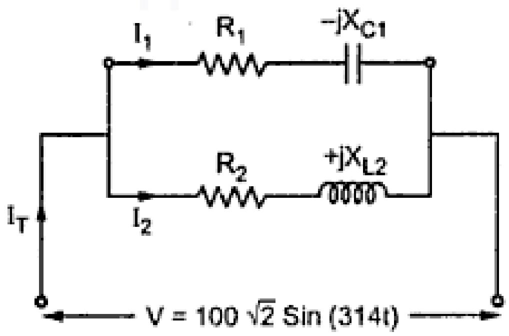


Fig. 7.86

$$i_1 = 10\sqrt{2} \sin (314\ t + \pi/4)$$

$$\therefore\ I_{m1} = 10\sqrt{2} \text{ A, i.e. } I_1 = \frac{10\sqrt{2}}{\sqrt{2}} = 10\text{A (rms)}$$

and $\theta_1 = \pi/4$ rad

$= 45°$ is its phase

$$\therefore\ I_1 = 10 \angle 45° \text{ A} = 7.071 + j\,7.071 \text{ A}$$

$$i_2 = 10\sqrt{2} \sin (314\ t - \pi/4)$$

$$\therefore\ I_{m2} = 10\sqrt{2} \text{ A, i.e. } I_2 = \frac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ A (r.m.s.)}$$

And $\theta_2 = -\pi/4$ rad $= -45°$ is its phase

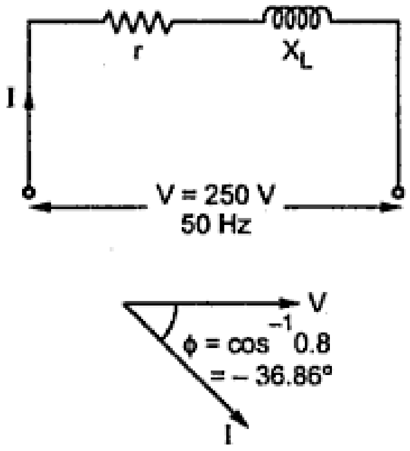


Fig. 7.87

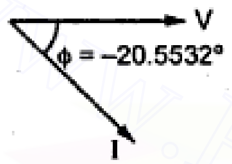


Fig. 7.88

$\therefore \quad Z_{coil} = 20 + j\,15\ \Omega = r + j\,X_L\ \Omega$

$\therefore \quad r = 20\ \Omega$ and $X_L = 15\ \Omega$

when $f = 50$ Hz

$\therefore\ 2\pi fL = 15$ when $f = 50$ Hz

$$\therefore \quad L = \frac{15}{2\pi \times 50} = 0.04774\ \text{H}$$

When connected across 25 Hz, V = 200 V

$$X_L = 2\pi f L = 2\pi \times 25 \times 0.04774$$

$$= 7.4989\ \Omega$$

$$\therefore \quad Z = r + j\,X_L = 20 + 7.4989\ \Omega$$

$$= 21.3596 \angle 20.5532^\circ\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{200\angle 0^\circ}{21.3596\angle 20.5532^\circ}$$

$$= 9.3634 \angle -20.5532^\circ\ \text{A}$$

$$\therefore \quad P = VI\cos\phi \quad \text{or} \quad I^2 r$$

$$= 200 \times 9.3634 \times \cos(-20.5532^\circ) \text{ or } [(9.3634)^2 \times 20]$$

$$= 1.7534\ \text{kW}$$

Example 7.48 : *An alternating voltage v = 141.4 sin (157.08 t + π/12) volts is applied to a circuit and an a.c. ammeter, wattmeter and power factor meter are connected to measure the respective quantities. Reading of ammeter is 5 Amp. and that of powre factor meter is 0.5 lagging, find (i) The expression for the instantaneous value of current, (ii) The wattmeter reading (iii) Impedance of the circuit in rectangular form* **(May-2005)**

Solution : Comparing given voltage with $v = V_m \sin(\omega t + \theta)$

$$V_m = 141.4\ \text{V},\ \omega = 157.08\ \text{rad/sec},\ \theta = \pi/12\ \text{rad} = 15^\circ$$

$$\therefore \quad V = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100\ \text{V (rms)},\ f = \frac{\omega}{2\pi} = \frac{157.08}{2\pi} = 25\ \text{Hz}$$

$$\therefore \quad V = 100 \angle 15^\circ\ \text{V}$$

Key Point : *All meters measure r.m.s. values*

$$\therefore \quad I = 5\ \text{A (r.m.s.) and } \cos\phi = 0.5 \text{ lagging}$$

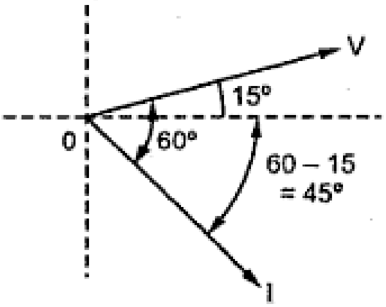


Fig. 7.89

I lags V by $\phi = \cos^{-1} 0.5 = 60°$

And phase of V is 15°. So phasor diagram is shown in the Fig. 7.89.

Thus phase of current is – 45° i.e. $\frac{\pi}{4}$ rad

and $I_m = \sqrt{2}\, I = 5\sqrt{2}$ A

i) Thus expression for instantaneous value of current is,

$$i = I_m \sin(\omega t + \theta_1) = 5\sqrt{2} \sin\left(157.08\, t - \frac{\pi}{4}\right) \text{A}$$

ii) Wattmeter reading is,

$$P = VI \cos\phi = 100 \times 5 \times 0.5 = 250 \text{ W}$$

iii) Now $\quad V = 100 \angle 15° \text{ V and } I = 5 \angle -45° \text{ A}$

$$\therefore \quad Z = \frac{V}{I} = \frac{100\angle 15°}{5\angle -45°} = 20 \angle 60° \,\Omega = 10 + j\, 17.3205 \,\Omega$$

Example 7.49 : *Two circuits, the impedances of which are given by $Z_1 = (12 + j15)\,\Omega$ and $Z_2 = (8 - j4)\,\Omega$, are connected in parallel across the potential difference of $(230 + j0)$ volt. Calculate i) total current drawn, ii) total power and branch powers consumed and iii) overall power factor of circuit.* **(Dec.-2005)**

Solution : The arrangement is shown in the Fig. 7.90.

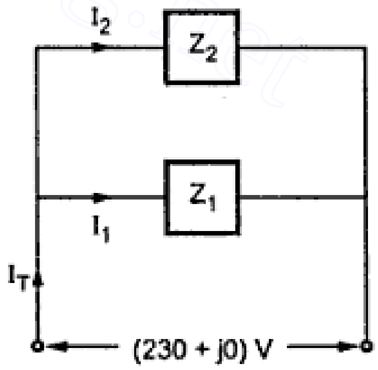


Fig. 7.90

$$Z_T = Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_1 = 12 + j5 = 13 \angle 22.62° \,\Omega$$

$$Z_2 = 8 - j4 = 8.944 \angle -26.565° \,\Omega$$

$$Z_1 + Z_2 = 20 + j = 20.025 \angle 2.862° \,\Omega$$

$$\therefore \quad Z_T = \frac{13 \angle 22.62° \times 8.944 \angle -26.565°}{20.025 \angle 2.862°}$$

$$= 5.8063 \angle -6.807° \,\Omega$$

i) $$I_T = \frac{V}{Z_T} = \frac{230 \angle 0°}{5.8063 \angle -6.807°}$$

$$= \mathbf{39.6121 \angle 6.807° \text{ A}}$$

ii) $$P_T = V I_T \cos\phi_T = 230 \times 39.6121 \times \cos(6.807°) = \mathbf{9.0465 \text{ kW}}$$

Now $\quad I_1 = \dfrac{V}{Z_1} = \dfrac{230 \angle 0°}{13 \angle 22.62°} = 17.6923 \angle -22.62° \text{ A}$

$\therefore \quad P_1 = V I_1 \cos\phi_1 = 230 \times 17.6923 \times \cos(-22.62°) = \mathbf{3.7562 \text{ kW}}$

And $\quad I_2 = \dfrac{V}{Z_2} = \dfrac{230 \angle 0°}{8.944 \angle -26.562°} = 25.715 \angle 26.562° \text{ A}$

$\therefore \quad P_2 = V I_2 \cos\phi_2 = 230 \times 25.715 \times \cos(26.562°) = \mathbf{5.29018 \text{ kW}}$

Cross-check : $P_T = P_1 + P_2$

iii) Overall p.f. = $\cos\phi_T$ = cos (+ 6.807) = **0.9929 leading** (positive ϕ_T)

Example 7.50 : *Show the waveforms of voltage, current and power if $v = V_m \sin \omega t$ volt is applied across a R - C series circuit.* **(Dec.-2005)**

Solution : The waveforms are shown in the Fig. 7.91.

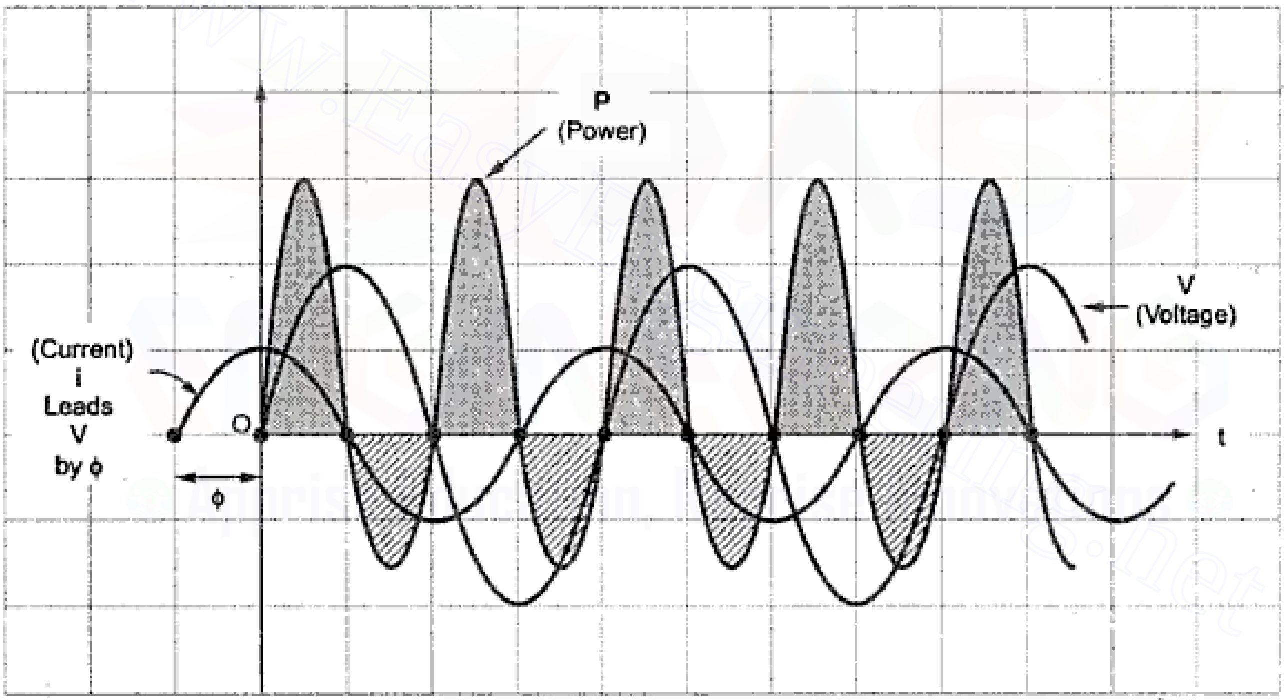


Fig. 7.91

Example 7.51 : *Two impedances, one inductive and the other capacitive are connected in series across the voltage 120 ∠ 30° volt and frequency of 50 Hz. The current flowing in circuit is 3 ∠ – 15°. If one of the impedances is (10 + j 48.3) Ω, find the other. Also calculate the value of L and C in the impedances.* **(Dec.-2005)**

Solution : The arrangement is shown in the Fig. 7.92.

$$Z_T = \frac{V}{I} = \frac{120 \angle 30°}{3 \angle -15°} = 40 \angle 45° \ \Omega$$

$$= 28.2842 + j\, 28.2842 \ \Omega$$

iii) $$Z_B = \frac{V}{I_B} = \frac{115\angle 0°}{8.5047\angle -37.52°} = 13.5219\angle +37.52°\,\Omega$$

$$= 10.7247 + j\,8.2353\,\Omega$$

iv) $$R_B = 10.7247\,\Omega$$

v) $$X_{LB} = 8.2353 = 2\pi f L_B$$

$$\therefore \quad L_B = \frac{8.2353}{100\pi} = 26.2137 \text{ mH}$$

Example 7.53 : *A coil of p.f. 0.6 is in series with 10 μF capacitor. When connected to a 50 Hz supply, the potential difference across the coil is equal to the potential difference across the capacitance. Find the resistance and inductance of the coil.* **(May-2006)**

Solution :

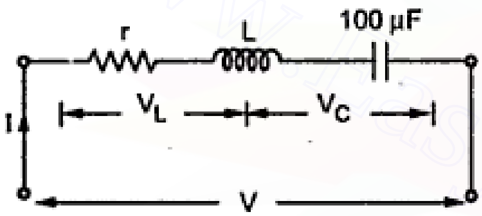


Fig. 7.94

$$V_L = I Z_L$$

$$\cos\phi_L = 0.6$$

$$V_C = I X_C = I \times \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= 31.83\ I$$

But $$V_L = V_C$$

$$\therefore \quad I Z_L = 31.83\ I$$

$$\therefore \quad Z_L = 31.83\ \Omega \quad \text{and} \quad \phi_L = \cos^{-1} 0.6 = 53.1301°$$

$$\therefore \quad Z_L = 31.83\angle 53.13° = 19.098 + j\,25.4639\,\Omega$$

$$\therefore \quad r = 19.098\,\Omega \quad \text{and} \quad X_L = 25.4639\,\Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = 0.08105 \text{ H}$$

Example 7.54 : *Sketch the waveforms of voltage, current and power if $v = V_m \sin \omega t$ volt is applied R-L series circuit and state expression for power.* **(May-2006)**

Solution : The waveforms are shown in the Fig. 7.95.

$$= \frac{61.03 \angle 1.869^\circ}{14.5602 \angle -15.945^\circ} = 4.1915 \angle 17.814^\circ \Omega = 4 + j\,1.2822\ \Omega$$

$$\therefore \quad Z_T = (Z_A \| Z_B) + Z_C = 4 + j\,1.2822 + 6 + j\,5 = 10 + j\,6.2822\ \Omega$$

$$= 11.8095 \angle 32.137^\circ \Omega$$

$$\therefore \quad I_T = \frac{V}{Z_T} = \frac{200 \angle 0^\circ}{11.8095 \angle 32.137^\circ} = \mathbf{16.9355 \angle -32.137^\circ\ A}$$

$$\therefore \quad I_A = I_T \times \frac{Z_B}{Z_A + Z_B} = \frac{16.9355 \angle -32.137^\circ \times 12.206 \angle -35^\circ}{14.5602 \angle -15.945^\circ} = \mathbf{14.1972 \angle -51.192^\circ\ A}$$

And $$I_B = I_T \times \frac{Z_A}{Z_A + Z_B} = \frac{16.9355 \angle -32.137^\circ \times 5 \angle 36.869^\circ}{14.5602 \angle -15.945^\circ} = \mathbf{5.8156 \angle 20.677^\circ\ A}$$

$$I_C = I_T = \mathbf{16.9355 \angle -32.137^\circ}$$

$$\cos \phi_T = \cos(-32.137^\circ) = \mathbf{0.8467\ lagging} \qquad \text{...as } I_T \text{ lags } V$$

Example 7.56 : *When an inductive coil is connected to a d.c. supply at 240 volt, the current in it is 16 amp. When the same coil is connected to an a.c. supply at 240 volt, 50 Hz, the current is 12.27 amp. Calculate (1) Resistance, (2) Impedance, (3) Reactance (4) Inductance of the coil.* **(May-2007)**

Solution : For a d.c. supply, f = 0 hence, $X_L = 2\pi f L = 0\,\Omega$

Hence only resistance of coil is effective.

$$\therefore \quad I_{dc} = \frac{V}{R} \quad \text{i.e.} \quad 16 = \frac{240}{R}$$

1) $$R = \mathbf{15\ \Omega} \qquad \text{... Resistance of coil}$$

2) When connected to a.c. supply,

$$Z = R + j2\pi f L \text{ and } I_{ac} = 12.27\ A$$

$$|Z| = \frac{|V|}{|I_{ac}|} = \frac{240}{12.27} = \mathbf{19.56\ \Omega} \qquad \text{...Impedance}$$

3) $$|Z| = \sqrt{R^2 + X_L^2}$$

$$\therefore \quad 19.56 = \sqrt{(15)^2 + (X_L)^2}$$

$$\therefore \quad X_L = \mathbf{12.5536\ \Omega} \qquad \text{...Reactance}$$

4) $$X_L = 2\pi f\, L$$

$$\therefore \quad L = \frac{12.5536}{2\pi \times 50} = \mathbf{0.04\ H} \qquad \text{...Inductance}$$

Example 7.57 : *A coil has inductance of 20 mH and resistance 5 ohm. It is connected across a supply voltage of v = 48 sin 314 t. Obtain the expression for current drawn by the coil.* **(May-2007)**

Solution : $L = 20 \text{ mH}, \quad R = 5\ \Omega$

$$V = 48 \sin 314t \text{ V}$$

Compare with,

$$V = V_m \sin \omega t$$

$$\therefore \quad V_m = 48 \quad \text{and} \quad \omega = 314 \text{ rad/sec}$$

$$\therefore \quad V = \frac{V_m}{\sqrt{2}} = 33.9411 \angle 0° \text{ V}$$

$$X_L = \omega L = 314 \times 20 \times 10^{-3} = 6.28\ \Omega$$

$$\therefore \quad Z = R + jX_L = 5 + j\,6.28\ \Omega = 8.0273 \angle 51.474°\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{33.9411 \angle 0°}{8.0273 \angle 51.474°} = 4.2282 \angle -51.474° \text{ A}$$

$$\therefore \quad I_m = \sqrt{2} \times I = \sqrt{2} \times 4.2282 = 5.9795 \text{ A}$$

Expression for the current is,

$$\mathbf{i = 5.9795 \sin (314\,t - 51.474°) \text{ A}}$$

Example 7.58 : *A series circuit consists of resistance of 10 ohm, an inductance of* $\frac{200}{\pi}$ *mH and capacitance of* $\frac{1000}{\pi}$ *μF. Calculate (1) Current flowing in the circuit if supply voltage is 200 V, 50 Hz (2) p.f. of the circuit, (3) Power drawn from the supply. Also draw the phasor diagram.* **(May-2007)**

Solution :

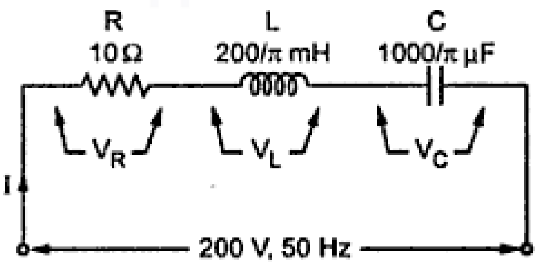


Fig. 7.97

$$L = 63.6619 \text{ mH}$$

$$\therefore \quad X_L = 2\pi f\,L = 20\ \Omega$$

$$C = 318.3098\ \mu\text{F}$$

$$\therefore \quad X_C = \frac{1}{2\pi f C} = 10\ \Omega$$

$$\therefore \quad Z = R + jX_L - jX_C = 10 + j20 - j10$$

$$= 10 + j10\ \Omega = 14.1421 \angle 45°\ \Omega$$

1) $$I = \frac{V}{Z} = \frac{200 \angle 0°}{14.1421 \angle 45°} = \mathbf{14.1421 \angle -45° \text{ A}} \quad \text{...Current}$$

2) $$P = I^2 R = (14.1421)^2 \times 10 = \mathbf{2000 \text{ W}} \quad \text{...Power}$$

Or $\qquad P = VI\cos\phi = 200 \times 14.1421 \times \cos(-45°) = \mathbf{2000\ W}$

3) $\qquad \text{p.f.} = \cos\phi = \cos(-45°) = \mathbf{0.7071\ lagging}$

The phasor diagram is shown in the Fig. 7.98.

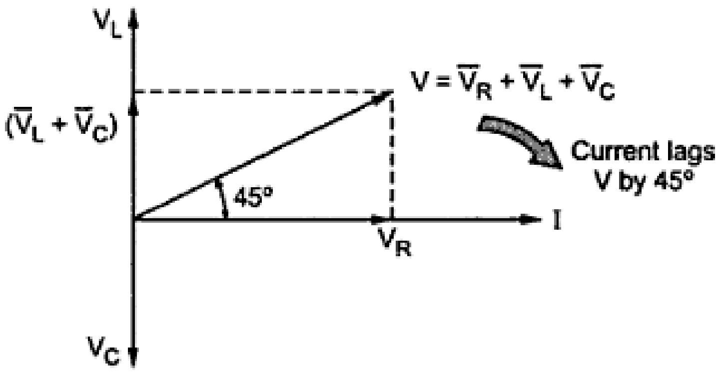


Fig. 7.98

Example 7.59 : *Two impedances $Z_1 = 40 \angle 30°$ ohm and $Z_2 = 30 \angle 60°$ ohm are connected in series across single phase, 230 V, 50 Hz supply. Calculate the (1) Current drawn, (2) p.f. and (3) Power consumed by the circuit.* **(May-2007)**

Solution :

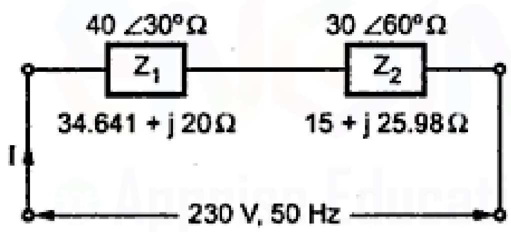


Fig. 7.99

$$Z_T = Z_1 + Z_2$$
$$= 34.641 + j20 + 15 + j25.98$$
$$= 49.641 + j\,45.98\ \Omega$$
$$= 67.663 \angle 42.807°\ \Omega$$

1) $\qquad I = \dfrac{V}{Z_T} = \dfrac{230 \angle 0°}{67.663 \angle 42.807°} = \mathbf{3.399 \angle -42.807°\ A}$ $\qquad$ ...current

2) $\qquad \text{p.f.} = \cos(\phi) = \cos(-42.807°) = \mathbf{0.7336\ lagging}$ $\qquad$ ...p.f.

3) $\qquad P = VI\cos\phi = 230 \times 3.399 \times 0.7336 = \mathbf{573.5064\ W}$ $\qquad$ ...power

Example 7.60 : *A 230 V, 50 Hz voltage is applied first to resistor of value 100 Ω and then to a capacitor of 100 μF. Obtain the expressions for the instantaneous currents for both the cases and draw the phasor diagram.* **(Dec.-2007)**

Solution : Case i) $\quad R = 100\ \Omega$

Let $V = 230$ V(rms) be reference

$$I_R = \frac{V}{R} = \frac{230}{100} = 2.3 \text{ A (r.m.s.)}$$

$$V = 230\angle 0^\circ \text{ V and } I_R = 2.3\angle 0^\circ \text{ A}$$

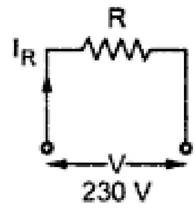


Fig. 7.100(a)

For pure resistance both V and I_R in phase.

$$I_{Rm} = \sqrt{2}I_R = \sqrt{2}\times 2.3 = 3.2526 \text{ A}$$

$$\therefore \quad i_R = I_{Rm}\sin(\omega t + \phi) \text{ where } \phi = 0 \text{ and } \omega = 2\pi f = 100\pi$$

$$\therefore \quad \mathbf{i_R = 3.2526 \sin(100\,\pi\, t)\ A} \qquad \text{...Instantaneous current}$$

The phasor diagram is shown in the Fig. 7.101 (a).

Case ii) $C = 100\ \mu F$

$$\text{Let } V = 230\angle 0^\circ \text{ V (r.m.s.)}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times 50\times 100\times 10^{-6}}$$

$$= 31.8309\ \Omega$$

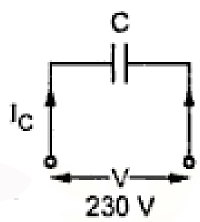


Fig. 7.100 (b)

The polar representation of capacitor is,

$$X_C\angle -90^\circ = 31.8309\angle -90^\circ \Omega$$

$$\therefore \quad I_C = \frac{V}{X_C} = \frac{230\angle 0^\circ}{31.8309\angle -90^\circ} = 7.2256\angle +90^\circ \text{ A (r.m.s.)}$$

$$\text{Hence} \quad I_{Cm} = \sqrt{2}\times I_C = \sqrt{2}\times 7.2256 = 10.2185 \text{ A}$$

$$\therefore \quad i_C = I_{Cm}\sin(\omega t + \phi) = \mathbf{10.2185 \sin(100\,\pi t + 90^\circ)\ A}$$

The phasor diagram is shown in the Fig. 7.101 (b).

(a) Case[i] **(b) Case[ii]**

Fig. 7.101

Example 7.61 : *A series R-L-C circuit has resistance of 50 Ω, inductance of 0.1 H and capacitance of 50 μF connected in series across single phase 230 V, 50 Hz supply, Calculate :*

i) Current drawn by circuit
ii) Power factor of the circuit
iii) Active and rective power consumed by circuit
iv) Draw the phasor diagram. **(Dec.-2007)**

Solution : The arrangement is shown in the Fig. 7.102 (a).

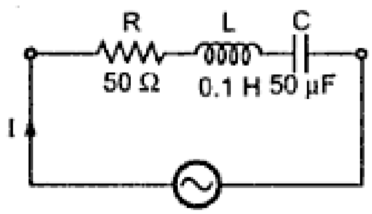


Fig. 7.102(a)

$$X_L = 2\pi fL = 2\pi\times50\times0.1 = 31.4159\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times50\times10^{-6}} = 63.662\ \Omega$$

$$\therefore \quad Z = R+jX_L - jX_C = 50 + j\,31.4159 - j63.662.$$

$$= 50 - j\,32.2461\ \Omega = 59.4963\ \angle -32.82°\ \Omega$$

Let voltage be reference i.e. $V = 230\ \angle 0°$ V

i) $$I = \frac{V}{Z} = \frac{230\angle 0°}{59.4963\angle -32.82°} = \mathbf{3.8657\angle +32.82°\ A}$$

ii) $\cos\phi = \cos(32.82°) = \mathbf{0.8403\ leading}$

iii) $P = VI\cos\phi = 230\times 3.8657\times 0.8403 = \mathbf{747.1888\ W}$

$Q = VI\sin\phi = 230\times 3.8657\times\sin(32.82°) = \mathbf{481.899\ VAR}$

iv) The phasor diagram is shown in the Fig. 7.102(b).

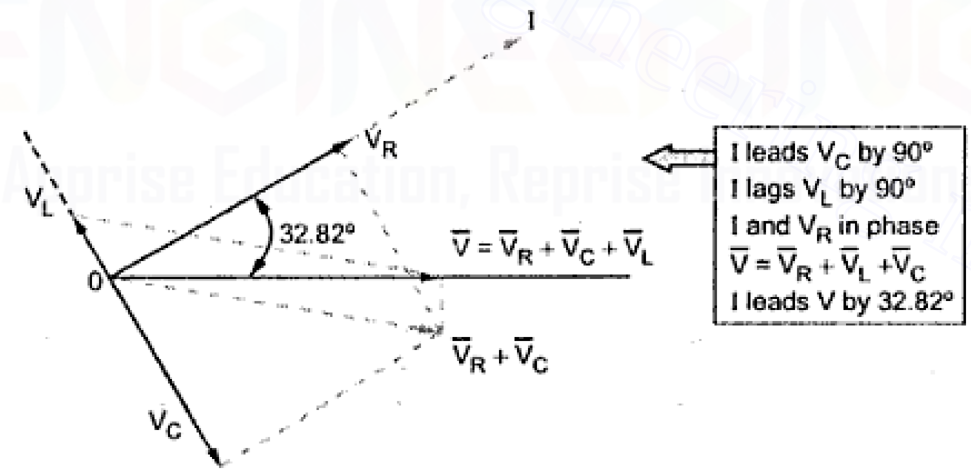


Fig. 7.102(b)

➠ **Example 7.62 :** *A parallel circuit consists of two branches. Branch (i) consists of R of 100 Ω connected in series with inductance of 1 H and branch (ii) consists of R of 50 Ω in series with capacitance of 79.5 μF. This parallel circuit is connected across single phase 200 V, 50 Hz supply, calculate :*

i) Branch currents

ii) Total current drawn by circuit

iii) Total power factor of circuit

iv) Total power drawn by circuit. **(Dec.-2007)**

Solution : The arrangement is shown in the Fig. 7.103.

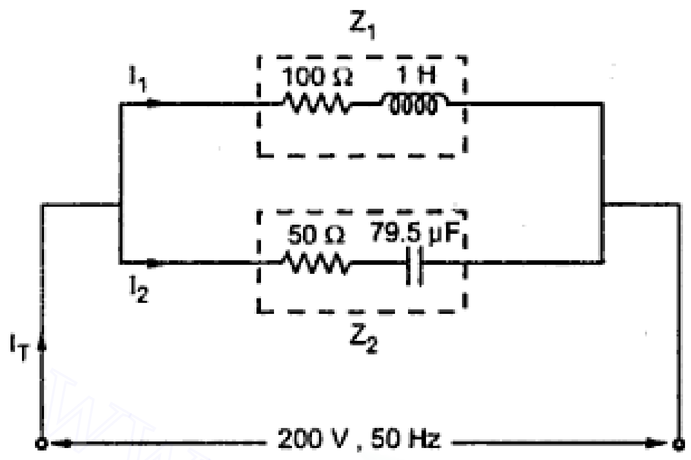


Fig. 7.103

Let voltage be reference hence

$V = 200\angle 0°$ V

$X_L = 2\pi fL = 2\pi\times 50\times 1 = 314.1592\ \Omega$

$\therefore\ Z_1 = 100 + j\,314.1592\ \Omega$

$= 329.6907\ \angle\ 72.343°\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi\times 50\times 79.5\times 10^{-6}}$

$= 40.039\ \Omega$

$\therefore\ Z_2 = 50 - j\,40.039\ \Omega$

$= 64.055\ \angle -38.687°\ \Omega$

$$\therefore \quad I_1 = \frac{V}{Z_1} = \frac{200\angle 0°}{329.6907\angle 72.343°}$$

$$= \mathbf{0.6066\ \angle -72.343°\ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{200\angle 0°}{64.055\angle -38.687°} = \mathbf{3.1223\angle +38.687°\,A}$$

$$I_T = \bar{I}_1 + \bar{I}_2 = (0.6066\angle -72.343°) + (3.1223\angle +38.687°)$$

$$= [0.18399 - j\,0.578] + [2.43718 + j\,1.9516]$$

$$= \mathbf{2.62117 + j\,1.3736\ A = 2.9592\ \angle\ 27.6563°\ A} \quad \text{...Total current}$$

$$\therefore \quad \phi_T = 27.6563°$$

$$\therefore \quad \cos\phi_T = \cos(27.6563°) = \mathbf{0.8857\ leading} \quad \text{...Total power factor}$$

$$P = V\,I_T\cos\phi_T = 200\times 2.9592\times 0.8857 = \mathbf{524.1926\ W} \quad \text{...Total power}$$

Example 7.63 : *The expression for voltage applied to a series circuit is e = 120 sin(314t) whereas current drawn is expressed as i = 10 sin* $\left(314\,t + \frac{\pi}{6}\right)$. *Calculate the values of components of the series circuit. Calculate the active and reactive power consumed by circuit. Draw the waveforms of instantaneous voltage, current and power consumed.* **(Dec.-2007)**

Solution : $e = 120\sin(314\,t)$ V, $i = 10\sin\left(314\,t + \frac{\pi}{6}\right)$A.

Comparing e with, $e = E_m \sin(\omega t)$

$$E_m = 120 \text{ V} \qquad \text{i.e. } E = \frac{E_m}{\sqrt{2}} = \frac{120}{\sqrt{2}} = 84.8528 \text{ V(r.m.s.)}$$

$$\omega = 314 \text{ rad/sec} \qquad \text{i.e. } 2\pi f = 314 \text{ hence } f = 50 \text{ Hz}$$

Comparing i with, $i = I_m \sin(314\,t + \phi)$

$$I_m = 10 \text{ A} \qquad \text{i.e. } I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07106 \text{ A(r.m.s.)}$$

$$\phi = \frac{\pi}{6} \text{ rad} = 30^\circ$$

In polar form, $E = 84.8528 \angle 0^\circ$ V, $I = 7.07106 \angle +30^\circ$ A

$$\therefore \quad Z = \frac{E}{I} = \frac{84.8528\angle 0^\circ}{7.07106\angle 30^\circ} = 12\angle -30^\circ \Omega = 10.3923 - j6\ \Omega$$

Comparing Z with $Z = R - j\,X_C$,

$$\therefore \quad R = 10.3923\ \Omega, \quad X_C = 6\ \Omega = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 6} = 530.5164\ \mu\text{F}$$

$$P = I^2 R = (7.07106)^2 \times 10.3923 = 519.6138 \text{ W}$$

$$\phi = 30^\circ \text{ is the power factor angle.}$$

$$\therefore \quad Q = VI \sin\phi = 84.8528 \times 7.07106 \times \sin 30^\circ = 300 \text{ VAR}$$

The waveforms of instantaneous voltage, current and power are shown in the Fig. 7.103(a).

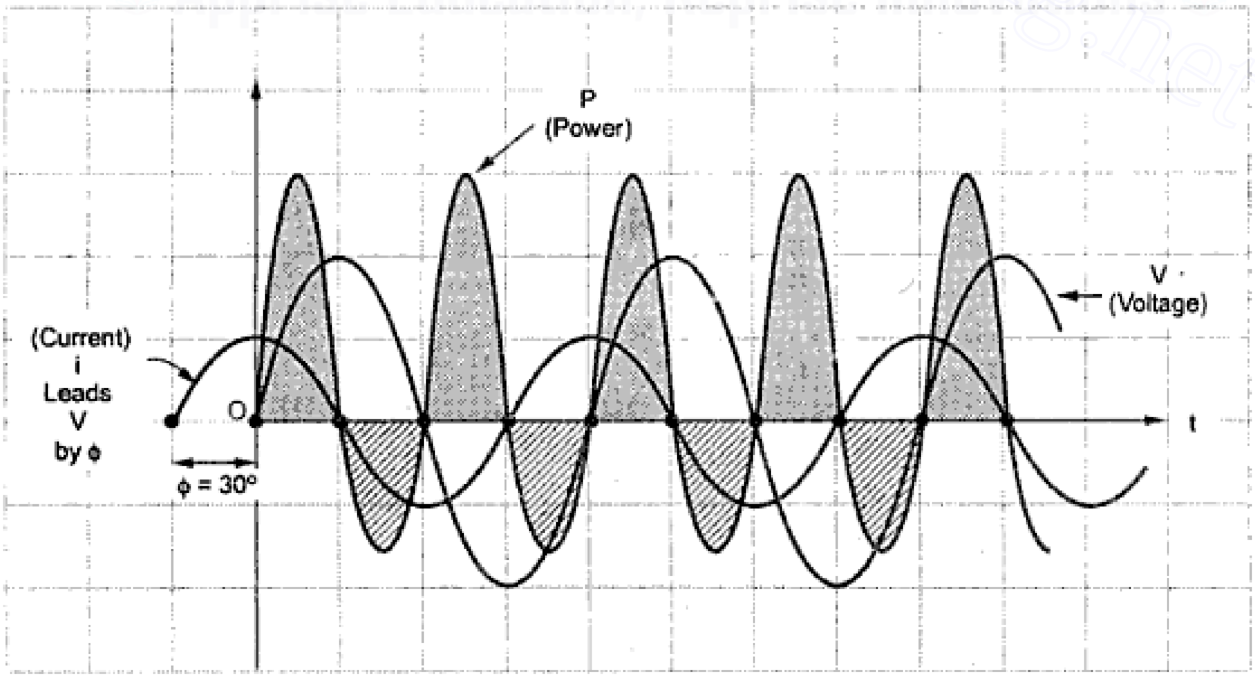


Fig. 7.103(a)

Example 7.64 : *Two impedances $Z_1 = 30 \angle 45°\Omega$ and $Z_2 = 45 \angle 30°\Omega$ are connected in praralle! across single phase 230 V, 50 Hz supply. Calculate the (i) Current drawn (ii) p.f. and (iii) Power consumed by circuit.* **(May-2008)**

Solution :

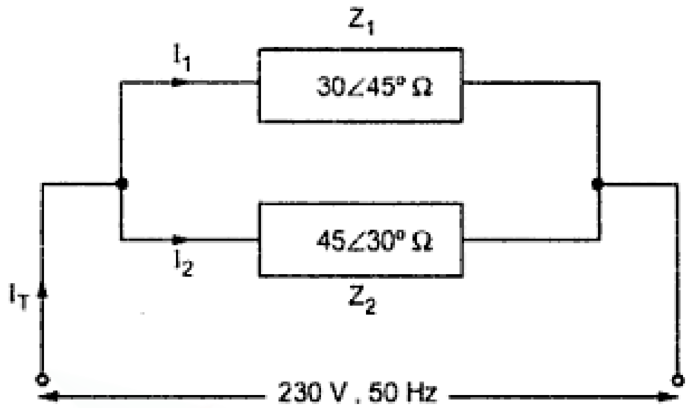


Fig. 7.104

$$Z_T = Z_1 \| Z_2 = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{30\angle 45° \times 45\angle 30°}{(30\angle 45°)+(45\angle 30°)}$$

$$= \frac{1350\angle 75°}{21.2132+j\,21.2132+38.9711+j\,22.5}$$

$$= \frac{1350\angle 75°}{60.1843+j\,43.7132} = \frac{1350\angle 75°}{74.3841\angle 35.9917°}$$

$\therefore \quad Z_T = 18.149\angle 39°\ \Omega$

i) $I_T = \dfrac{V}{Z_T} = \dfrac{230\angle 0°}{18.149\angle 39°} = \mathbf{12.6728\angle -39°\ A}$ ...Current drawn

ii) $\phi_T = -39°$, the negative sign indicates lagging

$\therefore \quad \cos\phi_T = \cos(-39°) = \mathbf{0.777\ lagging}$ ... Power factor

iii) $P = VI_T \cos\phi_T = 230\times 12.6728\times 0.777 = \mathbf{2265.047\ W}$ ...Power

Review Questions

1. *Show that current through purely resistive circuit is in phase with the applied voltage.*
2. *Show that current through pure inductance lags applied voltage by 90°.*
3. *Show that current through pure capacitor leads applied voltage by 90°.*
4. *Obtain an expression for the average power consumed by an a.c. circuit in terms of r.m.s. values of voltage, current and power factor.*
5. *What is power factor ? Explain its significance.*

6. *Show that average power consumed by pure inductor and pure capacitor is zero.*
7. *Show that*

   i) Current lags voltage in R-L series circuit

   ii) Current leads voltage in R-C series circuit
8. *Draw the phasor diagram for a series R-L-C circuit energized by a sinusoidal voltage showing the relative positions of the current, component voltage and the applied voltage for the following cases :-*

   (a) When $X_L > X_C$; (b) When $X_L < X_C$; and (c) When $X_L = X_C$.
9. *Explain the concept of admittance.*
10. *Define the following terms :*

    i) Admittance ii) Conductance iii) Susceptance
11. *Derive the expressions to calculate conductance and susceptance.*
12. *What is resonance ? State the characteristics of series resonant circuit.*
13. *What is Q factor ? How it is related to bandwidth and selectivity ?*
14. *Derive the expression for the resonating frequency of series resonant circuit.*
15. *Derive the expression for the resonating frequency of parallel resonant circuit.*
16. *Derive the expressions for upper and lower cut-off frequencies for series circuit.*
17. *State the characteristics of parallel resonating circuit.*
18. *Compare series and parallel resonating circuits.*
19. *Two voltage sources have equal emf's and a phase difference of α. When they are connected in series, the voltage is 200 V, when one source is reversed the voltage is 15 V. Find their emf's and phase angle α.* **(Ans. : 100.26 V, 8.58°)**
20. *The current in series circuit R = 5 Ω and L = 30 mH lags the applied voltage by 80°. Determine the source frequency and the impedance Z.* **(Ans. : 150.47 Hz, 28.8 $\angle$ 80° Ω)**
21. *A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500 pF, the current has its maximum value; which it is reduced to one half when capacitance is 600 pF. Find :*

    i) The resistance ii) The inductive reactance iii) The Q factor of the inductor.

    (Ans. : 30.627 Ω, 318.306 Ω, 10.393)
22. *A 1 ϕ parallel a.c. circuit consists of 3 branches connected in parallel. One branch consists of a pure resistance of 10 Ω. The second branch consists of a resistance of 20 Ω and an inductance of 0.1 H. The third branch consists of a resistance of 15 Ω and a capacitance of 150 μF. If the circuit is connected across a 230 V 50 Hz supply, calculate : -*

    i) Reactances/impedances of each branch ii) Total impedance of the circuit iii) Power factor of each branch iv) Phase angle of each branch v) The individual currents in the 3 branches with their phase angles vi) The resultant/total current drawn from supply vii) Power consumed in each branch viii) The total power drawn ix) Draw a neat circuit diagram showing all the relevant parameters x) Also, draw phasor/vector diagram.

The expression for current should be written in polar form with its phase angle. Note that the entire problem has to be solved by 3 (three) methods i.e. by using -

a) Complex algebra method b) Vector method c) Admittance method.

In each of the 3 methods, the answers should be the same.

(Ans. : 23 ∠ 0° A, 6.176 ∠ − 57.52° A, 8.851 ∠ + 54.74° A)

23. *A pure resistor, a pure capacitor and a pure inductor are connected in parallel, across a 50 Hz supply, find the impedance of the circuit as seen by the supply. Also find the resonant frequency.*

(Ans. : $\frac{1}{2\pi\sqrt{LC}}$, R)

24. *An a.c. circuit consists of a pure resistance and an inductive coil connected in series. The power dissipated in the resistance and in the coil are 1000 W and 200 W respectively. The voltage drops across the resistance and the coil are 200 V and 300 V respectively. Calculate the following :*

i) Draw a neat circuit diagram indicating all the relevant parameters. ii) Value of the pure resistance iii) Current through the circuit. iv) Resistance of the coil. v) Impedance of the coil. vi) Reactance of the coil. vii) Total impedance of the circuit. viii) Supply voltage.

(Ans. : 8 Ω, 59.46 Ω, 382.08 V)

25. *A current of 5 A flows through a non-inductive resistance in series with a coil when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V and across the coil is 200 V. Calculate :*

i) Impedance, reactance and resistance of the coil. ii) Power absorbed by the coil. iii) Total power iv) Draw the voltage vector diagram. **(Ans. : 5.5 Ω, 39.62 Ω, 762.5 W, 0.61 lag)**

26. *In parallel RC circuit shown in the Fig. 7.105,*

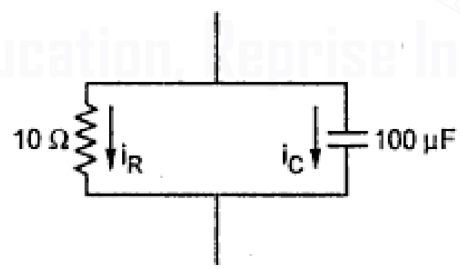


Fig. 7.105

i_R = 15 cos (5000 t − 30°) amperes. Obtain the current in capacitance.

(Ans. : 75 cos (5000 t + 60°) A)

27. *A choking coil and a pure resistor are connected in series across a supply of 230 V, 50 Hz. The voltage drop across the resistor is 100 V, and that across the choking coil is 150 V. Find graphically the voltage drop across the inductance and resistance of the choking coil. Hence find their values if the current is 1A.* **(Ans. : 92.5 Ω, 0.366 H)**

28. *In a particular circuit, a voltage of 10 V at 25 Hz produces a current of 100 mA, while the same voltage at 75 Hz, produces 60 mA. Draw the circuit diagram and insert the values of the components.* **(Ans. : 88.277 Ω, 0.3 H)**

29. *In the network shown in the Fig. 7.106 , source frequency is 500 rad/sec and current I_2 is $1.25 \angle 60°$ A.*

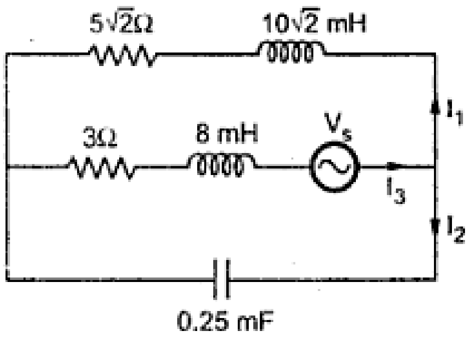


Fig. 7.106

Find,

a) Currents I_1 and I_3

b) Source voltage V_s

c) Source power factor

d) Draw phasor diagram

Find magnitudes and positions of all the quantities.

(Ans. : 1 $\angle$–75° A, 0.8914 $\angle$ 7.51° A, 10.9 $\angle$ –5.86° V, 0.97 lead)

30. *In the network shown in the Fig. 7.107,*

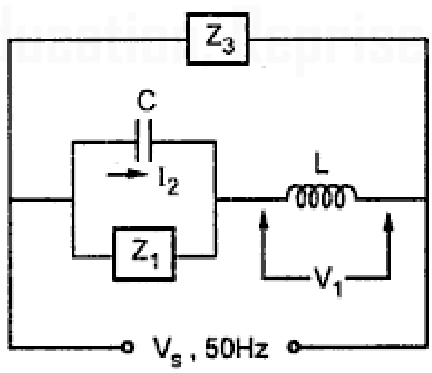


Fig. 7.107

$L = 0.153$ H, $C = 0.3183$ mF, $I_2 = 5\angle 60°$ A

and $V_1 = 250 \angle +90°$ V

when Z_3 is not connected, find :

1. *Z_1 and its components*
2. *V_s in the form $V_m \sin(\omega t + \theta)$*
3. *Power loss in the circuit*

Now calculate Z_3 with its components so that overall p.f. of the circuit is unity without adding to the circuit power loss.

Draw the phasor diagram.

(Ans. : 10 ∠ 30° Ω, 324.03 sin (100 (100πt +79.1°) V, 216.51 W, 68.2 μF)

❑❑❑

8 Polyphase A.C. Circuits

8.1 Introduction

We have seen that a single phase a.c. voltage can be generated by rotating a turn made up of two conductors, in a magnetic field. Such an a.c. producing machine is called single turn alternator. But voltage produced by such a single turn is very less and not enough to supply practical loads. Hence number of turns are connected in series to form one winding in a practical alternator. Such a winding is called armature winding. The sum of the voltages induced in all the turns is now available as a single phase a.c. voltage, which is sufficient to drive the practical loads.

But in practice there are certain loads which require polyphase supply. **Phase** means **branch, circuit** or **winding** while **poly** means **many**. So such applications need a supply having many a.c. voltages present in it simultaneously. Such a system is called **polyphase system.**

To develop polyphase system, the armature winding in an alternator is divided into number of phases required. In each section, a separate a.c. voltage gets induced. So there are many independent a.c. voltages present equal to number of phases of armature winding. The various phases of armature winding are arranged in such a manner that the magnitudes and frequencies of all these voltages is same but they have definite phase difference with respect to each other. The phase difference depends on number of phases in which armature is divided. For example, if armature is divided into three coils then three separate a.c. voltages will be available having same magnitude and frequency but they will have a phase difference of 360°/3 = 120° with respect to each other. All three voltages with a phase difference of 120° are available to supply a three phase load. Such a supply system is called **three phase system.** Similarly by dividing armature into various number of phases, a 2 phase, 6 phase supply system also can be obtained. A phase difference between such voltages is 360°/n where n is number of phases.

> **Key Point:** *In practice a three phase system is found to be more economical and it has certain advantages over other polyphase systems. Hence three phase system is very popularly used everywhere in practice.*

This chapter explains the various three phase circuits, the analysis of star and delta circuits and three phase active, reactive and apparent power. Let us see the advantages of three phase system first.

8.2 Advantages of Three Phase System

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 120° between each other. Such a three phase system has following advantages over single phase system.

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.

2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.

3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.

4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.

5) Three phase systems give steady output.

6) Single phase supply can be obtained from three phase but three phase cannot be obtained from single phase.

7) Power factor of single phase motors is poor than three phase motors of same rating.

8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard system throughout the world.

8.3 Generation of Three Phase Voltage System

It is already discussed that alternator consisting of one group of coils on armature produces one alternating voltage. But if armature coils are divided into three groups such that they are displaced by the angle 120° from each other, three separate alternating voltages get developed.

Consider armature of alternator divided into three groups as shown in the Fig. 8.1. The coils are named as R_1- R_2 , Y_1 -Y_2 and B_1- B_2 and mounted on same shaft. The ends of each coil are brought out through the slipring and brush arrangement to collect the induced e.m.f.

Mathematically this can be shown as :

$$e_R + e_Y + e_B$$

$$= E_m \sin \omega t + E_m \sin(\omega t - 120°) + E_m \sin(\omega t + 120°)$$

$$= E_m [\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°]$$

$$= E_m [\sin \omega t + 2 \sin \omega t \cos 120°] = E_m \left[\sin \omega t + 2 \sin \omega t \left(\frac{-1}{2}\right)\right] = 0$$

$$\therefore \quad \boxed{\bar{e}_R + \bar{e}_Y + \bar{e}_B = 0}$$

> **Key Point :** *The phasor addition of all the phase voltages at any instant in three phase system is always zero.*

8.4 Important Definitions Related to Three Phase System

Some terms are commonly used while analysing three phase system which are defined below

1) Symmetrical system : It is possible in polyphase system that magnitudes of different alternating voltages are different. But a three phase system in which the three voltages are of same magnitude and frequency and displaced from each other by 120° phase angle is defined as **symmetrical system.**

2) Phase sequence : The sequence in which the voltages in three phases reach their maximum positive values is called **phase-sequence.** Generally the phase sequence is R-Y-B.

> **Key Point :** *The phase sequence is important in determining direction of rotation of a.c. motors, parallel operation of alternators etc.*

8.5 Three Phase Supply Connections

In single phase system, two wires are sufficient for transmitting voltage to the load i.e. phase and neutral. But in case of three phase system, two ends of each phase i.e. R_1-R_2, Y_1-Y_2, and B_1-B_2 are available to supply voltage to the load. If all six terminals are used independently to supply voltage to load as shown in the Fig. 8.4, then total six wires will be required and it will be very much costly.

To reduce the cost by reducing the number of windings, the three windings are interconnected in a particular fashion. This gives different three phase connections.

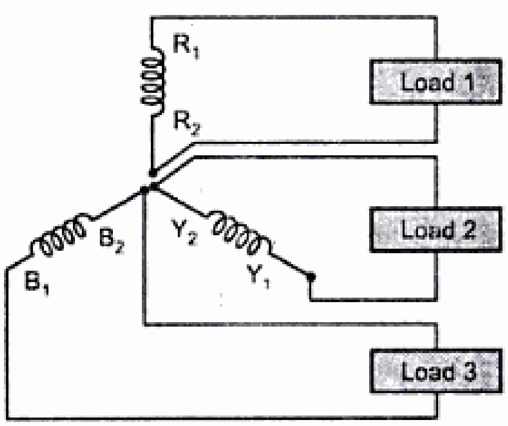


Fig. 8.4 Three phase connections

8.5.1 Star Connection

The star connection is formed by connecting starting or terminating ends of all the three windings together. The ends R_1 - Y_1 - B_1 are connected or ends R_2 - Y_2 - B_2 are connected together. This common point is called **Neutral Point**. The remaining three ends are brought out for connection purpose. These ends are generally referred as R-Y-B, to which load is to be connected.

The star connection is shown in the Fig. 8.5.

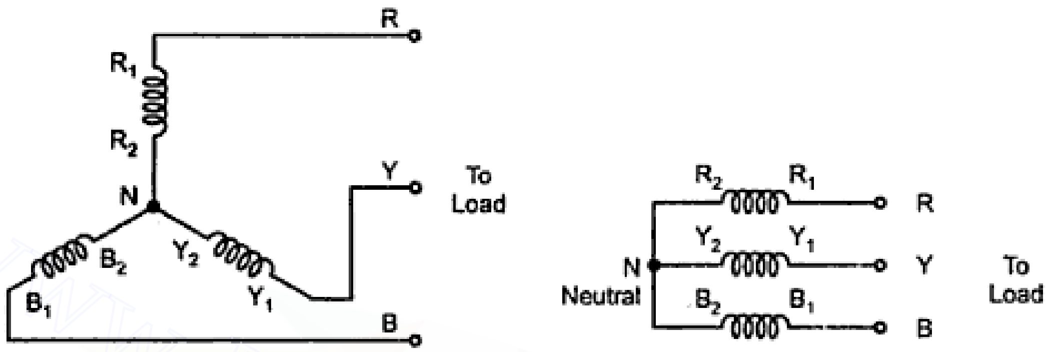


Fig. 8.5 Star connection

8.5.2 Delta Connection

The delta is formed by connecting one end of winding to starting end of other and connections are continued to form a closed loop. The supply terminals are taken out from the three junction points. The delta connection is shown in the Fig. 8.6.

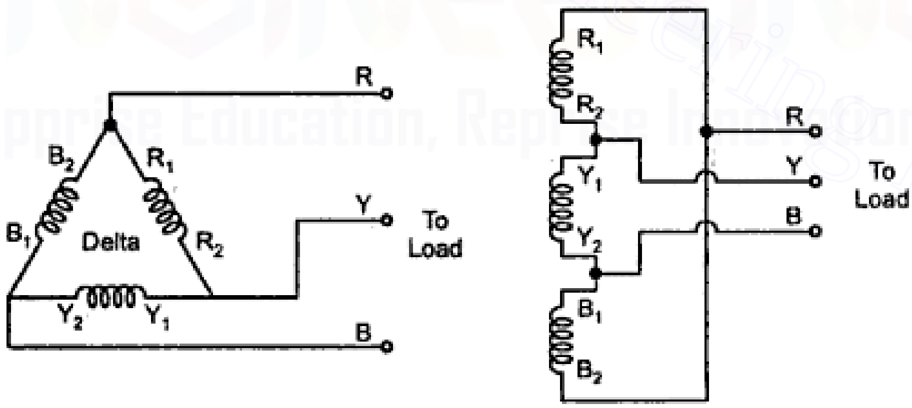


Fig. 8.6 Delta connection

8.6 Concept of Line Voltages and Line Currents

The potential difference between any two lines of supply is called **line voltage** and current passing through any line is called **line current**.

Consider a star connected system as shown in the Fig. 8.7.

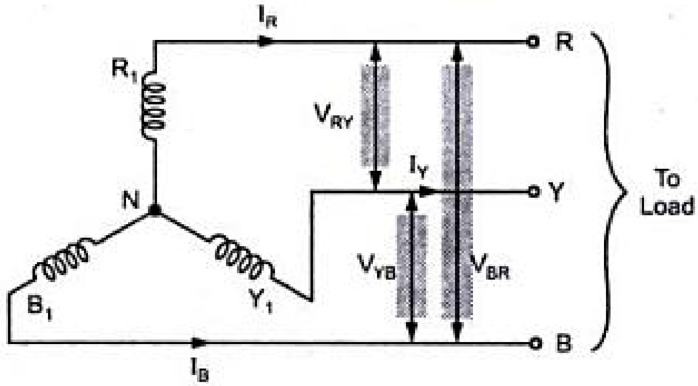


Fig. 8.7

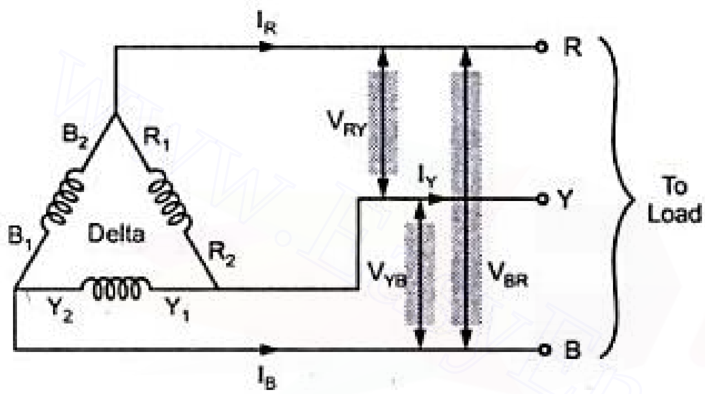


Fig. 8.8

Line voltages are denoted by V_L. These are V_{RY}, V_{YB} and V_{BR}. Line currents are denoted by I_L. These are I_R, I_Y and I_B.

Similarly for delta connected system we can show the line voltages and line currents as in the Fig. 8.8.

Line voltages V_L are V_{RY}, V_{BR}, V_{YB}.

While Line currents I_L are I_R, I_Y and I_B.

8.7 Concept of Phase Voltages and Phase Currents

Now to define the phase voltages and phase currents let us see the connections of the three phase load to the supply lines. Generally Red, Yellow and Blue coloured wires are used to differentiate three phases and hence the names given to three phases are R, Y and B.

The load can be connected in two ways, i) Star connection, ii) Delta connection

The **three phase load** is nothing but three different impedances connected together in star or delta fashion

i) Star connected load : There are three different impedances and are connected such that one end of each is connected together and other three are connected to supply terminalis R-Y-B. This is shown in the Fig. 8.9.

> **Key Point :** *The voltage across any branch of the three phase load i.e. across Z_{ph1}, Z_{ph2} or Z_{ph3} is called* ***phase voltage*** *and current passing through any branch of the three phase load is called* ***phase current.***

$$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY}$$

But $\quad \bar{V}_{NY} = -\bar{V}_{YN}$

Hence $\quad \bar{V}_{RY} = \bar{V}_R - \bar{V}_Y \qquad ...(1)$

Similarly, $\quad \bar{V}_{YB} = \bar{V}_{YN} + \bar{V}_{NB} = \bar{V}_{YN} - \bar{V}_{BN} = \bar{V}_Y - \bar{V}_B \qquad ...(2)$

and $\quad \bar{V}_{BR} = \bar{V}_B - \bar{V}_R \qquad ...(3)$

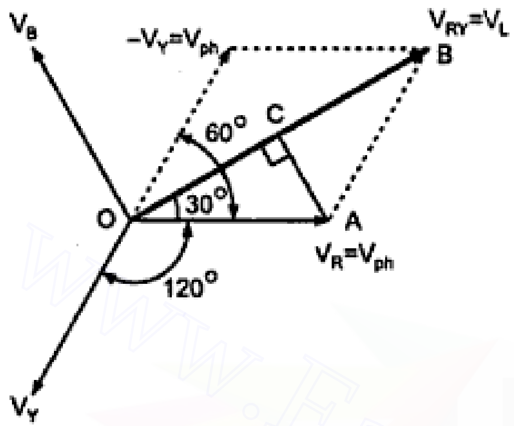


Fig. 8.12

The three phase voltage are displaced by 120° from each other. The phasor diagram to get V_{RY} is shown in the Fig. 8.12. The V_Y is reversed to get $-V_Y$ and then it is added to V_R to get V_{RY}.

The perpendicular is drawn from point A on vector OB representing V_L. In triangle OAB, the sides OA and AB are same as phase voltages. Hence OB bisects angle between V_R and $-V_Y$.

$$\therefore \angle BOA = 30°$$

And perpendicular AC bisects the vector OB.

$$\therefore \quad OC = CB = \frac{V_L}{2}$$

From triangle OAB, $\quad \cos 30° = \frac{OC}{OA} = \frac{(V_{RY}/2)}{V_R}$

$$\therefore \quad \frac{\sqrt{3}}{2} = \frac{(V_L/2)}{V_{ph}}$$

$\therefore \quad V_L = \sqrt{3}\, V_{ph}$ for star connection

Thus line voltage is $\sqrt{3}$ times the phase voltage in star connection.

Now lagging or leading nature of current depends on per phase impedance. If Z_{ph} is inductive i.e. $R+j\,X_L$ then current I_{ph} lags V_{ph} by angle ϕ where ϕ is $\tan^{-1}(X_L/R)$. If Z_{ph} is capacitive i.e. $R-j\,X_C$ then I_{ph} leads V_{ph} by angle ϕ If Z_{ph} is resistive i.e. $R+j\,0$ then I_{ph} is in phase with V_{ph}.

Key Point : *Remember that Z_{ph} relates I_{ph} and V_{ph} hence angle ϕ is always between I_{ph} and V_{ph}.*

And $\quad \boxed{|Z_{ph}| = \frac{|V_{ph}|}{|I_{ph}|}}$

Key Point : *The line values do not decide the impedance angle or power factor angle.*

$$\boxed{\phi = V_{ph} \wedge I_{ph} \neq V_L \wedge I_L}$$

The complete phasor diagram for lagging power factor load is shown in the Fig. 8.13.

$$\boxed{Z_{ph} = R_{ph} + j\, X_{Lph} = |Z_{ph}| \angle \phi \;\; \Omega}$$

Each I_{ph} lags corresponding V_{ph} by angle ϕ

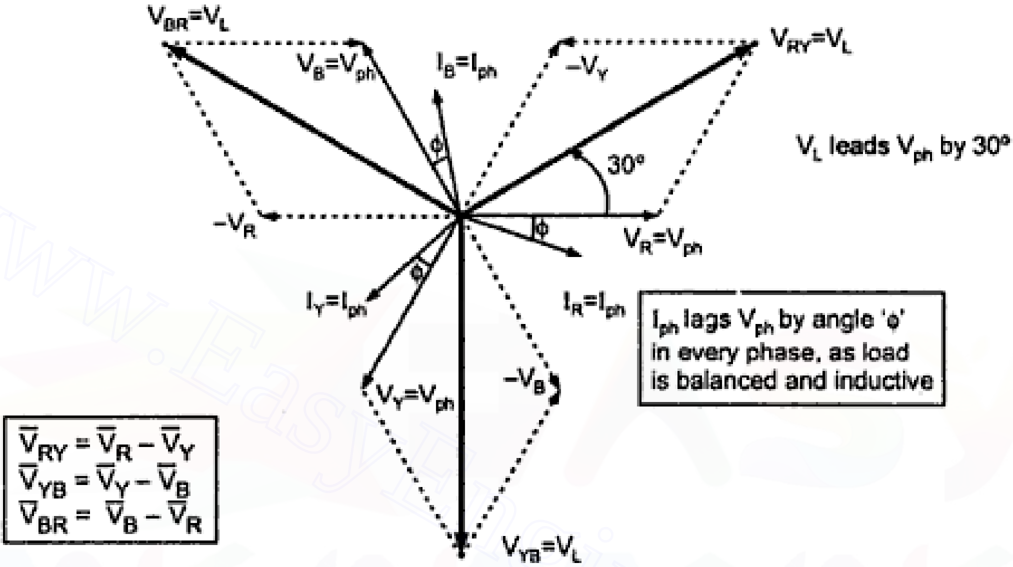


Fig. 8.13 Star and lagging p.f. load

All line voltages are also displaced by 120° from each other.

Key Point: *Every line voltage leads the respective phase voltage by 30°.*

Power : The power consumed in each phase is single phase power given by,

$$\boxed{P_{ph} = V_{ph}\, I_{ph}\, \cos\phi}$$

For balanced load, all phase powers are equal. Hence total three phase power consumed is,

$$P = 3P_{ph} = 3\, V_{ph}\, I_{ph}\, \cos\phi = 3\, \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$\therefore$ $$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

For star connection, to draw phasor diagram, use

$$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y, \quad \bar{V}_{YB} = \bar{V}_Y - \bar{V}_B \quad \text{and} \quad \bar{V}_{BR} = \bar{V}_B - \bar{V}_R$$

Example 8.1 : *Three inductive coils each having resistance of 16 ohm and reactance of 12 ohm are connected in star across a 400 V, three-phase 50 Hz supply. Calculate :*
i) Line voltage, ii) Phase voltage, iii) Line current,
iv) Phase current, v) Power factor, vi) Power absorbed.
Draw phasor diagram **(Dec. - 97)**

Solution : $R_{ph} = 16\ \Omega$, $X_L = 12\ \Omega$ per ph, Star connection $V_L = 400$ V

$$\therefore \quad Z_{ph} = R_{ph} + j\,X_L = 16 + j\,12\ \Omega = 20\ \angle +36.86°\ \Omega$$

Using rectangular to polar conversion on calculator.

i) Line voltage $\quad V_L = 400$ **V**

ii) Phase voltage $\quad V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = $ **230.94 V** $\quad$ ...Star $V_L = \sqrt{3}\,V_{ph}$

iii) $\quad I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{230.94}{20} = $ **11.547 A**

For star connection, $\quad I_L = I_{ph}$ $\quad$ i.e. Line current = **11.547 A**

iv) $\quad$ Phase current = **11.547 A**

v) Power factor $\quad \cos\phi = \dfrac{R_{ph}}{Z_{ph}} = \dfrac{16}{20} = $ **0.8 lagging**

...Inductive hence lagging

vi) Power absorbed $\quad P = \sqrt{3}\,V_L\,I_L\,\cos\phi = \sqrt{3}\times 400\times 11.547\times 0.8 = $ **6400 W**

The phasor diagram can be shown as in the Fig. 8.14.

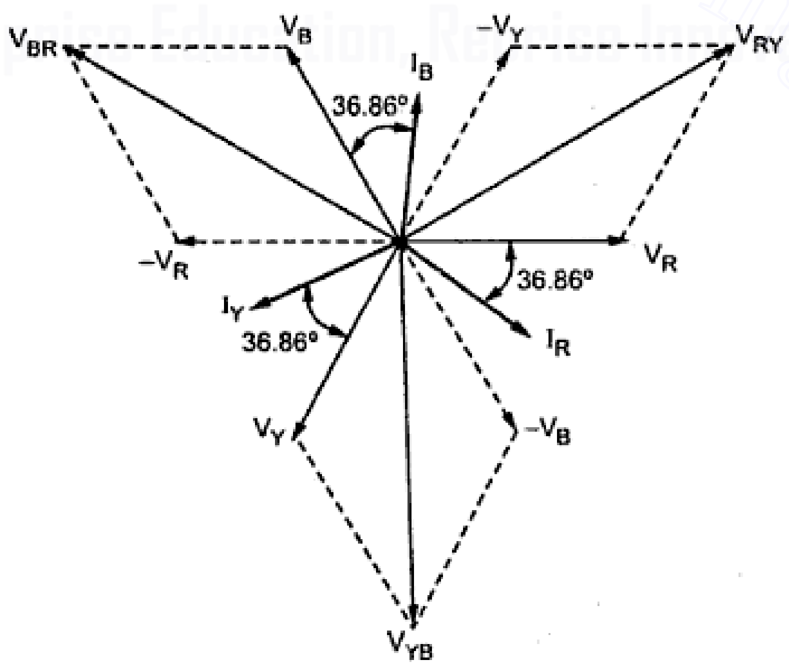


Fig. 8.14

8.9 Relations for Delta Connected Load

Consider the balanced delta connected load as shown in the Fig. 8.15.

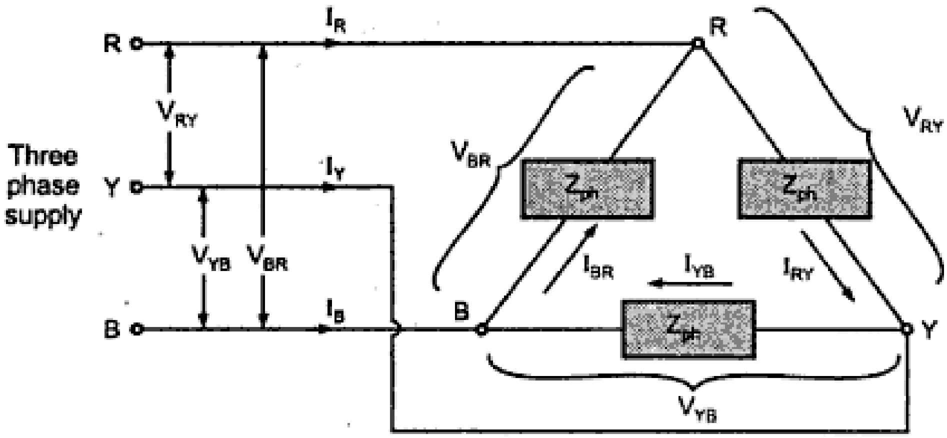


Fig. 8.15 Delta connected load

Line voltages $\quad V_L = V_{RY} = V_{YB} = V_{BR}$

Line currents $\quad I_L = I_R = I_Y = I_B$

Phase voltages $\quad V_{ph} = V_{RY} = V_{YB} = V_{BR}$

Phase currents $\quad I_{ph} = I_{RY} = I_{YB} = I_{BR}$

As seen earlier, $V_{ph} = V_L$ for delta connected load. To derive the relation between I_L and I_{ph}, apply the KCL at the node R of the load shown in the Fig. 8.15.

$$\sum I_{entering} = \sum I_{leaving} \text{ at node R}$$

$$\therefore \quad \bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\therefore \quad \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} \quad \text{...(1)}$$

Applying KCL at node Y and B, we can write equations for line currents I_Y and I_B as,

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} \quad \text{...(2)}$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} \quad \text{...(3)}$$

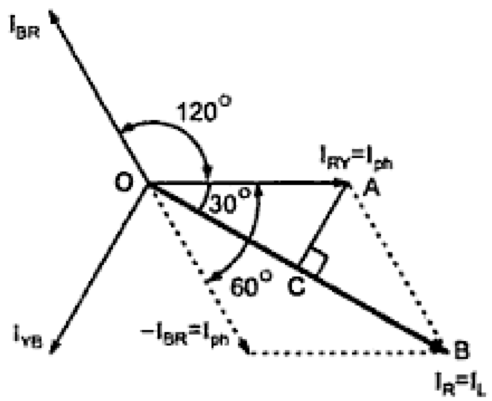


Fig. 8.16

The phasor diagram to obtain line current I_R by carrying out vector subtraction of phase currents I_{RY} and I_{YB} is shown in the Fig. 8.16.

The three phase currents are displaced from each other by 120°.

I_{BR} is reversed to get $-I_{BR}$ and then added to I_{RY} to get I_R.

The perpendicular AC drawn on vector OB, bisects the vector OB which represents I_L. Similarly OB bisects angle between $-I_{YB}$ and I_{RY} which is 60°

$$\therefore \quad \angle BOA = 30° \quad \text{and} \quad OC = CB = \frac{I_L}{2}$$

From triangle OAB,

$$\cos 30° = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$$

$$\therefore \quad \frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$$

$$\therefore \quad \boxed{I_L = \sqrt{3}\ I_{ph}} \quad \text{... for delta connection}$$

Again Z_{ph} decides whether I_{ph} has to lag, lead or remain in phase with V_{ph}. **Angle between V_{ph} and I_{ph} is ϕ.**

Thus for delta connection, to draw phasor diagram, use

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}, \quad \bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} \quad \text{and} \quad \bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB}$$

The complete phasor diagram for cos ϕ lagging power factor load is shown in the Fig. 8.17.

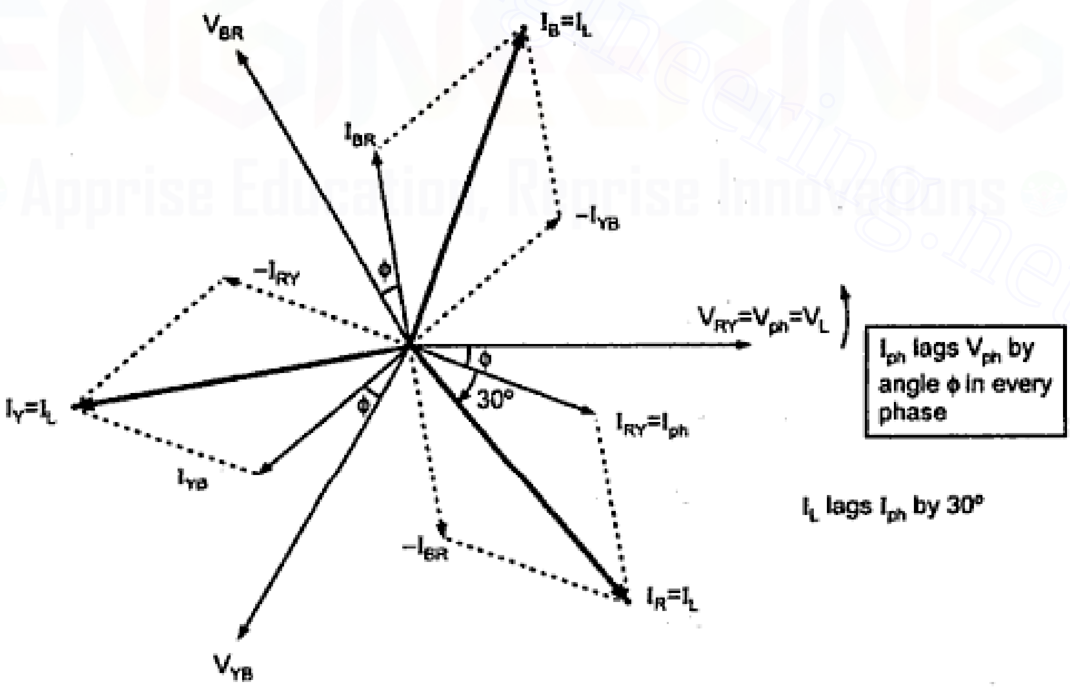


Fig. 8.17

$$Z_{ph} = R_{ph} + j\ X_{Lph} = |Z_{ph}| \angle \phi\ \Omega$$

Each I_{ph} lags respective V_{ph} by angle ϕ

Key Point: *Every line current lags the respective phase current by 30°.*

Power : Power consumed in each phase is single phase power given by,

$$\boxed{P_{ph} = V_{ph}\, I_{ph} \cos\phi}$$

Total power $\quad P = 3P_{ph} = 3V_{ph}I_{ph}\cos\phi = 3V_L \frac{I_L}{\sqrt{3}}\cos\phi$

$\therefore \quad P = \sqrt{3}\, V_L I_L \cos\phi$

Key Point : *The expression for power is same but values of line currents are different in star and delta connected load which must be correctly determined to obtain power.*

8.10 Power Triangle for Three Phase Load

Total apparent power $\quad S = 3 \times$ Apparent power per phase

$\therefore \quad S = 3\, V_{ph}I_{ph} = 3\, \frac{V_L}{\sqrt{3}} I_L = 3\, V_L \frac{I_L}{\sqrt{3}}$

$\therefore \quad S = \sqrt{3}\, V_L I_L$ volt-amperes (VA) or kVA

Total active power $\quad P = \sqrt{3}\, V_L I_L \cos\phi$ watts (W) or kW

Total reactive power $\quad Q = \sqrt{3}\, V_L I_L \sin\phi$ reactive volt amperes (VAR) or kVAR

Hence power triangle is as shown in the Fig. 8.18.

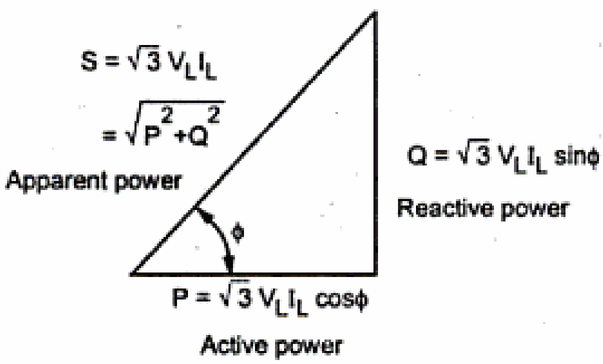


Fig. 8.18 Power triangle

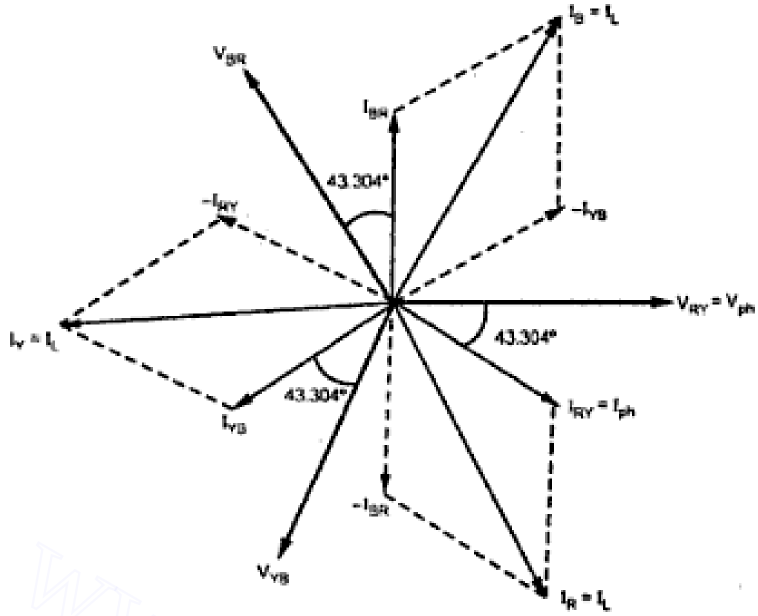


Fig. 8.19

For delta connection the relations for the phasor diagram are,

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$$

$$\bar{I}_y = \bar{I}_{YB} - \bar{I}_{RY},$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB}$$

and $V_L = V_{ph}$

The phasor diagram is shown in the Fig. 8.19.

Example 8.3 : *Prove that a three phase balanced load draws three times as much power when connected in delta, as it would draw when connected in star.*

Solution : Let load is three phase balanced with per phase impedance of Z_{ph} Ω. Let V_L be the line voltage available which remains same whether load is connected in star or delta. What changes is the phase voltage and hence phase and line current values depending on star and delta connection of the load.

Case 1 : Star connection of load

$$\therefore \quad V_{ph1} = \frac{V_L}{\sqrt{3}} = \text{Phase voltage for star connection}$$

$$\therefore \quad I_{ph1} = \frac{V_{ph1}}{Z_{ph}}$$

$$\text{and} \quad I_{L1} = I_{ph1} = \frac{V_{ph1}}{Z_{ph}} \qquad \text{... For star connection}$$

cos φ depends on components of Z_{ph} and remains same for any connection of the load.

$$\therefore \quad P_{star} = \sqrt{3}\, V_L\, I_{L1} \cos\phi = \sqrt{3} \times V_L \times \frac{V_{ph1}}{Z_{ph}} \cos\phi \qquad \text{... } V_L \text{ is constant}$$

$$= \sqrt{3}\, V_L \times \frac{(V_L / \sqrt{3})}{Z_{ph}} \cos\phi \qquad \text{... As } V_{ph1} = V_L / \sqrt{3}$$

$$= \frac{V_L^2}{Z_{ph}} \cos\phi \quad \text{watts}$$

Case 2 : Delta connection of load

$$V_{ph2} = V_L = \text{Phase voltage for delta connection}$$

$$\therefore \quad I_{ph2} = \frac{V_{ph2}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$$

and $$I_{L2} = \sqrt{3}\, I_{ph2} = \frac{\sqrt{3}\, V_L}{Z_{ph}} \quad \text{... For delta connection}$$

cos ϕ remains same for both star and delta connection

$$\therefore \quad P_{delta} = \sqrt{3}\, V_L\, I_{L2} \cos\phi \quad \text{... } V_L \text{ is constant}$$

$$= \sqrt{3}\, V_L \times \frac{\sqrt{3}\, V_L}{Z_{ph}} \cos\phi \quad \text{... As } I_{L2} = \sqrt{3}\, V_L / Z_{ph}$$

$$= 3\, \frac{V_L^2}{Z_{ph}} \cos\phi = 3\, P_{star} \quad \text{... Proved}$$

Key Point : *Thus three phase balanced load draws three times as much power when connected in delta, as it would draw when connected in star.*

Examples with Solutions

Example 8.4 : *Three identical choke coils are connected as a delta load to a three phase supply. The line current drawn from the supply is 15 A and total power consumed is 7.5 kW. The kVA input to the load is 10 kVA. Find out*

i) Line and phase voltage, ii) Impedance/phase, iii) Reactance/phase,

iv) Resistance/phase, v) Power factor vi) Phase current,

vii) Inductance (if frequency is 50 Hz)/phase. **(Dec. - 98)**

Solution : Coils are in delta connection

$$I_L = 15 \text{ A},\ P_T = 7.5 \text{ kW},\ \text{kVA} = 10 \text{ kVA}$$

Now $$VA = \sqrt{3}\, V_L\, I_L \quad \text{i.e. } 10 \times 10^3 = \sqrt{3}\, V_L \times 15$$

$$\therefore \quad V_L = \frac{10 \times 10^3}{15\sqrt{3}} = 385 \text{ V} = V_{ph}$$

i) $$V_L = V_{ph} = 385 \text{ V}$$

ii) $$|Z_{ph}| = \frac{V_{ph}}{I_{ph}}$$

but $\quad I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{15}{\sqrt{3}} = 8.66$ **A**

$\therefore \quad |Z_{ph}| = \frac{385}{8.66} = 44.456\ \Omega$

and $\quad P_T = \sqrt{3}\, V_L\, I_L \cos\phi$

$\therefore \quad 7.5 \times 10^3 = \sqrt{3} \times 385 \times 15 \times \cos\phi$

$\therefore \quad \cos\phi = 0.75$

$\therefore \quad \phi = 41.42°$ **lagging so + ve**

$\therefore \quad Z_{ph} = 44.456 \angle +41.42° = 33.33 + j\, 29.41\ \Omega$

iii) $\quad X_{Lph} = 29.41\ \Omega$

iv) $\quad R_{ph} = 33.336\ \Omega$

v) $\quad$ P.F. = **0.75 lagging**

vi) $\quad I_{ph} = 8.66$ **A**

vii) $\quad L_{ph} = \frac{X_{Lph}}{2\pi f} = \frac{29.4109}{2\pi \times 50} = 93.617$ **mH**

Example 8.5 : *Three equal impedances each of 10 ∠ 60° ohms are connected in star across 3-phase, 400 volts 50 Hz supply. Calculate*

i) Line voltage and phase voltage *ii) Line current and phase current*

iii) Power factor and active power consumed

iv) If the same three impedances are connected in delta to the same source of supply what is the active power consumed ? **(May - 99)**

Solution : $Z_{ph} = 10 \angle 60°\ \Omega$ in star

i) $\quad$ Line voltage = **400 V** = V_L

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\ \text{V}$$

ii) $\quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{10} = 23.094$ **A**

$I_L = I_{ph} = 23.094$ **A** ... Star connection

iii) $\quad$ Power factor = $\cos\phi = \cos 60° =$ **0.5 lagging**

$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 23.094 \times 0.5 = 8000$ **W**

iv) When delta connected, $V_L = V_{ph} = 400$ V

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40 \text{ A}$$

$$I_L = \sqrt{3} I_{ph} = 40\sqrt{3} \text{ A}$$

$$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 40\sqrt{3} \times 0.5 = 24000 \text{ W}$$

Example 8.6 : *A series combination of 3 ohms resistance and a 796.18 μF capacitor in each branch forms a three phase 'star' connected balanced load which is connected to a 415 V, 3 phase, 50 Hz, a.c. supply. Calculate (i) The power consumed and (ii) Current drawn by the load. If the same load is now connected as a 'delta', determine (i) The power consumed and (ii) Current drawn from the supply.* **(May - 2000)**

Solution : $R_{ph} = 3\ \Omega$ $C_{ph} = 796.18\ \mu F$

Case 1 : Star connection, $V_L = 415$ V, $f = 50$ Hz

$$X_{Cph} = \frac{1}{2\pi C_{ph} \times f} = \frac{1}{2\pi \times 796.18 \times 10^{-6} \times 50} = 4\ \Omega$$

$$\therefore \quad Z_{ph} = R_{ph} - j X_{Cph} = 3 - j\,4\ \Omega = 5 \angle -53.13°\ \Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

But, $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$ ...As star connection

$$\therefore \quad I_{ph} = \frac{239.6}{5} = 47.92 \text{ A}$$

$$\therefore \quad I_L = I_{ph} = 47.92 \text{ A}$$

$$\therefore \quad P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 415 \times 47.92 \times \cos(-53.13°)$$

$$= 20667.05 \text{ W}$$

Case 2 : Let load be now connected in delta

$$V_{ph} = V_L = 415 \text{ V}$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{5} = 83 \text{ A}$$

$$\therefore \quad I_L = \sqrt{3} \times I_{ph} = \sqrt{3} \times 83 = 143.7602 \text{ A}$$

$$\therefore \quad P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 415 \times 143.7602 \times \cos(-53.13°)$$

$$= 62001 \text{ W}$$

This is 3 times the power consumed in star connection.

Example 8.7 : *Two balanced 3 -ph loads are connected in parallel to a 415 V, 3-ph, 50 Hz a.c. supply. The loads are :-*

Load "A" : draws 10 A at 0.8 lag p.f., star connected.

Load "B" : has R = 6 Ω and C = 198 micro-farads per phase, delta connected.

Estimate :- i) Total line current, ii) Total power consumed,

iii) Impedance of load "A" in complex form , iv) Power factor of load "B" and its nature. **(May - 2001)**

Solution : The loads are shown in the Fig. 8.20. (a)

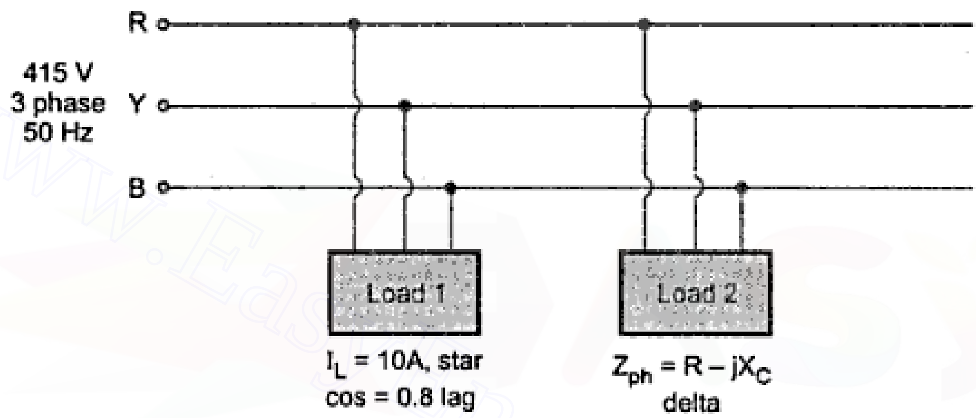


Fig. 8.20 (a)

$$V_L = 415 \text{ V}$$

For load 1 :

$$V_{ph} = V_L / \sqrt{3} = 239.6 \text{ V}$$

$$I_L = I_{ph} = 10\text{A}, \cos\phi = 0.8 \text{ lag}$$

$$P_1 = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times 415 \times 10 \times 0.8$$

$$= 5750.408 \text{ W}$$

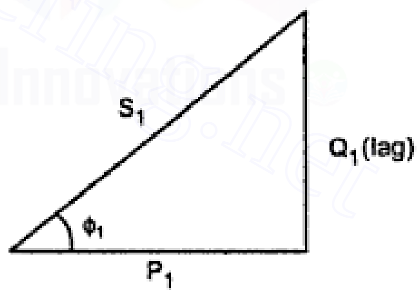


Fig. 8.20 (b)

$$Q_1 = \sqrt{3}\, V_L\, I_L \sin\phi$$

$$= \sqrt{3} \times 415 \times 10 \times 0.6$$

$$= 4312.8065 \text{ VAR}$$

For Load 2 :

$$Z_{ph} = R_{ph} - j\, X_{Cph}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 198 \times 10^{-6}}$$

$$= 16.076\ \Omega$$

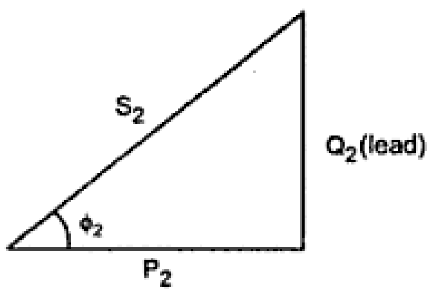


Fig. 8.20 (c)

Example 8.8 : *A balanced delta-connected load of impedance $60\angle 30°$ ohm per phase is connected to a 3-phase supply of 400 V, phase sequence A-B-C.*

Find :

i) The phase and line values of current

ii) Total power and reactive voltampere, and iii) The phase angle of line current I_A with respect ot line voltage V_{AB}, drawing a sketch of the relevant phasors. **(Dec. - 2002)**

Solution : Delta connected, $Z_{ph} = 60 \angle 30° \ \Omega$, $V_L = 400$ V.

$$V_{ph} = V_L = 400 \text{ V as delta connected}$$

i) $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400\angle 0°}{60\angle 30°} = \mathbf{6.667 \angle -30° A}$$

$\therefore$ $$I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\times 6.667 = \mathbf{11.547\ A}$$

ii) $$P_T = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3}\times 400\times 11.547\times \cos(-30°)$$

$$= \mathbf{6928.2\ W}$$

$$Q_T = \sqrt{3}\, V_L\, I_L \sin\phi = \text{reactive volt - amp}$$

$$= \sqrt{3}\times 400\times 11.547\times \sin(30°) = \mathbf{4000\ VAR}$$

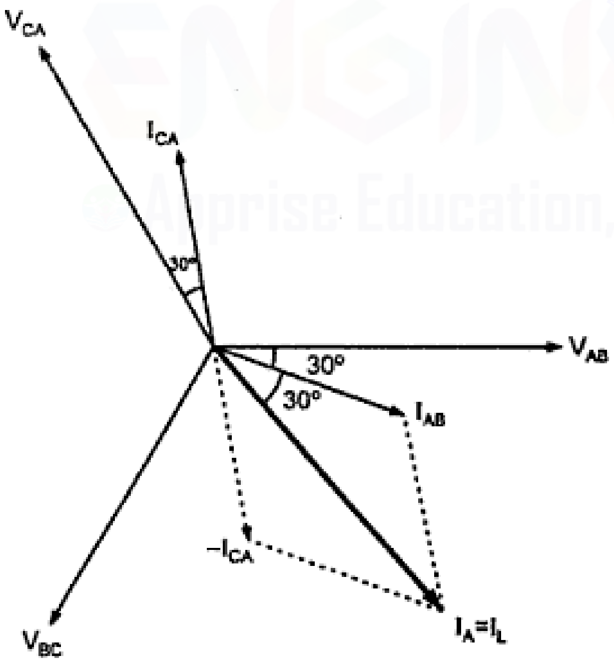


Fig. 8.21

iii) The phasor diagram is shown in the Fig. 8.21.

$$V_{AB} = V_{BC} = V_{CA} = V_{ph} = V_L$$

$$I_{AB} = I_{ph} = I_{CA}$$

$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA}$$

The angle between I_A and V_{AB} is 60°.

Example 8.9 : *A balanced star connected load of (8 + j 6) Ω/phase is connected to a 3 phase, 440 V supply. The line voltages are*

$V_{RY} = 440 \angle 0° \text{ V}, V_{YB} = 440 \angle -120° \text{ V}, V_{BR} = 440 \angle +120° \text{ V}$

Find the phasor expressions for the line currents I_R, I_Y and I_B. Draw the phasor diagram.

Solution : Given : Star connection, $Z_{ph} = 8 + j\,6\ \Omega$, $V_L = 440$ V

To find : Phasor expressions for I_R, I_Y and I_B.

$$Z_{ph} = 8 + j\,6 = 10 \angle 36.8698° \ \Omega$$

For star connection $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.034 \ \text{V}$$

$$\therefore \quad \bar{I}_{ph} = \frac{\bar{V}_{ph}}{\bar{Z}_{ph}}$$ But in this V_{ph} is not reference.

This indicates that angle of V_{ph} is not zero.

This is clear from the fact that line voltages are given by,

$$V_{RY} = 400 \angle 0\,° \text{ V}, \quad V_{YB} = 440 \angle -120° \text{ V}, \quad V_{BR} = 440 \angle +120° \text{ V}$$

Angle of V_{ph} can be calculated from phasor diagram as in the Fig. 8.22.

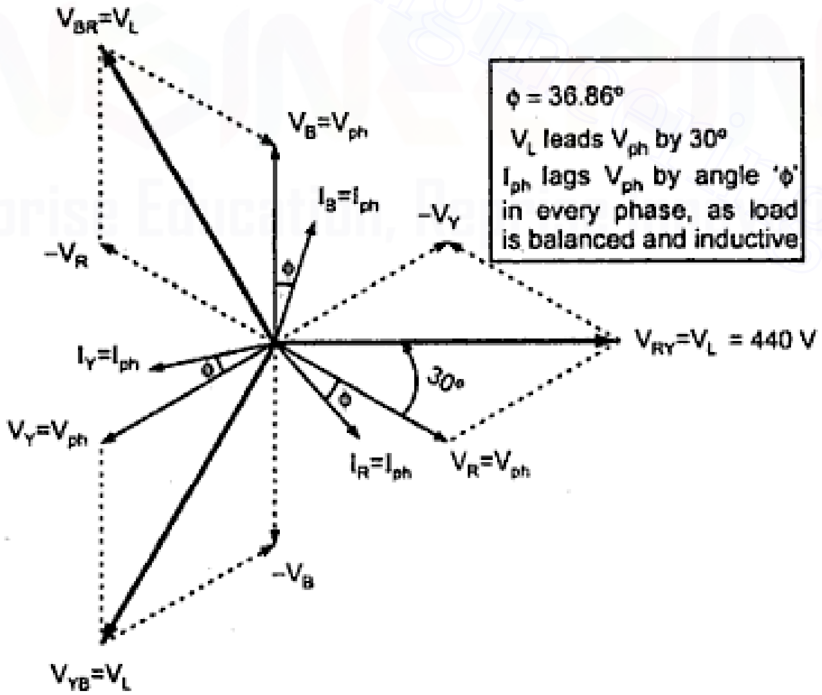


Fig. 8.22

$$I_{ph1} = \frac{\bar{V}_{ph1}}{\bar{Z}_{ph}} = \frac{254.034 \angle -30°}{10 \angle 36.8698°} = 25.4034 \angle \ 66.8698° \text{ A}$$

Calculate i) The capacitance to obtain the resultant power factor of 0.95 lagging.

ii) The line current taken by combined load.

Solution : Given : 3 coils R = 20 Ω, X_L = 15 Ω, Star connection, V_L = 400 V,

3 capacitors in delta connected to improve p.f. to 0.95 lag.

To find : I_L, active power and power factor of coils and value of capactance in each phase, combined line current.

Load 1 : $Z_{ph} = 20 + j\,15 = 25 \angle 36.86°$

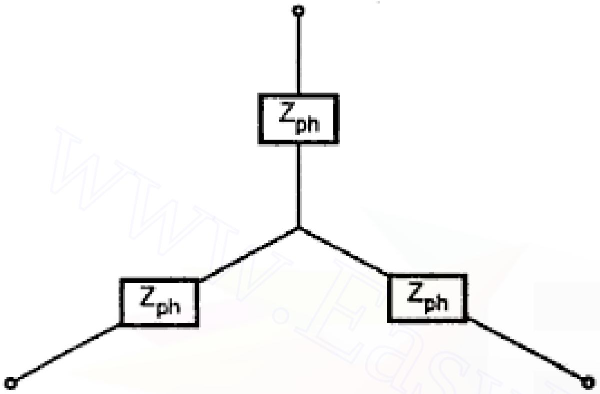


Fig. 8.25

star so $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9401$ V

$\therefore \quad \bar{I}_{ph} = \frac{\bar{V}_{ph}}{\bar{Z}_{ph}} = \frac{230.9401 \angle 0°}{25 \angle 36.86°}$

$\therefore \quad I_{ph} = 9.2376$ A at cos (– 36.86°)

Power factor lagging.

$\therefore \quad I_{ph} = I_L = 9.2376$ A and

Power factor = 0.8 lagging

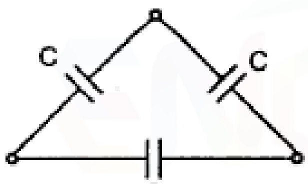


Fig. 8.26

Active power $= \sqrt{3}\, V_L\, I_L \cos\phi$

$= \sqrt{3} \times 440 \times 9.2376 \times 0.8$

= 5.1199 kW while sin ϕ = 0.6

Reactive power $= \sqrt{3}\, V_L\, I_L \sin\phi = 3.8399$ kVAR

Now, Delta capacitances are connected.

Now $Z_{ph} = X_{Cph}$ as $R_{ph} = 0$ for capacitors

∴ Active power consumption = 0 W by capacitors (cos ϕ = 0).

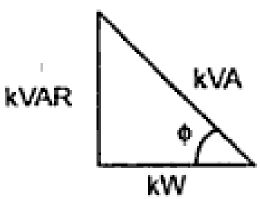


Fig. 8.27

Total active power = Active power by coils + Zero

$\therefore \quad P_T = 5.1199$ kW

Now new $\cos\phi_T = 0.95$ lag

$\therefore \quad \phi_T = 18.1948 \quad \therefore \tan\phi_T = 0.3286$

$$\tan\phi_T = \frac{\text{kVAR}}{\text{kW}}$$

$\therefore$ Total kVAR $= (\text{kW})_{total} \times \tan\phi_T$

$= 5.1199 \times 0.3286 = 1.682$ kVAR

$\therefore$ kVAR drawn by capacitor = 3.83399 kVAR – 1.682 kVAR

= 2.1570 kVAR

Now for capacitor bank as delta connected,

$$V_L = V_{ph} = 400 \text{ V}, \quad \sin\phi = 1, \quad \cos\phi = 0$$

$$\therefore \quad \text{kVAR of capacitors} = \sqrt{3}\, V_L\, I_L \sin\phi \qquad \ldots \sin\phi = 1$$

$$\therefore \quad 2.1570 \times 10^3 = \sqrt{3} \times 400 \times I_L$$

$$\therefore \quad I_L = 3.1134 \text{ A}$$

$$\therefore \quad I_{ph} = \frac{I_L}{\sqrt{3}} = 1.7975 \text{ A} \qquad \ldots \text{Delta connected}$$

$$\therefore \quad X_{Cph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{1.7975} = 222.524\ \Omega$$

$$\therefore \quad C = \frac{1}{2\pi f \times X_C} = 14.3045\ \mu\text{F}$$

$$\text{Total kW} = 5.1199 \text{ kW} = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$\text{Combination} \quad \cos\phi = 0.95$$

$$\therefore \quad 5.1199 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.95$$

$$\therefore \quad \text{Combined line current} = 7.7788 \text{ A} \qquad \ldots \text{Total line current}$$

Example 8.12 : *In a 3 phase, 4 wire system, the line voltage is 400 V and purely resistive loads of 5 kW, 11 kW are connected between lines and neutral. Draw the circuit diagram, calculate current in each line, current in the neutral.*

Solution : $\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$ (vector addition)

$\cos\phi$ for all loads is unity as all are resistive in nature.

$$V_{RN} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V} = V_{ph}$$

$$\therefore \quad P = V_{ph}\, I_{ph} \cos\phi$$

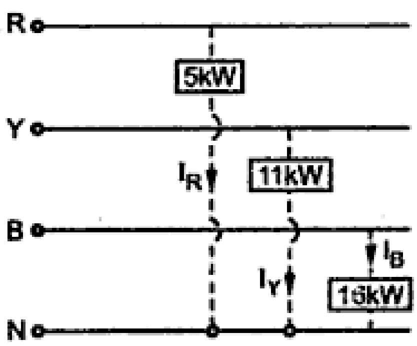


Fig. 8.28 Circuit diagram

Solution : $Z_{ph} = 30 + j\,40\Omega = 50\angle 53.13°\Omega$, $V_{ph} = 231$ V

The circuit diagram is shown in the Fig. 8.31.

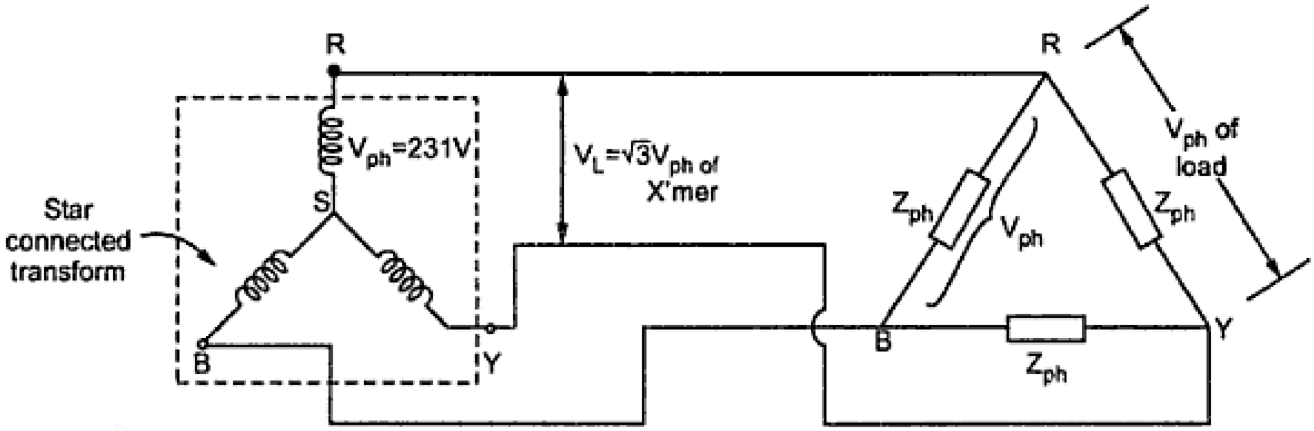


Fig. 8.31

As transformer is star connected with V_{ph} = 231 V, the line voltage available from the transformer is,

$$V_L = \sqrt{3}\, V_{ph} \text{ of x'mer} = \sqrt{3} \times 231 = 400.103 \text{ V}$$

From load point of view, as load is delta connected,

$$V_{ph} = V_L = 400.103 \text{ V}$$

i) $\therefore$ $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400.103}{50} = 8 \text{ A}$$ ... magnitude

ii) $$V_{ph} = 400.103 \text{ V}$$ ... potential difference across each phase

iii) $$I_L = \sqrt{3}\, I_{ph} = 13.856 \text{ A}$$ ... magnitude of current in X'mer winding

iv) $$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400.103 \times 13.856 \times \cos(53.13°)$$

$$= 5761.3279 \text{ W}$$

$$\text{p.f.} = \cos(53.13°) = 0.6 \text{ lagging}$$

Example 8.14 : *A balanced star connected load is supplied from a symmetrical 3 phase 400 volts, 50 Hz system. The current in each phase is 30 Amp and lags 30° behind the phase voltage. Find,*

i) phase voltage. *ii) resistance and reactance per phase.*

iii) load inductance per phase. *iv) total power consumed.*

Draw the phasor diagram showing the currents and voltages. **(Dec. - 2003)**

Solution : $V_L = 400$ V, f = 50 Hz, $I_{ph} = 30$ A lags V_{ph} by 30°.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \mathbf{230.94\ V} \quad \text{... Star connection}$$

i) $\therefore$ $V_{ph} = 230.94\ \angle 0°\ \text{V}$

and $I_{ph} = 30\ \angle -30°\ \text{A}$ lagging hence negative ϕ

ii) $\therefore$ $Z_{ph} = \frac{V_{ph}}{I_{ph}} = 7.698\ \angle +30°\ \Omega = 6.667 + j\,3.849\,\Omega$

Compare with, $R + j X_L$ hence

$$\mathbf{R_{ph} = 6.667\ \Omega,\quad X_{Lph} = 3.849\ \Omega}$$

iii) $f = 50$ Hz and $X_{Lph} = 2\pi f L_{ph}$

$\therefore$ $L_{ph} = \frac{3.849}{2\pi \times 50} = 0.01225\ \text{H} = \mathbf{12.25\ mH}$

iv) $P = \sqrt{3}\, V_L\, I_L\, \cos\phi = \sqrt{3} \times 400 \times I_L \times \cos(30°)$

Now $I_L = I_{ph} = 30$ A ... Star connection

$\therefore$ $P = \mathbf{18000\ W = 18\ kW}$

The phasor diagram is as shown in the Fig. 8.32.

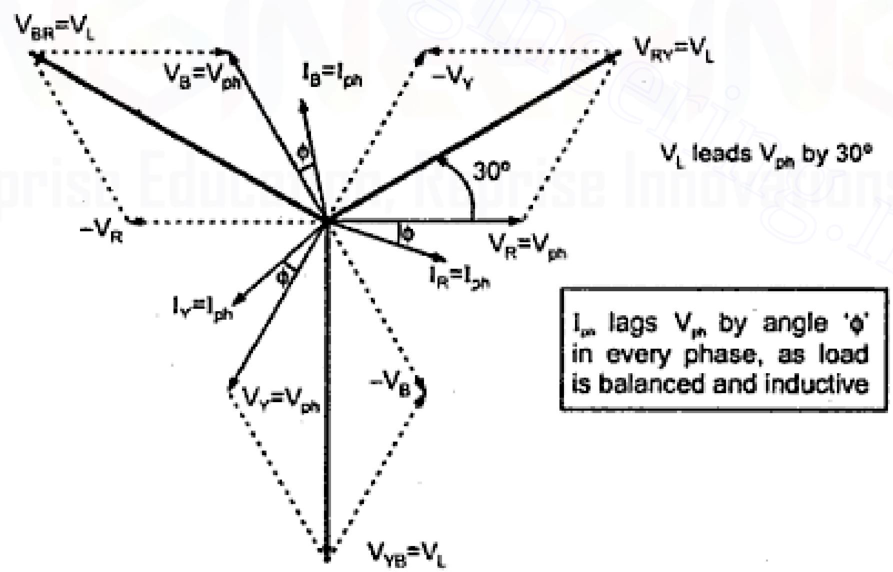


Fig. 8.32

➡ **Example 8.15 :** *A 220 V, 3-phase voltage is applied to a balanced delta-connected load of phase impedance (15+j20) ohm. Find the line current and power consumed by each phase. What is the phasor sum of the three line currents ? Why does it have to be this value ?*

(May - 2004)

Solution :

Given voltage is always line voltage unless and until specified clearly as the phase voltage.

$$V_L = 220\text{ V, delta load, } Z_{ph} = 15 + j\,20\Omega = 25\angle 53.13°\ \Omega$$

$$V_{ph} = V_L = 200\text{ V} \qquad \text{... As delta connected}$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{220}{25} = 8.8\text{ A}$$

and $$I_L = \sqrt{3}\, I_{ph} = \mathbf{15.242\ A}$$

$$P_{ph} = V_{ph}\, I_{ph}\, \cos\phi = 220 \times 8.8 \times \cos(53.13°) = \mathbf{1161.602\ W}$$

The phasor sum of the three line currents is **zero**. This is because the load is **balanced**. The three line currents are displaced by angle of 120° from each other and have same magnitude. The phasor sum of such currents is zero.

Example 8.16 : *A delta connected balanced load is connected to a 3 phase, 400 V supply. The load p.f. is 0.8 lagging. The line current is 34.64 ampere. Find (i) The resistance, reactance and impedance of the load per phase, (ii) Total power and (iii) Total reactive volt ampere. Draw the phasor diagram showing all the quantities.* **(May - 2004)**

Solution : $V_L = 400$ V, $\cos\phi = 0.8$ lag, $I_L = 34.64$ A, delta

$$V_{ph} = V_L = 400\text{ V}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = 20\text{ A}$$

$$\therefore \quad |Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{400}{20} = 20\ \Omega$$

$$\phi = \cos^{-1} 0.8 = 36.86°$$

$$\therefore \quad Z_{ph} = \mathbf{20\angle 36.86°\ \Omega = 16 + j\,12\ \Omega}$$

Comparing with $R_{ph} + j\,X_{L\,ph}$,

R_{ph} = **16 Ω/ph** and $X_{L\,ph}$ = **12 Ω/ph**

ii) $P = \sqrt{3}\, V_L\, I_L\, \cos\phi = \sqrt{3} \times 400 \times 34.64 \times 0.8 = \mathbf{20\ kW}$

iii) $Q = \sqrt{3}\, V_L\, I_L\, \sin\phi = \sqrt{3} \times 400 \times 34.64 \times \sin(36.869°) = \mathbf{14.399\ kVAR}$

The phasor diagram is shown in the Fig. 8.33.

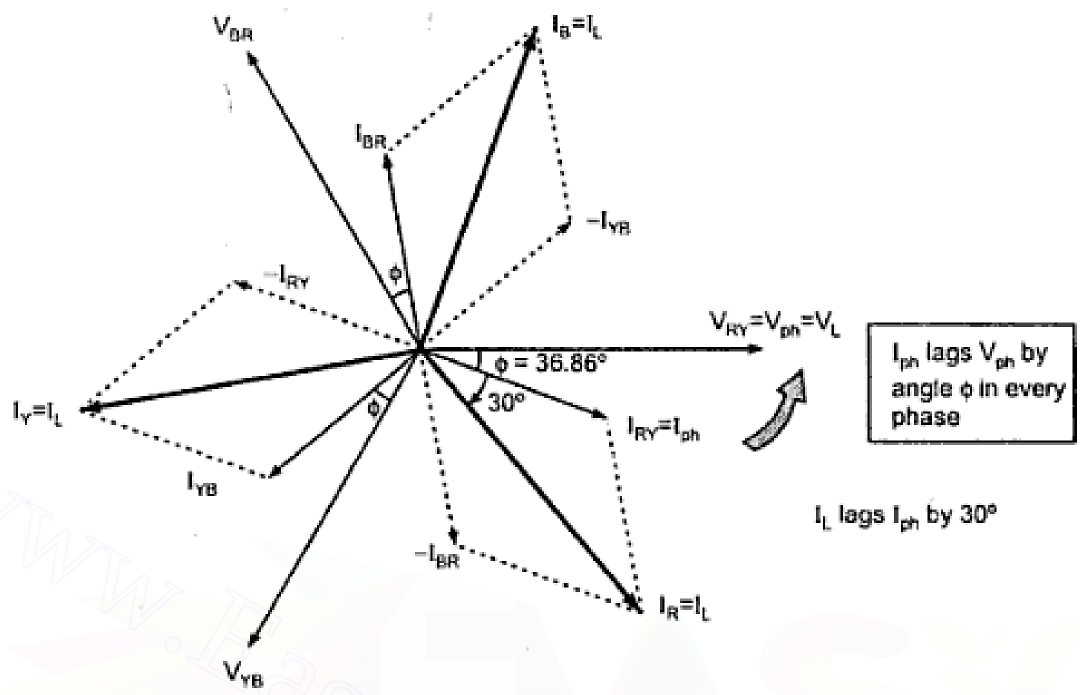


Fig. 8.33

Example 8.17 : *A symmetrical 3 phase, 400 volt system supplies a balanced load of 0.8 lagging power factor and connected in star. If the line current is 34.64 Amp, find*

(i) Impedance (ii) Resistance and reactance per phase

(iii) Total power and (iv) Total reactive voltamperes. **(May-2005)**

Solution : The circuit diagram is shown in the Fig. 8.34 .

$$\cos\phi = 0.8 \text{ lagging}$$

$$V_L = 400 \text{ V}$$

$$\therefore \quad V_{ph} = \frac{V_L}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_L = I_{ph} = 34.64 \text{ A}$$

$$\therefore \quad |Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{34.64}$$

$$= 6.667\ \Omega$$

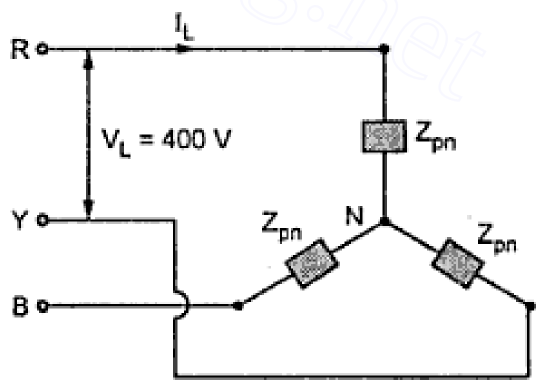


Fig. 8.34

and $\quad \phi = \cos^{-1} 0.8 = 36.8698°$ lagging so +ve for $|Z|$.

i) $\quad Z_{ph} = |Z_{ph}|\angle\phi = 6.667 \angle +36.8698°\ \Omega$

$$= 5.333 + j4\ \Omega = R_{ph} + j\,X_{Lph}$$

ii) $R_{ph} = 5.333\ \Omega$ and $X_{Lph} = 4\ \Omega$

iii) $P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 34.64 \times 0.8 = \mathbf{19.1999\ kW}$

iv) $Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 400 \times 34.64 \times 0.6 = \mathbf{14.399\ kVAR}$

Example 8.18 : *A balanced three-phase star connected load of 100 kW takes a leading current of 80 Amp, when connected across 3−φ, 1100 volt, 50 Hz supply. Find the value of resistance / phase and capacitance / phase of load and p.f. of load. If the same load is connected in delta, calculate power consumed.* **(Dec.-2005)**

Solution : $P_T = 100$ kW, $I_L = 80$ A, $V_L = 1100$ V, $f = 50$ Hz, star

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1100}{\sqrt{3}} = 635.0852\ V$$

$$I_{ph} = I_L = 80\ A$$

$$P_T = \sqrt{3}\ V_L I_L \cos\phi$$

$$\therefore \quad 100\times 10^3 = \sqrt{3}\times 1100\times 80\times \cos\phi$$

$$\therefore \quad \cos\phi = 0.656 \text{ leading i.e. } \phi = 48.9984°$$

$$\therefore \quad V_{ph} = 635.0852 \angle 0°\ V,$$

$$I_{ph} = 80 \angle +48.9984°\ A$$

$$\therefore \quad Z_{ph} = \frac{V_{ph}}{I_{ph}} = 7.9385 \angle -48.9984°\ \Omega = 5.2082 - j\,5.9911\ \Omega$$

$$\therefore \quad R_{ph} = \mathbf{5.2082\ \Omega},\ X_{Cph} = 5.9911\ \Omega$$

$$\therefore \quad C_{ph} = \frac{1}{2\pi\times f\times X_{Cph}} = \mathbf{531.313\ \mu F}$$

$$\text{p.f.} = \mathbf{0.656\ leading}$$

ii) Delta connection

$$V_{ph} = V_L = 1100\ V \text{ and } I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$\therefore \quad I_{ph} = \frac{1100 \angle 0°}{7.9385 \angle -48.9984°} = 138.5652 \angle +48.9984°\ A$$

$$\therefore \quad I_L = \sqrt{3}\ I_{ph} = \sqrt{3}\times 138.5652 = 240\ A$$

$$\therefore \quad P_T = \sqrt{3}\ V_L I_L \cos\phi = \mathbf{300\ kW} \qquad \text{... } \cos\phi \text{ remains same}$$

Key Point : *Power in delta connection is 3 times power in star.*

Example 8.19 : *Three inductive coils each with resistance of 15 ohm and an inductance of 0.03 H are connected 1) in star and 2) in delta across 3 phase, 400 volt, 50 Hz supply. Calculate in each case 1) Line current and 2) Power consumed by load.*

(May-2006, 2007)

Solution : For delta connection :

$$V_L = V_{ph} = 400 \text{ V}, \quad X_{Lph} = 2\pi fL = 2\pi\times 50\times 0.03 = 9.425\ \Omega$$

$$\therefore \quad Z_{ph} = 15 + j\ 9.425\ \Omega = 17.72\angle 32.14°\Omega$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{17.72} = 22.58 \text{ A}$$

$$\therefore \quad I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\times 22.58 = \mathbf{39.11\ A}$$

$$\therefore \quad P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3}\times 400\times 39.11\times \cos(32.14°) = \mathbf{22.943\ kW}$$

For star connection :

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$Z_{ph} = 17.72\angle 32.14°\ \Omega$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{17.72} = \mathbf{13.032\ A}$$

$$\therefore \quad I_L = I_{ph} = \mathbf{13.032\ A}$$

$$\therefore \quad P = \sqrt{3}\, V_L\, I_L\, \cos\phi = \sqrt{3}\times 400\times 13.032\times \cos(32.14°)$$

$$= \mathbf{7645.6047\ W}$$

Review Questions

1. *What are the advantages of a three phase system ?*
2. *Explain in short the three phase generation.*
3. *Define : Symmetrical system and phase sequence.*
4. *Explain the concept of balanced load.*
5. *Explain the difference between a line voltage and phase voltage, similarly line current and phase current.*
6. *Derive the relationship between a line current and a phase current and a line voltage and a phase voltage related to a star connected and delta connected load.*
7. *State the power relations for a three phase system and explain a power triangle.*
8. *A balanced 3 phase load connected in delta, draws a power of 10.44 kW at 200 V at a p.f. of 0.5 lead, find the values of the circuit elements and the reactive volt amperes drawn.*

(Ans. : 2.8736 Ω, 639.17 μF, 18.0825 kVAR)

9. *A balanced 3 phase star connected load of 100 kW takes a leading current of 80 A, when connected across a 3 phase, 1100 V, 50 Hz supply. Find the circuit constants of the load per phase.*

(Ans. : 5.208 Ω, 530.516 μF)

10. *Three similar choke coils are connected in star to a three phase supply. If the line current is 15 A, the total power consumed is 11 kW, and the volt ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.*

If these coils are now connected in delta calculate phase and line currents, active and reactive power. **(Ans. : 10.197 kVAR, 30.592 kVAR)**

11. *Three pure elements connected in star, draw -x kVAR. What will be the value of elements that will draw the same kVAR, when connected in delta across the same supply ?*

(Ans. : Three times than original)

12. *A 3 φ, star connected source feeds 1500 kW at 0.85 p.f. lag to a balanced mesh connected load. Calculate the current, it's active and reactive components in each phase of the source and the load. The line voltage is 2.2 kV.*

(Ans. : 463.112 A, 227.27 A, 140.91 A, 303.64 A, 244.06 A)

13. *For a balanced three phase wye connected load the phase voltage V_R is $100 \angle -45°$ and it draws a line current I_Y of $5 \angle +180°$. i) Find the complex impedance per phase ii) draw a power triangle and identify its all sides with magnitudes and appropriate units. Assume phase sequence R-Y-B.*

(Ans. : $20 \angle 15° \Omega$)

14. *Each leg of a balanced delta connected load consists 7 ohms resistance in series with 4 ohms inductive reactance. Line-to-line voltages are : $E_{ab} = 2360 \angle 0°$ V, $E_{bc} = 2360 \angle -120°$ V, $E_{ca} = 2360 \angle +120°$ V*

Determine : a) Phase current I_{ab}, I_{bc} and I_{ca} (both magnitudes and phase)

b) Each line current and its associated phase angle.

c) Load power factor.

d) Draw with the instruments, phasor diagram based on circuit diagram and clearly indicate on both circuit diagram and phasor diagram :

i) Line currents. ii) Phase currents iii) Line voltages.

iv) Phase voltages v) Load phase angle

e) Find the impedance per phase that draws the same power at the same power factor.

(Ans. : 292.724 $292.724 \angle -29.74°$ A, $292.724 \angle -149.74°$ A, $292.724 \angle +90.26°$ A, 507.012A, 0.8682lag, $8.0622 \angle -29.24° \Omega$)

□□□

9 Single Phase Transformers

9.1 Introduction

The main advantage of alternating currents over direct currents is that, the alternating currents can be easily transferable from low voltage to high or high voltage to low. Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called **transformer**. The transformer works on the principle of mutual induction. It transfers an electric energy from one circuit to other when there is no electrical connection between the two circuits. Thus we can define transformer as below.

> **Key Point** : *The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.*

The use of transformers in transmission system is shown in the Fig. 9.1.

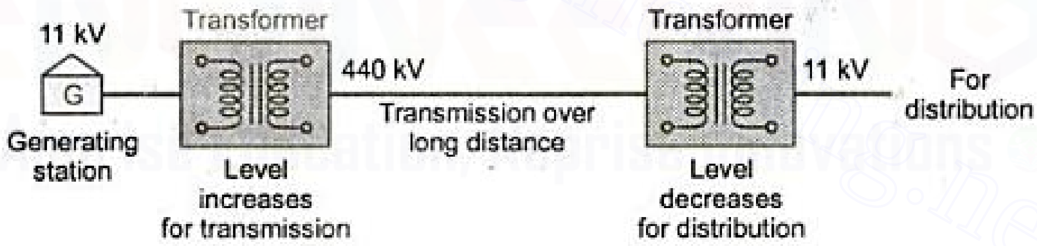


Fig. 9.1 Use of transformers in transmission system

9.2 Principle of Working

The principle of **mutual induction** states that when two coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f. can drive a current, when a closed path is provided to it. The transformer works on the same principle. In its elementary form, it consists of two inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in the Fig. 9.2.

One of the two coils is connected to a source of alternating voltage. This coil in which electrical energy is fed with the help of source is called **primary winding (P)**. The other

9.2.1 Can D.C. Supply be used for Transformers ?

The d.c. supply can not be used for the transformers.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If d.c. supply is given, the current will not change due to constant supply and transformer will not work.

Practically winding resistance is very small. For d.c., the inductive reactance X_L is zero as d.c. has no frequency. So total impedance of winding is very low for d.c. Thus winding will draw very high current if d.c. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.

There can be saturation of the core due to which transformer draws very large current from the supply when connected to d.c.

Thus d.c. supply should not be connected to the transformers.

9.3 Construction

There are two basic parts of a transformer i) Magnetic Core ii) Winding or Coils.

The core of the transformer is either square or rectangular in size. It is further divided into two parts. The vertical portion on which coils are wound is called **limb** while the top and bottom horizontal portion is called **yoke** of the core. These parts are shown in the Fig. 9.4 (a).

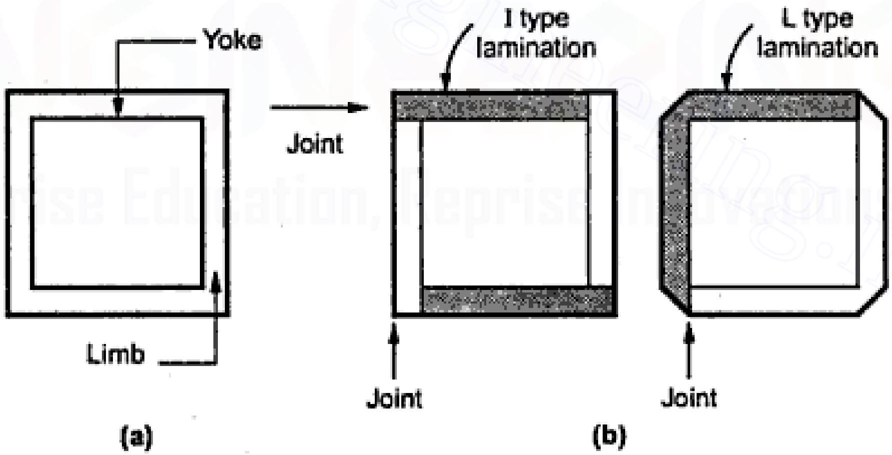


Fig. 9.4 Construction of transformer

Core is made up of laminations. Because of laminated type of construction, eddy current losses get minimised. Generally high grade silicon steel laminations [0.3 to 0.5 mm thick] are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the air gap at the joints. For this generally 'L' shaped or 'I' shaped laminations are used which are shown in the Fig. 9.4 (b).

The cross-section of the limb depends on the type of coil to be used either circular or rectangular. The different cross-sections of limbs, practically used are shown in the Fig. 9.5.

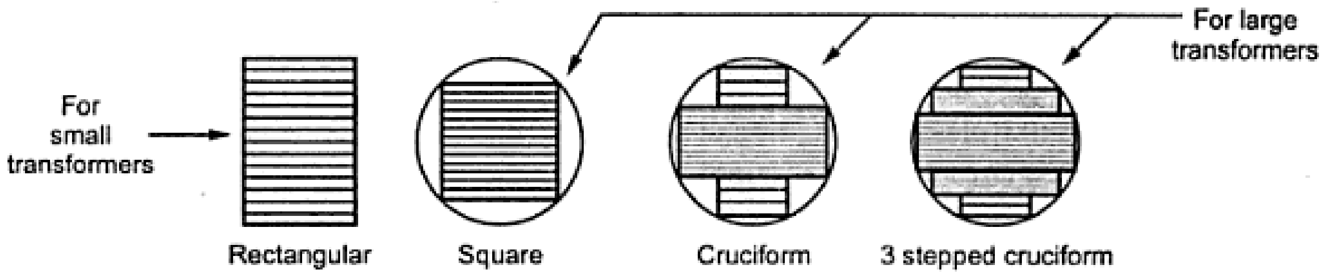


Fig. 9.5 Different cross-sections

9.3.1 Types of Windings

The coils used are wound on the limbs and are insulated from each other. In the basic transformer shown in the Fig. 9.2, the two windings wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to this leakage flux increases which affects the transformer performance badly. Similarly it is necessary that the windings should be very closes to each other to have high mutual inductance. To achieve this, the two windings are split into number of coils and are wound adjacent to each other on the same limb. A very common arrangement is cylindrical concentric coils as shown in the Fig. 9.6.

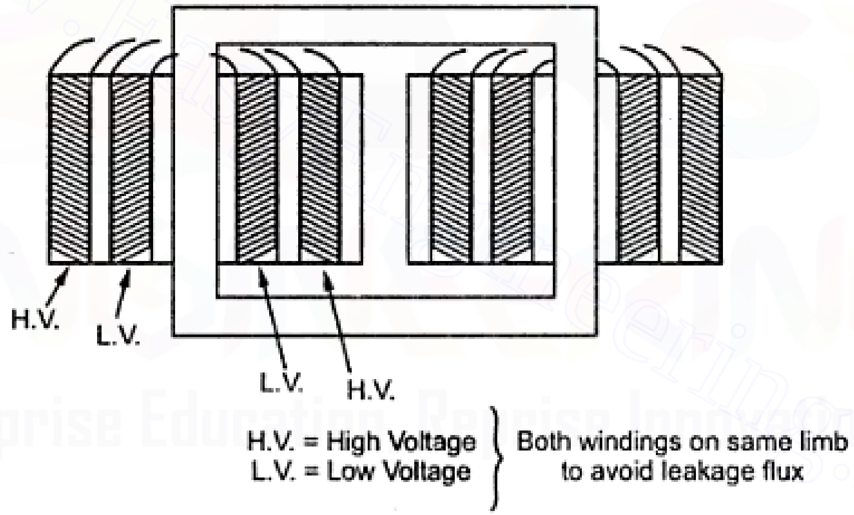


Fig. 9.6 Cylindrical concentric coils

Such cylindrical coils are used in the core type transformer. These coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

The other type of coils which is very commonly used for the shell type of transformer is **sandwich coils**. Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portions reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The **sandwich coil** is shown in the Fig. 9.7. The top and bottom coils are low voltage coils. All the portions are insulated from each other by paper.

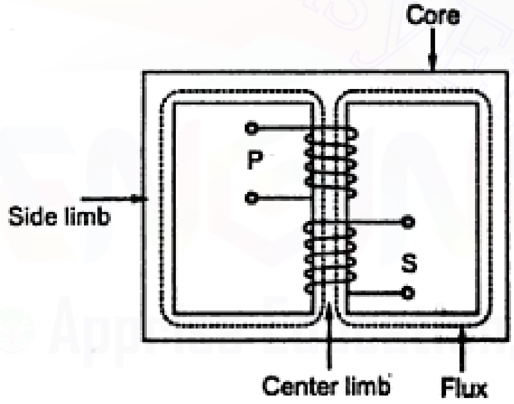
The Fig. 9.8 (a) shows the schematic representation of the core type transformer while the Fig. 9.8 (b) shows the view of actual construction of the core type transformer.

9.4.2 Shell Type Transformer

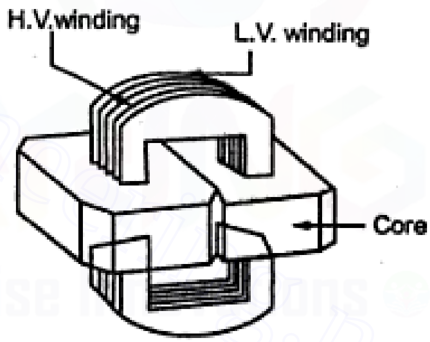
It has a double magnetic circuit. The core has three limbs. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils. As mentioned earlier, each high voltage coil is in between two low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called overlapped or imbricated joints. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laminations are required to be removed.

The Fig. 9.9 (a) shows the schematic representation while the Fig. 9.9 (b) shows the outaway view of the construction of the shell type transformer.



(a) Representation



(b) Construction

Fig. 9.9 Shell type transformer

9.4.3 Berry Type Transformer

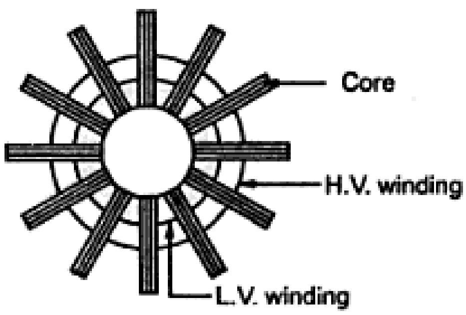


Fig. 9.10 Berry type transformer

This has distributed magnetic circuit. The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

Diagramatically it can be shown as in the Fig. 9.10.

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate cut, formed and welded into the rigid structures. All the joints are painted with a solution of light blue chalk which turns dark in the presence of oil, disclosing even the minutest leaks. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. The oil serves two functions : i) Keeps the coils cool by circulation and ii) Provides the transformers an additional insulation.

The oil should be absolutely free from alkalies, sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly. Hence the tanks are sealed air tight to avoid the contact of oil with atmospheric air and moisture. In large transformers, the chambers called **breathers** are provided. The breathers prevent the atmospheric moisture to pass on to the oil. The breathers contain the silica gel crystals which immediately absorb the atmospheric moisture. Due to long and continuous use, the sludge is formed in the oil which can contaminate the oil. Hence to keep such sludge separate from the oil in main tank, an air tight metal drum is provided, which is placed on the top of tank. This is called **conservator**.

9.4.4 Comparison of Core and Shell Type

	Core Type	Shell Type
1.	The winding encircles the core.	The core encircles most part of the winding.
2.	It has single magnetic circuit.	It has a double magnetic circuit.
3.	The core has two limbs.	The core has three limbs.
4.	The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
5.	The windings are uniformly distributed on two limbs hence natural cooling is effective.	The natural cooling does not exist as the windings are surrounded by the core.
6.	The coils can be easily removed from maintenance point of view.	The coils can not be removed easily.
7.	Preferred for low voltage transformers.	Preferred for high voltage transformers.

9.5 E.M.F. Equation of a Transformer

When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux ϕ The primary winding has N_1 number of turns. The alternating flux ϕ linking with the primary winding itself induces an e.m.f. in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

As ϕ is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity,

$$\text{Form Factor} = \frac{\text{R.M.S. value}}{\text{Average value}} = 1.11$$

$$\therefore \quad \text{R.M.S. value} = 1.11 \times \text{Average value}$$

$\therefore$ R.M.S. value of induced e.m.f. per turn

$$= 1.11 \times 4\, f\, \phi_m$$

$$= 4.44\, f\, \phi_m$$

There are N_1 number of primary turns hence the R.M.S value of induced e.m.f. of primary denoted as E_1 is,

$$E_1 = N_1 \times 4.44\, f\, \phi_m \quad \text{volts}$$

While as there are N_2 number of secondary turns the R.M.S value of induced e.m.f. of secondary denoted E_2 is,

$$E_2 = N_2 \times 4.44\, f\, \phi_m \quad \text{volts}$$

The expressions of E_1 and E_2 are called e.m.f. equations of a transformer.

Thus e.m.f. equations are,

$$\boxed{\begin{aligned} E_1 &= 4.44\, f\, \phi_m\, N_1 \quad \text{volts} \\ E_2 &= 4.44\, f\, \phi_m\, N_2 \quad \text{volts} \end{aligned}} \qquad \begin{aligned} &\ldots (1) \\ &\ldots (2) \end{aligned}$$

9.6 Ratios of a Transformer

Consider a transformer shown in Fig. 9.12 indicating various voltages and currents.

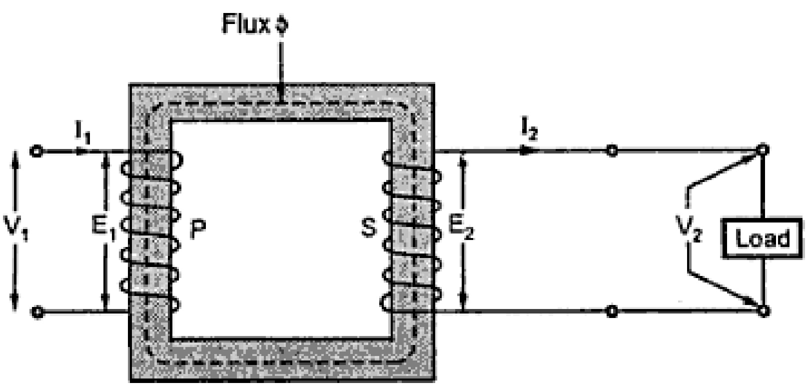


Fig. 9.12 Ratios of transformer

9.6.3 Current Ratio

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So $V_1 I_1 = \text{Input VA}$ and $V_2 I_2 = \text{Output VA}$

For an ideal transformer,

$$V_1 I_1 = V_2 I_2$$

$$\therefore \quad \boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

Key Point: *Hence the currents are in the inverse ratio of the voltage transformation ratio.*

9.6.4 Volt-Ampere Rating

When electrical power is transferred from primary winding to secondary, there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device. Now this temperature must be maintained below certain limiting value as it is always harmful from insulation point of view. As current is the main cause in producing heat, the output maximum rating is generally specified as the product of output voltage and output current i.e. $V_2 I_2$. This always indicates that when transformer is operated under this specified rating, its temperature rise will not be excessive. The copper losses depend on current and iron losses depend on voltage. These losses are independent of the load power factor $\cos\phi_2$. Hence though the output power depends on $\cos\phi_2$, the transformer losses are functions of V and I and the rating of the transformer is specified as the product of voltage and current called **VA rating.** This rating is generally expressed in kVA (kilovolt amperes rating).

$$\text{Now} \quad \frac{V_1}{V_2} = \frac{I_2}{I_1} = K$$

$$\therefore \quad V_1 I_1 = V_2 I_2$$

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

If V_1 and V_2 are the terminal voltages of primary and secondary then from specified kVA rating we can decide full load currents of primary and secondary, I_1 and I_2. This is the **safe maximum current limit** which may carry, keeping temperature rise below its limiting value.

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Key Point : *The full load primary and secondary currents indicate the safe maximum values of currents which transformer windings can carry.*

These values indicate, how much maximum load can be connected to a given transformer of a specified kVA rating.

Example 9.1 : *A single-phase, 50 Hz transformer has 80 turns on the primary winding and 400 turns on the secondary winding. The net cross-sectional area of the core is 200 cm². If the primary winding is connected to a 240V, 50 Hz supply, determine :*

i) The e.m.f. induced in the secondary winding

ii) The maximum value of the flux density in the core. **(May - 98)**

Solution : $N_1 = 80$, $f = 50$ Hz, $N_2 = 400$, $a = 200\ cm^2 = 200 \times 10^{-4}\ m^2$

$$E_1 = 240\ V$$

$$K = \frac{N_2}{N_1} = \frac{400}{80} = \frac{5}{1}$$

$$\therefore \quad K = \frac{E_2}{E_1} = \frac{E_2}{240} = \frac{5}{1}$$

$$\therefore \quad E_2 = 5 \times 240 = \mathbf{1200\ V}$$

Now $$E_1 = 4.44\ f\ \phi_m\ N_1$$

$$240 = 4.44 \times 50 \times \phi_m \times 80$$

$$\therefore \quad \phi_m = \frac{240}{4.44 \times 50 \times 80} = 0.01351\ Wb$$

$$\therefore \quad B_m = \frac{\phi_m}{a} = \frac{0.01351}{200 \times 10^{-4}} = \mathbf{0.6756\ Wb/m^2}$$

Example 9.2 : *For a single phase transformer having primary and secondary turns of 440 and 880 respectively, determine the transformer kVA rating if half load secondary current is 7.5 A and maximum value of core flux is 2.25 mWb.* **(May - 2000)**

Solution : $N_1 = 440$, $N_2 = 880$, $(I_2)_{H.L.} = 7.5$ A,

$$f_m = 2.25\ mWb, \quad E_2 = 4.44\ \phi_m\ f\ N_2$$

Assuming, $$f = 50\ Hz,$$

$$\therefore \quad E_2 = 4.44 \times 2.25 \times 10^{-3} \times 50 \times 880 = 439.56\ V$$

$$(I_2)_{F.L.} = \frac{\text{kVA rating}}{E_2}$$

And $$(I_2)_{H.L.} = \frac{1}{2}(I_2)_{F.L.}$$

Now $E_1 = 4.44 \, f \, \phi_m \, N_1$

$\therefore \quad 400 = 4.44 \times 50 \times \phi_m \times 500$

$\therefore \quad \phi_m = 3.6036 \text{ mWb}$

9.9 Transformer on Load

When the transformer is loaded, the current I_2 flows through the secondary winding. The magnitude and phase of I_2 is determined by the load. If load is inductive, I_2 lags V_2. If load is capacitive, I_2 leads V_2 while for resistive load, I_2 is in phase with V_2.

There exists a secondary m.m.f. $N_2 I_2$ due to which secondary current sets up its own flux ϕ_2. This flux opposes the main flux ϕ which is produced in the core due to magnetising component of no load current. Hence the m.m.f. $N_2 I_2$ is called **demagnetising ampere-turns**. This is shown in the Fig. 9.16 (a).

The flux ϕ_2 momentarily reduces the main flux ϕ, due to which the primary induced e.m.f. E_1 also reduces. Hence the vector difference $\overline{V}_1 - \overline{E}_1$ increases due to which **primary draws more current from the supply.** This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Fig. 9.16 (b).

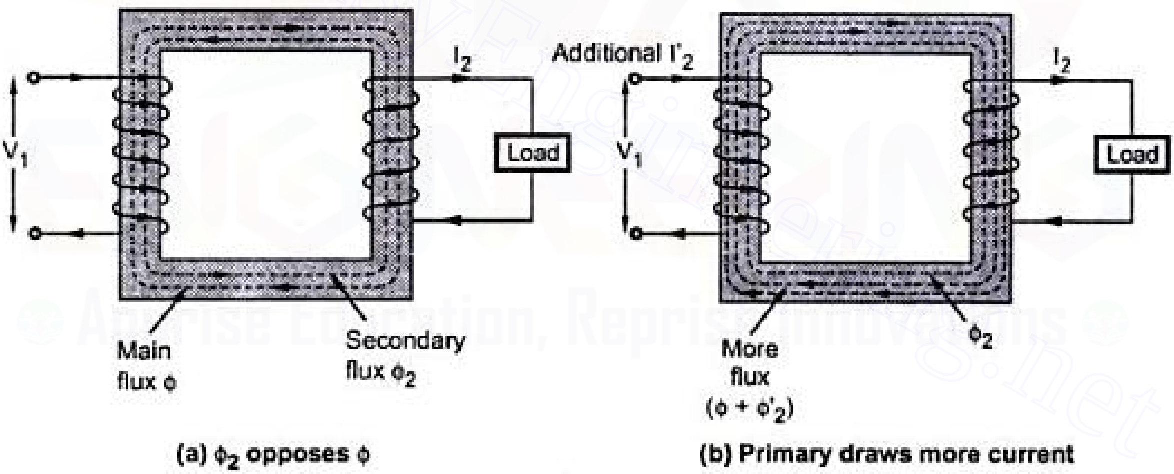


Fig. 9.16 Transformer on load

This current I_2' is in antiphase with I_2. The current I_2' sets up its own flux ϕ_2' which opposes the flux ϕ_2 and helps the main flux ϕ. This flux ϕ_2' neutralises the flux ϕ_2 produced by I_2. The m.m.f. i.e. ampere turns $N_1 I_2'$ balances the ampere turns $N_2 I_2$. Hence the net flux in the core is again maintained at constant level.

Key Point : *Thus for any load condition, no load to full load the flux in the core is practically constant.*

The load component current I_2' always neutralises the changes in the load. As practically flux in core is constant, the core loss is also constant for all the loads. Hence the transformer is called **constant flux machine.**

9.10 Effect of Winding Resistances

A practical transformer windings possess some resistances which not only cause the power losses but also the voltage drops. Let us see what is the effect of winding resistances on the performance of the transformer.

Let R_1 = Primary winding resistance in ohms

R_2 = Secondary winding resistance in ohms

Now when current I_1 flows through primary, there is voltage drop $I_1 R_1$ across the winding. The supply voltage V_1 has to supply this drop. Hence primary induced e.m.f. E_1 is the vector difference between V_1 and $I_1 R_1$.

$$\therefore \quad \overline{E}_1 = \overline{V}_1 - \overline{I_1 R_1} \qquad \text{... (1)}$$

Similarly the induced e.m.f. in secondary is E_2. When load is connected, current I_2 flows and there is voltage drop $I_2 R_2$. The e.m.f. E_2 has to supply this drop. The vector difference between E_2 and $I_2 R_2$ is available to the load as a terminal voltage V_2.

$$\therefore \quad \overline{V}_2 = \overline{E}_2 - \overline{I_2 R_2} \qquad \text{... (2)}$$

The drops $I_1 R_1$ and $I_2 R_2$ are purely resistive drops hence are always in phase with the respective currents I_1 and I_2.

9.10.1 Equivalent Resistance

The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer. The transfer of the resistances on any one side is advantageous as it makes the calculations very easy. Let us see how to transfer the resistances on any one side.

The total copper loss due to both the resistances can be obtained as,

$$\text{Total copper loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 \left[R_1 + \frac{I_2^2}{I_1^2} R_2 \right] = I_1^2 \left[R_1 + \frac{1}{K^2} R_2 \right] \qquad \text{... (3)}$$

where $\dfrac{I_2}{I_1} = \dfrac{1}{K}$ neglecting no load current.

Now the expression (3) indicates that the total copper loss can be expressed as $I_1^2 R_1 + I_1^2 \cdot \dfrac{R_2}{K^2}$. This means $\dfrac{R_2}{K^2}$ is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with I_2. This value of resistance R_2/K^2 which is the value of R_2 referred to primary is called **equivalent resistance of secondary referred to primary**. It is denoted as R_2'.

$$\therefore \quad \boxed{R_2' = \frac{R_2}{K^2}} \qquad \text{... (4)}$$

When resistances are transferred to primary, the secondary winding becomes zero resistance winding for calculation purpose. The entire copper loss occurs due to R_{1e}. Similarly when resistances are referred to secondary, the primary becomes resistanceless for calculation purpose. The entire copper loss occurs due to R_{2e}.

Important Note : *When a resistance is to be transferred from the primary to secondary, it must be multiplied by K^2. When a resistance is to be transferred from the secondary to primary, it must be divided by K^2. Remember that K is N_2/N_1.*

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

High voltage side → Low current side → High resistance side
Low voltage side → High current side → Low resistance side

Example 9.4 : *A 6600/400 V single phase transformer has primary resistance of 2.5 Ω and secondary resistance of 0.01 Ω. Calculate total equivalent resistance referred to primary and secondary.*

Solution : The given values are,

$$R_1 = 2.5\ \Omega, \quad R_2 = 0.01\ \Omega$$

$$K = \frac{400}{6600} = 0.0606$$

While finding equivalent resistance referred to primary, transfer R_2 to primary as R_2',

$$\therefore \quad R_2' = \frac{R_2}{K^2} = \frac{0.01}{(0.0606)^2} = 2.7225\ \Omega$$

$$\therefore \quad R_{1e} = R_1 + R_2' = 2.5 + 2.7225 = \mathbf{5.2225\ \Omega}$$

It can be observed that primary is high voltage hence high resistance side hence while transferring R_2 from low voltage to R_2' on high voltage its value increases.

To find total equivalent resistance referred to secondary, first calculate R_1',

$$R_1' = K^2 R_1 = (0.0606)^2 \times 2.5 = 0.00918\ \Omega$$

$$\therefore \quad R_{2e} = R_2 + R_1' = 0.01 + 0.00918 = \mathbf{0.01918\ \Omega}$$

9.11 Effect of Leakage Reactances

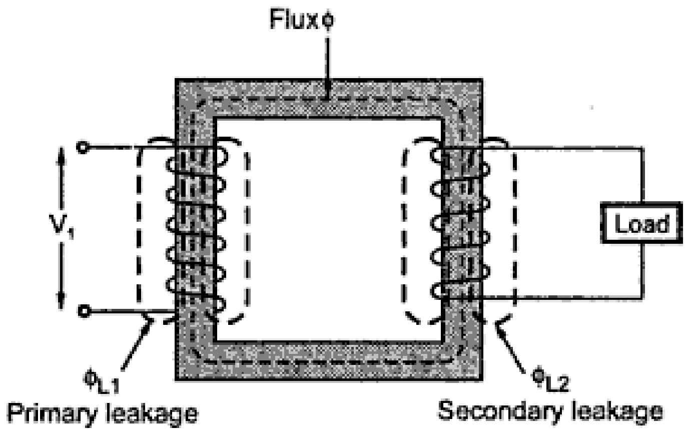


Fig. 9.19 Leakage fluxes

Uptill now it is assumed that the entire flux produced by the primary links with the secondary winding. But in practice it is not possible. Part of the primary flux as well as the secondary flux completes the path through air and links with the respecting winding only. Such a flux is called **leakage flux**. Thus there are two leakage fluxes present as shown in the Fig. 9.19.

The flux ϕ_{L1} is the primary leakage flux which is produced due to primary current I_1. It is in phase with I_1 and links with primary only.

The flux ϕ_{L2} is the secondary leakage flux which is produced due to current I_2. It is in phase with I_2 and links with the secondary winding only.

Due to leakage flux ϕ_{L1} there is self induced e.m.f. e_{L1} in primary. While due to leakage flux ϕ_{L2} there is self induced e.m.f. e_{L2} in secondary. The primary voltage V_1 has to overcome this voltage e_{L1} to produce E_1 while induced e.m.f. E_2 has to overcome e_{L2} to produce terminal voltage V_2. Thus the self induced e.m.f.s are treated as the voltage drops across the fictitious reactances placed in series with the windings. These reactances are called leakage reactances of the winding.

So $\quad X_1$ = Leakage reactance of primary winding

and $\quad X_2$ = Leakage reactance of secondary winding

The value of X_1 is such that the drop $I_1 X_1$ is nothing but the self induced e.m.f. e_{L1} due to flux ϕ_{L1}. The value of X_2 is such that the drop I_2X_2 is equal to the self induced e.m.f. e_{L2} due to flux ϕ_{L2}.

Leakage fluxes link with the respective windings only and not to both the windings. To reduce the leakage, as mentioned, in the construction both the winding's are placed on same limb rather than on separate limbs.

9.11.1 Equivalent Leakage Reactance

Similar to the resistances, the leakage reactances also can be transferred from primary to secondary or vice versa. The relation through K^2 remains same for the transfer of reactances as it is studied earlier for the resistances.

Let X_1 is leakage reactance of primary and X_2 is leakage reactance of secondary.

Then the total leakage reactance referred to primary is X_{1e} given by,

$$\boxed{X_{1e} = X_1 + X_2' \quad \text{where} \quad X_2' = \frac{X_2}{K^2}}$$

While the total leakage reactance referred to secondary is X_{2e} given by,

$$\boxed{X_{2e} = X_2 + X_1' \quad \text{where} \quad X_1' = K^2 X_1}$$

And $$K = \frac{N_2}{N_1} = \text{Transformation ratio}$$

9.12 Equivalent Impedance

The transformer primary has resistance R_1 and reactance X_1. While the transformer secondary has resistance R_2 and reactance X_2. Thus we can say that the total impedance of primary winding is Z_1 which is,

$$Z_1 = R_1 + j X_1 \, \Omega \quad \text{... (1)}$$

And the total impedance of the secondary winding is Z_2 which is ,

$$Z_2 = R_2 + j X_2 \, \Omega \quad \text{... (2)}$$

This is shown in the Fig. 9.20.

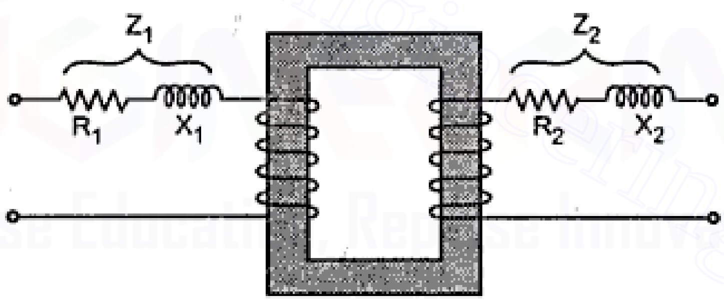


Fig. 9.20 Individual impedances

The individual magnitudes of Z_1 and Z_2 are,

$$Z_1 = \sqrt{R_1^2 + X_1^2} \quad \text{... (3)}$$

and $$Z_2 = \sqrt{R_2^2 + X_2^2} \quad \text{... (4)}$$

Similar to resistance and reactance, the impedance also can be referred to any one side.

Let Z_{1e} = Total equivalent impedance referred to primary

then $$\boxed{Z_{1e} = R_{1e} + j X_{1e}}$$

$$\therefore \quad Z_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2} \quad \text{... (5)}$$

d) $X_{2e} = X_2 + X_1' = X_2 + K^2 X_1 = 0.0075 + (0.05)^2 \times 2.6 = \mathbf{0.014\ \Omega}$

e) $Z_{1e} = R_{1e} + j\,X_{1e} = 3.55 + j\,5.6\ \Omega$

$\therefore \quad |Z_{1e}| = \sqrt{3.55^2 + 5.6^2} = \mathbf{6.6304\ \Omega}$

f) $Z_{2e} = R_{2e} + j\,X_{2e} = 0.00887 + j\,0.014\ \Omega$

$\therefore \quad |Z_{2e}| = \sqrt{(0.00887)^2 + (0.014)^2} = \mathbf{0.01657\ \Omega}$

g) To find full load copper loss, calculate full load current.

$$(I_1)\text{ F.L.} = \frac{\text{kVA} \times 1000}{V_1} = \frac{25 \times 1000}{2200} = 11.3636\text{ A}$$

$\therefore$ Total copper loss $= [(I_1)\text{ F.L.}]^2 R_{1e} = (11.3636)^2 \times 3.55 = 458.4194$ W

This can be cross checked as,

$$(I_2)\text{F.L.} = \frac{\text{kVA} \times 1000}{V_2} = \frac{25 \times 1000}{110} = 227.272\text{ A}$$

$$\begin{aligned}\text{Total copper loss} &= I_1^2 R_1 + I_2^2 R_2 \\ &= (11.3636)^2 \times 1.75 + (227.272)^2 \times 0.0045 \\ &= 225.98 + 232.4365 = \mathbf{458.419\ W}\end{aligned}$$

9.13 Voltage Regulation of Transformer

Because of the voltage drop across the primary and secondary impedances it is observed that the secondary terminal voltage drops from its no load value (E_2) to load value (V_2) as load and load current increases.

The **regulation** is defined as change in the magnitude of the secondary terminal voltage, when full load i.e. rated load of specified power factor supplied at rated voltage is reduced to no load, with primary voltage maintained constant expressed as the percentage of the rated terminal voltage.

Let E_2 = Secondary terminal voltage on no load

V_2 = Secondary terminal voltage on given load

then mathematically voltage regulation at given load can be expressed as,

$$\boxed{\%\text{ voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100}$$

The ratio ($E_2 - V_2$ / V_2) is called **per unit regulation.**

The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of the power factor of the load. If V_2 is determined for full load and specified power factor condition the regulation is called full load regulation.

9.14 Losses in a Transformer

In a transformer, there exists two types of losses.

i) The core gets subjected to an alternating flux, causing **core losses.**

ii) The windings carry currents when transformer is loaded, causing **copper losses.**

9.14.1 Core or Iron Losses

Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

It is given by, $\boxed{\text{Hysteresis loss} = K_h B_m^{1.67} f\, v \text{ watts}}$

where K_h = hysteresis constant depends on material

B_m = maximum flux density

f = frequency.

v = volume of the core

The induced e.m.f. in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses. The eddy current loss is given by,

$$\boxed{\text{Eddy current loss} = K_e B_m^2 f^2 t^2 \text{ watts/unit volume}}$$

where K_e = eddy current constant

t = thickness of the core

As seen earlier, the flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant. Hence the flux density B_m in the core and hence both hysteresis and eddy current losses are constants at all the loads. Hence the core or iron losses are also called **constant losses.** The iron losses are denoted as P_i.

The iron losses are minimised by using high grade core material like silicon steel having very low hysteresis loop and by manufacturing the core in the form of laminations.

9.14.2 Copper Losses

The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\text{Total Cu loss} = I_1^2 R_1 + I_2^2 R_2$$

$$= I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1')$$

$$= I_1^2 R_{1e} = I_2^2 R_{2e}$$

$$\therefore \qquad \% \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 \, R_{2e}} \times 100$$

This is full load percentage efficiency with,

$$I_2 = \text{Full load secondary current}$$

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

Let n = Fraction by which load is less than full load = $\dfrac{\text{Actual load}}{\text{Full load}}$

For example if transformer is subjected to half load then,

$$n = \frac{\text{Half load}}{\text{Full load}} = \frac{(1/2)}{1} = 0.5$$

When load changes, the load current changes by same proportion.

$$\therefore \qquad \text{New } I_2 = n\,(I_2)\text{F.L.}$$

Similarly the output $V_2 \, I_2 \cos \phi_2$ also reduces by the same fraction. Thus fraction of VA rating is available at the output.

Similarly as copper losses are proportional to square of current then,

$$\text{New } P_{cu} = n^2 \, (P_{cu})\text{F.L.}$$

So copper losses get reduced by n^2.

In general **for fractional load** the efficiency is given by,

$$\% \eta = \frac{n\,(\text{VA rating}) \cos\phi}{n\,(\text{VA rating}) \cos\phi + P_i + n^2 \, (P_{cu})\text{F.L.}} \times 100$$

Where n = Fraction by which load is less than full load

Key Point : *For all types of load power factors lagging, leading and unity the efficiency expression does not change.*

Example 9.7 : *3300 / 110 V, 50 Hz, 60 kVA single phase transformer has iron losses of 600 watts. Primary and secondary winding resistances are 3.3 ohms and 0.011 ohms respectively. Determine the efficiency of the transformer on full load at 0.8 lag p.f. load.*

(May-2000)

Solution : $V_1 = 3300$ V, $V_2 = 110$ V, kVA = 60, f = 50 Hz, $R_1 = 3.3\ \Omega$,

$R_2 = 0.011\ \Omega$, $P_i = 600$ W, $\cos\phi = 0.8$ lag

$$(I_1)_{\text{F.L.}} = \frac{\text{kVA}}{V_1} = \frac{60 \times 10^3}{3300} = 18.1818 \text{ A}$$

$$\text{Primary Cu loss} = I_1^2 R_1 = (18.1818)^2 \times 3.3 = 1090.90 \text{ W}$$

$$(I_2)_{F.L.} = \frac{kVA}{V_2} = \frac{60 \times 10^3}{110} = 545.4545 \text{ A}$$

$$\text{Secondary Cu loss} = I_2^2 R_2 = (545.4545)^2 \times 0.011 = 3272.7272 \text{ W}$$

$$\therefore \quad \text{Total Cu loss} = 1090.90 + 3272.7272 = 4363.6272 \text{ W}$$

$$\therefore \quad P_{cu} = 4363.6272 \text{ W on full load}$$

$$\therefore \quad \% \eta = \frac{kVA \times \cos\phi}{kVA \cos\phi + P_i + P_{cu}} \times 100$$

$$= \frac{60 \times 10^3 \times 0.8}{60 \times 10^3 \times 0.8 + 600 + 4363.6272} \times 100$$

$$= \mathbf{90.6282\ \%}$$

Example 9.8 : *A 4 kVA, 200/400 V, 50 Hz, single phase transformer has equivalent resistance referred to primary as 0.15 Ω. Calculate,*

i) The total copper losses on full load

ii) The efficiency while supplying full load at 0.9 p.f. lagging

iii) The efficiency while supplying half load at 0.8 p.f. leading.

Assume total iron losses equal to 60 W.

Solution : $V_1 = 200$ V, $V_2 = 400$ V, $S = 4$ kVA, $R_{1e} = 0.15\ \Omega$, $P_i = 60$ W

$$K = \frac{400}{200} = 2$$

$$\therefore \quad R_{2e} = K^2 R_{1e} = (2)^2 \times 0.15 = 0.6\ \Omega$$

$$(I_2) \text{ F.L.} = \frac{kVA}{V_2} = \frac{4 \times 10^3}{400} = 10 \text{ A}$$

(i) Total copper losses on full load,

$$(P_{cu}) \text{ F.L.} = [(I_2) \text{ F.L.}]^2 R_{2e} = (10)^2 \times 0.6 = \mathbf{60\ W}$$

(ii) $\cos\phi = 0.9$ lagging and full load

$$\therefore \quad \% \eta = \frac{\text{VA rating} \cos\phi}{\text{VA rating} \cos\phi + P_i + (P_{cu})\text{F.L.}} \times 100$$

$$\therefore \quad \eta = \frac{4 \times 10^3 \times 0.9}{4 \times 10^3 \times 0.9 + 60 + 60} \times 100 = \mathbf{96.77\%}$$

(iii) $\cos\phi = 0.8$ leading, half load

As half load, $n = 0.5$

$$(P_{cu})H.L. = n^2 \times (P_{cu})F.L. = (0.5)^2 \times 60 = \mathbf{15\ W}$$

$$\therefore \quad \%\eta = \frac{n \times (\text{VA rating}) \cos\phi}{n \times (\text{VA rating}) \cos\phi + P_i + (P_{cu})\,H.L.} \times 100$$

$$= \frac{0.5 \times 4 \times 10^3 \times 0.8}{0.5 \times 4 \times 10^3 \times 0.8 + 60 + 15} \times 100 = \mathbf{95.52\%}$$

9.16 Condition for Maximum Efficiency

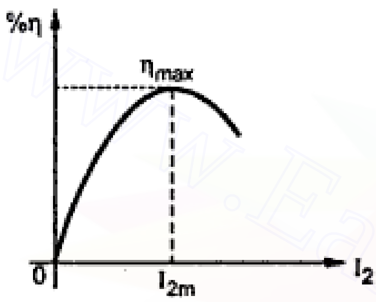


Fig. 9.23

When a transformer works on a constant input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing. The graph of efficiency against load current I_2 is shown in the Fig. 9.23.

The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_{max}.

Let us determine,

1. Condition for maximum efficiency.
2. Load current at which η_{max} occurs.
3. kVA supplied at maximum efficiency.

The efficiency is a function of load i.e. load current I_2 assuming $\cos\phi_2$ constant. The secondary terminal voltage V_2 is also assumed constant. So for maximum efficiency,

$$\frac{d\eta}{dI_2} = 0$$

Now
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}$$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) \frac{d}{dI_2}(V_2 I_2 \cos\phi_2)$$

$$-(V_2 I_2 \cos\phi_2) \cdot \frac{d}{dI_2}(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) = 0$$

$$\therefore \text{ Saving in copper } = K \times \text{Weight of copper in two winding transformer} \quad \text{... Step down}$$

In step up autotransformer this expression becomes,

$$\text{Saving in copper } = \frac{1}{K} \times \text{Weight of copper in two winding transformer} \quad \text{... Step up}$$

Key Point : *As transformation ratio K approaches to unity, greater is the saving in copper.*

9.18.2 Advantages of Autotransformer

1) Copper required is very less.
2) The efficiency is higher compared to two winding transformer.
3) The size and hence cost is less compared to two winding transformer.
4) The resistance and leakage reactance is less compared to two winding transformer.
5) The copper losses I^2R, are less.
6) Due to less resistance and leakage reactance, the voltage regulation is superior than the two winding transformer.

9.18.3 Limitations of Autotransformer

1) Low impedance hence high short circuit currents for short circuits on secondary side.
2) If a section of winding common to primary and secondary is opened, full primary voltage appears across the secondary resulting in higher voltage on secondary and danger of accidents.
3) No electrical separation between primary and secondary which is risky in case of high voltage levels.

9.18.4 Applications of Autotransformer

1) For interconnecting systems which are operating roughly at same voltage.
2) For starting rotating machines like induction motors, synchronous motors.
3) To give a small boost to a distribution cable to correct for the voltage drop.
4) As a furnace transformer for getting required supply voltage.
5) As a variac, to vary the voltage to the load, smoothly from zero to the rated value. Such variacs are commonly used for dimming the lights in cinema halls. Hence the variacs are also called dimmerstats. The principle of dimmerstat is shown in the Fig. 9.27.

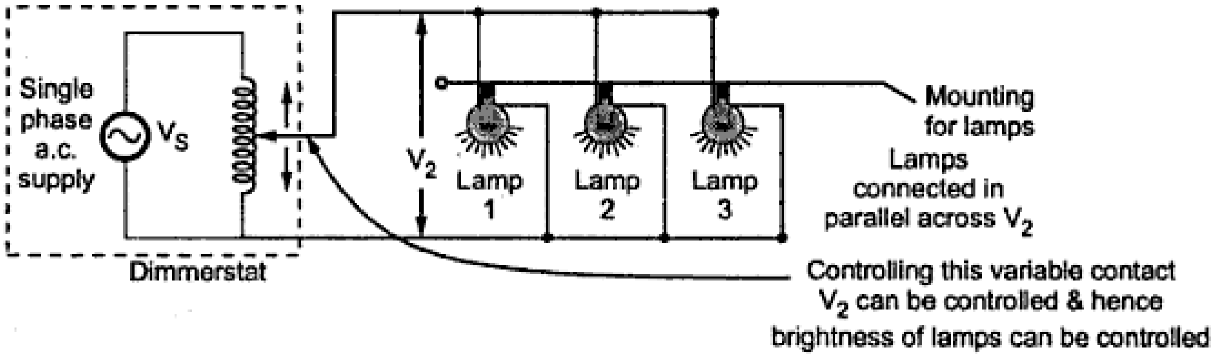


Fig. 9.27 Autotransformer as dimmerstat

Examples with Solutions

➠ **Example 9.11 :** *A 250 kVA, single phase transformer has 98.135 % efficiency at full load and 0.8 lagging p.f. The efficiency at half load and 0.8 lagging p.f. is 97.751 %. Calculate the iron loss and full load copper loss.*

Solution : a) The output power at full load

$$= 250\times10^3\times0.8 = 200\times10^3 \text{ watt}$$

$$\text{The input power at full load,} = \frac{200\times10^3}{0.98135}$$

$$\text{The total loss} = \text{Input} - \text{Output} = \frac{200\times10^3}{0.98135} - 200\times10^3$$

$$P_i + P_{cu} = 3800.88 \text{ watt} \qquad \text{...(i)}$$

Where P_i = Iron loss and P_{cu} = Full load copper loss

The power output at half load = $125\times10^3\times0.8 = 100\times10^3$

$$\text{The power input at half load} = \frac{100\times10^3}{0.97751}$$

$$\text{Total loss} = \frac{100\times10^3}{0.97751} - 100\times10^3$$

$$P_i + (0.5)^2 P_c = 2300.74 \quad \text{i.e.} \quad P_i + 0.25\, P_c = 2300.74 \qquad \text{...(ii)}$$

From equations (i) and (ii),

$$P_i = \mathbf{2000.18 \text{ watt}}$$

$$P_{cu} = \mathbf{1800.69 \text{ watt}} \qquad \text{... full load}$$

Example 9.12 : *A single phase transformer has 500 turns on primary and 1000 turns on secondary. The voltage per turn in the primary winding is 0.2 volts. Calculate*

i) Voltage induced in the primary winding.

ii) Voltage induced in the secondary winding.

iii) The maximum value of the flux density if the cross-sectional area of the core is 200 cm².

iv) kVA rating of the transformer if the current in primary at full load is 10 A, the frequency is 50 Hz.

Solution : The data given is,

$$N_1 = 500, \quad N_2 = 1000, \quad f = 50 \text{ Hz}, \quad A = 200\times10^{-4} \text{ m}^2$$

$$\frac{\text{Volt}}{\text{Turn}} = \left(\frac{E_1}{N_1}\right) = 0.2 \text{ volts}$$

i) Voltage induced in the primary

$$E_1 = \frac{\text{Volt}}{\text{Turn}}\times N_1 = 0.2\times 500 = \mathbf{100\ volts}$$

ii) Voltage induced in the secondary

$$E_2 = \frac{N_2}{N_1}\times E_1 = \frac{100}{500}\times 1000 = \mathbf{200\ volts}$$

iii) The maximum value of the flux in the core

$$\phi_m = \frac{E_1}{4.44\times f\times N_1} = \frac{100}{4.44\times50\times500} = 9.009\times10^{-4} \text{ Wb}$$

The maximum value of flux density B_m can be calculated as

$$B_m = \frac{\phi_m}{A} = \frac{9.009\times10^{-4}}{200\times10^{-4}} = \mathbf{0.045\ Wb/m^2} \text{ or tesla}$$

iv) kVA rating of the transformer $= V_1\times I_1\times 10^{-3} = V_2\times I_2\times 10^{-3}$

$$= 100\times10\times10^{-3} = \mathbf{1\ kVA}$$

Example 9.13 : *A 500 kVA transformer has iron losses of 2 kW and full load copper losses of 5 kW Calculate the efficiency at 75 % of full load and unity power factor.*

(Dec.-2007)

Solution : S = 500 kVA, P_{cu} (F.L.) = 5 kW, P_i = 2 kW

To find η at 75 % of full load, cos ϕ = 1

$\therefore \quad n = 75 \text{ \% of full load} = 0.75$

$$\therefore \quad \% \eta = \frac{n \times VA \cos\phi}{n \times VA \cos\phi + P_i + n^2 P_{cu}(F.L.)} \times 100$$

... New $P_{cu} = n^2 \times P_{cu}$ (F.L.)

$$= \frac{0.75 \times 500 \times 10^3 \times 1}{0.75 \times 500 \times 10^3 \times 1 + 2 \times 10^3 + (0.75)^2 \times 5 \times 10^3} \times 100 = \mathbf{98.7329\ \%}$$

Example 9.14 : *A 200 kVA, single phase transformer has primary voltage of 2000 V and secondary voltage 500 volts. The supply frequency is 50 Hz. The total effective resistance and reactance referred to the primary are 0.5 Ω and 2 Ω respectively. Calculate the voltage regulation of the transformer at full load unity p.f.*

Solution : S = 200 kVA, V_1 = 2000 V, V_2 = 500 V, f = 50 Hz, R_{1e} = 0.5 Ω , X_{1e} = 2 Ω

The full load current on primary side is,

$$I_1 = \frac{kVA \times 10^3}{V_1} = \frac{200 \times 10^3}{2000} = \mathbf{100\ A}$$

The voltage regulation at unity p.f. is,

$$\% \text{ Reg} = \frac{I_1 R_{1e} \cos\phi + I_1 X_{1e} \sin\phi}{V_1} \times 100$$

$$= \frac{100 \times 0.5 \times 1 + 100 \times 2 \times 0}{2000} \times 100 = \mathbf{2.5\ \%}$$

Example 9.15 : *The iron loss of 100 kVA, 1000 V/250 V, single phase 50 Hz transformer is 1000. The copper loss when primary carries current of 50 A is 500 watt. Calculate :*

i) Area of cross-section of limb if working flux density is 0.9 T and primary has 1000 turn ii) Primary and secondary currents. iii) Efficiency at full-load and 0.8 power factor.

(Dec.-97, Dec. - 2006)

Solution : Iron loss P_i = 1000 W, V_1 = 1000 V, V_2 = 250 V

$$K = \frac{V_2}{V_1} = \frac{250}{1000} = \frac{1}{4}$$

Copper loss $\quad P_{cu} = 500 \text{ W at } I_1 = 50 \text{ A}$

i) $\quad V_1 = 4.44 f \phi_m N_1$

$\therefore \quad 1000 = 4.44 \times 50 \times \phi_m \times 1000$

$\therefore \quad \phi_m = 4.5045 \times 10^{-3} \text{ Wb and } B_m = 0.9 \text{ T}$

Now $B_m = \dfrac{\phi_m}{a}$

$\therefore \quad a = \dfrac{\phi_m}{B_m} = \dfrac{4.5045 \times 10^{-3}}{0.9} = 5.005 \times 10^{-3}\ m^2 = 50.05\ cm^2$

ii) $I_1 = \dfrac{kVA \times 10^{+3}}{V_1} = \dfrac{100 \times 10^3}{1000} = \mathbf{100\ A}$

$I_2 = \dfrac{kVA \times 10^3}{V_2} = \dfrac{100 \times 10^3}{250} = \mathbf{400\ A}$

iii) $P_i = 1000\ W$

$P_{cu} \propto I_1^2$

$\therefore \quad \dfrac{P_{cu1}}{P_{cu2}} = \left(\dfrac{I_1}{I_2}\right)^2$

$P_{cu1} = 500\ W$ at $I_1 = 50\ A$

But on full load $I_2 = 100\ A$

$\therefore \quad \dfrac{500}{P_{cu2}} = \left(\dfrac{50}{100}\right)^2$

$\therefore \quad P_{cu2} = 2000\ W$ on full load

$\therefore \quad \%\eta = \dfrac{kVA \cos\phi}{kVA \cos\phi + P_i + P_{cu}} \times 100$

Now $\cos\phi = 0.8$ given

$\therefore \quad \%\eta = \dfrac{100 \times 10^3 \times 0.8}{100 \times 10^3 \times 0.8 + 1000 + 2000} \times 100$

$= \mathbf{96.38\ \%}$ ... full load efficiency

Example 9.16 : *A single phase 100 kVA, 3.3 kV/230 V, 50 Hz transformer has 89.5 % efficiency at 0.85 lag p.f. both at full load and also at half load. Determine the efficiency of the transformer at 75% load and 0.9 lead p.f.* **(Dec. - 98)**

Solution : The efficiency is given by,

$$\%\eta = \frac{n\ kVA \cos\phi}{n\ kVA \cos\phi + P_i + P_{cu}}$$

n = Fraction of load

P_i = Iron loss

P_{cu} = Full load copper loss

For n = 1 and n = 0.5 with cos ϕ = 0.85 the efficiency is 89.5 %.

$$\therefore \quad 0.895 = \frac{1\times100\times10^3\times0.85}{1\times100\times10^3\times0.85\times P_i + P_{cu}}$$

$$\therefore \quad P_i + P_{cu} = 9972.067 \quad \ldots (1)$$

$$\text{and} \quad 0.895 = \frac{0.5\times100\times10^3\times0.85}{0.5\times100\times10^3\times0.85+P_i+\frac{P_{cu}}{4}}$$

As for half load, $P_{cu} \propto I^2$

$$\therefore \quad \frac{P_{cu\,F.L.}}{P_{cu\,H.L.}} = \left(\frac{I_{F.L.}}{I_{H.L.}}\right)^2 = 4$$

$$\therefore \quad P_{cu} = \frac{P_{cu}}{4} \quad \text{at half load}$$

$$P_i + \frac{P_{cu}}{4} = 4986.033 \quad \ldots (2)$$

Subtracting equation (2) from equation (1) we get,

$$P_{cu} = 6648.044 \text{ W}$$

$$P_i = 3324.022 \text{ W}$$

Now for 75% load, n = 0.75

$$\therefore \quad \text{New } P_{cu}' = P_{cu} \times (0.75)^2 \text{ at 75 \% load}$$

$$\therefore \quad \text{New } P_{cu} = 6648.044 \times (0.75)^2 = 3739.52 \text{ W}$$

$$\cos\phi = 0.9 \text{ leading}$$

$$\therefore \quad \%\eta = \frac{0.75\times100\times10^3\times0.9}{0.75\times100\times10^3\times0.9+3324.022+3739.52}\times100 = 90.5268 \%$$

Example 9.17 : *Observation table and some of the calculated values of a direct loading test on 4 kVA, 200 / 400 V, 50 Hz, 1-ph, transformer are given below : -*

Input side					Output side						
Obs. No.	V_1 volts	I_1 amps	W_1 watts	p.f.	V_2 volts	I_2 amps	W_2 watts	Losses Constant/ Variable watts		% efficiency	% Regulation
1	400	10.0	-	-	190	20	3040	64	-	92	-
2	400	7.82	-	-	-	12.9	2000	64	-	-	3 %

Fill up the blank space (marked–) with appropriate values in the following form :-

Obs. No.	Quantity	Formula	Substitution	Value (unit)	Reason (if any)

(May-2001)

Solution : For obs. no. 1, $\% \eta = \frac{W_2}{W_1} \times 100$

$$\therefore \quad 92 = \frac{W_2}{W_1} \times 100$$

$$\therefore \quad W_1 = 3304.348 \text{ W}$$

Now $\quad W_1 = V_1 I_1 \cos \phi_1$

$$\therefore \quad \cos \phi_1 = \frac{3304.348}{400 \times 10} = 0.826 \text{ lag}$$

No load voltage = 200 V

$$\therefore \quad \% R = \frac{\text{No load} - \text{Load voltage}}{\text{No Load}} \times 100 = \frac{200-190}{200} \times 100 = 5\%$$

$$\text{Total losses} = W_1 - W_2 = 3304.348 - 3040$$

$$= 264.348 \text{ W} = \text{Constant} + \text{Variable}$$

$$\therefore \quad \text{Variable} = 264.348 - 64 = 200.348 \text{ W}$$

For obs. no. 2, $= \frac{\text{no load} - \text{Load voltage}}{\text{no Load}} \times 100$

$$\therefore \quad 0.03 = \frac{200 - \text{Load voltage}}{200}$$

$$\therefore \quad V_2 = 194 \text{ V}$$

Now for $\quad I_2 = 20$ A, Copper loss = Variable loss

$$= 200.348 \text{ W}$$

and $\quad I_{2 \text{ F.L.}} = \frac{4 \times 10^3}{200} = 20 \text{ A}$

$$\therefore \quad (P_{cu})_{\text{F.L.}} = 200.348 \text{ W}$$

Now $\quad$ New $I_2 = 12.9$ A, $\quad$ and $P_{cu} \propto I^2$

$$\therefore \quad \frac{(P_{cu}) \text{ F.L.}}{(P_{cu}) \text{ new}} = \left(\frac{I_{2 \text{ F.L.}}}{I_{2 \text{ new}}} \right)^2$$

$$\therefore \quad \frac{200.348}{(P_{cu}) \text{ new}} = \left(\frac{20}{12.9} \right)^2$$

Solution :

$V_1 = 6600$ V, $V_2 = 220$ V, $N_1 = 1500$, $B_m = 1.2$ T

i) $$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{i.e.} \quad \frac{6600}{220} = \frac{1500}{N_2}$$

$$\therefore \quad N_2 = 50$$

ii) $$V_1 = 4.44 \ \phi_m \ f N_1 \quad \text{i.e.} \quad 6600 = 4.44 \times \phi_m \times 50 \times 1500$$

$$\therefore \quad \phi_m = 19.8198 \ \text{mWb}$$

Now $$\phi_m = B_m \times a$$

$$\therefore \quad 19.8198 \times 10^{-3} = 1.2 \times a$$

$$\therefore \quad a = 16.5165 \times 10^{-3} \ m^2$$

Example 9.21 : *The no-load and full-load unity power factor readings of direct loading test on single phase transformer are as given below. Find its efficiency and regulation at full load, unity p.f.* **(May - 2004)**

No load test	*Primary side*			*Secondary side*	
	V_1	I_1	W_1	V_2	I_2
	220 V	*1.65 A*	*75 W*	*110 V*	*00 A*
Full load unity p.f. test	*220 V*	*18 A*	*3700 W*	*102.5 V*	*35 A*

Solution : From given table,

On full load, $$W_1 = 3700 \text{ W}$$

and $$W_2 = V_2 \ I_2 \cos\phi = 102.5 \times 35 \times 1 \qquad \ldots \cos\phi = 1$$

$$= 3587.5 \text{ W}$$

$$\therefore \quad \%\eta = \frac{W_2}{W_1} \times 100 = 96.95\%$$

On no load, $$E_2 = 110 \text{ V}$$

$$\% R = \frac{E_2 - V_2}{E_2} \times 100 = \frac{110 - 102.5}{110} \times 100 = +6.8181\%$$

iv) $n = 0.75$ and $\cos\phi = 1$

$\therefore \quad P_{cu}(\text{new}) = n^2 P_{cu}(\text{F.L.}) = (0.75)^2 \times 1024 = 576 \text{ W}$

$$\therefore \quad \%\eta = \frac{n \text{ kVA} \cos\phi}{n \text{ kVA} \cos\phi + P_i + P_{cu}(\text{new})} \times 100$$

$$= \frac{0.75 \times 80 \times 10^3 \times 1}{0.75 \times 80 \times 10^3 \times 1 + 500 + 576} \times 100 = \mathbf{98.238\ \%}$$

Example 9.23 : *A single phase transformer when connected to a lamp load gave following results :*

Sr. No.	V_1	I_1	W_1	V_2	I_2	W_2
1.	200	1.5	60	100	0	0
2.	200	12.9	2510	97	25	2425

Calculate efficiency and % Regulation of transformer at I_2 = 25 amp (second reading).

(May - 2005)

Solution : At I_2 = 25 A, from the given table,

$W_1 = 2510 \text{ W}$ and $W_2 = 2425 \text{ W}$

$$\therefore \quad \%\eta = \frac{W_2}{W_1} \times 100 = \frac{2425}{2510} \times 100 = \mathbf{96.6135\ \%}$$

Ans $\quad E_2 = V_2$ on no load = 100 V

While $\quad V_2 = 97$ V on $I_2 = 25$ A

$$\therefore \quad \%R = \frac{E_2 - V_2}{E_2} \times 100 = \frac{100 - 97}{100} \times 100 = \mathbf{3\ \%}$$

Example 9.24 : *A 50 kVA, 2200/220 V, 50 Hz transformer has an iron loss of 300 watt. The resistances of low and high voltage windings are 0.005 ohm and 0.5 ohm respectively. If the load power factor is 0.8 lagging, calculate its efficiency on full load and half load.*

(Dec.-2005)

Solution : $P_i = 300 \text{ W}, R_2 = 0.005\ \Omega, R_1 = 0.5\ \Omega, \cos\phi = 0.8$ lag

$$K = \frac{V_2}{V_1} = \frac{220}{2200} = 0.1$$

$$\therefore \quad R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 0.5 + \frac{0.005}{(0.1)^2} = 1\Omega$$

$$\therefore \quad I_{1(\text{F.L.})} = \frac{\text{kVA}}{V_1} = \frac{50 \times 103}{2200} = 22.7272 \text{ A}$$

$$\therefore \qquad P_{cu}(F.L.) = [I_{1(F.L)}]^2 \times R_{1e} = 516.5289 \text{ W}$$

i) Full load

$$\% \eta = \frac{VA \cos\phi}{VA \cos\phi + P_i + P_{cu}(F.L)} \times 100$$

$$= \frac{50 \times 10^3 \times 0.8}{50 \times 10^3 \times 0.8 + 300 + 516.5289} \times 100 = 97.999\ \% \approx 98\ \%$$

ii) Half load, n = 0.5

$$\therefore \qquad \% \eta_{HL} = \frac{n \times VA \cos\phi}{nVA \cos\phi + P_i + n^2 [P_{cu}(F.L)]} \times 100$$

$$= \frac{0.5 \times 50 \times 10^3 \times 0.8}{0.5 \times 50 \times 10^3 \times 0.8 + 300 + (0.5)^2 \times 516.5289} \times 100 = \mathbf{97.89\ \%}$$

Review Questions

1. *Explain the principle of working a single phase transformer.*
2. *Explain the construction of a single phase transformer.*
3. *Discuss the difference between core type and shell type of construction.*
4. *Derive from the first principles, the e.m.f. equation for a transformer.*
5. *State the relationships between voltages and currents on primary side and secondary side of a single phase transformer.*
6. *What is kVA rating of a transformer ?*
7. *Explain the various features of an ideal transformer.*
8. *What is the difference between ideal transformer and practical transformer ?*
9. *Draw a no load phasor diagram of a transformer and explain.*
10. *Write a note on various winding parameters of a transformer.*
11. *Define regulation, stating an expression to obtain it.*
12. *Enumerate the various losses in a transformer. How these losses can be minimised ?*
13. *What do you understand by efficiency of a transformer ?*
14. *Derive the condition for maximum efficiency.*
15. *What is an autotransformer ? State its advantages, limitations and applications.*
16. *Derive the expression for copper saving in an autotransformer.*
17. *Transformer is a constant flux machine. Explain with reasons.*
18. *A 600 kVA, single phase transformer has an efficiency of 94 % both at full load and half load at unity power factor. Determine its efficiency at 75 % of full load at 0.9 power factor .*

    (Ans. : 93.722 %)
19. *The magnetic material used for a transformer has an iron loss of 75 W at 25 Hz while 165 W at 40 Hz when flux density in the core remains same. If the transformer is connected to 60 Hz supply, what will be the new eddy current loss, hysteresis loss and iron loss ?*

    (Ans. : 270 W, 67.5 W, 337.5 W)

20. *A 5 kVA, 200/100 V, 1 ϕ, 50 Hz transformer has a rated secondary voltage at full load. When the load is removed the secondary voltage is found to be 110 V ? Determine percentage regulation.*

(Ans. : 9.09 %)

21. *In a 25 kVA transformer the iron loss and full load copper loss are 350 W and 400 W respectively. Calculate the efficiency of the transformer at i) Half load at unity p.f. and ii) 3 / 4^{th} th full load at 0.8 p.f. lagging.* **(Ans. : 96.525 %, 96.308 %)**

22. *The required no load ratio in a single-phase 50 Hz core type transformer is 6000/150 V. Find the number of turns per limb on the high and low voltage side if the flux is to be about 0.06 Wb.*

(Ans. : 225.225 turns, 5.63 turns)

23. *A single-phase 25 kVA 1000/2000 V, 50 Hz transformer has maximum efficiency of 98 % at full-load u.p.f. Determine its efficiency at,*

(a) 3/4^{th} full-load u.p.f. (b) 3/4^{th} full-load 0.8 p.f. (c) 1.25 full-load 0.9 p.f.

(Ans. : 97.918 %, 96.909 %, 97.728 %)

24. *A 2200 / 220 V , single phase transformer draws a no load current of 0.5 A and consumes 300 W on no load. Find magnetising and active components of no load current.*

(Ans. : 0.1363 A, 0.4810 A)

25. *A 50 kVA single phase transformer has 600 turns on primary and 40 turns on the secondary. The primary winding is conencted to the 2.2 kV, 50 Hz supply. Calculate :*

i) Secondary voltage on no load ii) Full load primary and secondary currents.

(Ans. : 146.67 V, 22.727 A, 340.909 A)

26. *A single phase transformer has 500 turns on primary and 250 turns on secondary winding. The maximum value of flux is 9 mWb. Calculate the primary voltage, secondary voltage, kVA rating of the transformer if full load primary current is 10 A and the supply frequency is 50 Hz.*

(Ans. : 999 V, 499.5 V, 9.99 kVA)

27. *The reading of direct load test on a transformer at unity p.f. are,*

	Primary			Secondary		
	V_1	I_1	W_1	V_2	I_2	W
No Load	230	2	20	115	0	0
Full Load	230	11	1265	110	10	-

Calculate regulation and efficiency of transformer at full load. **(Ans. : 86.95 %, 4.347 %)**

28. *A 10 kVA, 3300/240 V, single phase, 50 Hz transformer has a core area of 300 sq. cm. The permissible flux density is 1.3 T. Calculate,*

i) Number of primary turns ii) Number of secondary turns iii) Primary full load current

(Ans.: 382, 28, 3.0303 A)

A

B-H Curve and Magnetic Losses

A.1 B-H Curve or Magnetization Curve

We have already seen that magnetic field strength H is $\frac{NI}{l}$. As current in coil changes, magnetic field strength also changes. Due to this flux produced and hence the flux density also changes. So there exists a particular relationship between B and H for a material which can be shown on the graph.

Key Point: *The graph between the flux density (B) and the magnetic field strength (H) for the magnetic material is called as its magnetization curve or B-H curve.*

Let us obtain the B-H curve experimentally for a magnetic material. The arrangement required is shown in the Fig. A.1.

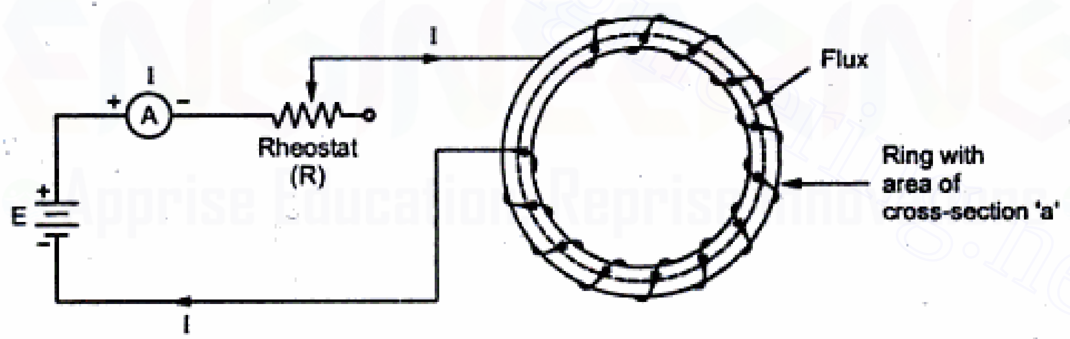


Fig. A.1 Experimental set up to obtain B-H curve

The ring specimen as a mean length of 'l' metres with a cross-sectional area of 'a' square metres. Coil is wound for 'N' turns carrying a current 'I' which can be varied by changing the variable resistance 'R' connected in series. Ammeter is connected to measure the current. For measurement of flux produced, fluxmeter can be used which is not shown in the Fig. A.1.

So H can be calculated as $\frac{NI}{l}$ while B can be calculated as $\frac{\phi}{a}$ for various values of current and plotted.

With the help of resistance R, I can be changed from zero to maximum possible value.

The B-H curve takes the following form, as shown in the Fig. A.2.

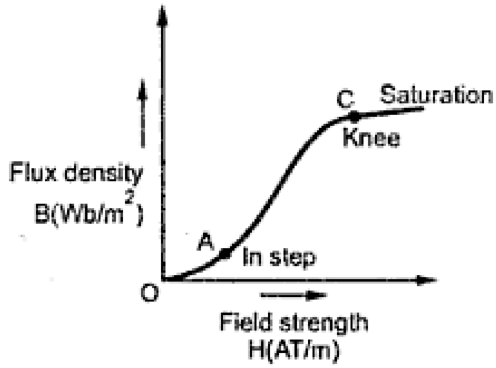


Fig. A.2 B-H curve

The graph can be analysed as below :

i) **Initial portion :** Near the origin for low values of 'H', the flux density does not increase rapidly. This is represented by curve OA. The point A is called as **instep.**

ii) **Middle portion :** In this portion as 'H' increases, the flux density B increases rapidly. This is almost straight line curve. At point 'C' it starts bending again. The point 'C' where this portion bends is called as **knee point.**

iii) **Saturation portion :** After the knee point, rate of increase in 'B' reduces drastically. Finally the curve becomes parallel to 'X' axis indicating that any increase in 'H' hereafter is not going to cause any change in 'B'. The ring is said to be **saturated** and region as **saturation region.**

We have seen already that according to molecular theory of magnetism, when all molecular magnets align themselves in the same direction due to application of H, saturation occurs.

Such curves are also called **saturation curves.**

A.1.1 B-H Curve and Permeability

From B-H curve, a curve of relative permeability μ_r and H can also be obtained.

As $$B = \mu H$$

$$= \mu_r \mu_0 H$$

$$\therefore \quad \boxed{\mu_r = \frac{B}{\mu_0 H}}$$

μ_0 is constant which is $4\pi \times 10^{-7}$. So B/H is nothing but slope of B-H curve.

Key Point: *So slope of B-H curve at various points decide the value of relative permeability at that point.*

Initially the slope is low so value of μ_r is also low. At knee point, the value of slope of B-H curve is maximum and hence μ_r is maximum. But in saturation region the value of very small. When atoms combine to form molecules such magnetic fields balance each other and hence molecule becomes magnetically neutral in nature.

However in addition to such orbital motions, electrons, revolve about their own axes. A spinning electron also sets up its own magnetic field around it axis of spin. Direction of field depends on the direction of spin. In an atom all electrons are not rotating in same direction but they are revolving in such directions that the magnetic fields produced, due to spinning neutralize each other. In ferromagnetic materials like iron, nickel etc. have number of uncompensated electron spins and hence such materials show the magnetic properties.

After the applications of external m.m.f. all electrons start revolving in same direction and hence gives strong magnetic effect. The theory of electrons is exactly similar to the theory of molecular magnets present inside the materials, discussed earlier in the chapter three.

A.3 Magnetic Hysteresis

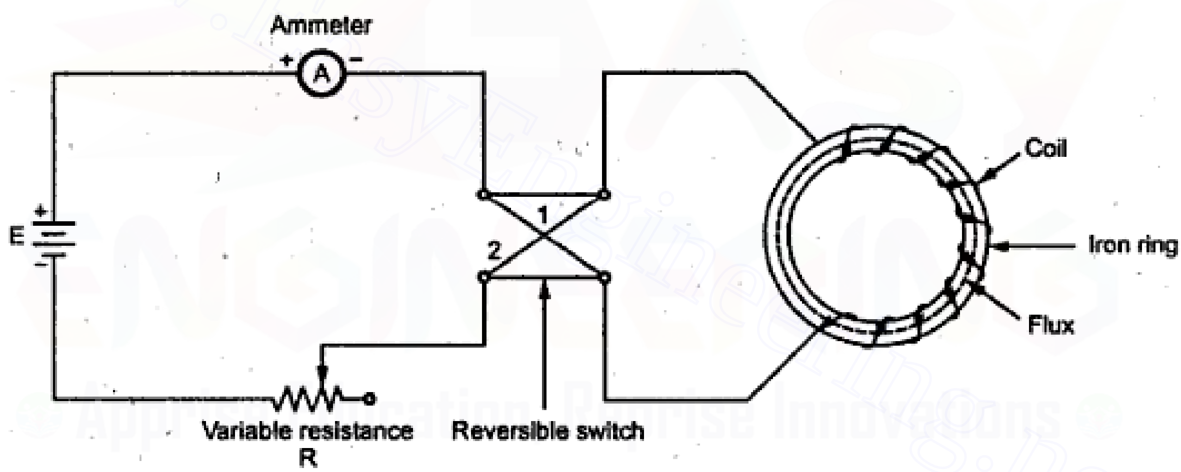


Fig. A.5 Experimental set up to obtain hysteresis loop

Earlier in chapter three we have studied B-H curve for a magnetic material. Magnetic hysteresis is extension to the same concept of B-H curve.

Instead of plotting B-H curve only for increase in current if plotted for one complete cycle of magnetization (increase in current) and demagnetization (decrease in current) then it is called **hysteresis curve or hysteresis loop.**

Consider a circuit consisting of a battery 'E', an ammeter, variable resistance R and reversible switch shown in the Fig. A.5.

A.3.1 Steps in Obtaining Hysteresis Loop

i) Initially variable resistance is kept maximum so current through the circuit is very low. The field strength $H = \frac{NI}{l}$ is also very low. So as current is increased, for low values of field strengths, flux density do not increase rapidly. But after the knee point flux density increases rapidly upto certain point. This point is called **point of saturation.** There-after any change in current do not have an effect on the flux density. This curve is nothing but the magnetization curve (B-H curve) discussed in earlier chapter. This is the initial part of hysteresis loop.

ii) After the saturation point, now current is again reduced to zero. Due to this field strength also reduces to zero. But it is observed that flux density do not trace the same curve back but falls back as compared to previous magnetization curve. This phenomenon of falling back of flux density while demagnetization cycle is called **hysteresis.** Hence due to this effect, when current becomes exactly zero, there remains some magnetism associated with a coil and hence the flux density. The core does not get completely demagnetized though current through coil becomes zero. This value of flux density when exciting current through the coil and magnetic field strength is reduced to zero is called **residual flux density or remanent flux density.** This is also called **residual magnetism** of the core. The magnitude of this residual flux or magnetism depends on the nature of the material of the core. And this property of the material is called **retentivity.**

iii) But now if it is required to demagnetize the core entirely then it is necessary to reverse the direction of the current through the coil. This is possible with the help of the intermediate switch.

> **Key Point :** *The value of magnetic field strength required to wipe out the residual flux density is called the coercive force. It is measured interms of coercivity.*

iv) If now this reversed current is increased, core will get saturated but in opposite direction. At this point flux density is maximum but with opposite direction.

v) If this current is reduced to zero, again core shows a hysteresis property and does not get fully demagnetized. It shows same value of residual magnetism but with opposite direction.

vi) If current is reversed again, then for a certain magnitude of field strength, complete demagnetization of the core is possible.

vii) And if it is increased further, then saturation in the original direction is achieved completing one cycle of magnetization and demagnetization.

The curve plotted for such one cycle turns out to be a closed loop which is called hysteresis loop. Its nature is shown in the Fig. A.6.

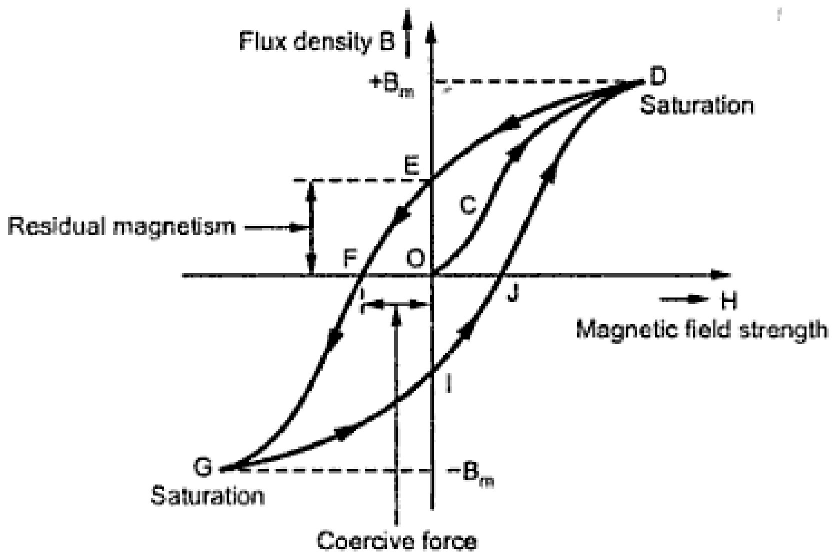


Fig. A.6 Hysteresis loop

Part of Curve	Represents What ?
O-C-D :	Region corresponding to normal magnetization curve increased form 'O' to 'I_{max}' corresponding to 'B_m'. Maximum flux density is + B_m.
D-E :	Current reduced to zero, but core cannot be completely demagnetized. O-E represents residual magnetism and residual flux density, denoted by + B_r.
E-F :	Current is reversed and increased in reversed direction to get complete demagnetization of the core. O-F represent coercive force required to completely wipe out + B_r.
F-G :	Current is increased in reversed direction till saturation in opposite direction is achieved. Maximum flux density same but with opposite direction i.e. – B_m.
G-I :	Current is reduced to zero but again flux density lags and core cannot be completely demagnetized. O-I represents residual flux density in other direction i.e. – B_r.
I-J :	Current is again reversed and increased till complete demagnetization is achieved.
J-D :	Current is again increased in original direction till saturation is reached. Corresponding flux density is again + B_m.

A.3.2 Theory Behind Hysteresis Effect

As seen from the loop 'O-C-D-E-F-G-I-J-D' shown in the Fig. A.6, the flux density B always lags behind the values of magnetic field strength H. When H is zero, corresponding flux density is + B_r. This effect is known as **hysteresis**.

This can be explained with the help of theory of molecular magnets inside a material. When a ferromagnetic material is subjected to a magnetic field strength, all the molecular magnets inside, align themselves in the direction of the applied m.m.f. If this applied force is removed or reduced some of the molecular magnets remain in the aligned state and magnetic neutralization of the material is not fully possible. So it continues to show some magnetic properties which is defined as the **residual magnetism**.

The value of the residual magnetism as said earlier depends on the quality of the magnetic material and the treatment it receives at the time of manufacturing. This property is called as **retentivity**.

Key Point : *Higher the value of retentivity, higher the value of the power of the magnetic material to retain its magnetism. For high retentivity, higher is the coercive force required.*

It can be measured in terms **coercivity** of the material.

A.4 Hysteresis Loss

According to the molecular theory of magnetism groups of molecules acts like elementary magnets, which are magnetized to saturation. This magnetism is developed because of the magnetic effect of electron spins, which are known as '**domains**'.

When the material is unmagnetized, the axis of the different domains are in various direction. Thus the resultant magnetic effect is zero.

When the external magnetomotive force is applied the axes of the various domains are oriented. The axes coincide with the direction of the magnetomotive force. Hence the resultant of individual magnetic effects is a strong magnetic field.

When a magnetic material is subjected to repeated cycles of magnetization and demagnetization, it results into disturbance in the alignment of the various domain. Now energy gets stored when magnetic field is established and energy is returned when field collapses. But due to hysteresis, all the energy is never returned though field completely collapses. This loss of energy appears as heat in the magnetic material. This is called as **hysteresis loss**. So disturbance in the alignment of the various domains causes hysteresis loss to take place. This hysteresis loss is undesirable and may cause undesirable high temperature rise due to heat produced. Due to such loss overall efficiency also reduces.

Such hysteresis loss depends on the following factors

1. The hysteresis loss is directly proportional to the area under the hysteresis curve i.e. area of the hysteresis loop.
2. It is directly proportional to frequency i.e. number of cycles of magnetization per second.

$$\therefore \qquad P = N \times a \times \frac{\Delta B}{dt} \times I$$

$\therefore$ Energy supplied in time dt = P × dt Joules

$$\Delta E = N \times a \times \Delta B \times l$$

$$\Delta E = \frac{NI}{l} \times l \times a \times \Delta B$$

$$\Delta E = H \times a \times l \times \Delta B \quad \text{Joules}$$

This is energy supplied within time dt.

$\therefore$ Energy supplied for one cycle can be obtained by integrating above expression.

$$\therefore \qquad E = a \times l \times \oint H \Delta B \quad \text{Joules}$$

Here $\oint$ is nothing but integration of the areas enclosed by strip H Δ B for one cycle i.e. the area enclosed by hysteresis loop for one cycle.

And $\qquad a \times l$ = Volume of the material.

$\therefore$ Energy supplied during one cycle in Joules = Volume × area of the hysteresis loop.

$\boxed{\therefore \textbf{ Energy supplied per unit volume = area of the hysteresis loop.}}$

When 'H' is increased from zero to maximum, energy is supplied while when 'H' is reduced energy is recovered. But all the energy is not recovered.

So net energy absorbed by material during one cycle appears as hysteresis loss.

$\therefore$ Energy per unit volume per cycle = hysteresis loss per unit volume.

$\therefore$ Hysteresis loss in Joules/cycle/m^3 = Area of the hysteresis loop.

If the hysteresis curve is drawn with scale as,

1 cm = x ampere-turns/metre of H

1 cm = y teslas for B

Then the hysteresis loss can be calculated as,

Hysteresis loss/cycle/m^3 = [x × y × Area of hysteresis curve in cm^2]

In practice the hysteresis loss is calculated with reasonable accuracy by experimentally determined mathematical expression devised by **Steinmetz**, which is as follows

$$\boxed{\text{Hysteresis loss} = K_h (B_m)^{1.6} f \times \text{volume} \quad \text{watts}}$$

where K_h = characteristic constant of the material

B_m = maximum flux density

f = frequency in cycles per second

A.4.2 Practical Use of Hysteresis Loop

As we have seen that hysteresis loss is undesirable as it produces heat which increases temperature and also reduces the efficiency.

In machines where the frequency of the magnetization and demagnetization cycle is more, such hysteresis loss is bound to be more.

So selection of the magnetic material in such machines based on the hysteresis loss. Less the hysteresis loop area for the material, less is the hysteresis loss.

> **Key point :** *So generally material with less hysteresis loop area are preferred for different machines like transformer cores, alternating current machines, telephones.*

Shapes of hysteresis loops for different materials are shown in the Fig. A.8.

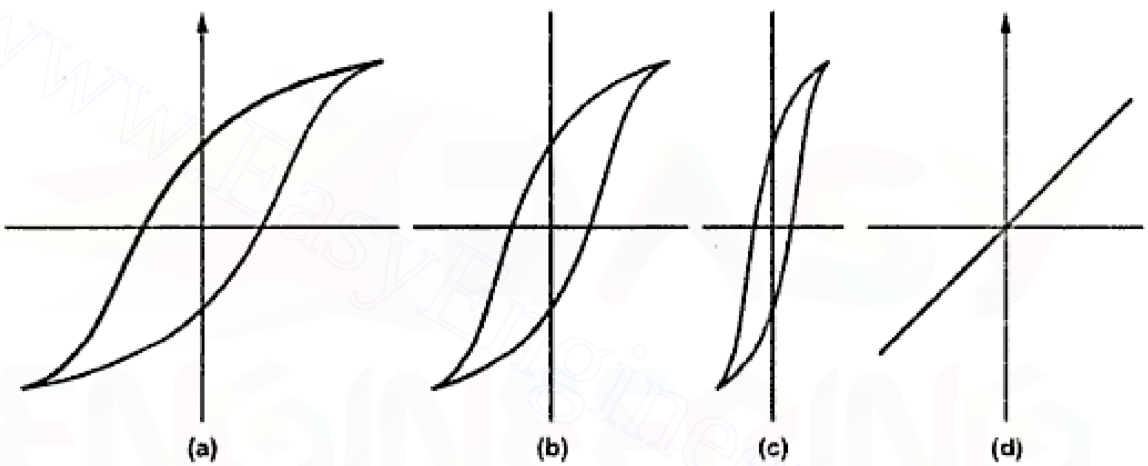


Fig. A.8 Practical importance of hysterisis loop

The Fig. A.8 (a) shows loop of hard steel, which is magnetic material.

The Fig. A.8 (b) shows loop of cast steel.

The Fig. A.8 (c) shows loop of permalloy (Alloy of nickel and iron) i.e. ferromagnetic materials.

The Fig. A.8 (d) shows loop for air or non magnetic material.

The materials iron, nickel, cobalt and some of their alloys and compounds show a strong tendency to move from weaker to stronger portion of a non-uniform magnetic field. Such substances are called **ferromagnetic materials.**

The hysteresis loss is proportional to the area of the hysteresis loop. For ferromagnetic materials the hysteresis loop area is less as shown in the Fig. A.8 (c) thus hysteresis loss is less in such materials.

In nonmagnetic materials, the hysteresis loop is straight line having zero area hence hysteresis loss is also zero in such materials.

A.5 Eddy Current Loss

Consider a coil wound on a core. If this coil carries an alternating current i.e. current whose magnitude varies with respect to time, then flux produced by it is also of alternating nature. So core is under the influence of the changing flux and under such condition according to the Faraday's law of electromagnetic induction, e.m.f. gets induced in the core. Now if core is solid, then such induced e.m.f. circulates currents through the core. Such currents in the core which are due to induced e.m.f. in the core are called as eddy currents. Due to such currents there is power loss (I^2R) in the core. Such loss is called as **eddy current loss**. This loss, similar to hysteresis loss, reduces the efficiency. For solid core with less resistance, eddy currents are always very high.

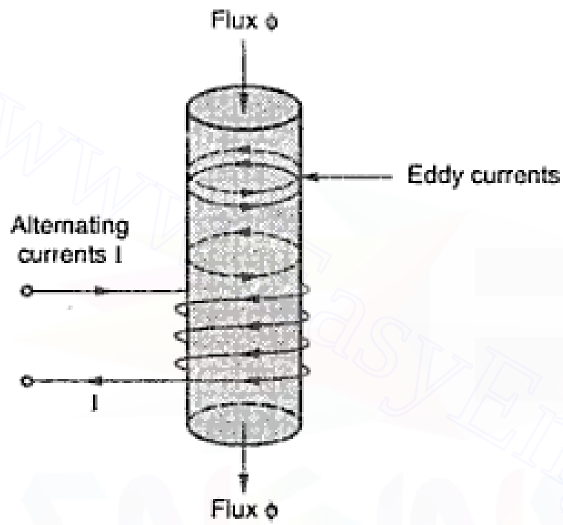


Fig. A.9 Eddy currents

The Fig. A.9 shows a core carrying the eddy currents.

Eddy current loss depends on the various factors which are

i) Nature of the material

ii) Maximum flux density

iii) Frequency

iv) Thickness of laminations used to construct to core

v) Volume of magnetic material.

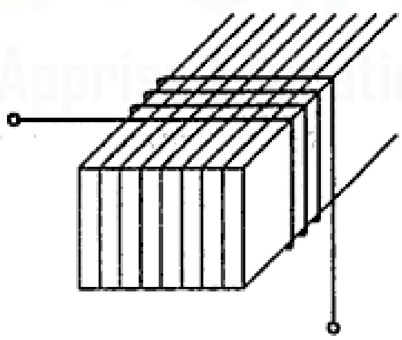


Fig. A.10 Laminated core

It has been found that loss can be considerably reduced by selecting high resistivity magnetic material like silicon. Most popular method used to reduce eddy current loss is to use laminated construction to construct the core. Core is constructed by stacking thin pieces known as laminations as shown in the Fig. A.10. The laminations are insulated from each other by thin layers of insulating material like varnish, paper, mica. This restricts the paths of eddy currents, to respective laminations only. So area through which currents flow decreases, increasing the resistance and magnitude of currents gets reduced considerably.

The loss may also be reduced by grinding the ferromagnetic material to a powder and mixing it with a binder that effectively insulates the particles one from other. This mixture is then formed under pressure into the desired shape and heat treated. Magnetic cores for use in communication equipment are frequently made by this process.

This loss is quantified by using the expression,

$$\boxed{\text{Eddy current loss} = K_e\,(B_m)^2\, f^2\, t^2 \times \text{volume} \quad \text{watts}}$$

where

K_e = A characteristic constant of material

B_m = maximum flux density

f = frequency

t = thickness of the lamination

A.6 Magnetic Loss [Core Loss or Iron Loss]

The magnetic losses can be classified as

i) Hysteresis loss

ii) Eddy current loss

The magnetic loss occurs in the core hence they are known as core losses. Since core material is generally iron or its alloy this loss is also referred as iron loss. The magnetic loss will result in the following effects,

i) It reduces the efficiency of the electrical equipment.

ii) It increases the temperature because of heating of the core.

> **Key Point :** *The hysteresis loss can be reduced by selecting good quality magnetic material.*

The area of hysteresis loop should be narrow. Silicon steel is employed for the core material so that hysteresis loss can be minimised.

> **Key Point :** *The eddy current loss can be reduced by using thin laminations for the core.*

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Basic Electrical Engineering

- Chapterwise important formulae
- Chapterwise University Questions with Answer

  Dec. - 03
  May - 04
  Dec. - 04
  May - 05
  Dec. - 05
  May - 06
  Dec. - 06
  May - 07
  Dec. - 07
  May - 08
- University Solved Papers

  Dec. - 2008
  May - 2009

Important Formulae

Chapter 1 Fundamentals of Electricity

1) $I = \frac{Q}{t}$ Amperes

2) Electrical Potential $= \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$

3) $R = \frac{\rho l}{a}$

4) $G = \frac{1}{R} = \frac{a}{\rho l} = \frac{1}{\rho}\left(\frac{a}{l}\right) = \sigma\left(\frac{a}{l}\right)$

where σ = Conductivity

5) $\alpha_1 = \frac{\text{Change in resistance per } ^\circ C}{\text{Resistance at } t_1\,^\circ C} = \frac{(R_2 - R_1 / t_2 - t_1)}{R_1}$

6) $R_t = R_0 (1 + \alpha_0 t)$

7) $R_2 = R_1 [1 + \alpha_1 \Delta t]$

8) $\alpha_1 = \frac{\alpha_2}{1 + \alpha_2 (t_1 - t_2)} = \frac{1}{\frac{1}{\alpha_2} + (t_1 - t_2)}$

9) $\alpha_2 = \frac{\alpha_1}{1 + \alpha_1 (t_2 - t_1)} = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$

10) $\alpha_t = \frac{\alpha_0}{1 + \alpha_0 (t - 0)} = \frac{\alpha_0}{1 + \alpha_0 t}$

11) $\rho_t = \rho_0 (1 + \alpha_0 t)$

12) $\rho_{t2} = \rho_{t1} [1 + \alpha_{t1} (t_2 - t_1)]$

13) $R_i = \frac{\rho}{2\pi l} \text{Log}_e \left(\frac{R_2}{R_1}\right) \Omega$

Note : If two cables are joined in series, their conductor resistances come in series but their insulation resistances are in parallel while if two cables are connected in parallel then conductor resistances are in parallel but insulation resistances are in series.

14) $1\ W = 1\ J/sec$

15) $P = T \times \omega, \quad \omega = \frac{2\pi N}{60}$ = Angular velocity in rad/sec

16) $W = VIt \quad J$

17) $P = VI \quad$ J/sec i.e. watts

18) $P = VI = I^2R = \frac{V^2}{R}$ where R = Resistance in Ω

19) Electrical energy E = Power $\times$ Time = VIt joules

20) $1\ Wh = 1\ watt \times 1\ hour = 1\ watt \times 3600\ sec = 3600$ watt-sec i.e. J

21) $1\ kWh = 1000\ Wh = 1 \times 10^3 \times 3600\ J = 3.6 \times 10^6\ J$

22) Heat energy $H = VIt = I^2 R t = \frac{V^2}{R} t$ joules

23) Sensible heat $= m\, C\, \Delta t$ joules

24) 1 calorie = 4.186 joules

25) 1 joule $= \frac{1}{4.186}$ calorie = 0.2389 calorie

26) $1\ kWh = 3.6 \times 10^6\ J$

$= 860$ kcal

27) Total heat in heating process = Sensible heat + Latent heat

where Latent heat = mL

Chapter 2 D. C. Circuits

1) Ohm's Law is, $I = \frac{V}{R}$ amperes

2) $V = I\,R$ volts

3) $\frac{V}{I}$ = Constant = R ohms

4) **For n resistances in series,** $R = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

For n = 2, $R = \frac{R_1 R_2}{R_1 + R_2}$

5) Current distribution in parallel circuit,

$$I_2 = \left[\frac{R_1}{R_1 + R_2}\right] I_T$$

$$I_1 = \left[\frac{R_2}{R_1 + R_2}\right] I_T$$

6) Applying KCL, $\sum I$ at junction O = 0

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

7) Around a closed path $\sum V = 0$

Note :

1) **Potential rise** i.e. travelling from negative to positively marked terminal, must be considered as **Positive.**
2) **Potential drop** i.e. travelling from positive to negatively marked terminal, must be considered as **Negative.**
3) While tracing a closed path, select any one direction clockwise or anticlockwise. This selection is totally independent of the directions of currents and voltages of various branches of that closed path.

Note : If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, **the loops should not be considered involving current source. The KVL equations must be written to those loops which do not include any current source.** This is because drop across current source is unknown.

Easy way to remember Delta-Star :

The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

Easy way to remember star-delta

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

Chapter 3 Magnetic Circuits

1) Coulomb's law $F = \frac{K M_1 M_2}{d^2}$

where M_1, M_2 = Pole strengths

d = Distance between poles

K = Depends on surrounding

2) 1 weber = 10 lines of force

3) $B = \frac{\phi}{a} \frac{Wb}{m^2}$ or tesla

4) $H = \frac{NI}{l}$ AT/m

5) For infinitely long conductor,

$$H = \frac{I}{2\pi d} \quad A/m$$

If there are N turns,

$$H = \frac{NI}{2\pi d} \; AT/m$$

6) For a long solenoid, $H = \frac{NI}{l}$ AT/m

7) $F = B I l$ newtons

8) $F = B I l \sin\theta$ newtons

9) Force between two parallel conductors,

$$F = 2 \times 10^{-7} I_1 I_2 \frac{l}{r} \text{ newton}$$

10) $\mu = \frac{B}{H}$

11) $B = \mu H$

12) $\mu_0 = \frac{B}{H}$ in vacuum $= 4\pi \times 10^{-7}$ H/m

13) $\mu = \mu_0 \mu_r$ H / m

14) m.m.f. = N I ampere turns

5) $L = \frac{N\phi}{I}$

6) $e = -L\frac{dI}{dt}$ volts

7) $L = \frac{N^2}{S}$ henries

8) $L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l}$ henries

9) Magnitude of mutually induced e.m.f. $e_2 = -M\frac{dI_1}{dt}$

10) $M = \frac{N_2 \phi_2}{I_1}$

11) $M = \frac{N_2 K_1 \phi_1}{I_1}$

12) $M = \frac{K_1 N_1 N_2}{S}$

13) $M = \frac{N_1 N_2 a \mu_0 \mu_r}{l}$

14) $M = \frac{K_2 N_1 N_2}{S}$

15) $K = \sqrt{K_1 K_2}$

16) $K = \frac{M}{\sqrt{L_1 L_2}}$

17) **Series aiding :**

$$L_{eq} = L_1 + L_2 + 2M$$

18) **Series opposition :**

$$L_{eq} = L_1 + L_2 - 2M$$

19) $E = \frac{1}{2} L I^2$ joules ... Energy stored in magnetic field.

20) E / unit volume $= \frac{1}{2}\frac{B^2}{\mu}$ joules / m^3

Chapter 5 Electrostatics

1) $$F = \frac{K Q_1 Q_2}{d^2} \quad \text{newtons}$$

2) $$K = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_r \varepsilon_0}$$

3) $$\varepsilon_0 = \frac{1}{36\pi\times10^9} = 8.854 \times 10^{-12} \quad \text{F/m}$$

4) Electric flux, ψ = Q coulombs (numerically)

5) $$D = \frac{\psi}{A} = \frac{Q}{4\pi r^2} \quad \text{C/m}^2$$

6) $$\delta = \frac{Q}{A} \quad \text{C/m}^2$$

7) $$E = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r d^2}$$

8) $$D = E\,\varepsilon_0\,\varepsilon_r \quad \text{C/m}^2$$

9) $$\varepsilon = \frac{D}{E} \quad \text{F/m}$$

10) $$\varepsilon_0 = \frac{D}{E} \quad \text{F/m in vacuum}$$

11) $$\varepsilon_0 = \frac{1}{36\pi\times10^9} = 8.854 \times 10^{-12} \quad \text{F/m}$$

12) $$\varepsilon = \varepsilon_r \varepsilon_0$$

13) $$\text{Electric potential } V = \frac{\text{Work done (W)}}{\text{Charge (Q)}} \quad \text{... volts}$$

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

$$V_{AB} = V_A - V_B = \frac{W_A - W_B}{q} \quad \text{volts}$$

14) **Potential at P :**

$$V_P = W = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r} \quad \text{volts}$$

11) **Average value :**

$$I_{av} = \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}$$

While $$V_{av} = \frac{V_1 + V_2 + \ldots\ldots + V_n}{n}$$

$$I_{av} = 0.637\ I_m$$

Similarly, $$V_{av} = 0.637\ V_m$$

12) Form factor, $$K_f = \frac{\text{R.M.S. value}}{\text{Average value}}$$

13) Peak factor $$K_p = \frac{\text{Maximum value}}{\text{R.M.S. value}}$$

14) $$e = E_m \sin(\omega t \pm \phi)$$

where ϕ = Phase of the alternating quantity

> 1. The phase is measured with respect to reference direction i.e. positive X-axis direction.
> 2. The phase measured in anticlockwise direction is positive while the phase measured in clockwise direction is negative.

15) Polar system, $r \angle \pm \phi$ while Rectangular system, $x \pm j\,y$

and $x = r\cos\phi$, $y = r\sin\phi$ ...(1)

While $r = \sqrt{x^2 + y^2}$, $\phi = \tan^{-1}\left(\frac{y}{x}\right)$ ... (2)

16) The polar form of an alternating quantity can be easily obtained from its equation or phase as,

If $e = E_m \sin(\omega t \pm \phi)$ then

E in polar $= E \angle \pm \phi$ where E = r.m.s. value

> **Note :** The r.m.s. vlaue of an alternating quantity exists in its polar form and not in rectangular form. Thus to find r.m.s. value of an alternating quantity express it in polar form.

17) In general, $e = E_m \cos(\omega t \pm \phi)$

then $e = E_m \sin(\omega t + 90 \pm \phi)$

$\therefore$ The phase $= 90 \pm \phi$

18) **Product and division of phasors :**

$$P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2] = [r_1 \times r_2] \angle \phi_1 + \phi_2$$

$$\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2$$

Chapter 7 Single Phase A. C. Circuits

No.	Circuit	Impedance (Z)		ϕ	p.f. $\cos\phi$	Remark
		Polar	Rectangular			
1.	Pure R	$R \angle 0° \Omega$	$R + j0 \Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90° \Omega$	$0 + j X_L \Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90° \Omega$	$0 - j X_C \Omega$	– 90°	0	Zero leading
4.	Series RL	$\lvert Z\rvert \angle +\phi° \Omega$	$R + j X_L \Omega$	$0° \angle \phi \angle 90°$	$\cos\phi$	Lagging
5.	Series RC	$\lvert Z\rvert \angle -\phi° \Omega$	$R - jX_C \Omega$	$-90° \angle \phi \angle 0°$	$\cos\phi$	Leading
6.	Series RLC	$\lvert Z\rvert \angle \pm \phi° \Omega$	$R + j X \Omega$ $X = X_L - X_C$	ϕ	$\cos\phi$	$X_L > X_C$ Lagging $X_L < X_C$ Leading $X_L = X_C$ Unity

1) **Series response :**

$$\omega_r = \frac{1}{\sqrt{LC}} \quad \text{rad/sec}$$

$$\text{Selectivity} = \frac{f_r}{\text{B. W.}} = \frac{f_2 - f_1}{\text{B. W.}}$$

$$f_r = \sqrt{f_1 f_2}$$

$$\text{B.W.} = \frac{R}{2\pi L}$$

$$\Delta f = \frac{R}{4\pi L}$$

$$f_1 = f_r - \Delta f$$

and $$f_2 = f_r + \Delta f$$

$$Q = \frac{\omega_r L}{R} \qquad \text{but } \omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{\omega_r}{\text{B. W.}} \qquad \text{as B.W.} = (\omega_2 - \omega_1) = \frac{R}{L}$$

2) **Parallel A.C. circuit :**

$$Y = G \mp jB$$

In the above expression, $G = \text{Conductance} = \frac{R}{Z^2}$

and $B = \text{Susceptance} = \frac{X}{Z^2}$

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

B is negative if inductive and B is positive if capacitive.

3) **Parallel response :**

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

where $Z_D = \frac{L}{RC}$ = Dynamic impedance

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Chapter 8 Polyphase A. C. Circuits

1) **Three phase supply :**

$$e_R = E_m \sin(\omega t)$$

$$e_Y = E_m \sin(\omega t - 120^\circ)$$

$$e_B = E_m \sin(\omega t - 240^\circ) = E_m \sin(\omega t + 120^\circ)$$

2) **Star connection :**

$I_L = I_{ph}$ **for star connection**

$V_L = \sqrt{3}\ V_{ph}$ for star connection

Thus line voltage is $\sqrt{3}$ times the phase voltage in star connection.

$$|Z_{ph}| = \frac{|V_{ph}|}{|I_{ph}|}$$

$$\phi = V_{ph} \wedge I_{ph} \neq V_L \wedge I_L$$

$$Z_{ph} = R_{ph} + j X_{Lph} = |Z_{ph}| \angle \phi \ \ \Omega$$

$$P_{ph} = V_{ph}\ I_{ph} \cos\phi$$

$$P = \sqrt{3}\ V_L I_L \cos\phi$$

For star connection, to draw phasor diagram, use

$$\boxed{\overline{V}_{RY} = \overline{V}_R - \overline{V}_Y,\quad \overline{V}_{YB} = \overline{V}_Y - \overline{V}_B \quad \text{and} \quad \overline{V}_{BR} = \overline{V}_B - \overline{V}_R}$$

3) **Delta connection :**

$$V_L = V_{ph}$$

$$I_L = \sqrt{3}\; I_{ph} \qquad \text{for delta connection}$$

Thus for delta connection, to draw phasor diagram, use

$$\boxed{\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR},\quad \overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY} \quad \text{and} \quad \overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB}}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$S = \sqrt{3}\, V_L I_L \quad \text{volt-amperes (VA) or kVA}$$

Total active power $P = \sqrt{3}\, V_L I_L \cos\phi$ watts (W) or kW

Total reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi$ reactive volt amperes (VAR) or kVAR

Chapter 9 Single Phase Transformers

1) $E_1 = 4.44\, f\, \phi_m\, N_1$ volts ... (1)

2) $E_2 = 4.44\, f\, \phi_m\, N_2$ volts ... (2)

3) $$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

$$E_2 = K\, E_1 \qquad \text{where} \quad K = \frac{N_2}{N_1}$$

4) $$\frac{E_2}{E_1} = \frac{V_2}{V_1} = K$$

5) $$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

6) $$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \qquad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

7) $\overline{I}_o = \overline{I}_m + \overline{I}_c$ ... (1)

$I_m = I_o \sin\phi_o$ ... (2)

$I_c = I_o \cos\phi_o$ ... (3)

$$I_o = \sqrt{I_m^2 + I_c^2} \quad \text{... (4)}$$

8) $W_o = V_1 I_o \cos \phi_o = V_1 I_c$ ... (5)

9) $W_o = V_1 I_o \cos \phi_o = P_i =$ Iron loss ... (6)

The load component current I_2' always neutralises the changes in the load. As practically flux in core is constant, the core loss is also constant for all the loads. Hence the transformer is called **constant flux machine.**

10) $R_2' = \dfrac{R_2}{K^2}$

11) $R_{1e} = R_1 + R_2' = R_1 + \dfrac{R_2}{K^2}$

12) $R_1' = K^2 R_1$

13) $R_{2e} = R_2 + R_1' = R_2 + K^2 R_1$

High voltage side → Low current side → High resistance side

Low voltage side → High current side → Low resistance side

14) $X_{1e} = X_1 + X_2'$ where $X_2' = \dfrac{X_2}{K^2}$

15) $X_{2e} = X_2 + X_1'$ where $X_1' = K^2 X_1$

16) $Z_{1e} = R_{1e} + j X_{1e}$

17) $Z_{2e} = R_{2e} + j X_{2e}$

18) $Z_{2e} = K^2 Z_{1e}$ and $Z_{1e} = \dfrac{Z_{2e}}{K^2}$

19) % voltage regulation $= \dfrac{E_2 - V_2}{E_2} \times 100$

20) $\% R = \dfrac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{E_2} \times 100$ ... Referring to secondary

21) $\% R = \dfrac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{E_1} \times 100$ ... Referring to primary

22) Hysteresis loss $= K_h B_m^{1.67} f v$ Watts

23) Eddy current loss $= K_e B_m^2 f^2 t^2$ Watts/unit volume

24) $P_{cu} \propto I^2 \propto (kVA)^2$

1 Fundamentals of Electricity

Q.1 *Sketch a graph showing the variation of resistance of a given metallic conductor with change in its temperature over the normal range of 0 to 100 °C. Hence, define its temperature coefficient of resistance, and state its unit.* **[Dec.-2003, 5 Marks]**

Ans. : Refer section 1.11.

Q.2 *A bucket contains 15 - litres of water at 20 °C. A 2 kW immersion heat is used to raise the temperature of water to 95 °C. The overall efficiency of the process is 90% and the specific heat capacity of water is 4187 J/kg °K. Find the time required for the process.* **[Dec.-2003, 6 Marks]**

Ans. : Refer Ex. 1.43.

Q.3 *At 0 °C, a specimen of copper wire has its resistance equal to 4 milli ohm and its temperature coefficient of resistance equal to (1/234.5) per °C. Find the values of its resistance and temperature coefficient of resistance at 70 °C.* **[Dec.-2003, 5 Marks]**

Ans. : Refer Ex. 1.44.

Q.4 *An electric pump lifts 12 m^3 of water per minute to a height of 15 m. If its overall efficiency is 60 %, find the input power. If the pump is used for 4 hours a day, find the daily cost of energy at Rs. 2.25 per unit.* **[Dec.-2003, 6 Marks]**

Ans. : Refer Ex. 1.45.

Q.5 *Enlist the aspects of maintenance and the precautions to be observed of lead-acid batteries, so that these give efficient service over a longer life.* **[Dec.-2003, 6 Marks]**

Ans. : Refer section 1.19.5.

Q.6 *Describe the methods of charging storage batteries. What are the indications which confirm that a lead-acid cell is fully charged ?* **[Dec.-2003, 6 Marks]**

Ans. : Refer section 1.24 and 1.19.4.

Q.7 *How are various materials classified as conductors, resistors and insulators ? Give two examples of each. What is the effect of temperature and moisture on the resistance of an insulating material ?* **[May-2004, 6 Marks]**

Ans. : Refer section 1.10.

Q.17 *The effective heat of 100 MW power station is 220 m. Station supplies full load for 12 hours a day. The overall efficiency of power station is 86.4%. Find the volume of water used.* **[Dec.-2004, 6 Marks]**

Ans. : Refer Ex. 1.49.

Q.18 *Compare lead acid cell and nickel cadmium cell.*

[Dec.-2004, Dec.-2005, Dec.-2006, Dec.-2007, 5 Marks]

Ans. : Refer section 1.29.

Q.19 *A single-core insulated cable of length 'L' metre has its conductor diameter 'd' metre and the thickness of insulation surrounding the conductor is 't' metre. Derive the expression for its insulation resistance, if the resistivity of the insulating material is 'ρ' ohm metre.* **[May-2005, 6 Marks]**

Ans. : Refer section 1.12.

Q.20 *An electric furnace is used to melt aluminium. Initial temperature of the solid aluminium is 32 °C and its melting point is 680 °C. Specific heat capacity of aluminium is 0.95 kJ/kg °K, and the heat required to melt 1 kg of aluminium at its melting point is 450 kJ. If the input power drawn by the furnace is 20 kW and its overall efficiency is 60 %, find the mass of aluminium melted per hour.*

[May-2005, 8 Marks]

Ans. : Refer Ex. 1.50.

Q.21 *Show how four cells, each rated 1.5 V, 0.1 A, can be connected as batteries in three different ways to obtain different voltage and current ratings. State the voltage and current ratings of each type.* **[May-2005, 2 Marks]**

Ans. : Refer Ex. 1.51.

Q.22 *At 0 °C, the resistances and their temperature coefficients of resistance of two resistors 'A' and 'B' are 80 ohm and 120 ohm, and 0.0038 per °C and 0.0018 per °C, respectively. Find the temperature-coefficient of resistance at 0 °C of their series combination.* **[May-2005, 8 Marks]**

Ans. : Refer Ex. 1.52.

Q.23 *In a thermal generating station the heat energy obtained by burning 1 kg of coal is 16,000 kJ. Find the mass of coal required to get an output electrcial energy of 1 kWh from the station, if its overall efficiency is 18 %.* **[May-2005, 4 Marks]**

Ans. : Refer Ex. 1.53.

Q.24 *Enlist the various types of storage batteries. State the application of each.*

[May-2005, 4 Marks]

Ans. : Refer section 1.19, 1.27, 1.28.

Q.25 *Obtain an expression for insulation resistance of the single core cable.* **[Dec.-2005, 5 Marks]**

Ans. : Refer section 1.12.

Q.26 *An electric furnace is used in order to melt 50 kg of tin per hour. Melting temperature of tin is 235°C and room temperature is 15°C. Latent heat of fusion for tin is 13.31 kcal/kg. Specific heat of tin is 0.055 kcal/kg° K. If the input to furnace is 5 kW find the efficiency of the furnace.* **[Dec.-2005, 6 Marks]**

Ans. : Refer Ex. 1.28

Q.27 *With usual notations prove that* $(\alpha_1 - \alpha_2) = \alpha_1 \alpha_2 (t_2 - t_1)$ **[Dec.-2005, Dec.-2007, 5 Marks; May-2008, 6 Marks]**

Ans. : Refer Ex. 1.19.

Q.28 *Determine the current flowing at the instant of switching a 100 watt lamp on 230 V supply. The ambient temperature is 25 °C. The filament temperature is 2000 °C and the resistance temperature coefficient is 0.005/°C at 0°C.* **[Dec.-2005, 6 Marks]**

Ans. : Refer example 1.23 for the procedure.

$$I = 4.25 \text{ A}$$

Q.29 *Discuss the effect of temperature on the resistance of various materials and hence define resistance temperature coefficient.* **[May-2006, 6 Marks]**

Ans. : Refer sections 1.10 and 1.11.

Q.30 *What is the function of separators in lead acid battery ? Write down chemical reactions during first charging and recharging lead acid cell.* **[May-2006, 5 Marks]**

Ans. : Refer sections 1.19.1 and 1.19.2.

Q.31 *A D.C. shunt motor, after running several hours on constant voltage of 400 V, takes field current of 1.6 A. If temperature rise is 40°C, what value of extra resistance is required in field circuit to maintain field current equal to 1.6 A. Assume motor started from cold at 20°C and* $\alpha_{20} = 0.0043/°C$. **[May-2006, 6 Marks]**

Ans. : Refer Ex. 1.55.

Q.32 *Define insulation resistance of the coil and hence discuss effect of moisture on it.* **[May-2006, 4 Marks]**

Ans. : Refer sections 1.12 and 1.12.3.

Q.33 *What is trickle charging ? Explain the methods used for charging storage batteries.* **[May-2006, 6 Marks]**

Ans. : Refer section 1.24.

Q.34 *Calculate the current required by a 1500 V D.C. locomotive when driving a total load of 100×10^3 kg at 25 km per hour up on incline of 1 in 100. Assume tractive resistance of 0.069 N/kg and efficiency of motor's gearing as 70 %.*
[May-2006, 7 Marks]

Ans. : Refer Ex. 1.54.

Q.35 *Define resistance temperature coefficient. If α_1 and α_2 are the resistance temperature coefficients of material at t_1°C and t_2°C respectively, then prove that $\frac{\alpha_1}{\alpha_2} = 1 + \alpha_1 (t_2 - t_1)$.*
[Dec.-2006, 5 Marks]

Ans. : Refer section 1.11.

Q.36 *An electrical driven pump lifts 80 m³ of water per minute through a height of 12 metres. The efficiencies of motor and pump are 70 % and 80 % respectively. Calculate,*
1) Current drawn by motor if it works on 400 V supply and
2) Energy consumption in kWh and cost of energy at the rate of Rs. 5/kWh if pump operates 5 Hrs. per day for 30 days.
[Dec.-2006, 7 Marks]

Ans. : Refer example 1.32.

Note : The cost given is Rs. 5/kWh instead of 75 paise/kWh in the example 1.32.

$\therefore$ Total cost = 16817.136×5 = Rs. **84085.68**

Q.37 *Explain the maintenance procedure for lead acid battery.*
[Dec.-2006, May-2008, 4 Marks]

Ans. : Refer section 1.19.5.

Q.38 *An immersion heater is used for heating 9 litres of water. Its resistance is 50 ohm and has efficiency of 83.6 %. How much time is required to heat water from 20 °C to 70 °C, when connected to 250 V supply. Specific heat capacity of water is 4180 J/kg-°K.*
[Dec.-2006, 6 Marks]

Ans. : Refer Ex. 1.56.

Q.39 *With neat sketch explain the construction and working of lead acid cell.*
[May-2007, 6 Marks]

Ans. : Refer section 1.19.

Q.40 *With usual notations derive the expression, $\alpha_2 = \frac{\alpha_1}{1 + \alpha_1(t_2 - t_1)}$.*
[May-2007, 5 Marks]

Ans. : Refer section 1.11.3.

Q.47 *Define resistance Temperature Coefficient (RTC). State its unit and state if it is true that RTC can have (i) Zero value, (ii) Negative value and (iii) Positive value.* **(May-2008, 6 Marks)**

Ans. : Refer section 1.11.

Q.48 *A resistance element having cross-sectional area of 10 mm² and length of 10 metre takes a current of 4 Amp from 220 V supply at temperature of 20 °C. Find out (i) the resistivity of the material and (ii) current it will take when temperature rises to 60 °C. Assume α_{20} = 0.0003 /°C.* **[May-2008, 6 Marks]**

Ans. : Refer Ex. 1.59.

Q.49 *With neat sketch explain construction and working of nickel cadmium cell.* **[May-2008, 5 Marks]**

Ans. : Refer section 1.28

Q.50 *Define insulation resistance and derive expression for it for single core cable.* **[May-2008, 5 Marks]**

Ans. : Refer section 1.12.

Q.13 *Write the Kirchhoff's voltage equations for the circuit shown in the Fig. 6 and hence find current flowing through 4 Ω resistance.* **[Dec.-2004, 6 Marks]**

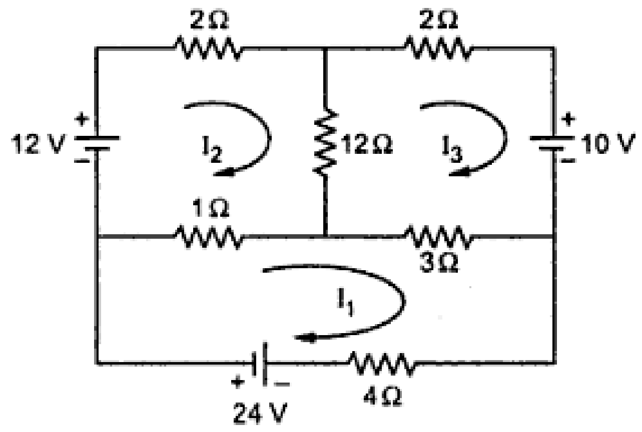

Fig. 6

Ans. : Refer Ex. 2.57.

Q.14 *For the network shown in Fig. 7 find the current flowing through 5 Ω resistance using Superposition theorem.* **[Dec.-2004, 9 Marks]**

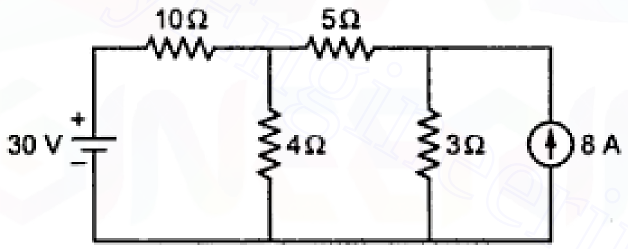

Fig. 7

Ans. : Refer Ex. 2.58.

Q.15 *For the Fig. 7 of Q. 14 find the current flowing through 5 Ω resistance using Thevenin's theorem.* **[Dec.-2004, 8 Marks]**

Ans. : Refer Ex. 2.59.

Q.16 *State and explain with a suitable example, the Superposition theorem as applied to d.c. resistive networks.* **[May-2005, 6 Marks]**

Ans. : Refer section 2.17.

3 Magnetic Circuits

Q.1 *A magnetic circuit has the mean length of flux path of 20 cm, and cross-sectional area of 1 cm^2. Relative permeability of its material is 2400. Find the m.m.f. required to produce a flux density of 2 tesla in it.*
If an air gap of 1 mm is introduced in it, find the m.m.f. required for the air gap as a fraction of the total m.m.f. to maintain the same flux-density. **[Dec. - 2003, 6 Marks]**

Ans. : Refer Ex. 3.19.

Q.2 *The length of mean flux path and the cross-sectional area of a magnetic circuit are 'l' metre and 'a' - sq. metre respectively. The relative permeability is 'μr'. A coil of 'T' - turns, uniformly wound around it, carries a current of 'I' - ampere. Define, at a point on the mean flux path, the magnetising force and the flux density. Derive the expression for the flux set up and hence explain the terms 'm.m.f.' and 'reluctance' of the magnetic circuit.* **[Dec. - 2003, 8 Marks]**

Ans. : Refer section 3.8, 3.9, 3.15, 3.18.

Q.3 *With reference to a magnetic circuit, explain the terms 'magnetic flux', 'magnetomotive force', 'magnetic field intensity', 'magnetic flux density', 'reluctance', 'permeability of free space' and 'relative permeability'. State their units.* **[May - 2004, 8 Marks]**

Ans. : Refer section 3.6, 3.15, 3.9, 3.8, 3.16, 3.14.

Q.4 *Compare the magnetic circuit with the electric circuit.* **[May - 2004, 4 Marks]**

Ans. : Refer section 3.20.

Q.5 *A coil is wound uniformly with 300 turns over a steel ring of relative permiability 900, having a mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 5 A is passed through the coil, find i) m.m.f. ii) reluctance of the ring and iii) flux.* **[Dec. - 2004, 6 Marks]**

Ans. : Refer Ex. 3.20.

Q.6 *For the iron ring solenoid having a magnetic path of 'l'm, area of cross-section Am^2, and coil of N turns carrying I amp. Find the expression for flux (ϕ).*

[Dec. - 2004, 4 Marks]

Ans. : Refer section 3.18.

Q.7 *Compare a magnetic circuit with the electric circuit. Bring out clearly the dissimilarities between them.* **[May - 2005, 4 Marks]**

Q.15 *An iron ring has circular cross-section 4 cm in radius and the average circumference of 100 cm. The ring is wound with 700 turns.*

Calculate :

1) Current required to produce 2 mWb in the ring if μ_r for the iron is 900.

2) If a saw cut of 1 mm wide is made in the ring, calculate the current which will give same value of flux in the ring. **[May-2006, 6 Marks]**

Ans. : Refer Ex. 3.11.

Q.16 *The mean diameter of steel ring is 40 cm and flux density of 0.9 Wb/m^2 is produced by 3500 Aturns/meter. If the cross-section of the ring is 15 cm^2 and number of turns 440, calculate:*

i) the exciting current ii) the self inductance in henry and iii) exciting current and inductance when air gap of 1 cm is cut in the ring. **[Dec.-2006, 8 Marks]**

Ans. : Refer Ex. 3.24.

Q.17 *Compare electric and magnetic circuit.* **[Dec.-2005, 5 Marks; Dec.-2006, 4 Marks May-2007, Dec.-2007, 5 Marks; May-2008, 6 Marks]**

Ans. : Refer section 3.20.

Q.18 *Define as related to magnetic circuit-*

i) Flux density, ii) Permeability, iii) Reluctance and iv) M.M.F. **[Dec.-2006, 4 Marks]**

Ans. : i) Refer section 3.8.

ii) Refer section 3.14.

iii) Refer section 3.16.

iv) Refer section 3.15.

Q.19 *Define and state the units for the following :*

1) Magnetic flux density, *2) Magnetic field strength,*

3) Reluctance, *4) Permeance,*

5) Permeability and *6) Relative permeability.* **[May-2007, 6 Marks, Dec.-2007, 6 Marks]**

Ans. : 1) **Magnetic flux density :** Refer section 3.8.

2) **Magnetic field strength :** Refer section 3.9.

3) **Reluctance :** Refer section 3.16.

4) **Permeance :** Refer section 3.17.

5) **Permeability :** Refer section 3.14.

6) **Relative permeability :** Refer section 3.14.3.

Q.20 *An iron ring of mean length 50 cm has air gap of 1 mm and a winding of 200 turns. If the relative permeability of iron is 300, find the flux density when a current of 1 amp flows through the coil.* **[Dec.-2007, 6 Marks]**

Ans. : Refer Ex. 3.25.

Q.21 *Write a short note on Magnetic Leakage and Fringing.* **[May-2007, 6 Marks; Dec.-2007, 5 Marks]**

Ans. : Refer Ex. 3.21.

Q.22 *Derive the expression for force experienced by two long straight conductors carrying current and placed in vacuum.* **[May-2008, 5 Marks]**

Ans. : Refer section 3.13.

Q.23 *A ring has diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semi-circular sections of cast iron and cast steel with each joint having a reluctance equal to an air gap of 0.2 mm. Find the Amp-Turns required to produce a flux of 8×10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively.* **[May-2008, 8 Marks]**

Ans. : Refer Ex. 3.26.

Q.24 *Define as related to magnetism : (i) Absolute Permeability (ii) Magnetic Field Intensity (iii) Permeance (iv) Relative permeability.* **[May-2008, 5 Marks]**

Ans. : Refer section 3.14.1, 3.17, 3.14.3.

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4 Electromagnetic Induction

Q.1 *An electric conductor of effective length of 0.3 metre is made to move with a constant velocity of 5 metre per second perpendicular to a magnetic field of uniform flux density 0.5 tesla. Find the e.m.f. induced in it. If this e.m.f. is used to supply a current of 25 A, find the force on the conductor, and state its direction w.r.t. motion of conductor. Ignoring friction, find the power required to keep the conductor moving across the field.* **[Dec. - 2003, 4 Marks]**

Ans. : Refer Ex. 4.23.

Q.2 *Two identical coils P and Q, each with 1500 turns, are placed in parallel planes near to each other, so that 70% of the flux produced by current in coil P links with coil Q. If a current of 4 A is passed through any one coil, it produces a flux of 0.04 mWb linking with itself. Find the self inductances of the two coils, the mutual inductance and coefficient of coupling between them.* **[Dec. - 2003, 4 Marks]**

Ans. : Refer Ex. 4.24.

Q.3 *A straight conductor 1.5 m long lies in a plane perpendicular to a uniform magnetic field of flux density 1.2 tesla. When a current of 'I' ampere is passed through it, it makes the conductor move across the magnetic field with a velocity of 1 m/s. Ignoring resistance of the conductor and friction, find the current 'I', if the power of the moving conductor is 90 watt. Find the e.m.f. induced in the conductor and the force on it. State the sense of the force w.r.t. the velocity, and sense of the e.m.f. induced w.r.t. current.* **[May - 2004, 6 Marks]**

Ans. : Refer Ex. 4.26.

Q.4 *A coil M is wound around a magnetic circuit. Explain the phenomenon of self induced e.m.f. in it. Define its self inductance and state its unit. Another coil N is wound around the same magnetic circuit. Explain the phenomenon of mutual inductance between the coils and define 'coefficient of coupling' between them.* **[May - 2004, 6 Marks]**

Ans. : Refer section 4.7, 4.8.

Q.5 *Obtain an expression for coefficient of coupling for two magnetically coupled coils.* **[Dec.- 2004, 6 Marks]**

Ans. : Refer section 4.8.4.

Q.6 *Obtain an expression for energy stored in magnetic field.* **[Dec. - 2004, 6 Marks]**

Ans. : Refer section 4.10.

Q.8 *An uncharged capacitance of 30 μF, connected in series with a resistance of 500-ohm, is suddenly connected across 100 V d.c. supply. Find (i) time constant of the circuit (ii) initial current (iii) current after 0.05 second (iv) final energy stored in the capacitor.* **[May-2004, 6 Marks]**

Ans. : Refer Ex. 5.18.

Q. 9 *Derive, mathematical expression for a capacitor voltage and current at any instant during charging of capacitor through resistance 'R'. Also sketch the graph of capacitor voltage and current against time.* **[Dec.-2004, 7 Marks]**

Ans. : Refer section 5.24.

Q.10 *Obtain an expression for capacitance of parallel plate capacitor with composite medium of three materials.* **[Dec.-2004, 5 Marks]**

Ans. : Refer section 5.20.

Q.11 *Derive the expression for parallel plate capacitor when medium is composite.* **[May-2005, 6 Marks]**

Ans. : Refer section 5.20.

Q.12 *A capacitor is made of two parallel plates with an area of 11 cm² and are separated by mica sheet 2 mm thick. If for mica $\varepsilon_r = 6$, find its capacitance. If now, one plate of capacitor is moved further to give an air gap of 0.5 mm wide between the plates and mica. Find the new value of capacitance.* **[May-2005, 8 Marks]**

Ans. : Refer Ex. 5.19.

Q.13 *Three capacitors of values 2 μF, 4 μF and 6 μF have an applied voltage of 60 V across their series combination. Determine the voltage on each of the capacitors.* **[Dec.-2005, 5 Marks]**

Ans. : Refer Ex. 5.4.

Q.14 *Define as related with electrostatics i) Electric flux, ii) Electric flux density, iii) Electric field, iv) Permittivity.* **[Dec.-2005, 4 Marks]**

Ans. : Refer sections 5.5, 5.6, 5.4 and 5.8.

Q.15 *State and explain how capacitances are classified on the basis of nature of dielectric used.* **[Dec.-2005, 5 Marks]**

Ans. : Refer section 5.23.

Q.16 *Derive an expression for energy stored in capacitor.* **[Dec.-2005, 4 Marks, May-2006, May-2008, 5 Marks]**

Ans. : Refer section 5.21.

Q. 17 *Define as related to electrostatics :*

1) Electric Field Strength

2) Permittivity

3) Dielectric Strength and

4) Electromotive Force **[May-2006, 4 Marks]**

Ans. : Refer sections 5.7, 5.8, 5.16.

Q.18 *A capacitor of 2 μF capacitance charged to p.d. of 200 V is discharged through a resistor of 2 MΩ.*

Calculate :

1) The initial value of discharged current

2) Its value 4 seconds later, and

3) Initial rate of decay of the capacitor voltage. **[May-2006, 5 Marks]**

Ans. : Refer Ex. 5.20.

Q.19 *Explain what do you understand by dielectric strength and dielectric breakdown in capacitors.* **[Dec.-2006, 4 Marks; May-2008, 5 Maks]**

Ans. : Refer section 5.16.

Q.20 *Show that the capacitance of a composite dielectric capacitor is given by*

$$C_{eq} = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{t_1} + \frac{\varepsilon_0 A_2}{\frac{(t_1 - t_2)}{\varepsilon_{r2}} + \frac{t_2}{\varepsilon_{r3}}}$$

where $\varepsilon_{r1}, \varepsilon_{r2}$ and ε_{r3} are the relative permittivities of the three dielectrics used in capacitor and t_1 and t_2 are dielectric thicknesses. **[Dec.-2006, 5 Marks]**

Ans. : Refer Type 2 of section 5.20.

Q.21 *A capacitor consist of two parallel rectangular plates each 120 mm square separated by 1 mm in air. When a voltage of 1000 V is applied between the plates an average current of 12 mA flows for 5 second.Calculate : i) the charge on the capacitor, ii) the electric flux density and*
iii) the electric field strength in the dielectric. **[Dec.-2006, 5 Marks]**

Ans. : Refer Ex. 5.21.

Q.22 *Derive the expression for energy stored in electric field.* **[May-2007, 5 Marks]**

Ans. : Refer Ex. 5.21.

Q.23 *Two capacitors of 8 μF and 2 μF are connected in series across 400 volt supply. Calculate 1) Resultant Capacitance, 2) Charge on each capacitor and 3) p.d. across each capacitor.* **[May-2007, 5 Marks]**

Ans. : Refer Ex. 5.22.

Q.24 *Define as related to electrostatic field (i) Electric flux (ii) Electric flux Density (iii) Electric field strength (iv) Absolute permittivity and (v) Relative permittivity.*
(Dec.-2007, 5 Marks)

Ans. : Refer sections 5.5, 5.6, 5.7, 5.8.

Q.25 *Derive the expressions for instantaneous voltage, charge and charging current for an R-C charging circuit. Hence define time constant for R-C charging circuit.*
(Dec.-2007, 6 Marks)

Ans. : Refer sections 5.24, 5.25.

Q.26 *Two capacitors of 2 μF and 4 μF are connected in (i) parallel and ii) series across 100V D.C. supply.*
Determine : i) Energy stored in each capacitor and ii) Equivalent capacitance of their combination in each case. **(May-2008, 6 Marks)**

Ans. : Refer Ex. 5.23.

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Q.7 *Derive an expression for r.m.s. value of sinusoidal alternating current interms of its peak value.* **[Dec-2004, 5 Marks]**

Ans. : Refer section 6.7.

Q.8 *Find the resultant of three voltages given by $v_1 = 10 \sin \omega t$, $v_2 = 20 \sin (\omega t - \pi/4)$ and $v_3 = 30 \cos (\omega t + \pi/6)$.* **[Dec-2004, 7 Marks]**

Ans. : Refer Ex. 6.28.

Q.9 *In a certain circuit supplied from 50 Hz mains, the potential difference has a maximum value of 500 volt and the current has a maximum value of 10 Amp. At the instant t = 0, the instantaneous values of potential difference and current are 400 volt and 4 Amp respectively both increasing in positive direction. State expressions for instantaneous values of potential difference and current at time 't'. Calculte the instantaneous values at time t=0.015 second. Find phase angle between potential difference and current.* **[May-2005, 8 Marks]**

Ans. : Refer Ex. 6.29.

Q.10 *A 50 Hz sinusoidal current has peak factor 1.4 and form factor 1.1. Its average value is 20 Amp. The instantaneous value of curent is 15 Amp at t = 0 sec. write the equation of current and draw its waveform.* **[May-2005, 6 Marks]**

Ans. : Refer Ex. 6.30.

Q.11 *Define r.m.s value, form factor and peak factor of sinusoidal current.* **[May-2005, 4 Marks]**

Ans. : Refer sections 6.7, 6.8, 6.9, 6.10.

Q.12 *Define Form factor and Peak factor as related with a.c. supply.* **[Dec.-2005, 4 Marks]**

Ans. Refer sections 6.9 and 6.10.

Q.13 *An alternating current varying sinusoidally with a frequency of 50 Hz has a r.m.s. value of current of 20 Amp. At what time, measured from negative maximum value, instantaneous current will be $10\sqrt{2}$ Amp ?* **[Dec.-2005, 4 Marks]**

Ans. : Refer Ex. 6.31.

Q.14 *Define i) Average value and ii) RMS value of sinusoidally varying quantity and hence derive expressions for RMS value in terms of its peak value and average value in terms of its peak value.* **[Dec.-2005, 8 Marks]**

Ans. : Refer sections 6.7 and 6.8.

Q.15 *Define the following terms related to alternating quantities :* **[May-2006, 6 Marks]**

1) Wave Form *2) Periodic Time*

3) Frequency *4) Average Value*

5) Maximum Value 6) R.M.S Value

Ans. Refer sections 6.5, 6.7 and 6.8.

Q.16 *Draw a neat sketch in each case, of the waveform and write expression of Instantaneous Value for the following :* **[May-2006, 6 Marks]**

1) Sinusoidal current of amplitude 10 A, 50 Hz passing through its zero value at $\omega t = \pi/3$ and rising positively.

2) Sinusoidal current of amplitude 8 A, 50 Hz passing through its zero value at $\omega t = -\pi/6$ and rising positively.

Ans. : Refer Ex. 6.32.

Q.17 *Define*

1) Average value and 2) R.M.S. value of sinusoidally varying quantity and hence derive the expressions for each in terms of their peak values. **[May-2006, 8 Marks]**

Ans. : Refer sections 6.7 and 6.8.

Q.18 *The instantaneous current is given by,*

$i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the waveform from $t = 0$ over one complete cycle. **[Dec.-2006, 8 Marks]**

Ans. : Refer Ex. 6.15.

Q.19 *Explain the concepts of phase and phase difference in alternating quantities.*

[Dec.-2006, 4 Marks]

Ans. : Refer section 6.13.

Q.20 *Prove that in an alternating quantity varying sinusoidally the maximum value is $\sqrt{2}$ times the effectives value. Similarly maximum value is also equal to 1.569 times the average value.* **[Dec.-2006, 8 Marks]**

Ans. : Refer sections 6.7.2 and 6.8.2.

$$I_{max} = \sqrt{2}\, I_{rms}$$

$$I_{av} = 0.637\, I_{max} \text{ for sinusoidal quantity}$$

$$\therefore \quad I_{max} = \frac{1}{0.637}\, I_{av} = \mathbf{1.569\, I_{av}}.$$

Q.21 *Define the following terms as related to alternating quantities*
(1) Frequency, (2) Periodic time, (3) Amplitude, (4) Peak factor, (5) Form factor (5) Instantaneous value. **[May-2007, 6 Marks]**

Ans. : Refer sections 6.5, 6.9 and 6.10.

Q.22 *An alternating current is given by i = 14.14 sin 377 t.*
Find (i) R.M.S. value of current, (ii) Frequency, (iii) Instantaneous value of current, when t = 3 ms and (iv) Time taken by current to reach 10 amp for 1 st time after passing through zero. **[May-2007, 6 Marks]**

Ans. : Refer Ex. 6.33.

Q.23 *Define as related to an alternating quantity (i) Instantaneous value (ii) Wave form (iii) Cycle (iv) Frequency and (v) Amplitude.* **(Dec.2007, 5 Marks)**

Ans. : Refer section 6.5.

Q.24 *For an A.C. circuit e = 100 sin ωt, calculate the value of e at t = 0.005 sec for (i) 50 Hz and (ii) 150 Hz. Sketch the waveform for e from t = 0 sec to t = 0.01 sec for both cases on the same time axis.* **(Dec.-2007, 6 Marks)**

Ans. : Refer Ex. 6.34.

Q.25 *Derive the expression for average value of sinusoidally varying current in terms of its peak value.* **(Dec.-2007, 4 Marks)**

Ans. : Refer section 6.8.

Q.26 *Define R.M.S. value of alternating quantity and derive its expression for sinusoidal current.* **(May-2008, 5 Marks)**

Ans. : Refer section 6.7.

Q.27 *Define average value of alternatiing quantity and derive it's expression for sinusoidal current ?* **(May-2008, 5 Marks)**

Ans. : Refer section 6.8.

Q.28 *Determine the average value, effective value and form factor of a sinusoidally varying half wave rectified alternating current as shown in Fig. 1.* **(May-2008, 6 Marks)**

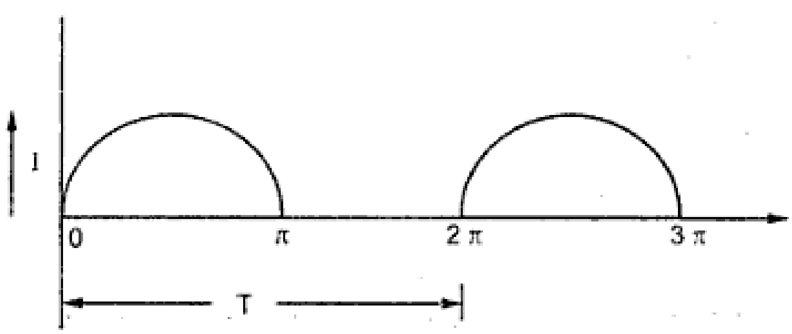

Fig. 1

Ans. : Refer Ex. 6.21.

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Find the value of capacitor C_1 to be connected across the above series circuit, so that current drawn from supply is the minimum. **[May - 2004, 10 Marks]**

Ans. : Refer section 7.42.

Q.10 *What is meant by resonance in series R-L-C circuit connected across sinusoidal a.c. supply ? Deduce the equation for resonant frequency.* **[Dec. - 2004, 5 Marks]**

Ans. : Refer section 7.9.

Q.11 *What is admittance of an a.c. circuit ? What are its two components ? State unit of these quantities. How the admittance is expressed in rectangular and polar form ?* **[Dec. - 2004, 5 Marks]**

Ans. : Refer section 7.10.

Q.12 *An e.m.f. given by $\vartheta = 100 \sin 100 \pi . t$ is impressed across a circuit consists of resistance of 40 Ω in series with 100 μF capacitor and 0.25 H inductor. Determine -*

i) r.m.s. value of current ii) Power consumed iii) Power factor

[Dec. - 2004, 7 Marks]

Ans. : Refer Ex. 7.43.

Q.13 *A sinusoidal voltage of $\vartheta = V_m \sin \omega t$ is applied across a series R-L circuit. Derive the expression for current and average power consumed by the circuit. Also sketch the waveform of voltage, current and power against time as a common X-axis.*

[Dec. - 2004, 8 Marks]

Ans. : Refer section 7.5.

Q.14 *Two circuits having impedances $Z_1 = 10 + j\ 15\ \Omega$ and $Z_2 = 6 - j\ 8\ \Omega$ are connected in parallel. If total current supplied is 15 A,*

Find,

i) Branch currents. ii) Power consumed in each branch. iii) Phasor diagram. **[Dec. - 2004, 9 Marks]**

Ans. : Refer Ex. 7.44.

Q.15 *A series circuit consisting of a coil and a variable capacitance having realance X_C. The coil has resistance of 10 Ω, inductive reactance of 20 Ω. It is observed that at certain value of capacitance current in the circuit is maximum, find (1) This value of capacitance (2) Impedance of the circuit (3) Power factor (4) Current, if applied voltage is 100 V, 50 Hz.* **[May - 2005, 8 Marks]**

Ans. : Refer Ex. 7.45.

Q.16 *Two impedances ($R_1 - jX_{C1}$) and ($R_2 + jX_{L2}$) are connected in parallel across supply voltage $v = (100\sqrt{2}) \sin (314\ t)$. The current flowing through the two impedances are given by $i_1 = 10\sqrt{2} \sin (314\ t + \pi/4)$ and $i_2 = 10\sqrt{2} \sin (314\ t - \pi/4)$ respectively.*

Find equation for instantaneous value of total current drawn from supply. Also find values of R_1, R_2, X_{C1} and X_{L2}. **[May - 2005, 8 Marks]**

Ans. : Refer Ex. 7.46.

Q.17 *A coil connected across a 250 volt, 50 Hz supply takes a current of 10 Amp at 0.8 power factor lag. What will be the power taken by the choke coil, when connected across a 200 volt, 25 Hz supply ? Also calculate resistance and inductance of the coil.* **[May - 2005, 8 Marks]**

Ans. : Refer Ex. 7.47.

Q.18 *An alternating voltage $v = 141.4 \sin (157.08\ t + \pi/12)$ volts is applied to a circuit and an a.c. ammeter, wattmeter and power factor meter are connected to measure the respective quantities. Reading of ammeter is 5 Amp. and that of power factor meter is 0.5 lagging, find (i) The expression for the instantaneous value of current, (ii) The wattmeter reading (iii) Impedance of the circuit in rectangular form.*

[May - 2005, 8 Marks]

Ans. : Refer Ex. 7.48.

Q.19 *Prove that current in purely resistive circuit is in phase with applied A.C. voltage and current in purely capacitive circuit leads applied A.C. voltage by 90°.*

[Dec.-2005, 8 Marks]

Ans. : Refer sections 7.2 and 7.4.

Q.20 *Two circuits, the impedances of which are given by $Z_1 = (12 + j15)\ \Omega$ and $Z_2 = (8 - j4)\ \Omega$, are connected in parallel across the potential difference of $(230 + j0)$ volt. Calculate i) total current drawn, ii) total power and branch powers consumed and iii) overall power factor of circuit.* **[Dec.-2005, 9 Marks]**

Ans. : Refer Ex. 7.49.

Q.21 *Show the waveforms of voltage, current and power if $v = V_m \sin \omega t$ volt is applied across a R - C series circuit.* **[Dec.-2005, 5 Marks]**

Ans. : Refer section 7.6.

Q.22 *Draw the admittance triangle and define admittance, susceptance and conductance as related with A.C. circuit.* **[Dec.-2005, 5 Marks]**

Ans. : Refer section 7.10.

Q.23 *Two impedances, one inductive and the other capacitive are connected in series across the voltage $120 \angle 30°$ volt and frequency of 50 Hz. The current flowing in circuit is $3 \angle -15°$. If one of the impedances is $(10 + j\,48.3)\ \Omega$, find the other. Also calculate the value of L and C in the impedances.* **[Dec.-2005, 7 Marks]**

Ans. : Refer Ex. 7.51.

Q.24 *Derive the expression for instantaneous current and power consumed when voltage of $\upsilon = V_m \sin \omega t$ is applied to a circuit consisting of pure capacitance of 'C' farad. Sketch the relevant waveforms.* **[May-2006, 8 Marks]**

Ans. : Refer section 7.4.

Q.25 *Two circuits A and B are connected in parallel to a 115 V, 50 Hz supply. The total current taken by the combinations is 10 A at unity p.f. Circuit A consists of a $10\ \Omega$ resistance and 200×10^{-6} F capacitor connected in series. Circuit B consists of a resistance and an inductance reactor in series. Determine the following data for circuit B :*

1) Current *2) p.f.*

3) Impedance *4) Resistance, and*

5) Reactance **[May-2006, 9 Marks]**

Ans. : Refer Ex. 7.52.

Q.26 *A coil of p.f. 0.6 is in series with 10 μF capacitor. When connected to a 50 Hz supply, the potential difference across the coil is equal to the potential difference across the capacitance. Find the resistance and inductance of the coil.*

[May-2006, 6 Marks]

Ans. : Refer Ex. 7.53.

Q.27 *Sketch the waveforms of voltage current and power if $\upsilon = V_m \sin \omega t$ volt is applied R-L series circuit and state expression for power.* **[May-2006, 6 Marks]**

Ans. : Refer Ex. 7.54.

Q.28 *Define the terms :*

1) Admittance

2) Susceptance and Conductance as related to A.C. circuit.

[May-2006, May-2007, 5 Marks]

Ans. : Refer section 7.10.

Q.29 *Define following terms and hence state their units :*
i) Active power, ii) Reactive power, iii) Conductance and iv) Susceptance.

[Dec.-2006, 4 Marks]

Ans. : i) Refer section 7.5.4.

ii) Refer section 7.5.5.

iii) Refer section 7.10.4.

iv) Refer section 7.10.5.

Q.30 *What is meant by resonance in series R-L-C circuit connected across sinusoidal A.C. supply? Derive the equation for resonant frequency.* **[Dec.-2006, 5 Marks]**

Ans. : Refer section 7.9.

Q.31 *Two circuits having same numerical value ohmic impedance are connected in parallel. The power factor of one circuit is 0.8 and for other it is 0.6. What is the power factor of the combination when*

i) Both the impedances are inductive ?

ii) One is inductive and another is capacitive ? **[Dec.-2006, 8 Marks]**

Ans. : Refer Ex. 7.37.

Q.32 *If a sinusoidal voltage of $v = V_m \sin \omega t$ is applied across purely capacitive circuit, derive the expression for current drawn and power consumed and draw their waveforms.* **[Dec.-2006, 5 Marks]**

Ans. : Refer section 7.4.

Q.33 *Draw and explain phasor diagram of an R-L-C series circuit when (i) $X_L > X_C$ and ii) $X_C > X_L$.* **[Dec.-2006, 4 Marks]**

Ans. : Refer section 7.7.

Q.34 *Two impedances $Z_A = (4 + 3j)$ and $Z_B = (10 - j7)$ are connected in parallel and impedance $Z_C = (6 + j5)$ is connected in series with parallel combination of Z_A and Z_B. If the voltage applied across circuit is 200 volt at 50 Hz. Calculate :*

i) currents flowing in Z_A, Z_B and Z_C and

ii) the total power factor of the circuit **[Dec.-2006, 8 Marks]**

Ans. : Refer Ex. 7.55.

Q.35 *When an inductive coil is connected to a d.c. supply at 240 volt, the current in it is 16 amp. When the same coil is connected to an a.c. supply at 240 volt, 50 Hz, the current is 12.27 amp. Calculate (1) Resistance, (2) Impedance, (3) Reactance (4) Inductance of the coil.* **[May-2007, 6 Marks]**

Ans. : Refer Ex. 7.56.

Q.36 *A coil has inductance of 20 mH and resistance 5 ohm. It is connected across a supply voltage of $v = 48 \sin 314\, t$. Obtain the expression for current drawn by the coil.* **[May-2007, 4 Marks]**

Ans. : Refer Ex. 7.57.

Q.37 *A series circuit consists of resistance of 10 ohm, an inductance of* $\frac{200}{\pi}$ *mH and capacitance of* $\frac{1000}{\pi}$ μ*F. Calculate (1) Current flowing in the circuit if supply voltage is 200 V, 50 Hz (2) p.f. of the circuit, (3) Power drawn from the supply. Also draw the phasor diagram.* **[May-2007, 8 Marks]**

Ans. : Refer Ex. 7.58.

Q.38 *Two circuits, the impedances of which are given by* $Z_1 = (10 + j15)$ *ohm, and* $Z_2 = (6 - j8)$ *ohm, are connected in parallel across A.C. supply. If the total current supplied is 15 amp, what is the power taken by each branch?* **[May-2007, 9 Marks]**

Ans. : Refer example 7.44 for the procedure and verify the answers :

$P_1 = 737.7092$ **W and** $P_2 = 1438.5441$ **W**

Q.39 *Two impedances* $Z_1 = 40 \angle 30°$ *ohm and* $Z_2 = 30 \angle 60°$ *ohm are connected in series across single phase, 230 V, 50 Hz supply. Calculate the (1) Current drawn, (2) p.f. and (3) Power consumed by the circuit.* **[May-2007, 8 Marks]**

Ans. : Refer Ex. 7.59.

Q.40 *A 230 V, 50 Hz voltage is applied first to resistor of value 100 Ω and then to a capacitor of 100 μF. Obtain the expressions for the instantaneous currents for both the cases and draw the phasor diagrams.* **[Dec.-2007, 6 Marks]**

Ans. : Refer Ex. 7.60.

Q.41 *A series R-L-C circuit has resistance of 50 Ω, Inductance f 0.1 H and capacitance of 50 μF connected in series across single phase 230 V, 50 Hz supply, calculate :*

i) Current drawn by circuit

ii) Power factor of the circuit

iii) Active and rective power consumed by circuit

iv) Draw the phasor diagram. **[Dec.-2007, 8 Marks]**

Ans. : Refer Ex. 7.61.

Q.42 *A parallel circuit consists of two branches. Branch (i) Consists of R of 100 Ω connected in series with inductance of 1 H and branch (ii) consists of R of 50 Ω in series with capacitance of 79.5 μF. This parallel circuit is connected across single phase 200 V, 50 Hz supply, calculate :*

i) Branch currents *ii) Total current drawn by circuit*

iii) Total power factor of circuit *iv) Total power drawn by circuit.*

[Dec.-2007, 8 Marks]

Ans. : Refer Ex. 7.62.

Ans. : Refer section 8.9.

Q.13 *A balanced 3 phase star connected load of 120 kW takes a leading current of 100 A when connected across 3 phase 3.3 kV, 50 Hz supply. Determine the impedance, resistance, capacitance and p.f. of load.* **[Dec.-2006, 8 Marks]**

Ans. :

Refer answer of Q.11 for procedure. The answers are,

$$Z_{ph} = 19.0525\ \Omega$$

$$R_{ph} = 4\ \Omega,\ C_{ph} = 170.88\ \mu F,\ \cos\phi = 0.21 \text{ leading.}$$

Q.14 *Define :*

1) Phase sequence 2) Balanced load and

3) Symmetrical system as reference to three phase circuits.

[Dec.-2006, May-2007, 4 Marks; May-2008, 6 Marks]

Ans. : 1) Refer section 8.4.

2) Refer section 8.7.3.

3) Refer section 8.4.

Q.15 *Prove that a three phase balanced load draws three times as much power when connected in delta, as it would draw when connected in star.* **[Dec.-2006, 4 Marks]**

Ans. : Refer example 8.3.

Q.16 *Three inductive coils each with resistance of 15 ohm and an inductance of 0.03 H are connected (1) in star and (2) in delta across 3 phase 400 volt, 50 Hz supply. Calculate in each case (1) Line current and (2) Power consumed by load.*

[May-2007, 8 Marks]

Ans. : Refer Ex. 8.19.

Q.17 *Derive the relationship between the line current and phase current, line voltage and phase voltage for a balanced three phase delta connected load connected across three phase supply. Also derive the expression for power consumed interms of line values.* **[Dec.-2007, 8 Marks]**

Ans. : Refer section 8.9.

Q.18 *Prove that power consumed by 3 phase delta connected load is equal to three times power consumed by the same load connected in star across three phase supply.* **[Dec.-2007, 6 Marks]**

Ans. : Refer Ex. 8.3.

Q.19 *Three coils each having resistance of 10 Ω and inductance of 0.03 H are connected in delta across a 3 phase 400 volt 50 Hz supply. Calculate the current drawn and power consumed by load.* **(May-2008, 4 Marks)**

Ans. : Refer Ex. 8.2.

Q.20 *State the relations between line and phase values of voltage and current for 3 phase star connected and delta connected load. State the equation for active and reactive power consumed by three phase load.* **[May-2008, 6 Marks]**

Ans. : Refer section 8.8 and 8.9.

❑❑❑

Applying kVL to the two loops shown in the Fig. 2(a).

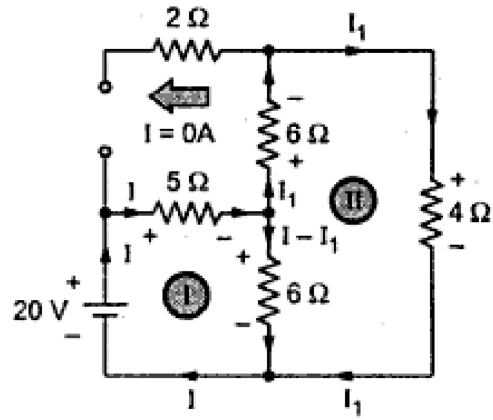

Fig. 2(a)

$$-5I - 6(I - I_1) + 20 = 0$$

i.e. $$11I - 6I_1 = 20 \quad \ldots(1)$$

$$-6I_1 - 4I_1 + 6(I - I_1) = 0$$

i.e. $$6I - 16I_1 = 0 \quad \ldots(2)$$

Solving (1) and (2), $I_1 = \mathbf{0.8571\ A} \downarrow$

... Due to 20 V alone

Step 2 : Consider 5A source alone, short 20 V source

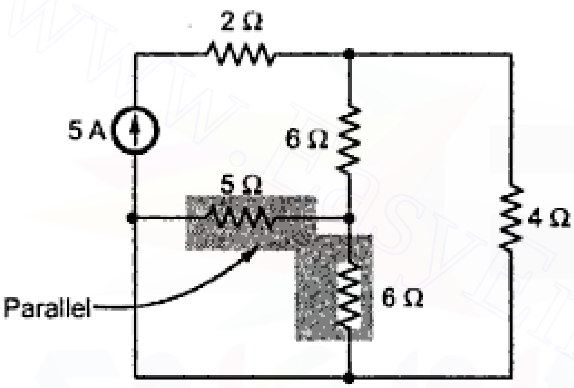

Fig. 2(b)

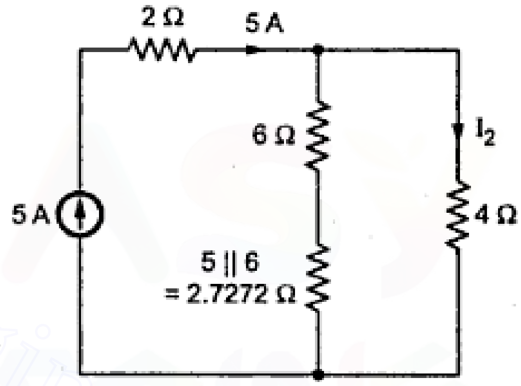

Fig. 2(c)

Using current distribution rule,

$$I_2 = \frac{5 \times 8.7272}{8.7272 + 4} = \mathbf{3.4285\ A} \downarrow$$

... Due to 5 A alone

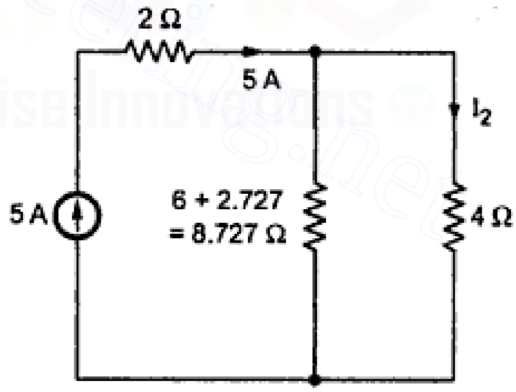

Fig. 2(d)

$$\therefore \quad I_{4\Omega} = I_1 + I_2$$

$$= 0.8571\ A \downarrow + 3.4285\ A \downarrow$$

$$= \mathbf{4.2856\ A} \downarrow$$

II) Thevenin's theorem :

Step 1 : Remove 4 Ω branch.

Step 2 : Calculate $V_{OC} = V_{TH}$

Key Point : *As current source exists, obtain current distribution considering current source and then apply KVL to the loops without current source. Thus apply KVL to loop I,*

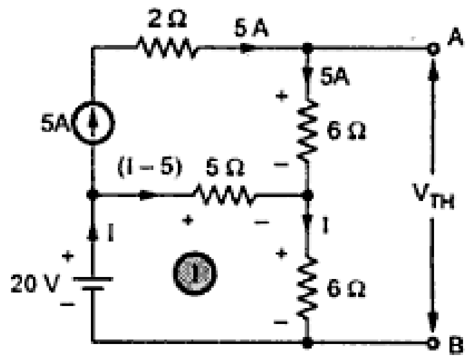

Fig. 2(e)

$\therefore \quad -5(I-5) - 6I + 20 = 0$

i.e. $-11\,I = -45$

$\therefore \quad I = 4.0909$ A

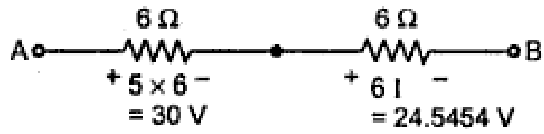

Fig. 2 (f)

Trace the path from A to B as shown in the Fig. 2 (f)

$\therefore \quad V_{TH} = V_{AB} = 30\ 24.5454$

$= \mathbf{54.5454\ V}$

Step 3 : Calculate R_{eq}, opening current source and shorting voltage source.

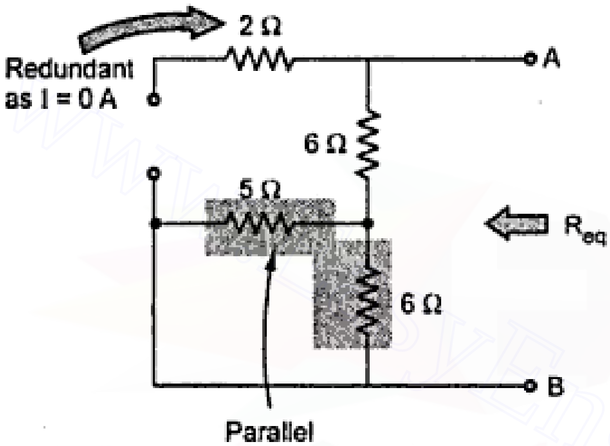

Fig. 2(g)

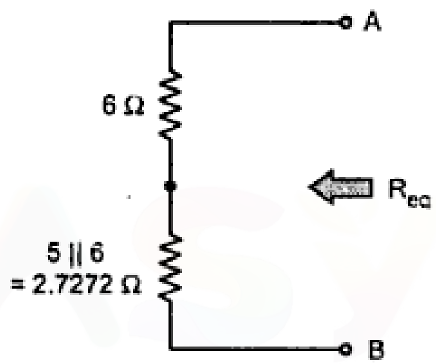

Fig. 2(h)

$\therefore \quad R_{AB} = R_{eq} = 6 + 2.7272 = \mathbf{8.7272\ \Omega}$

Step 4 : The Thevenin's equivalent is shown in the Fig. 2(i).

Step 5 : Hence the current through 4 Ω is,

$$I_L = \frac{54.5454}{8.7272+4} = \mathbf{4.2856\ A} \downarrow$$

This is same as obtained by the Superposition theorem.

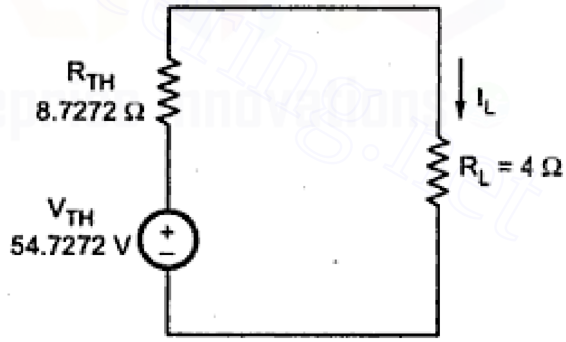

Fig. 2(i)

OR

Q. 4 **a)** *State and explain maximum power transfer theorem.* **(4)**

Ans. : Refer section 2.20.

b) *Find the equivalent resistance across the terminals A-B for the network as shown in Fig. 3. All resistances are in ohms.* **(6)**

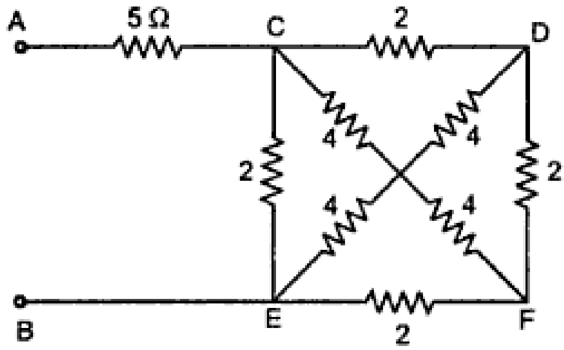

Fig. 3

Ans. : Name the various nodes as shown in the Fig. 3(a).

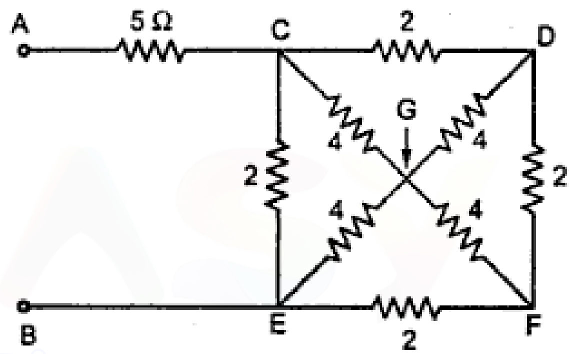

Fig. 3(a)

Assume that the various resistances are connected at the junction point G. Convert the delta CGE and DGF to equivalent star.

$$R_1 = \frac{2 \times 4}{2+4+4} = 0.8\ \Omega$$

$$R_2 = \frac{4 \times 4}{2+4+4} = 1.6\ \Omega$$

$$R_3 = \frac{2 \times 4}{2+4+4} = 0.8\ \Omega$$

The delta DGF is similar to delta CGE.

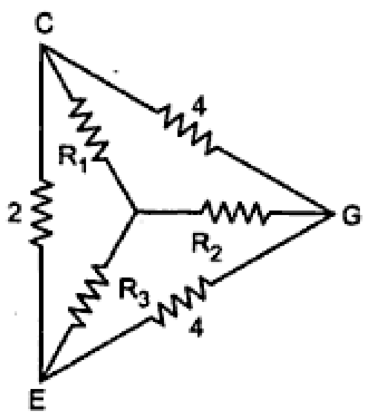

Fig. 3(b)

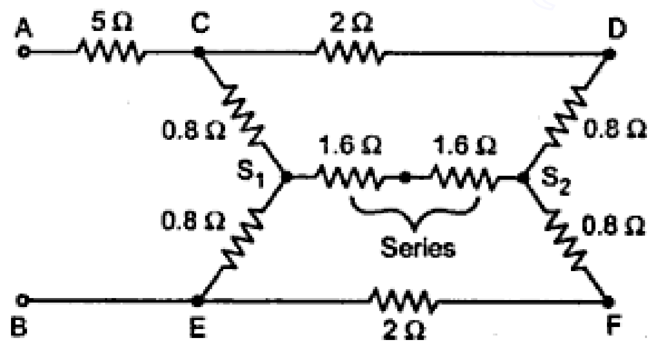

Fig. 3(c)

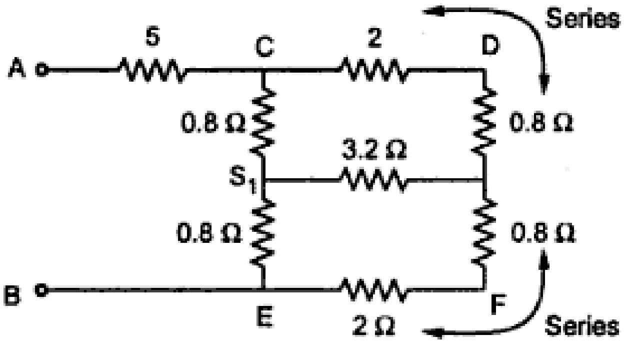

Fig. 3(d)

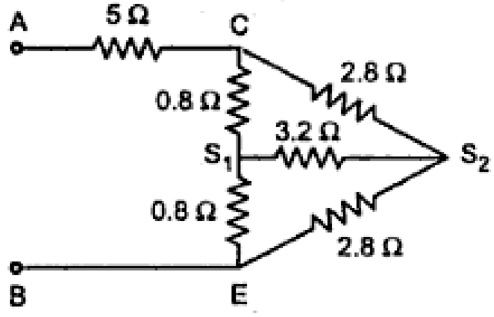

Fig. 3(e)

Convert the ΔCS_1S_2 to star.

$$R_1 = \frac{0.8 \times 2.8}{0.8 + 2.8 + 3.2} = 0.3294\ \Omega$$

$$R_2 = \frac{2.8 \times 3.2}{0.8 + 2.8 + 3.2} = 1.3176\ \Omega$$

$$R_3 = \frac{3.2 \times 0.8}{0.8 + 2.8 + 3.2} = 0.365\ \Omega$$

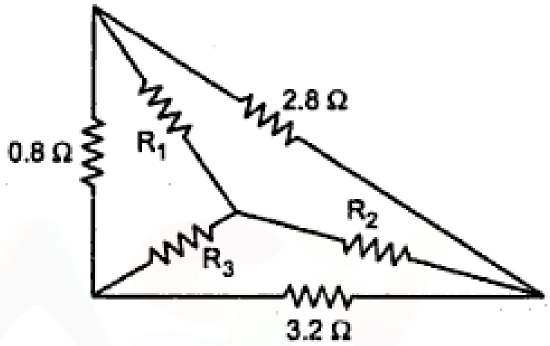

Fig. 3(f)

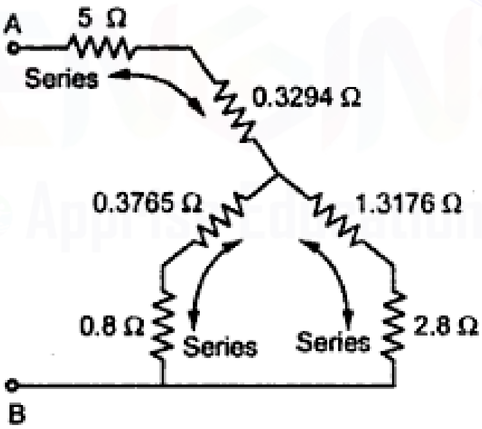

Fig. 3(g)

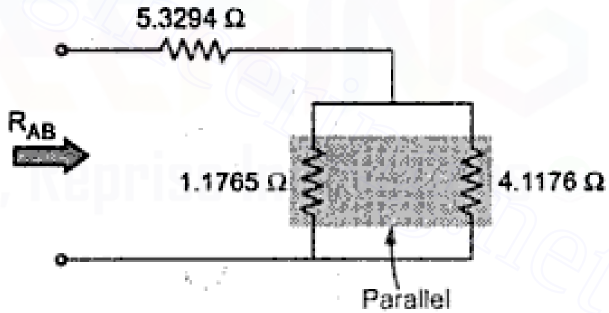

Fig. 3(h)

$\therefore \quad R_{AB} = 5.3294 + 0.9151$

$= \mathbf{6.2444\ \Omega}$

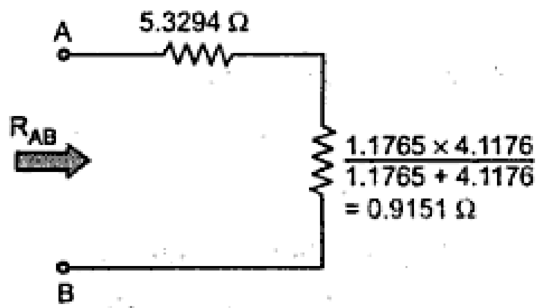

Fig. 3(i)

c) *Use Kirchhoff's laws to find the value of unknown resistance R such that 2 A current flows through it. The direction of the current is shown in Fig. 4. All the resistances are in ohms.* **(7)**

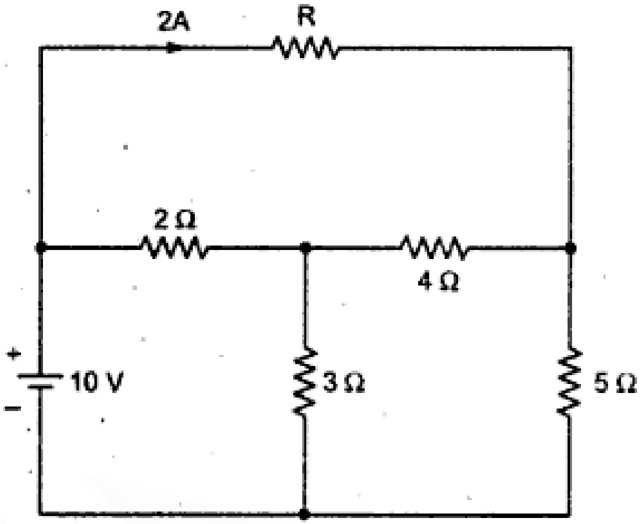

Fig. 4

Ans. : The current distribution considering 2 A current is shown in the Fig. 4 (a).

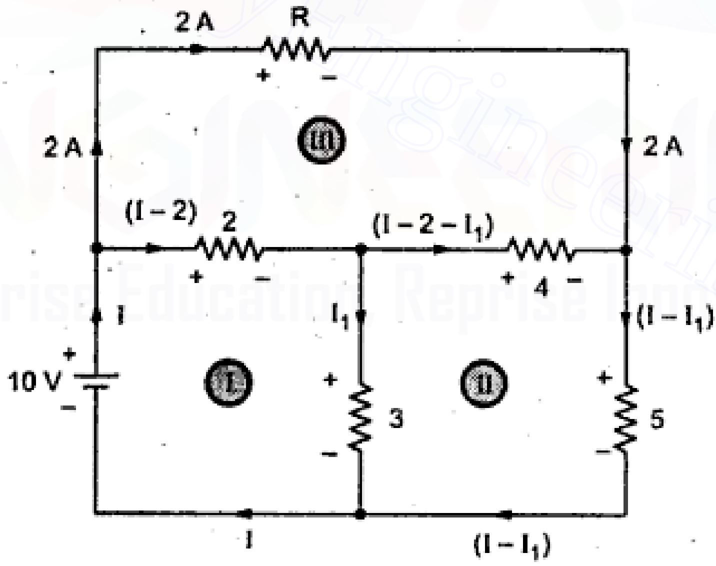

Fig. 4(a)

Applying kVL to the two loops I and II,

$$-2(I-2)-3\,I_1+10 \;=\; 0 \qquad \text{i.e.} \qquad 2\,I+3\,I_1=14 \qquad \ldots(1)$$

$$-4(I-2-I_1)-5(I-I_1)+3\,I_1 \;=\; 0 \qquad \text{i.e.} \quad -9\,I+12\,I_1=-8 \qquad \ldots(2)$$

Solving equations (1) and (2), $I = 3.7647$ A, $I_1 = 2.1568$ A

Now apply KVL to loop III,

$-2\,R+4(I-2-I_1)+2(I-2)$ and use values of I and I_1

$\therefore \quad -2R + 4(3.7647 - 2 - 2.1568) + 2(3.7647 - 2) = 0$

$\therefore \quad -2R - 1.5684 + 3.5294 = 0$ i.e. $R = \mathbf{0.98\ \Omega}$

Q. 5 **a)** *With usual notations show that the magnitude of the dynamically induced e.m.f. is $e = Blv$.*
where $v \rightarrow$ velocity component perpendicular to the magnetic field.. **(4)**

Ans. : Refer section 4.5.1.

b) *A ring of cast steel has on external diameter of 25 cm and a square cross-section of 4 cm side. An ordinary steel bar 17 cm × 4 cm × 0.5 cm is fitted with negligible gap inside and across this ring. A coil of 500 turns and carrying a D.C. current of 1.5 A is placed on one half of the ring. Find the flux in the other half of the ring. Neglect leakage. Assume relative permeability of cast steel as 850 and that for ordinary steel as 700.* **(7)**

Ans. : The ring is shown in the Fig. 5(a).

Inner diameter of ring.

= length of steel bar

= 17 cm.

$\therefore$ Mean diameter of ring

$= \dfrac{\text{outer} + \text{inner}}{2}$

$= \dfrac{25 + 17}{2} = 21$ cm

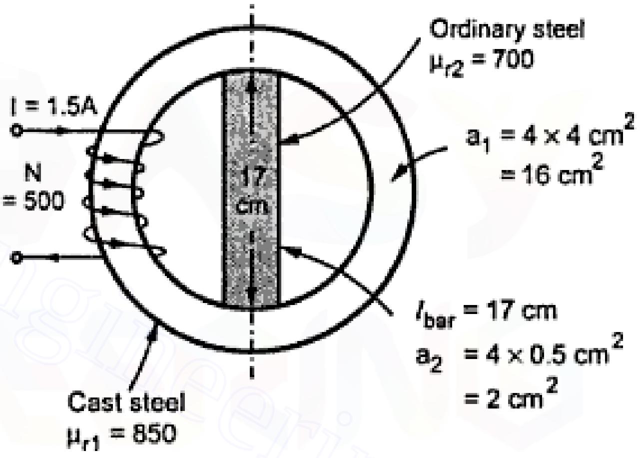

Fig. 5(a)

$\therefore$ Mean circumference

$= \pi \times 21 = 65.9734$ cm.

$\therefore$ Length of half section of ring

$= l_{i1} = \dfrac{65.9734}{2} = 32.9867$ cm

$\therefore$ Length of other section of ring

$= l'_{i1} = l_{i1} = 32.9867$ cm

This is a parallel magnetic circuit as shown in the Fig. 5(b).

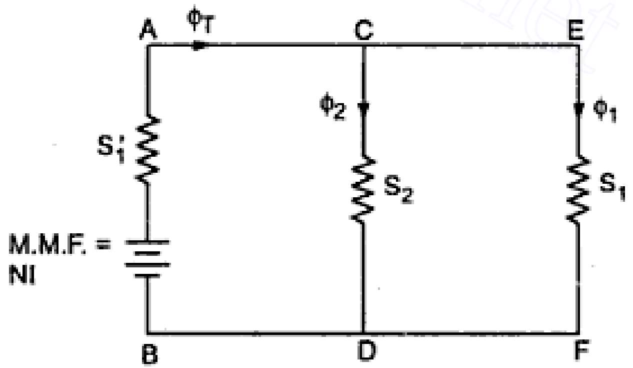

Fig. 5(b)

$\therefore$ Total m.m.f. $= NI = 500 \times 1.5 = 750$ AT

Now $\phi_T = \phi_1 + \phi_2$

Key Point: *M.M.F across CD and EF is same as both are in parallel.*

$\therefore$ Total m.m.f = m.m.f. for path AB + m.m.f. for path CD or path EF.

$$\text{m.m.f. for path AB} = \phi_T \times S_1'$$

$$\text{m.m.f. for path CD} = \phi_2 \times S_2 = \phi_T \times S_1$$

$$\therefore \quad \text{Total m.m.f.} = \phi_T \times S_1' + \phi_T \times S_1$$

$$S_1 = \frac{l_{i1}}{\mu_0 \mu_{r1} a_1} = \frac{32.9867 \times 10^{-2}}{4\pi \times 10^{-7} \times 850 \times 16 \times 10^{-4}} = 193.0145 \times 10^3 \text{ AT/Wb}$$

$$S_1' = \frac{l_{i1}'}{\mu_0 \mu_{r1} a_1} \text{ but } l_{i1}' = l_{i1} \text{ hence } S_1' = 193.0145 \times 10^3 \text{ AT/Wb}$$

$$\therefore \text{ Total m.m.f.} = (\phi_1 + \phi_T) 193.0145 \times 10^3 \quad \text{... (1)}$$

But $\phi_2 S_2 = \phi_1 S_1$ and $S_2 = \dfrac{l_{bar}}{\mu_0 \mu_{r2} a_2}$

$$\therefore \quad S_2 = \frac{17 \times 10^{-2}}{4\pi \times 10^{-7} \times 700 \times 2 \times 10^{-4}} = 966.297 \times 10^3 \text{ AT/Wb}$$

$$\therefore \quad \phi_2 \times 966.297 \times 10^3 = \phi_1 \times 193.0145 \times 10^3$$

$$\therefore \quad \phi_2 = 0.19974 \phi_1 \quad \text{... (2)}$$

But $\phi_T = \phi_1 + \phi_2$ hence using in (1),

$$\text{Total m.m.f.} = (\phi_1 + \phi_1 + \phi_2) \times 193.0145 \times 10^3$$

$$\therefore \quad 750 = (2\phi_1 + 0.19974 \phi_1) \times 193.0145 \times 10^3$$

$$\therefore \quad 750 = 2.19974 \phi_1 \times 193.0145 \times 10^3$$

$\therefore \phi_1 =$ **1.7664 mWb** ... Flux through other half of ring

c) *Two coils A and B in a magnetic circuit have 600 and 500 turns respectively. A current of 8 A in coil A produces a flux of 0.04 Wb. If the coefficient of coupling is 0.2. Calculate (1) Self inductance of coil A when B is open circuit. (2) Flux linkage with coil B. (3) Mutual inductance. (4) Em.f. induced in B when flux changes from zero to full value in 0.02 seconds.* **(5)**

Ans. : $N_A = 600$, $N_B = 500$, $K = 0.2$,

For coil A, $I_A = 8\text{ A}$, $\phi_A = 0.04$ Wb

1) $$L_A = \frac{N_A \phi_A}{I_A} = \frac{600 \times 0.04}{8} = \mathbf{3\ H}$$

SECTION - II

Q. 7 **a)** *Define as related to electrostatic :* **(4)**

1) Electric flux density 2) Permittivity 3) Dielectric strength 4) Capacitance

Ans. : Refer sections 1) 5.6. 2) 5.8. 3) 5.16. 4) 5.12.

b) *Derive an expression for energy stored in capacitor.* **(4)**

Ans. : Refer section 5.21.

c) *Three capacitor A, B, C have capacitances 20, 50 and 25 μF respectively. Calculate (1) Charge on each when connected in parallel to a 250 V supply. (2) Total capacitance and (3) Potential difference across each when connected in series.* **(6)**

Ans. : $C_A = 20\ \mu F$, $C_B = 50\ \mu F$, $C_C = 25\ \mu F$

1. Connected in parallel across V = 250 V. The voltage across each remains same as in parallel.

$$\therefore \quad V = \frac{Q_A}{C_A} = \frac{Q_B}{C_B} = \frac{Q_C}{C_C} = 250$$

$$\therefore \quad Q_A = 250 \times C_A = 250 \times 20 \times 10^{-6} = \mathbf{5\ mC}$$

$$\therefore \quad Q_B = 250 \times C_B = 250 \times 50 \times 10^{-6} = \mathbf{12.5\ mC}$$

$$\therefore \quad Q_C = 250 \times C_C = 250 \times 25 \times 10^{-6} = \mathbf{6.25\ mC}$$

2. Total capacitance when connected in series is,

$$\frac{1}{C_{eq}} = \frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_C} = \frac{1}{20 \times 10^{-6}} + \frac{1}{50 \times 10^{-6}} + \frac{1}{25 \times 10^{-6}}$$

$$\therefore \quad C_{eq} = \mathbf{9.0909\ \mu F}$$

3. When connected in series, the charge Q remains same for all of them.

$$\therefore \quad Q = C_A V_A = C_B V_B = C_C V_C$$

and $$Q = C_{eq} \times V = 9.099 \times 10^{-6} \times 250 = 2.27272\ mC$$

$$\therefore \quad V_A = \frac{Q}{C_A} = \mathbf{113.6363\ V}, \quad V_B = \frac{Q}{C_B} = \mathbf{45.4544\ V}, \quad V_C = \frac{Q}{C_C} = \mathbf{90.909\ V}$$

Key Point : *Check that* $V = V_A + V_B + V_C$

OR

Q. 8 **a)** *Define related to sinusoidal quantity.*

1) R.M.S. value 2) Average value 3) From factor 4) Peak factor

Ans. : Refer sections 1) 6.7. 2) 6.8. 3) 6.9. 4) 6.10.

b) *Explain the concepts of phase and phase difference in an alternating quantities.* **(4)**

Ans. : Refer section 6.13.

c) *Three voltages represented by*
$e_1 = 20 \sin \omega t$, $e_2 = 30\sin(\omega t - \pi/4)$, $e_3 = 40\cos(\omega t + \pi/6)$
act together in a circuit. Find an expression for the resultant voltage. Represent them by appropriate vectors. **(8)**

Ans. : From e_1,

$$\bar{E}_1 = \frac{E_{1m}}{\sqrt{2}} \angle 0° = \frac{20}{\sqrt{2}} \angle 0° = 14.1421 \angle 0° \text{ V}$$

From e_2,

$$\bar{E}_2 = \frac{E_{2m}}{\sqrt{2}} \angle -\frac{\pi}{4} = \frac{30}{\sqrt{2}} \angle -45°$$

$$= 21.2132 \angle -45° \text{ V} = 15 - j\,15 \text{ V}$$

$$e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right) = 40\sin(90° + \omega t + 30°) = 40\sin(\omega t + 120°)$$

$$\therefore \quad \bar{E}_3 = \frac{E_{3m}}{\sqrt{2}} \angle +120° = \frac{40}{\sqrt{2}} \angle +120°$$

$$= 28.2842 \angle +120° = -14.14.21 + j\,24.4948 \text{ V}$$

$$\therefore \quad \bar{E}_R = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 = 14.1421 + j\,0$$
$$+ 15 - j\,15 - 14.1421 + j\,24.4948$$

$$= 15 + j\,9.4948 \text{ V}$$

$$= 17.7524 \angle 32.333° \text{ V}$$

$$\therefore \quad E_R(\text{RMS}) = 17.7524 \text{ V}$$

i.e. $$E_{Rm} = \sqrt{2} \times 17.7524 = 25.1056 \text{ V}$$

Hence expression for the resultant is,

$$e_R = \mathbf{25.1056 \sin(\omega t + 32.333°) \text{ V}}$$

The phasor diagram is shown in the Fig. 7.

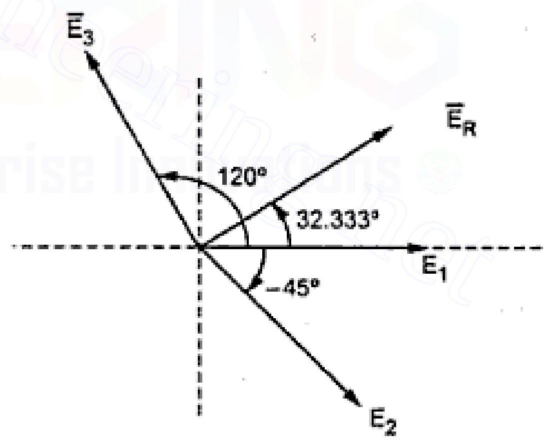

Fig. 7

Q. 9 a) *Define the following terms with their units :* **(4)**

1) Admittance 2) Susceptance 3) Power 4) Power factor

Ans. : Refer sections 1) 7.10.2 2) 7.10.5. 3) 7.5.4. 4) 7.5.6.

b) *Derive an expression for instantaneous current and power consumed when voltage of $v = V_m \sin \omega t$ is applied through pure inductance alone.* **(6)**

For load 2, $S_2 = \sqrt{P_2^2 + Q_2^2}$ i.e. $Q_2 = 48$ kVAR

For load 3, $S_3 = \sqrt{P_3^2 + Q_3^2}$ i.e. $Q_3 = 28.5657$ kVAR

$\therefore$ Q_T = total kVAR drawn = $Q_1 = Q_2 = Q_3$ = 76.5657 kVAR

The power triangle for the combined load is shown in the Fig. 8(b).

$$\therefore \quad \tan\phi_T = \frac{Q_T}{P_T} = \frac{76.5657}{102} = 0.75064$$

$$\therefore \quad \phi_T = 36.8935°$$

$$\therefore \quad \cos\phi_T = \mathbf{0.799\ lagging}$$

... Combined load p.f.

$$\therefore \quad S_T = \text{Combined kVA} = \sqrt{P_T^2 + Q_T^2}$$

$$= \sqrt{102^2 + 76.5657^2} = \mathbf{127.5394\ kVA}$$

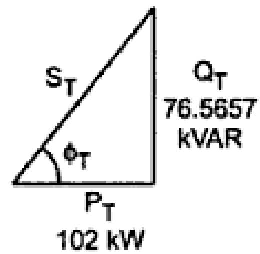

Fig. 8(b)

Q. 11 a) *Derive the relationship between the line current and phase current, line voltage and phase voltage, for a balanced three phase star-connected load connected across three phase supply. Derive also power consumed by load.* **(8)**

Ans. : Refer section 8.8.

b) *The 100 Ω non-inductive resistances are connected in (1) star (2) delta across a 400 V, 50 Hz, 3 phase supply. Calculate power taken from supply system in each case. In the event of one of the three resistances getting open - circuited, what should be the value of total power taken from the mains in each of the two cases ?* **(10)**

Ans. : The circuits for two cases are shown in the Fig. 9.

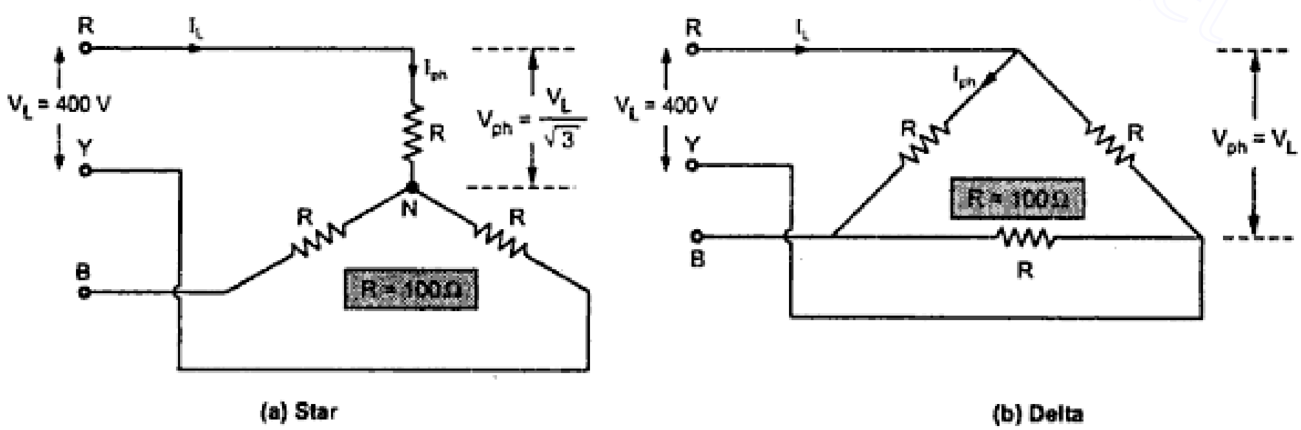

Fig. 9

a) Star :

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9401 \text{ V}$$

$$Z_{ph} = R_{ph} = 100\ \Omega,\ \cos\phi = 1$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{R_{ph}} = \frac{230.9401}{100} = 2.3094\ A$$

$$I_L = I_{ph} = 2.3094\ A$$

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 2.3094 \times 1 = \mathbf{1600\ W}$$

b) Delta :

$$V_{ph} = V_L = 400\ V$$

$$\therefore \quad I_{ph} = \frac{V_{ph}}{R_{ph}} = \frac{400}{100} = 4\ A$$

$$\therefore \quad I_L = \sqrt{3} I_{ph} = 4\sqrt{3}\ A$$

$$\therefore \quad P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 4\sqrt{3} \times 1 = \mathbf{4800\ W}$$

If now one of the three resistances is open circuited,

a) Star :

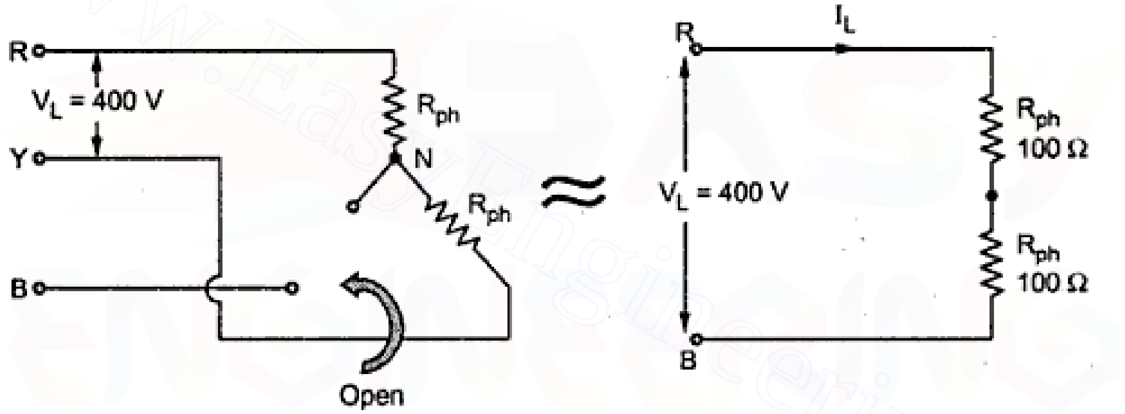

Fig. 9(a)

$$\therefore \quad I_L = \frac{V_L}{2R_{ph}} = \frac{400}{2 \times 100} = \mathbf{2\ A}$$

$$\therefore \quad P = I_L^2 R_{ph} + I_L^2 R_{ph} = 4 \times 100 + 4 \times 100 = \mathbf{800\ W}$$

b) Data :

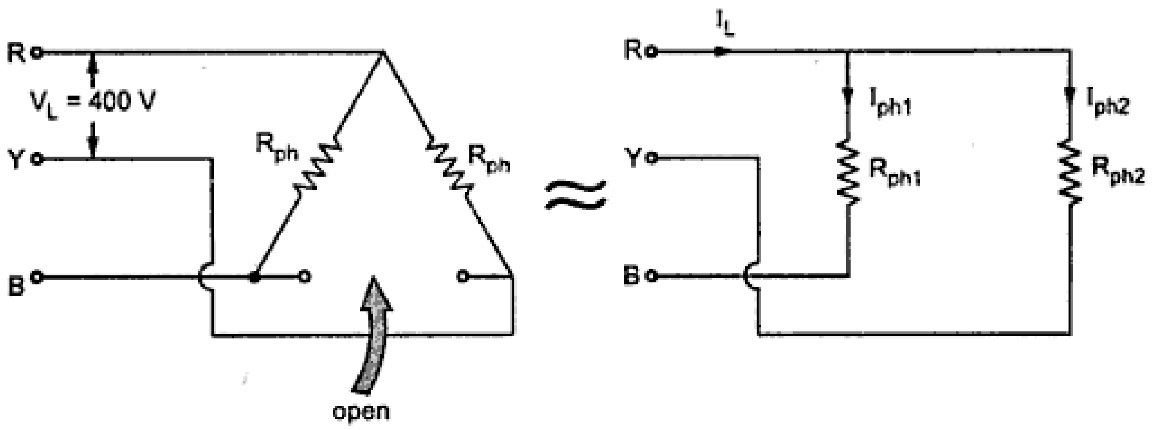

Fig. 9(b)

Basic Electrical Engineering

May 2009

F.E. (Common to all Branches) Semester - I

[Time : 3 Hours] **[Maximum Marks : 100]**

Instructions :

1) *Attempt Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6, Q. 7 or 8, Q. No. 9 or 10, Q. 11 or 12.*
2) *Answer to the two sections must be written in separate answer-books.*
3) *Figures to the right indicate full marks.*
4) *Use of non-programmable pocket size scientific calculator is permitted.*
5) *Neat diagrams must be drawn wherever necessary.*
6) *Assume suitable data, if necessary.*

SECTION - I

Q. 1 **a)** *Define insulation resistance and obtain an expression for insulation resistance of a single core cable.* **(8)**

Ans. : Refer section 1.12.

b) *Determine the current flowing at the instant of switching a 60 watt lamp on a 240 V supply. The ambient temperature is 24 °C. The filament temperature is 2000 °C and R.T.C. of a filament material is 0.005 per °C at 0 °C.* **(6)**

Ans. : Refer example 1.23 for the procedure and use the given data.

The answer is, **I = 2.4597 A.**

c) *Compare lead acid cell and nickel cadmium cell.* **(4)**

Ans. : Refer section 1.29

OR

Q. 2 **a)** *Write down chemical equations during charging of lead acid battery.* **(4)**

Ans. : Refer section 1.19.2.

b) *if α_1 and α_2 are RTCs of material at t_1 °C and t_2°C then prove that,*

$$\frac{\alpha_1}{\alpha_2} = 1 + \alpha_1 (t_2 - t_1)$$ **(6)**

Ans. : Refer section 1.11.3.

c) *An electric pump lifts 80 m^3 of water per hour to a height of 30 m. The pump efficiency is 85% and the motor efficiency is 75%. If the pump is used for 4 hours*

Step 3 : Find $R_{eq} = R_{TH}$, shorting the voltage sources.

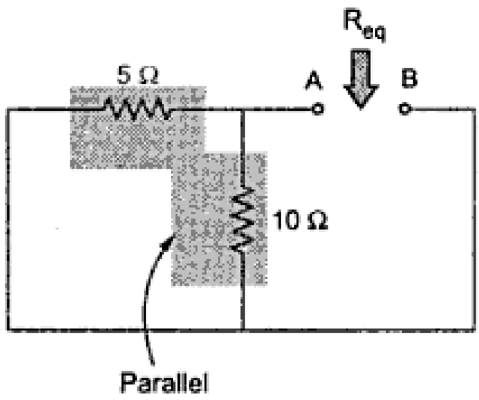

Fig. 1(c)

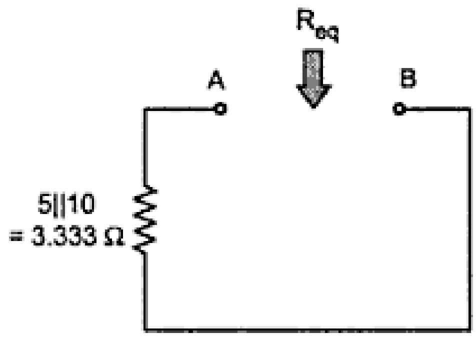

Fig. 1(d)

$$\therefore \qquad R_{eq} = 3.333\ \Omega$$

Step 4 : The Thevenin's equivalent is shown in the Fig. 1(e).

Step 5 : The current through branch AB is,

$$I_L = \frac{V_{TH}}{R_{eq} + R_L} = \frac{23.3333}{3.333 + 2}$$

$$= 4.375\ A \downarrow \quad \text{(From A to B)}$$

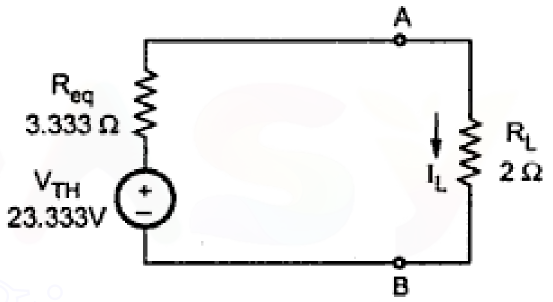

Fig. 1(e)

c) *State the formula to convert the star connected network into its equivalent delta connected network.* **(4)**

Ans. : Refer section 2.16.2.

OR

Q. 4 a) *State and explain maximum power transfer theorem.* **(6)**

Ans. : Refer section 2.20.

b) *Using Superposition theorem, calculate current flowing in* ***Brahcn A-B*** *for the circuit shown in Fig. 1.* **(10)**

Ans. : Step 1 : Consider 40 V source, short other sources.

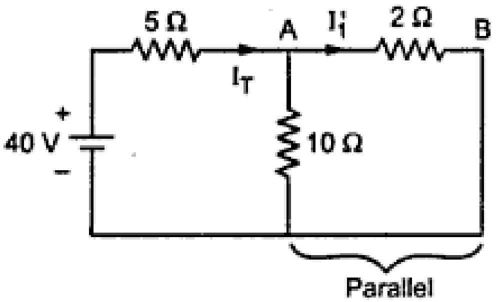

Fig. 2(a)

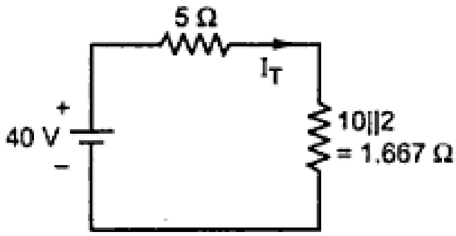

Fig. 2(b)

$$\therefore \quad I_T = \frac{40}{4+1.667} = 6 \text{ A}$$

Using current distribution rule,

$$I_1' = I_T \times \frac{10}{2+10} = \frac{6\times10}{2} = \mathbf{5 \text{ A (A to B)}}. \quad \text{... Due to 40 V only}$$

Step 2 : Consider 20 V source, short other sources.

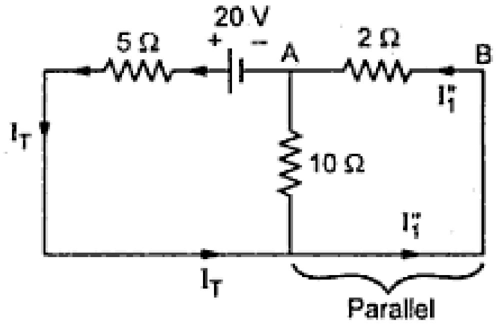

Fig. 2(c)

$$I_T = \frac{20}{5+(2||10)} = \frac{20}{5+1.667}$$

$$= 3 \text{ A}$$

$$\therefore \quad I_1'' = I_T \times \frac{10}{10+2}$$

$$= \frac{3\times10}{12}$$

$$= \mathbf{2.5 \text{ A (B to A)}} \quad \text{... Due to 20 V only}$$

Step 3 : Consider 10 V source, short other sources.

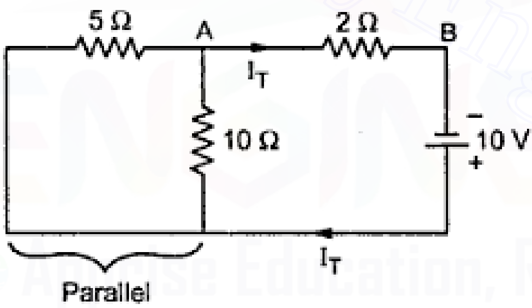

Fig. 2(d)

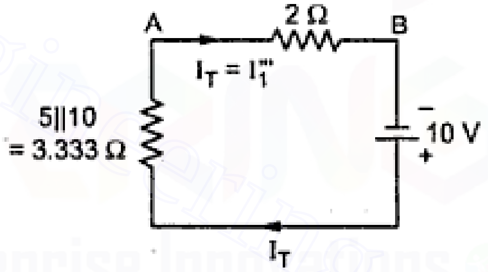

Fig. 2(e)

$$\therefore \quad I_1''' = I_T = \frac{10}{2+3.333} = \mathbf{1.875 \text{ A (A to B)}} \quad \text{... Due to 10 V only}$$

$$\therefore \quad I_{2\Omega} = I_1' + I_1'' + I_1''' = 5\text{(A to B)} + 2.5\text{(B to A)} + 1.875 \text{ (A to B)}$$

$$= 5 - 2.5 + 1.875 = \mathbf{4.375 \text{ A (From A to B)}}$$

This is same as obtained by Thevenin's theorem in Q. 3(B).

Q. 5 a) *Compare electric circuit and magnetic circuit.* **(5)**

Ans. : Refer section 3.20.

b) *Write a short note on magnetic leakage and fringing.* **(5)**

Ans. : Refer section 3.21.

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Contents

- **General**
  Concepts of e.m.f., p.d. and current, Resistance, Effect of temperature on resistance, Resistance temperature coefficient, Insulation resistance, S.I. units of work, Power and energy, Conversion of energy from one form to another in electrical, Mechanical and thermal systems, Batteries and cells, Their types, Primary cells and secondary cells, Lead Acid, Ni-Cd and Ni-MH batteries, Current capacity and cell ratings, Charging, importance of initial charging and discharging of batteries, Series and parallel battery connections, Maintenance procedure.
- **D.C. Circuits**
  Classification of electrical networks, Ohm's law, Kirchhoff's law and their applications for network solutions, Simplifications of networks using series and parallel combinations and star-delta conversions, Superposition theorem, Thevenin's theorem and maximum power transfer theorem.
- **Electromagnetism**
  Magnetic effect of an electric current, Cross and dot conventions, Right hand thumb rule and cork screw rule, Nature of magnetic field of long straight conductor, Solenoid and toroid, Concept of m.m.f. Flux, Flux density, Reluctance, Permeability and field strength, Their units and relationships, Simple series and parallel magnetic circuits, Comparison of electrical and magnetic circuit, Force on current carrying conductors placed in magnetic field, Fleming's left hand rule.
  Faradays laws of electromagnetic induction, Statically and dynamically induced e.m.f., Self and mutual inductance, Coefficient of couplings, Energy stored in magnetic field.
- **Electrostatics and A.C. Fundamentals**
  Electrostatics field, Electric flux density, Electric field strength, Absolute permittivity, Relative permittivity, Capacitance and capacitor, Composite dielectric capacitors, Capacitors in series and parallel, Energy stored in capacitors, Charging and discharging of capacitors and time constant.
  Sinusoidal voltages and currents, Their mathematical and graphical representation, Concept of instantaneous, Peak (maximum), Average and r.m.s. values, Frequency, Cycle period, Peak factor and form factor, Phase difference, Lagging, Leading and in phase quantities and phasor representation, Rectangular and polar representation of phasors.
- **Single Phase A.C. Circuits**
  Study of A.C. circuits consisting of pure resistance, Pure inductance, Pure capacitance and corresponding voltage-current phasor diagrams and waveforms, Development of concept of reactance, Study of series R-L, R-C, R-L-C circuit and resonance, Study of parallel R-L, R-C and R-L-C circuit, Concept of impedance, Admittance, Conductance and susceptance in case of above combinations and relevant voltage-current phasor diagrams, Concept of active, Reactive and apparent power and power factor.
- **Polyphase A.C. Circuits and Single Phase Transformers**
  Polyphase A.C. circuits : Concpet of three-phase supply and phase sequence. Voltages, Currents and power relations in three phase balanced star-connected loads and delta-connected loads along with phasor diagrams.
  Single phase transformers : Construction, Principle of working, e.m.f. equation, Voltage and current ratios, Losses, Definition of regulation and efficiency, Determination of these by direct loading method, Descriptive treatment of autotransformers and dimmerstats.

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Technical Publications Pune
1, Amit Residency, 412 Shaniwar Peth, Pune - 411030, M.S., India.
Telefax : +91 (020) 24495496/97, Email : technical@vtubooks.com
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