MULTICOLOUR ILLUSTRATIVE EDITION

A Textbook of
Refrigeration and
Air Conditioning

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CONTENTS

(v)

(vii)

(xii)

1

Introduction

1.1 Definition

The term 'refrigeration' may be defined as the process of removing heat from a substance under controlled conditions. It also includes the process of reducing and maintaining the temperature of a body below the general temperature of its surroundings. In other words, the refrigeration means a continued extraction of heat from a body whose temperature is already below the temperature of its surroundings.

For example, if some space (say in cold storage) is to be kept at $- 2°C$ (271K), we must continuously extract heat which flows into it due to leakage through the walls and also the

heat which is brought into it with the articles stored after the temperature is once reduced to $-2°C$ (271K). Thus in a refrigerator, heat is virtually being pumped from a lower temperature to a higher temperature. According to *Second Law of Thermodynamics, this process can only be performed with the aid of some external work. It is thus obvious that supply of power (say electric motor) is regularly required to drive a refrigerator. Theoretically, a refrigerator is a reversed heat engine or a heat pump which pumps heat from a cold body and delivers it to a hot body. The substance which works in a heat pump to extract heat from a cold body and to deliver it to a hot body is called a *refrigerant*.

The refrigeration system is known to the man since the middle of nineteenth century. The scientists, of the time, developed a few stray machines to achieve some pleasure. But it paved the way by inviting the attention of scientists for proper studies and research. They were able to build a reasonably reliable machine by the end of nineteenth century for refrigeration jobs. But with the advent of efficient rotary compressors and gas turbines, the science of refrigeration reached the present height. Today it is used for the manufacture of ice and similar products. It is also widely used for the cooling of storage chambers in which perishable foods, drinks and medicines are stored. The refrigeration has also wide applications in submarine ships, aircraft and rockets.

1.2 Fundamental Units

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called *fundamental units*.

1.3 Derived Units

Some units are expressed in terms of other units, which are derived from fundamental units, are known as *derived units* e.g. the unit of area, velocity, acceleration, pressure etc.

1.4 System of Units

There are only four systems of units, which are commonly used and universally recognised. These are known as:

1. C.G.S. units, 2. F.P.S. units, 3. M.K.S. units and 4. S.I. units.

Since the present courses of studies are conducted in S.I. system of units, therefore we shall discuss this system of unit only.

1.5 S.I. Units (International System of Units)

The 11th General Conference** of Weights and Measures have recommended a unified and systematically constituted system of fundamental and derived units for international use. This system is now being used in many countries. In India, the standards of Weights and Measures Act, 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.
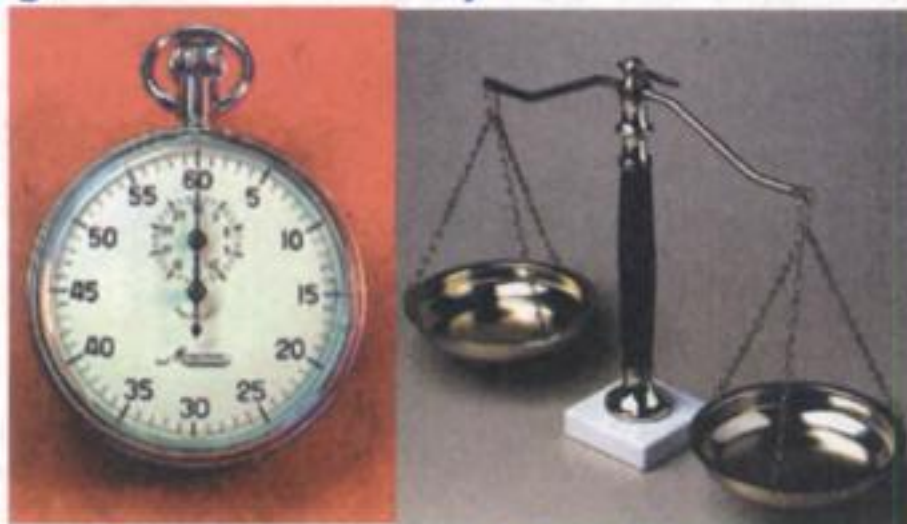
Stopwatch   . Simple balance

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*   Refer Art. 1.37.

**   It is known as General Conference of Weights and Measures (G.C.W.M.). It is an international organisation, of which most of the advanced and developing countries (including India) are members. The conference has been entrusted with the task of prescribing definitions for various units of weights and measures, which are the very basis of science and technology today.

In this system of units, there are seven fundamental units and two supplementary units, which cover the entire field of science and engineering. These units are shown in Table 1.1.

Table 1.1. Fundamental and supplementary units.

S.No.	Physical quantity	Unit
	Fundamental units	
1.	Length (*l*)	Metre (m)
2.	Mass (*m*)	Kilogram (kg)
3.	Time (*t*)	Second (s)
4.	Temperature (*T*)	Kelvin (K)
5.	Electric current (*I*)	Ampere (A)
6.	Luminous intensity (I_v)	Candela (cd)
7.	Amount of substance (*n*)	Mole (mol)
	Supplementary units	
1.	Plane angle (α, β, θ, ϕ)	Radian (rad)
2.	Solid angle (Ω)	Steradian (sr)

The derived units, which will be commonly used in this book, are given in Table 1.2.

Table 1.2. Derived units.

S.No.	Quantity	Symbol	Unit
1.	Linear velocity	*V*	m/s
2.	Linear acceleration	*a*	m/s^2
3.	Angular velocity	ω	rad/s
4.	Angular acceleration	α	rad/s^2
5.	Mass density	ρ	kg/m^3
6.	Force, Weight	*F, W*	N ; $1N = 1kg\text{-}m/s^2$
7.	Pressure	*p*	N/m^2
8.	Work, Energy, Enthalpy	*W,E,H*	J ; $1J = 1N\text{-}m$
9.	Power	*P*	W ; $1W = 1J/s$
10.	Absolute or dynamic viscosity	μ	N–s/m^2
11.	Kinematic viscosity	v	m^2/s
12.	Frequency	*f*	Hz ; $1Hz = 1$ cycle/s
13.	Gas constant	*R*	J/kg K
14.	Thermal conductance	*h*	W/m^2 K
15.	Thermal conductivity	*k*	W/m K
16.	Specific heat	*c*	J/kg K
17.	Molar mass or Molecular mass	*M*	kg/mol

1.6 Metre

The metre is defined as the length equal to 1 650 763.73 wave lengths in vacuum of the radiation corresponding to the transition between the levels 2 p_{10} and $5d_5$ of the Krypton – 86 atom.

1.7 Kilogram

The kilogram is defined as the mass of the international prototype (standard block of platinum–iridium alloy) of the kilogram, kept at the International Bureau of Weights and Measures at Sevres, near Paris.

1.8 Second

The second is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium – 133 atom.

1.9 Kelvin

The kelvin is defined as the fraction 1 / 273.16 of the thermodynamic temperature of the triple point of water.

Note: The triple point of water is taken as a fundamental fixed point having a temperature 273.16 K.

1.10 Presentation of Units and their Values

The frequent changes in the present day life are facilitated by an international body known as International Standard Organisation (ISO) which makes recommendations regarding international standard procedures. The implementation of ISO recommendations, in a country, is assisted by its organisation appointed for the purpose. In India, Bureau of Indian Standards (BIS) previously known as Indian Standards Institution (ISI) has been created for this purpose. We have already discussed that the fundamental units in S.I. units for length, mass and time are metre, kilogram and second respectively. But in actual practice, it is not necessary to express all lengths in metres, all masses in kilograms and all times in seconds. We shall, sometimes, use the convenient units, which are multiples or divisions of our basic units in tens. As a typical example, although the metre is the unit of length, yet a small length of one-thousandth of a metre proves to be more convenient unit, especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in front of the basic units to indicate the multiplier. The full list of these prefixes is given in the following table.

Table 1.3. Prefixes used in basic units.

Factor by which the unit is multiplied	Standard form	Prefix	Abbreviation
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

* These prefixes are generally becoming obsolete probably due to possible confusion. Moreover, it is becoming a conventional practice to use only those powers of ten which conform to 10^{3x}, where x is a positive or negative whole number.

With rapid development of Information Technology, computers are playing a major role in analysis. synthesis and design of machines.

1.11 Rules for S.I. Units

The eleventh General Conference of Weights and Measures recommended only the fundamental and derived units for S.I. system. But it did not elaborate the rules for the usage of the units. Later on many scientists and engineers held a number of meetings for the style and usage of S.I. units. Some of the decisions of the meetings are as follows:

1. For numbers having five or more digits, the digits should be placed in groups of three separated by spaces* (instead of commas) counting both to the left and right to the decimal point.

2. In a four digit number,** the space is not required unless the four digit number is used in a column of numbers with five or more digits.

3. A dash is to be used to separate units that are multiplied together. For example, newton × metre is written as N-m. It should not be confused with mN, which stands for millinewton.

4. Plurals are never used with symbols. For example, metre or metres are written as m.

5. All symbols are written in small letters except the symbols derived from the proper names. For example, N for newton and W for watt.

6. The units with names of scientists should not start with capital letter when written in full. For example, 90 newton and not 90 Newton.

At the time of writing this book, the authors sought the advice of various international authorities, regarding the use of units and their values. Keeping in view the international reputation of the authors, as well as international popularity of their books, it was decided to present units*** and their values as per recommendations of ISO and BIS. It was decided to use :

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*   In certain countries, comma is still used as the decimal mark.

**   In certain countries, a space is used even in a four digit number.

***   In some of the question papers of the universities and other examining bodies, standard values are not used. The authors have tried to avoid such questions in the text of the book. However, at certain places, the questions with sub-standard values have to be included, keeping in view the merits of the question from the reader's angle.

4500	not	4 500	or	4,500
7 589 000	not	7589000	or	7,58,90,00
0.012 55	not	0.01255	or	.01255
30×10^6	not	3,00,00,000	or	3×10^7

The above mentioned figures are meant for numerical values only. Now let us discuss about the units. We know that the fundamental units in S.I. system of units for length, mass and time are metre, kilogram and second respectively. While expressing these quantities, we find it time consuming to write the units such as metres, kilograms and seconds, in full, every time we use them. As a result of this, we find it quite convenient to use some standard abbreviations.

We shall use :

m	for metre or metres
km	for kilometre or kilometres
kg	for kilogram or kilograms
t	for tonne or tonnes
s	for second or seconds
min	for minute or minutes
N-m	for newton × metres (*e.g.* work done)
kN-m	for kilonewton × metres
rev	for revolution or revolutions
rad	for radian or radians

1.12 Mass and Weight

Sometimes much confusion and misunderstanding is created, while using the various systems of units in the measurement of force and mass. This happens, because of the lack of clear understanding of the difference between mass and weight. The following definitions of mass and weight should be clearly understood.
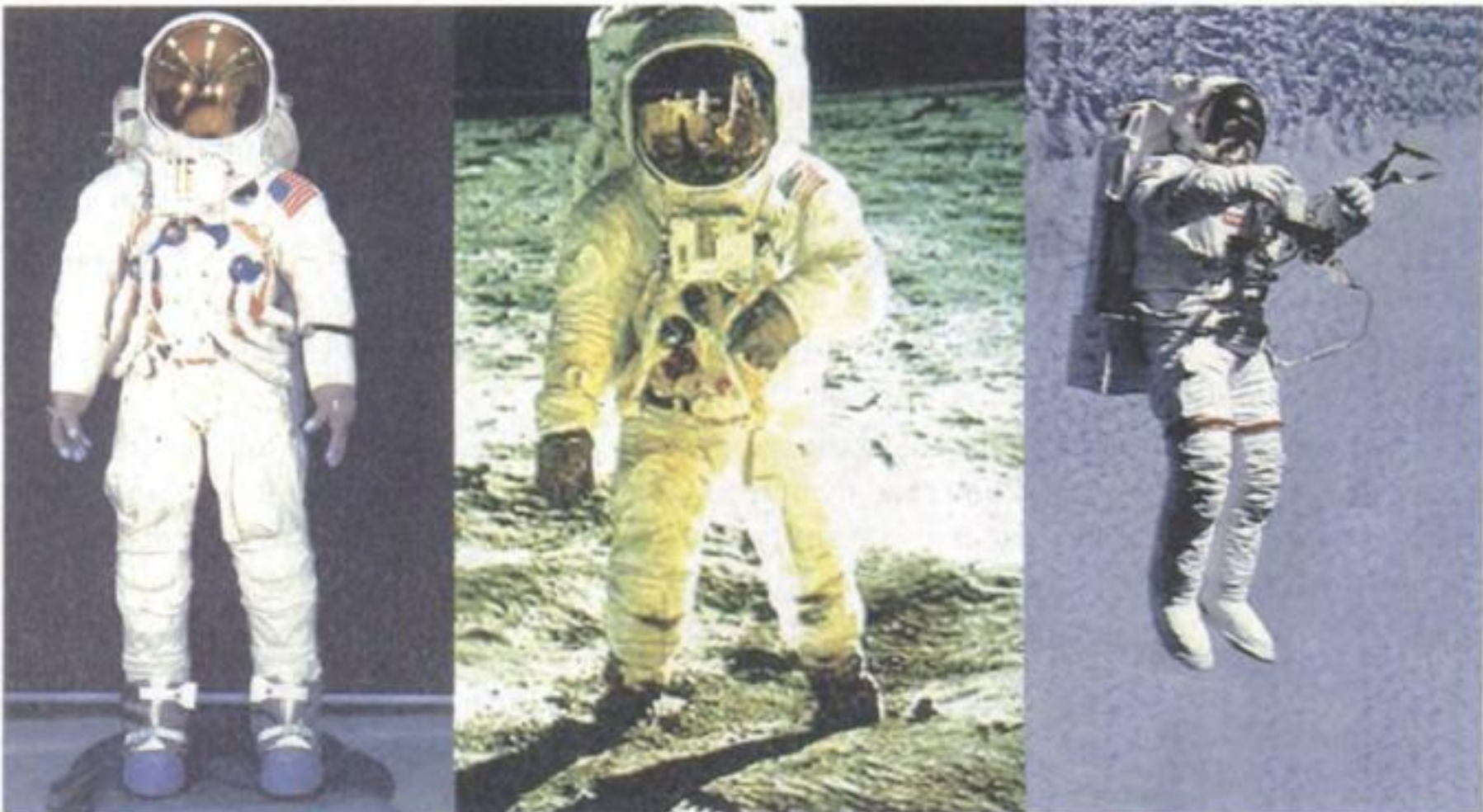
A man whose mass is 60 kg weighs 588.6 N (60 × 9.81 m/s²) on earth, approximately 96 N (60 × 1.6 m/s²) on moon and zero in space. But mass remains the same everywhere.

1. *Mass.* It is the amount of matter contained in a given body, and does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance.

2. *Weight.* It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with the distance of the body from the centre of the earth, therefore weight of the body will also vary with its position on the earth's surface (say latitude and elevation). It is thus obvious, that the weight is a *force.*

The earth's pull in metric units, at sea level and 45° latitude, has been adopted as one force unit and named as one kilogram of force. Thus it is a definite amount of force. But, unfortunately, it has the same name as the unit of mass. The weight of a body is measured by the use of a spring balance, which indicates the varying tension in the spring as the body is moved from place to place.

Note: The confusion in the units of mass and weight is eliminated, to a great extent, in S.I. units. In this system, mass is taken in kg and weight in newtons. The relation between the mass (m) and the weight (W) of a body is

$$W = m g \quad \text{or} \quad m = W/g$$

where W is in newtons, m is in kg and g is the acceleration due to gravity in m/s².

1.13 Force

It is an important factor in the field of Engineering science, which may be defined as an agent which produces or tends to produce, destroy or tends to destroy the motion. According to Newton's Second Law of Motion, the applied force or impressed force is directly proportional to the rate of change of momentum. We know that

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

Let
$$m = \text{Mass of the body,}$$
$$u = \text{Initial velocity of the body,}$$
$$v = \text{Final velocity of the body,}$$
$$a = \text{Constant acceleration, and}$$
$$t = \text{Time required to change the velocity from } u \text{ to } v.$$

∴ Change of momentum $= m v - m u$

and rate of change of momentum $= \dfrac{mv - mu}{t} = \dfrac{m(v - u)}{t} = ma$ $\qquad \ldots \left(\because \dfrac{v - u}{t} = a \right)$

or Force, $F \propto m a$ or $F = k m a$

where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces a unit acceleration to a body of unit mass.

∴ $$F = m a = \text{Mass} \times \text{Acceleration}$$

In S.I. system of units, the unit of force is called newton (briefly written as N). A *newton may be defined as the force while acting upon a mass of one kg produces an acceleration of 1 m/s² in the direction of which it acts.* Thus

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m/s}^2 = 1 \text{ kg-m/s}^2$$

1.14 Absolute and Gravitational Units of Force

We have already discussed that when a body of mass 1 kg is moving with an acceleration of 1 m/s², the force acting on the body is 1 newton (briefly written as 1 N). Therefore, when the same body is moving with an acceleration of 9.81 m/s², the force acting on the body is 9.81 N. But we denote 1 kg mass attracted towards the earth with an acceleration of 9.81 m/s² as 1 kilogram-

force (briefly written as kgf) or 1 kilogram-weight (briefly written as kg-wt). It is thus obvious, that

$$1 \text{ kgf} = 1 \text{ kg} \times 9.81 \text{ m/s}^2 = 9.81 \text{ kg-m/s}^2 = 9.81 \text{ N}$$

$$\ldots (\because 1\text{N} = 1 \text{ kg-m/s}^2)$$

The above unit of force *i.e.* kilogram force (kgf) is called *gravitational* or *engineer's unit of force*, whereas newton is the *absolute* or *scientific* or *S.I. unit of force*. It is thus obvious, that the gravitational or engineer's units of force are '*g*' times the unit of force in the absolute or S.I. units.

It will be interesting to know that the *mass of the body in absolute units is numerically equal to the weight of the same body in gravitational units*. For example, consider a body whose mass,

$$m = 100 \text{ kg}$$

Therefore the force, with which the body will be attracted towards the centre of the earth,

$$F = ma = mg = 100 \times 9.81 = 981 \text{ N}$$

Now, as per definition, we know that the weight of a body is the force, by which it is attracted towards the centre of the earth. Therefore weight of the body,

$$W = 981 \text{ N} = 981 / 9.81 = 100 \text{ kgf} \qquad \ldots (\because 1\text{kgf} = 9.81 \text{ N})$$

In brief, the weight of a body of mass *m* kg at a place where gravitational acceleration is '*g*' m/s^2 is *m.g* newtons.

1.15 Thermodynamic Systems

The *thermodynamic system* (or simply known as *system*) may be broadly defined as a *definite area* or *a space* where some thermodynamic process is taking place. It is a region where our attention is focussed for studying a thermodynamic process. A little observation will show that a thermodynamic system has its boundaries and anything outside the boundaries is called its *surroundings* as shown in Fig. 1.1. These boundaries may be *fixed* like that of a tank enclosing a certain mass of compressed gas, or *movable* like the boundary of a certain volume of liquid in a pipe line.

The thermodynamic systems may be classified into the following three groups :

1. Closed system ; 2. Open system ; and 3. Isolated system.

These systems are discussed, in detail, as follows :

1. *Closed system.* This is a system of fixed mass and identity whose boundaries are determined by the space of the matter (working substance) occupied in it.
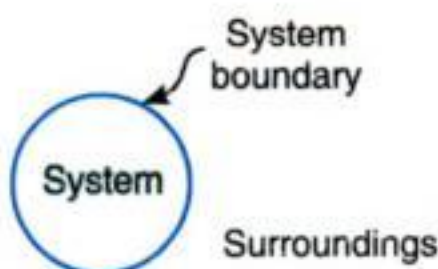
Fig. 1.1. Thermodynamic system.
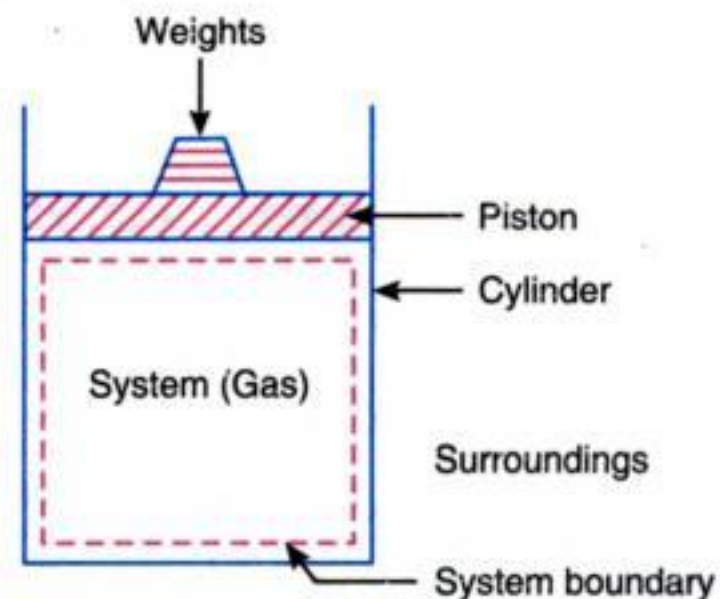
Fig. 1.2. Closed thermodynamic system.

A closed system is shown in Fig. 1.2. The gas in the cylinder is considered as a system. If heat is supplied to the cylinder from some external source, the temperature of the gas will increase and the piston will rise.

As the piston rises, the boundary of the system moves. In other words, the heat and work energy crosses the boundary of the system during this process, but there is no addition or loss of the original mass of the working substance. It is thus obvious, that the mass of the working substance, which comprises the system, is fixed.

Thus, a closed system does not permit any mass transfer across its boundary, but it permits transfer of energy (heat and work).

2. *Open system.* In this system, the mass of the working substance crosses the boundary of the system. Heat and work may also cross the boundary. Fig. 1.3 shows the diagram of an air compressor which illustrates an open system.

The working substance crosses the boundary of the system as the low pressure (L.P.) air enters the compressor and leaves the high pressure (H.P.) air. The work crosses the boundary of the system through the driving shaft and the heat is transferred across the boundary from the cylinder walls.

Thus, an open system permits both mass and energy (heat and work) transfer across the boundaries and the mass within the system may not be constant.
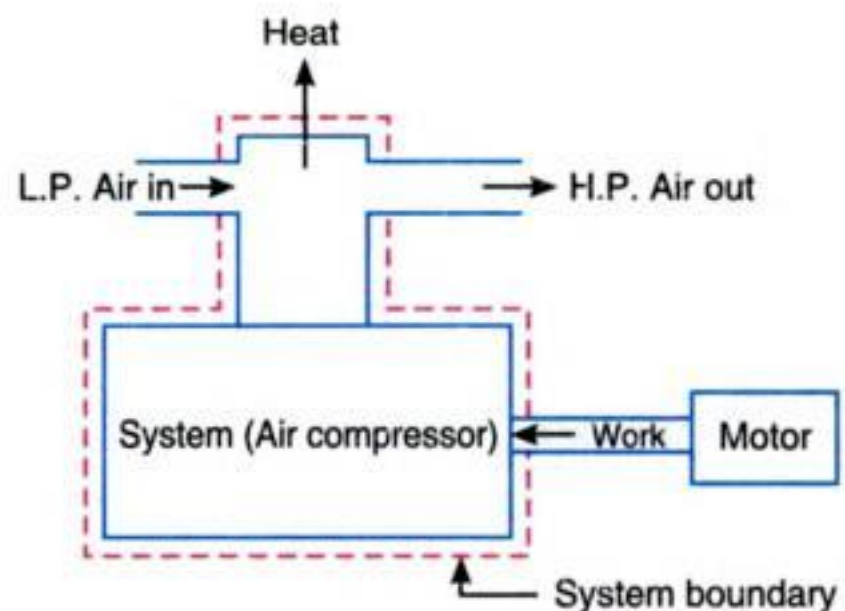
Fig. 1.3. Open thermodynamic system.

Note: An open system may be referred to as *control volume.* An open system is equivalent in every respect to a control volume, but the term open system is used throughout this text as it specifically implies that the system can have mass and energy crossing the system boundary.

3. *Isolated system.* A system which is completely uninfluenced by the surroundings is called an isolated system. It is a system of fixed mass and no heat or work energy cross its boundary. In other words, an isolated system does not have transfer of either mass or energy (heat or work) with the surroundings. An open system with its surroundings (known as an universe) is an example of an isolated system.

Note: The practical examples of isolated system are rare. The concept of this system is particularly useful in formulating the principles derived from the Second Law of Thermodynamics.

1.16 Properties of a System

The state of a system may be identified or described by certain observable quantities such as volume, temperature, pressure and density etc. All the quantities, which identify the state of a system, are called *properties*.

Note: Thermodynamics deals with those quantities also which are not properties of any system. For example, when there is a flow of energy between a system and its surroundings, the energy transferred is not a property of the system or its surroundings.

The thermodynamic properties of a system may be divided into the following two general classes:

1. *Extensive properties.* A quantity of matter in a given system is divided, notionally into a number of parts. The properties of the system, whose value for the entire system is equal to the sum of their values for the individual parts of the system are called *extensive properties, e.g.* total volume, total mass and total energy of a system are its extensive properties.

2. *Intensive properties.* It may be noticed that the temperature of the system is not equal to the sum of the temperatures of its individual parts. It is also true for pressure and density of the system. Thus properties like temperature, pressure and density are called *intensive properties*.

Note: The ratio of any extensive property of a system to the mass of the system is called an average specific value of that property (also known as intensive property) *e.g.* specific volume of a system (v_s) is the ratio of the total volume (v) of the system to its total mass (m). Mathematically,

$$v_s = v/m$$

The specific volume is an intensive property.

1.17 State of a System

The state of a system (when the system is in thermodynamic equilibrium) is the condition of the system at any particular moment which can be identified by the statement of its properties, such as pressure, volume, temperature etc. The number of properties which are required to describe a system depends upon the nature of the system.

Consider a system (gas) enclosed in a cylinder and piston arrangement as shown in Fig. 1.4. Let the system is initially in equilibrium when the piston is at position 1, represented by its properties p_1, v_1 and T_1. When the system expands, the piston moves towards right and occupies the final position at 2. At this, the system is finally in the equilibrium state represented by the properties p_2, v_2 and T_2.

Fig. 1.4. State of a system.

The initial and final states, on the pressure-volume diagram, are shown in Fig. 1.4.
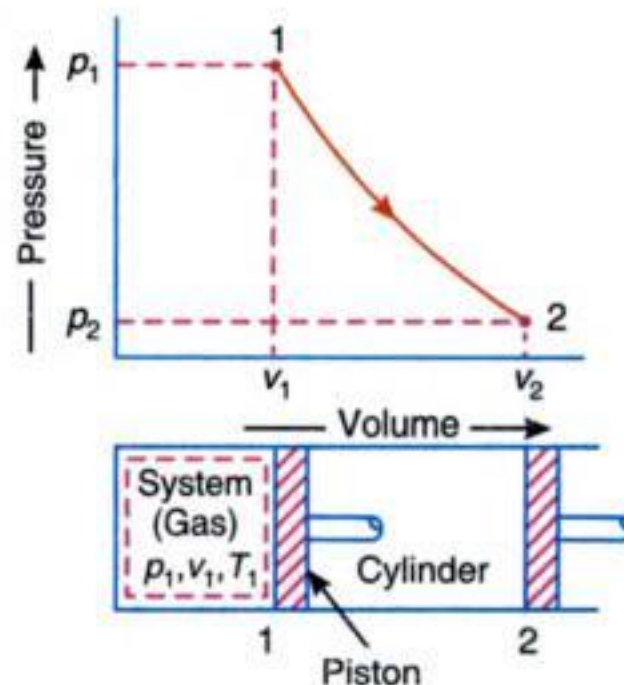
1.18 Temperature

It is an intensive thermodynamic property, which determines the degree of hotness or the level of heat intensity of a body. A body is said to be at a *high temperature* or *hot*, if it shows high level of heat intensity in it. Similarly, a body is said to be at a *low temperature* or *cold*, if it shows a low level of heat intensity.

The temperature of a body is measured with the help of an instrument known as *thermometer* which is in the form of a glass tube containing mercury in its stem. Following are the two commonly used scales for measuring the temperature of a body :

1. Celsius or centigrade scale, and 2. Fahrenheit scale.

Each of these scales is based on two fixed points known as *freezing point of water* under atmospheric pressure or *ice point* and the *boiling point of water* or *steam point*.

1. Celsius or centigrade scale. This scale was first used by Celsius in 1742. This scale is mostly used by engineers and scientists. The freezing point of water on this scale is marked as zero, and the boiling point of water as 100. The space between these two points has 100 equal divisions, and each division represents one degree Celsius (written as °C).

2. Fahrenheit scale. This scale was first used in 1665. In this scale, the freezing point of water is marked as 32 and the boiling point of water as 212. The space between these two points has 180 equal divisions and each division represents one degree Fahrenheit (written as °F).

Note: The relation between Celsius scale and Fahrenheit scale is given by :

$$\frac{C}{100} = \frac{F-32}{180} \quad \text{or} \quad \frac{C}{5} = \frac{F-32}{9}$$

1.19 Absolute Temperature

As a matter of fact, the zero readings of Celsius and Fahrenheit scales are chosen arbitrarily for the purpose of simplicity. It helps us in our calculations, when changes of temperature in a

process are known. But, whenever the value of temperature is used in equations relating to fundamental laws, then the value of temperature, whose reference point is true zero or absolute zero, is used. The temperature, below which the temperature of any substance can not fall, is known as *absolute zero temperature*.

The absolute zero temperature, for all sorts of calculations, is taken as –273°C in case of Celsius scale and – 460°F in case of Fahrenheit scale. The temperatures measured from this zero are called *absolute temperatures*. The absolute temperature in Celsius scale is called degrees Kelvin (briefly written as K)*; such that K = °C + 273. Similarly, absolute temperature in Fahrenheit scale is called degrees Rankine (briefly written as °R); such that °R = °F + 460.
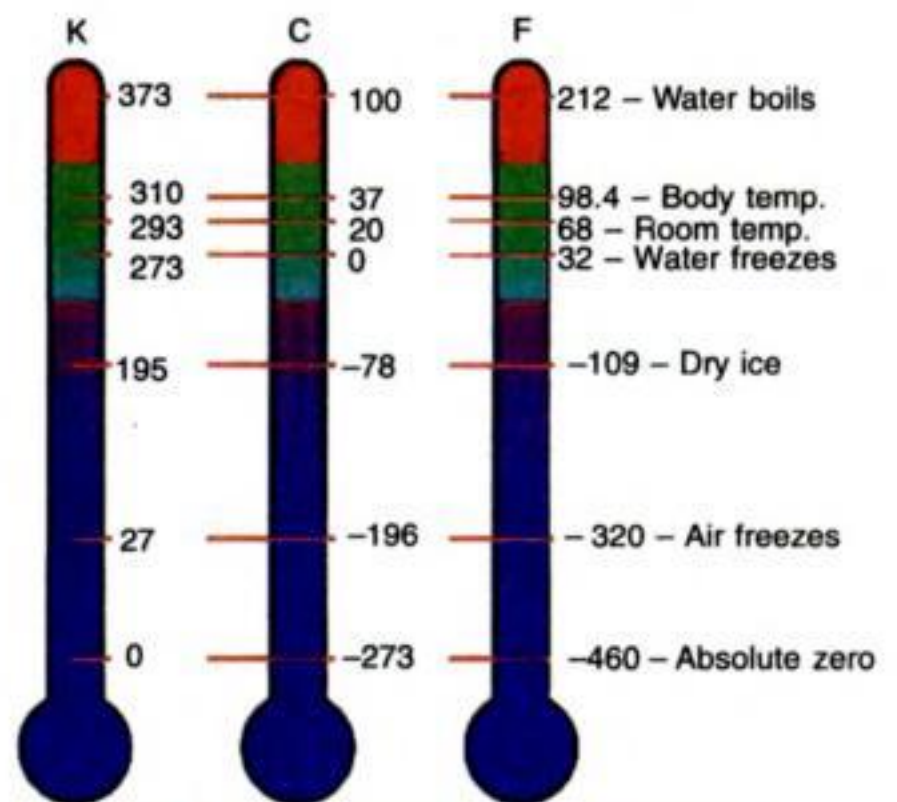
Relation between various temperatures.

1.20 Thermodynamic Equilibrium

A system is said to be in thermodynamic equilibrium, if it satisfies the following three requirements of equilibrium.

1. Mechanical equilibrium. A system is said to be in mechanical equilibrium, when there is no unbalanced forces acting on any part of the system or the system as a whole.

2. Thermal equilibrium. A system is said to be in thermal equilibrium, when there is no temperature difference between the parts of the system or between the system and the surroundings.

3. Chemical equilibrium. A system is said to be in chemical equilibrium, when there is no chemical reaction within the system and also there is no movement of any chemical constituent from one part of the system to the other.

1.21 Equality of Temperature

Consider two bodies of the same or different materials, one hot and the other cold. When these bodies are brought in contact, the hot body becomes colder, and the cold body becomes warmer. If these bodies remain in contact for some time, a state reaches when there is no further observable change in the properties of the two bodies. This is a state of thermal equilibrium and at this stage, the two bodies have the equal temperatures. It thus follows that when two bodies are in thermal equilibrium with each other, their temperatures are equal.

1.22 Pressure

The term 'pressure' may be defined as the normal force per unit area. The unit of pressure depends upon the units of force and area.

In S.I. system of units, the practical unit of pressure is N/mm^2, N/m^2, kN/m^2, MN/m^2 etc. But sometimes a bigger unit of pressure (known as bar) is used, such that,

$$1 \text{ bar } = 1 \times 10^5 \text{ N/m}^2 = 0.1 \times 10^6 \text{ N/m}^2 = 0.1 \text{ MN/m}^2$$

Sometimes the pressure is expressed in another unit, called Pa (named after Pascal) and kPa, such that

$$1 \text{ Pa } = 1 \text{ N/m}^2 \quad \text{and} \quad 1 \text{ kPa} = 1 \text{kN/m}^2$$

* In S.I. units, degrees Kelvin is not written as °K but only K.

1.23 Gauge Pressure and Absolute Pressure

All the pressure gauges read the difference between the actual pressure in any system and the atmospheric pressure. The reading of the pressure gauge is known as *gauge pressure*, while the actual pressure is called *absolute pressure*. Mathematically,

<div align="center">Absolute pressure = Atmospheric pressure + Gauge pressure</div>

This relation is used for pressures above atmospheric, as shown in Fig. 1.5 (*a*). For pressures below atmospheric, the gauge pressure will be negative. This negative gauge pressure is known as *vacuum pressure*. Therefore

<div align="center">Absolute pressure = Atmospheric pressure – Vacuum pressure</div>

This relation is shown in Fig. 1.5 (*b*).
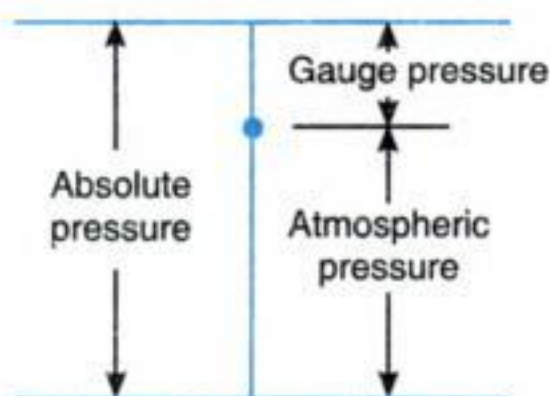
(*a*) Relation between absolute, atmospheric and gauge pressure.
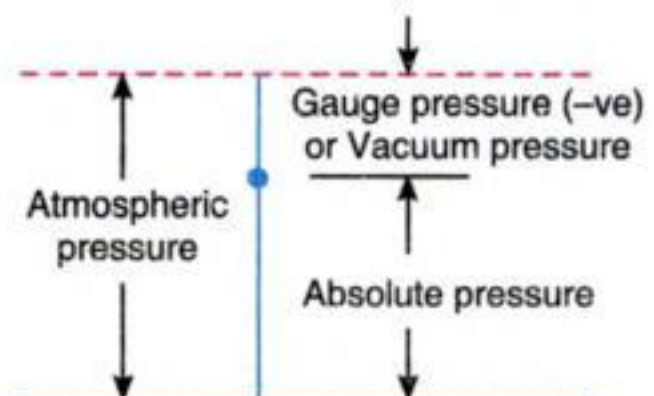
(*b*) Relation between absolute, atmospheric and vacuum pressure.

<div align="center">Fig. 1.5</div>

The standard value of atmospheric pressure is taken as 1.013 bar (or 760 mm of Hg) at sea level.

Note: We know that

$$1 \text{ bar} = 10^5 \text{ N/m}^2$$

$\therefore$ Atmospheric pressure $= 1.013 \times 10^5 = 1013 \times 10^2 \text{ N/m}^2$

We also know that atmospheric pressure

$$= 760 \text{ mm of Hg}$$

$\therefore$ $1 \text{ mm of Hg} = 1013 \times 10^2 / 760 = 133.3 \text{ N/m}^2$

or $1 \text{ N/m}^2 = 760 / 1013 \times 10^2 = 7.5 \times 10^{-3} \text{ mm of Hg}$

1.24 Normal Temperature and Pressure (N.T.P.)

The conditions of temperature and pressure at 0°C (273K) temperature and 760 mm of Hg pressure are termed as *normal temperature* and *pressure* (briefly written as N.T.P.).

1.25 Standard Temperature and Pressure (S.T.P.)

The temperature and pressure of any gas, under standard atmospheric conditions, is taken as 15°C (288K) and 760 mm of Hg respectively.

1.26 Energy

The energy is defined as the capacity to do work. In other words, a system is said to possess energy when it is capable of doing work. The energy possessed by a system is of the following two types:

1. Stored energy, and 2. Transit energy (or energy in transition)

The *stored energy* is the energy possessed by a system within its boundaries. The potential energy, kinetic energy and internal energy are the examples of stored energy.

The *transit energy* (or energy in transition) is the energy possessed by a system which is capable of crossing its boundaries. The heat, work and electrical energy are the examples of transit energy.

It may be noted that only the stored energy is a thermodynamic property whereas the transit energy is not a thermodynamic property as it depends upon the path.

1.27 Types of Stored Energy

We have discussed above that the potential energy, kinetic energy and internal energy are the different types of stored energy. These energies are discussed, in detail, as follows:

1. Potential energy. It is the energy possessed by a body or a system, for doing work, by virtue of its position above the ground level. For example, a body raised to some height above the ground level possesses potential energy because it can do some work by falling on earth's surface.

Let $\qquad W$ = Weight of the body,

$\qquad m$ = Mass of the body,

$\qquad z$ = Distance through which the body falls, and

$\qquad g$ = Acceleration due to gravity = 9.81 m/s^2.

∴ Potential energy, $\qquad PE = Wz = mgz$

It may be noted that

(*a*) When W is in newtons and z in metres, then potential energy will be in N-m.

(*b*) When m is in kg and z in metres, then the potential energy will also be in N-m, as discussed below:

We know that potential energy,

$$PE = mgz = \text{kg} \times \frac{\text{m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \dots \left(\because 1\,\text{N} = \frac{1\,\text{kg-m}}{\text{s}^2} \right)$$

2. Kinetic energy. It is the energy possessed by a body or a system, for doing work, by virtue of its mass and velocity of motion.

Let $\qquad m$ = Mass of the body, and

$\qquad V$ = Velocity of the body.

When m is in kg and V is in m/s, then kinetic energy will be in N-m, as discussed below :

We know that kinetic energy,

$$KE = \frac{1}{2} m V^2 = \text{kg} \times \frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg-m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \dots \left(\because 1\,\text{N} = \frac{1\,\text{kg-m}}{\text{s}^2} \right)$$

3. Internal energy. It is the energy possessed by a body or a system due to its molecular arrangement and motion of the molecules. It is usually represented by U.

In the study of thermodynamics, we are mainly concerned with the change in internal energy (dU) which depends upon the change in temperature of the system.

Notes: 1. The total energy of the system (E) is equal to the sum of the above three types of energies. Mathematically,

$$E = PE + KE + U = mgz + \frac{1}{2} \times m V^2 + U$$

Any other form of the energy such as chemical energy, electrical energy etc. is neglected. For unit mass, the above expression is written as

$$e = pe + ke + u = gz + \frac{V^2}{2} + u$$

2. When the system is stationary and the effect of gravity is neglected, then $PE = 0$, and $KE = 0$. In such a case,

$$E = U \quad \text{or} \quad e = u$$

1.28 Heat

The heat is defined as the energy transferred, without transfer of mass across the boundary of a system because of a temperature difference between the system and the surroundings. It is usually represented by Q and is expressed in joule (J) or kilo-joule (kJ).

The heat can be transferred in three distinct ways, *i.e.* conduction, convection and radiation. The transfer of heat through solids takes place by *conduction*, while the transfer of heat through fluids is by *convection*. The *radiation* is an electromagnetic wave phenomenon in which energy can be transported through transparent substances and even through a vacuum. These three modes of heat transfer are quite different, but they have one factor in common. All these modes occur across the surface area of a system because of a temperature difference between the system and the surroundings.

The following points are worth noting about heat :

1. The heat is transferred across a boundary from a system at a higher temperature to a system at a lower temperature by virtue of the temperature difference.
2. The heat is a form of transit energy which can be identified only when it crosses the boundary of a system. It exists only during transfer of energy into or out of a system.
3. The heat flowing into a system is considered as *positive* and the heat flowing out of a system is considered as *negative*.

1.29 Sensible Heat

When a substance is heated and the temperature rises as the heat is added, the increase in heat is called *sensible heat*. Similarly, when heat is removed from a substance and the temperature falls, the heat removed (or subtracted) is called sensible heat. It is usually denoted by h_f.

Thus, the sensible heat may be defined as the heat which causes a change in temperature in a substance. For example, the heat absorbed in heating of water upto the boiling temperature is the sensible heat.

1.30 Latent Heat

All pure substances are able to change their state. Solids become liquids and liquids become gas. These changes of state occur at the same temperature and pressure combinations for any given substance. It takes the addition of heat or the removal of heat to produce these changes. The heat which brings about a change of state with no change in temperature is called *latent* (or hidden) *heat*. It is usually denoted by h_{fg}.

The latent heat of ice is 335 kJ/kg. This means that the heat absorbed by 1 kg of ice to change it into water at 0°C and at atmospheric pressure is 335 kJ. This heat is called *latent heat of fusion* (or melting) *of ice*. The water starts vaporising at 100°C (*i.e.* boiling temperature) and changes its state from water to steam (*i.e.* gaseous form). The heat absorbed during this change of state from liquid to gas is called *latent heat of vaporisation* or *condensation*. The latent heat of vaporisation of water at 100°C and at atmospheric pressure is 2257 kJ/kg.

1.31 Specific Heat

The specific heat of a substance may be broadly defined as the amount of heat required to raise the temperature of a unit mass of any substance through one degree. It is generally denoted by c. In S.I. system of units, the unit of specific heat (c) is taken as kJ/kg K. If m kg of a substance

of specific heat c is required to raise the temperature from an initial temperature of T_1 to a final temperature of T_2, then heat required,

$$Q = m\, c\, (T_2 - T_1)\ kJ$$

where T_1 and T_2 may be either in °C or K.

Thermometer

Thermometer

Object # 1

Object # 2

Heat **Q**

Heat is transferred from warmer object to cooler object.
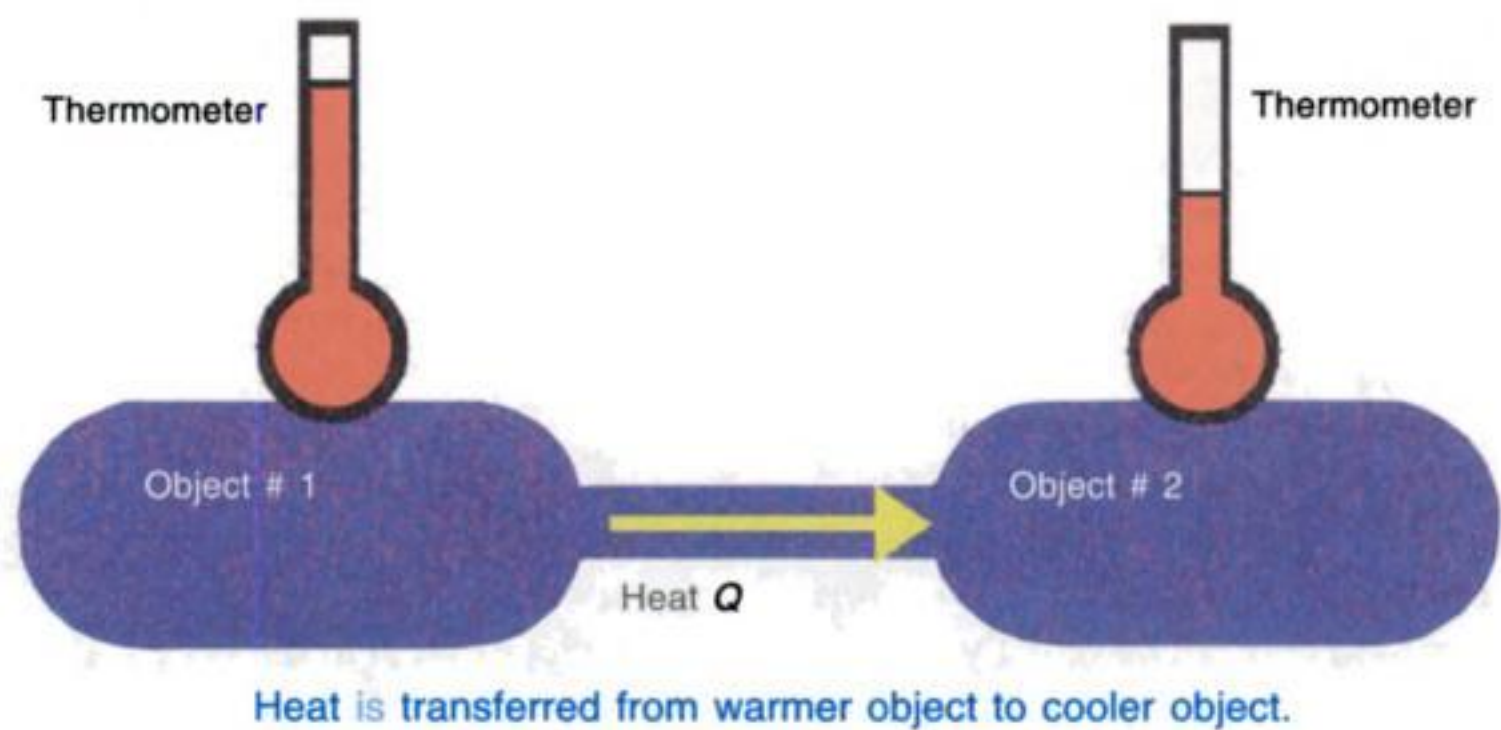
The average values of specific heats for some commonly used substances are given in following table :

Table 1.4. Values of specific heats for some commonly used substances.

Substance	Specific heat (kJ/kg K)	Substance	Specific heat (kJ/ kg K)
Steel	0.490	Air	1.000
Copper	0.406	Nitrogen	1.010
Glass	0.783	Oxygen	0.925
Mercury	0.138	R-12 (Dichlorodifluoromethane)	0.892
Brick	0.837	R-22 (Monochlorodifluoromethane)	1.089
Water	4.187	R-717 (Ammonia)	4.606
Ice	2.110	R-744 (Carbon dioxide)	2.512
Steam	2.094	Salt brine 20%	3.560

1.32 Mechanical Equivalent of Heat

It was established by Joule that heat and mechanical energies are mutually convertible. He established, experimentally, that there is a numerical relation between the unit of heat and the unit of work. This relation is denoted by J (named after Joule) and is known as Joule's equivalent or mechanical equivalent of heat.

Note : In S.I. system of units, the unit of work done is joule or kilo joule (such that 1J = 1 N-m or 1kJ = 1kN-m). The unit of heat is also joule or kilo joule. So we can straightway convert heat units into mechanical units and vice versa.

1.33 Work

In mechanics, work is defined as the product of the force (F) and the distance moved (x) in the direction of the force. Mathematically,

$$\text{Work done} = F \times x$$

The unit of work depends upon the unit of force and the distance moved. In S.I. system of units, the practical unit of work is newton-metre (briefly written as N-m). The work of 1 N-m is known as joule (briefly written as J) such that 1 N-m = 1 J.

In thermodynamics, work may be defined as follows:

1. According to Obert, work is defined as *the energy transferred (without the transfer of mass) across the boundary of a system because of an intensive property difference other than temperature that exists between the system and the surroundings.*

In engineering practice, the intensive property difference is the pressure difference. The pressure difference (between the system and the surrounding) at the surface of the system gives rise to a force and the action of this force over a distance is called mechanical work.

In some cases, the intensive property difference may be the electrical potential difference between the system and the surrounding. In this case, the resulting energy transfer across the system and boundary is known as electrical work.

2. According to Keenan, *work is said to be done by a system during a given operation if the sole effect of the system on things external to the system (surroundings) can be reduced to the raising of a weight.*

The weight may not be actually raised but the net effect external to the system should be the raising of a weight.

For example, consider a system consisting of a storage battery, as shown in Fig. 1.6. The terminals connected to a resistance through a switch constitute external to the system (*i.e.* surroundings). When the switch is closed for a certain period of time, then the current will flow through the battery and the resistance, as a result the resistance becomes warmer. This clearly shows that the system (battery) has interaction with the surroundings. In other words, the energy transfer (electrical energy) has taken place between the system and the surroundings because of potential difference (not the temperature).
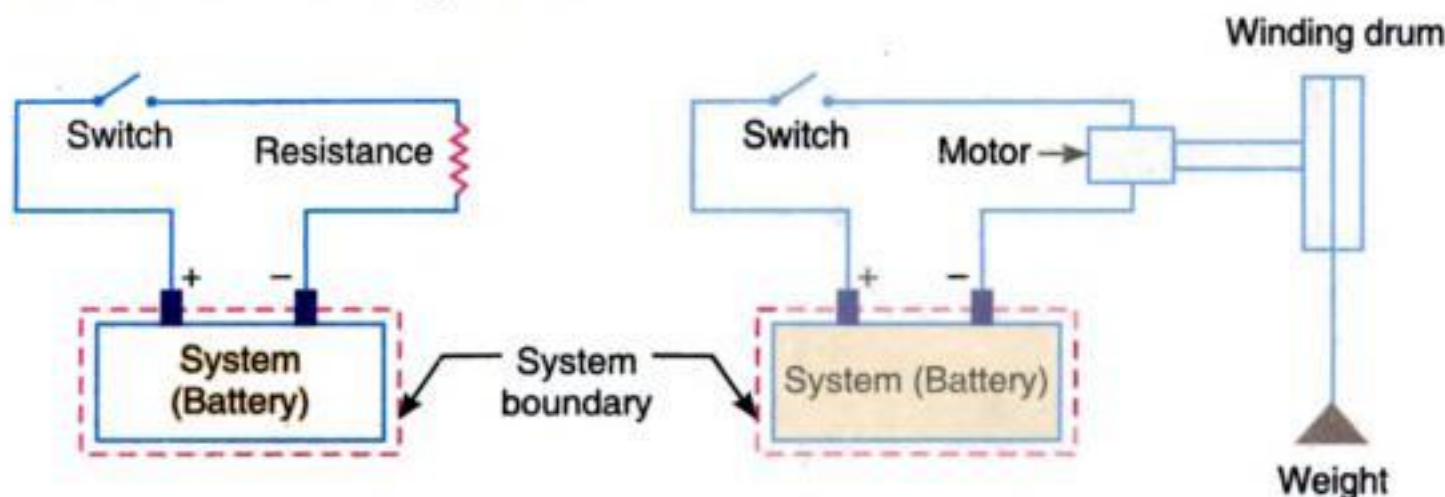
Fig. 1.6. Thermodynamic work.

Now according to the mechanics definition of work, there is no force which moves through a distance. Thus no work is done by the system. However, according to the thermodynamic definition, the work is done by the system because the resistance can be replaced by an ideal motor (100% efficient) driving a winding drum, thereby raising weight. Thus, the sole effect external to the system (surroundings) has been reduced to the raising of a weight. Hence, thermodynamic work is done by the system.

Note : The work done by the system is considered as *positive* work, while the work done on the system is considered as *negative* work.

1.34 Heat and Work - A Path Function

Consider that a system from an initial equilibrium state 1 reaches to a final equilibrium state 2 by two different paths 1-*A*-2 and 1-*B*-2, as shown in Fig. 1.7. The processes are quasi-static.

When the system changes from its initial state 1 to final state 2, the quantity of heat transfer will depend upon the intermediate stages through which the system passes, *i.e.* its path. In other words, heat is a path function. Thus, heat is an inexact differential and is written as δQ. On integrating, for the path 1-A-2,

$$\int_1^2 \delta Q = [Q]_1^2 = Q_{1-2} \text{ or } {}_1Q_2$$

It may be noted that $\int_1^2 \delta Q \neq Q_2 - Q_1$, because heat is not

a point function. Thus, it is meaningless to say 'heat in a system or heat of a system'. The heat can not be interpreted similar to temperature and pressure.*

The work, like heat, is not a thermodynamic property, therefore it is a path function as its value depends upon the particular path followed during the process. Since the areas under the curves 1-A-2 and 1-B-2 are different, therefore work done by these two processes will also be different**. Hence, work is an inexact differential and is written as δW. On integration, for the path 1-A-2,

Fig. 1.7. Heat and work–a path function.

$$\int_1^2 \delta W = [W]_1^2 = W_{1-2} \text{ or } {}_1W_2$$

As discussed above, $\int_1^2 \delta W \neq W_2 - W_1$, because work is not a point function. Thus, it is

meaningless to say 'work in a system or work of a system'. Since the work can not be interpreted similar to temperature and pressure of the system, therefore it is a path function and it depends upon the process. It is not a point function as the temperature and pressure. The work done in taking the system from state 1 to state 2 will be different for different paths.

1.35 Comparison of Heat and Work

There are many similarities between heat and work. These are

1. The heat and work are both transient phenomena. The systems do not possess heat or work. When a system undergoes a change, heat transfer or work done may occur.
2. The heat and work are boundary phenomena. They are observed at the boundary of the system.
3. The heat and work represent the energy crossing the boundary of the system.
4. The heat and work are path functions and hence they are inexact differentials. They are written as δQ and δW.

1.36 Power

It may be defined as the rate of doing work or work done per unit time. Mathematically,

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

---

* Heat is not a thermodynamic property whereas the temperature and pressure are thermodynamic properties.

** The area under the pressure-volume (pv) diagram represents the work done during the process and is given by $p\, dv$.

In S.I. system of units, the unit of power is watt (briefly written as W) which is equal to 1 J/s or 1 N-m/s. Generally, a bigger unit of power called kilowatt (briefly written as kW) is used which is equal to 1000 W.

Notes: 1. If T is the torque transmitted in N-m or J and ω is the angular speed in rad/s, then

$$\text{Power, } P = T\omega = T \times 2\pi N/60 \text{ watt} \qquad \qquad \dots (\because \omega = 2\pi N/60)$$

where N is the speed in r.p.m.

2. The ratio of power output to power input is known as *efficiency*. It is denoted by a Greek letter eta (η) . It is always less than unity and is represented as percentage. Mathematically,

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}}$$

1.37 Laws of Thermodynamics

The following three laws of thermodynamics are important from the subject point of view:

1. *Zeroth Law of Thermodynamics.* This law states, "*When two systems are each in thermal equilibrium with a third system, then the two systems are also in thermal equilibrium with one another.*"

This law provides the basis of temperature measurement.

2. *First Law of Thermodynamics.* This law may be stated as follows :

(a) *The heat and mechanical work are mutually convertible.* According to this law, when a closed system undergoes a thermodynamic çycle, the net heat transfer is equal to the net work transfer. In other words, the cyclic integral of heat transfers is equal to the cyclic integral of work transfers. Mathematically,

$$\oint \delta Q = \oint \delta W$$

where symbol $\oint$ stands for cyclic integral (integral around a complete cycle), and δQ and δW represent infinitesimal elements of heat and work transfers respectively. It may be noted that δQ and δW are expressed in same units.

(b) *The energy can neither be created nor destroyed though it can be transformed from one form to another.* According to this law, when a system undergoes a change of state (or a thermodynamic process), then both heat transfer and work transfer takes place. The net energy transfer is stored within the system and is known as stored energy or total energy of the system. Mathematically,

$$\delta Q - \delta W = dE$$

The symbol δ is used for a quantity which is inexact differential and symbol d is used for a quantity which is an exact differential. The quantity E is an extensive property and represents the total energy of the system at a particular state.

3. *Second Law of Thermodynamics* : The second law of thermodynamics may be defined in many ways, but the two common statements according to Kelvin-Planck and Clausius are as follows :

According to Kelvin-Planck '*It is impossible to construct an engine working in a cyclic process, whose sole purpose is to convert heat energy from a single thermal reservoir into an equivalent amount of work*'. In other words, no actual heat engine, working on a cyclic process, can convert whole of the heat supplied to it, into mechanical work. It means that there is a degradation of energy in the process of producing mechanical work from the heat supplied. Thus the Kelvin-Planck statement of the second law of thermodynamics, is sometimes known as *law of degradation of energy.*

According to Clausius statement "*It is impossible for a self acting machine, working in a cyclic process, to transfer heat from a body at a lower temperature to a body at a higher temperature without the aid of an external agency*". In other words, heat cannot flow itself from a cold body to a hot body without the help of an external agency (*i.e.* without the expenditure of mechanical work).

1.38 Perfect Gas

A perfect gas (or an ideal gas) may be defined as a state of a substance, whose evaporation from its liquid state is complete,* and strictly obeys all the gas laws under all conditions of temperature and pressure. In actual practice, there is no real or actual gas which strictly obeys the gas laws over the entire range of temperature and pressure. But, the real gases which are ordinarily difficult to liquify, such as oxygen, nitrogen, hydrogen and air, within certain temperature and pressure limits, may be regarded as perfect gases.

1.39 Laws of Perfect Gases

The physical properties of a gas are controlled by the following three variables :

1. Pressure exerted by the gas, 2. Volume occupied by the gas, and 3. Temperature of the gas.

The behaviour of a perfect gas, undergoing any change in the above mentioned variables, is governed by the following laws, which have been established from experimental results.

(a) Boyle's Law. This law was formulated by Robert Boyle in 1662. It states, "*The absolute pressure of a given mass of a perfect gas varies inversely as its volume, when the temperature remains constant.*" Mathematically,

$$p \propto \frac{1}{v} \quad \text{or} \quad p\,v = \text{Constant}$$

The more useful form of the above equation is

$$p_1\,v_1 = p_2\,v_2 = p_3\,v_3$$
$$= \ldots = \text{Constant}$$
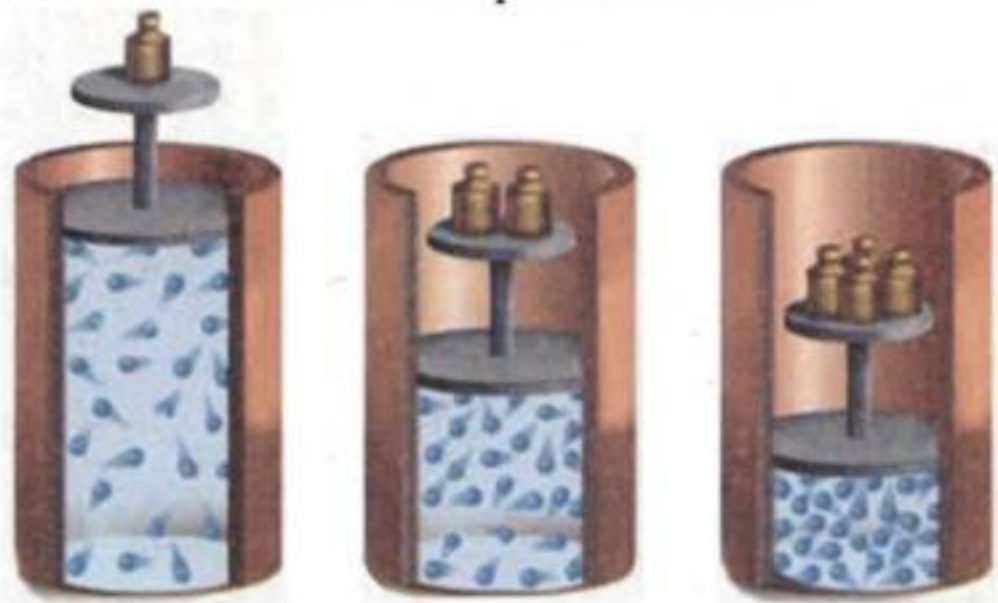
Boyle's law

where suffixes 1, 2 and 3 ... refer to different sets of conditions.

(b) Charles' Law. This law was formulated by a Frenchman Jacques A.C. Charles in about 1787. It may be stated in the following two different forms :

(i) "*The volume of a given mass of a perfect gas varies directly as its absolute temperature, when the absolute pressure remains constant.*" Mathematically,

$$v \propto T \quad \text{or} \quad \frac{v}{T} = \text{Constant}$$

or

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} = \frac{v_3}{T_3} = \ldots = \text{Constant}$$

where suffixes 1, 2 and 3 ... refer to different sets of conditions.

---

* If its evaporation is partial, the substance is called vapour. A vapour, therefore, contains some particles of liquid in suspension. It is thus obvious that steam, carbon dioxide, sulphur dioxide and ammonia are regarded as vapours. It may be noted that a vapour becomes dry, when it is completely evaporated. If the dry vapour is further heated, the process is called *super heating* and the vapour is called *superheated vapour*. The behaviour of superheated vapour is similar to that of a perfect gas.

(ii) *"All perfect gases change in volume by 1/273th of its original volume at 0°C for every 1° change in temperature, when the pressure remains constant."*

Let v_0 = Volume of a given mass of gas at 0°C, and

v_T = Volume of the same mass of gas at t°C.

Then, according to the above statement,

$$v_T = v_0 + \frac{1}{273} \times v_0 t = v_0\left(\frac{273+t}{273}\right) = v_0 \times \frac{T}{T_0}$$

or

$$\frac{v_T}{T} = \frac{v_0}{T_0}$$

where T = Absolute temperature corresponding to t°C, and

T_0 = Absolute temperature corresponding to 0°C.

A little consideration will show that the volume of a gas goes on decreasing by 1/273 th of its original volume for every 1°C decrease in temperature. It is thus obvious, that at a temperature of –273°C, the volume of the gas would become *zero. The temperature at which the volume of a gas becomes zero is called *absolute zero temperature.*

Note: In all calculations of a perfect gas, the pressure and temperature values are expressed in absolute units.

(c) Gay-Lussac Law. This law states, *"The absolute pressure of a given mass of a perfect gas varies directly as its absolute temperature, when the volume remains constant".* Mathematically,

$$p \propto T \quad \text{or} \quad \frac{p}{T} = \text{Constant}$$

or

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} = \frac{p_3}{T_3} \qquad = ... = \text{Constant}$$

where suffixes 1, 2 and 3 ... refer to different sets of conditions.

1.40 General Gas Equation

In the previous section we have discussed the gas laws which give us the relation between the two variables when the third variable is constant. But in actual practice, all the three variables *i.e.* pressure, volume and temperature, change simultaneously. In order to deal with all practical cases, the Boyle's law and Charles' law are combined together, which give us a *general gas equation*.

According to Boyle's law

$$p \propto \frac{1}{v} \quad \text{or} \quad v \propto \frac{1}{p} \qquad \qquad ...(\text{Keeping } T \text{ constant})$$

and according to Charles' law,

$$v \propto T \qquad \qquad ...(\text{Keeping } p \text{ constant})$$

It is thus obvious that

$$v \propto \frac{1}{p} \text{ and } T \text{ both} \quad \text{or} \quad v \propto \frac{T}{p}$$

---

* It is only theoretical. Its exact value is – 273.16° C. But for all practical purposes, this value is taken as – 273° C.

$$\therefore \qquad p\,v \propto T \quad \text{or} \quad p\,v = CT$$

where C is a constant, whose value depends upon the mass and properties of the gas concerned. The more useful form of the general gas equation is :

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3} = \dots = \text{Constant}$$

where suffixes 1, 2 and 3 .... refer to different sets of conditions.

1.41 Joule's Law

It states, "*The change of internal energy of a perfect gas is directly proportional to the change of temperature*". Mathematically,

$$dE \propto dT \quad \text{or} \quad dE = m\,c\,dT = m\,c\,(T_2 - T_1)$$

where

$$m = \text{Mass of the gas, and}$$

$$c = \text{A constant of proportionality, known as specific heat.}$$

An important consequence of this law is that if the temperature of a given mass m of a gas changes from T_1 to T_2, then the internal energy will change from E_1 to E_2 and the change in internal energy $(E_2 - E_1)$ will be same irrespective of the manner how the pressure (p) and volume (v) of the gas have changed.

Note: From the Joule's law, we see that whenever a gas expands, without doing any external work and without taking in or giving out heat, its internal energy as well as temperature does not change.

1.42 Characteristic Equation of a Gas

It is a modified form of general gas equation. If the volume (v) in the general gas equation is taken as that of 1 kg of gas (known as its specific volume, and denoted by v_s), then the constant C (in the general gas equation) is represented by another constant R (in the characteristic equation of gas). Thus the general gas equation may be rewritten as :

$$p\,v_s = RT$$

where R is known as *characteristic gas constant* or *simply gas constant*.

For any mass m kg of a gas, the characteristic gas equation becomes :

$$m\,p\,v_s = m\,R\,T$$

or

$$p\,v = m\,R\,T \qquad \dots (\because m\,v_s = v)$$

Notes: 1. The units of gas constant (R) may be obtained as discussed below :

$$R = \frac{pv}{mT} = \frac{\text{N/m}^2 \times \text{m}^3}{\text{kg} \times \text{K}} = \text{N-m/kg K} = \text{J/kg K} \qquad \dots (\because 1 \text{ N-m} = 1\text{J})$$

2. The value of gas constant (R) is different for different gases. In S.I. units, its value for atmospheric air is taken 287 J/kg K or 0.287 kJ/kg K.

3. The equation $p\,v = m\,R\,T$ may also be expressed in another form *i.e.*

$$p = \frac{m}{v}R\,T = \rho\,R\,T \qquad \dots \left(\because \frac{m}{v} = \rho\right)$$

where ρ (rho) is the density of the given gas.

1.43 Specific Heats of a Gas

The specific heat of a substance may be broadly defined as the amount of heat required to raise the temperature of its unit mass through one degree. All the liquids and solids have one specific heat only. But a gas can have any number of specific heats (lying between zero and

infinity) depending upon the conditions, under which it is heated. The following two types of specific heats of a gas are important from the subject point of view:

1. *Specific heat at constant volume.* It is the amount of heat required to raise the temperature of a unit mass of gas through one degree when it is heated at a constant volume. It is generally denoted by c_v.

Consider a gas contained in a container with a fixed lid as shown in Fig. 1.8. Now, if this gas is heated, it will increase the temperature and pressure of the gas in the container. Since the lid of the container is fixed, therefore the volume of the gas remains unchanged.

Fig. 1.8. Heat being supplied at constant volume.

Let m = Mass of the gas,

T_1 = Initial temperature of the gas, and

T_2 = Final temperature of the gas.

∴ Total heat supplied to the gas at constant volume,

Q_{1-2} = Mass × Sp. heat at constant volume × Rise in temperature

= $m c_v (T_2 - T_1)$

It may be noted that whenever a gas is heated at constant volume, no work is done by the gas*. The whole heat energy is utilised in increasing the temperature and pressure of the gas. In other words, all the amount of heat supplied remains within the body of the gas, and represents the *increase in internal energy of the gas.*

2. *Specific heat at constant pressure.* It is the amount of heat required to raise the temperature of a unit mass of gas through one degree, when it is heated at constant pressure. It is generally denoted by c_p.

Consider a gas contained in a container with a movable lid as shown in Fig. 1.9. Now if this gas is heated, it will increase the temperature and pressure of the gas in the container. Since the lid of the container is movable, therefore it will move upwards, in order to counterbalance the tendency for pressure to rise.

Fig. 1.9. Heat being supplied at constant pressure.

Let m = Mass of the gas,

T_1 = Initial temperature of the gas,

v_1 = Initial volume of the gas, and

T_2, v_2 = Corresponding values for the final condition of the gas.

∴ Total heat supplied to the gas at constant pressure,

Q_{1-2} = Mass × Sp. heat at constant pressure × Rise in temperature

= $m c_p (T_2 - T_1)$

Whenever a gas is heated at a constant pressure, the heat supplied to the gas is utilised for the following two purposes :

---

* We know that work done by the gas,

$W = p \, dv = p (v_2 - v_1)$

where p = Pressure of the gas, and

dv = Change in volume = $v_2 - v_1$

When there is no change in volume, then $dv = 0$. Therefore $W = 0$.

1. To raise the temperature of the gas. This heat remains within the body of the gas, and represents the increase in internal energy. Mathematically, increase in internal energy,

$$dU = m c_v (T_2 - T_1)$$

2. To do some external work during expansion. Mathematically, work done by the gas,

$$W_{1-2} = p (v_2 - v_1) = m R (T_2 - T_1)$$

It is thus obvious, that the specific heat at constant pressure is higher than the specific heat at constant volume.

From above, we may write as

$$Q_{1-2} = dU + W_{1-2} \quad \text{or} \quad Q_{1-2} - W_{1-2} = dU$$

$$\dots \text{(First Law of Thermodynamics)}$$

1.44 Enthalpy of a Gas

In Thermodynamics, one of the basic quantities most frequently recurring is the sum of the internal energy (U) and the product of pressure and volume (pv). This sum ($U + pv$) is termed as enthalpy and is written as H. Mathematically,

$$\text{Enthalpy, } H = U + pv$$

Since ($U + pv$) is made up entirely of properties, therefore enthalpy (H) is also a property. For a unit mass, specific enthalpy,

$$h = u + p v_s$$

where

$$u = \text{Specific internal energy, and}$$

$$v_s = \text{Specific volume.}$$

Note: We know that

$$Q_{1-2} = dU + W_{1-2} = dU + p \, dv$$

When gas is heated at constant pressure from an initial condition 1 to a final condition 2, then change in internal energy,

$$dU = U_2 - U_1$$

and work done by the gas,

$$W_{1-2} = p \, dv = p (v_2 - v_1)$$

$$\therefore \quad Q_{1-2} = (U_2 - U_1) + p (v_2 - v_1)$$

$$= (U_2 + pv_2) - (U_1 + p v_1) = H_2 - H_1$$

and for per unit mass,

$$q_{1-2} = h_2 - h_1$$

Thus, for a constant pressure process, the heat supplied to the gas is equal to the change of enthalpy.

1.45 Ratio of Specific Heats

The ratio of two specific heats (*i.e.* c_p/c_v) of a gas is an important constant in the field of Thermodynamics and is represented by a Greek letter gamma (γ). It is also known as *adiabatic index*. Since c_p is always greater than c_v, therefore the value of γ is always greater than unity.

We know that

$$c_p - c_v = R \quad \text{or} \quad c_p = c_v + R$$

Dividing both sides by c_v,

$$\frac{c_p}{c_v} = 1 + \frac{R}{c_v} \quad \text{or} \quad \gamma = 1 + \frac{R}{c_v}$$

The values of c_p, c_v and γ for some common gases are given below :

Table 1.5. Values of c_p and c_v for some common gases.

S.No.	Name of gas	C_p (kJ/kgK)	C_v (kJ/kgK)	$\gamma = \dfrac{C_p}{C_v}$
1.	Air	1.000	0.720	1.40
2.	Carbon dioxide (CO_2)	0.846	0.657	1.29
3.	Oxygen (O_2)	0.913	0.653	1.39
4.	Nitrogen (N_2)	1.043	0.745	1.40
5.	Ammonia (NH_3)	2.177	1.692	1.29
6.	Carbon monoxide (CO)	1.047	0.749	1.40
7.	Hydrogen (H_2)	14.257	10.133	1.40
8.	Argon (A)	0.523	0.314	1.67
9.	Helium (He)	5.234	3.153	1.66
10.	Methane (CH_4)	2.169	1.650	1.31

1.46 Entropy

The term 'entropy' which literally means transformation, was first introduced by Clausius. It is an important thermodynamic property of a working substance, which increases with the addition of heat, and decreases with its removal. As a matter of fact, it is tedious to define the term entropy. But it is comparatively easy to define change of entropy of a working substance. In a reversible process, over a small range of temperature, the increase or decrease of entropy, when multiplied by the absolute temperature, gives the heat absorbed or rejected by the working substance. Mathematically, heat absorbed by the working substance,

$$\delta Q = TdS \quad \text{or} \quad dS = \frac{\delta Q}{T} \qquad \ldots (i)$$

where
T = Absolute temperature, and
dS = Increase in entropy.

Note: The above relation also holds good for heat rejected by the working substance. In that case, dS will be decrease in entropy.

The engineers and scientists use it for providing quick solution to problems dealing with adiabatic expansion. The entropy is usually represented by S.

The adiabatic expansion on the temperature-entropy (T-S) diagram is shown by the curve 1-2 in Fig. 1.10.

The total change in entropy may be obtained by integrating the equation (i) from state 1 to state 2.

$$\therefore \quad \int_1^2 dS = \int_1^2 \frac{\delta Q}{T} \qquad \ldots (ii)$$

The unit of entropy depends upon the unit of heat employed and the absolute temperature.

Therefore, if the heat supplied or rejected is in kJ and the temperature is in K, then the unit of entropy is kJ/K. The entropy may be expressed in so many units of entropy without assigning any dimensional units. Since the entropy is expressed per unit mass of the working substance, it
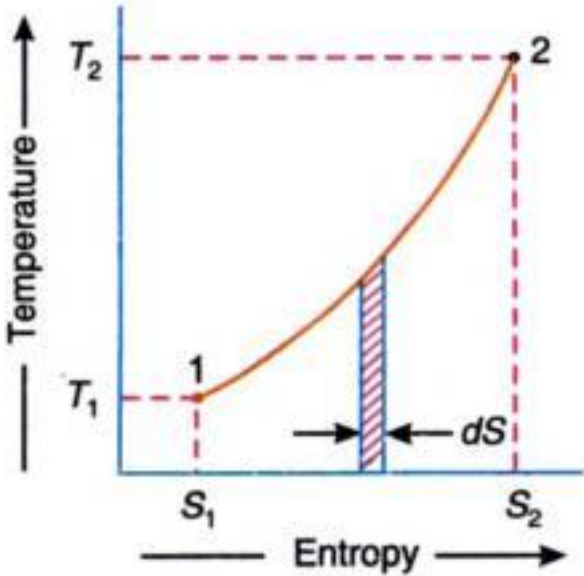
Fig. 1.10. Temperature-entropy diagram.

would be more correct to speak *specific entropy*. The absolute values of entropy cannot be determined, but only the change in entropy may be obtained by using equation (*ii*).

Theoretically, the entropy of a substance is *zero* at *absolute zero temperature*. Hence, in entropy calculations, some convenient datum should be selected from which measurement may be made.

It may be noted that water at 0°C is assumed to have zero entropy, and changes in its entropy are reckoned from this temperature.

Notes: 1. The area under the *T-S* diagram of any thermodynamic process represents the heat absorbed or rejected during that process.

2. Since $\int \dfrac{\delta Q}{T}$ is same for all reversible paths between states 1 and 2, so we conclude that this

quantity is independent of a path and is a function of end states only. In other words, the entropy is a point function and thus it is a property of the system. The entropy may be expressed as a function of other thermodynamic properties of the system, such as the pressure and temperature or pressure and volume.

3. We know that according to First Law of Thermodynamics,

$$\delta Q \; = \; dU + \delta W = dU + p\, dv \; ... \; (\because \; \delta W = p\, dv) \qquad ... (iii)$$

and
$$\delta Q \; = \; T\, dS \qquad ... (iv)$$

From equations (*iii*) and (*iv*),

$$T\, dS \; = \; dU + p\, dv \qquad ... (v)$$

It is very interesting to note that in equations (*iii*) and (*iv*), δQ and δW are path functions, therefore these equations are true only for reversible processes. But in equation (*v*), *ds*, *dU* and *dv* are point functions as they depend upon the initial and final equilibrium states, therefore equation (*v*) is true for reversible as well as irreversible processes.

4. The entropy remains constant in a reversible process and increases in an irreversible process.

5. The change of entropy is *positive* when heat is absorbed by the gas and there is an increase of entropy.

6. The change of entropy is *negative* when heat is removed from the gas and there is a decrease of entropy.

1.47 Thermodynamic Processes

When a system changes its state from one equilibrium state to another equilibrium state, then the path of successive states through which the system has passed, is known as a *thermodynamic process*. Strictly speaking, no system is in true equilibrium during the process because the properties (such as pressure, volume, temperature etc.) are changing. However, if the process is assumed to take place sufficiently slowly so that the deviation of the properties at the intermediate states is infinitesimally small, then every state passed through by the system will be in equilibrium. Such a process is called *quasi-static* or *reversible process* and it is represented by a continuous curve on the property diagram (*i.e.* pressure-volume diagram) as shown in Fig. 1.11(*a*).

If the process takes place in such a manner that the properties at the intermediate states are not in equilibrium state (except the initial and final state), then the process is said to be *non-equilibrium* or *irreversible process*. This process is represented by the broken lines on the property diagram as shown in Fig. 1.11 (*b*).

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*   The entropy is an extensive property of the system. The ratio of the extensive property of the system to the mass of the system is the specific value of that property.

(a) Reversible process.   (b) Irreversible process.
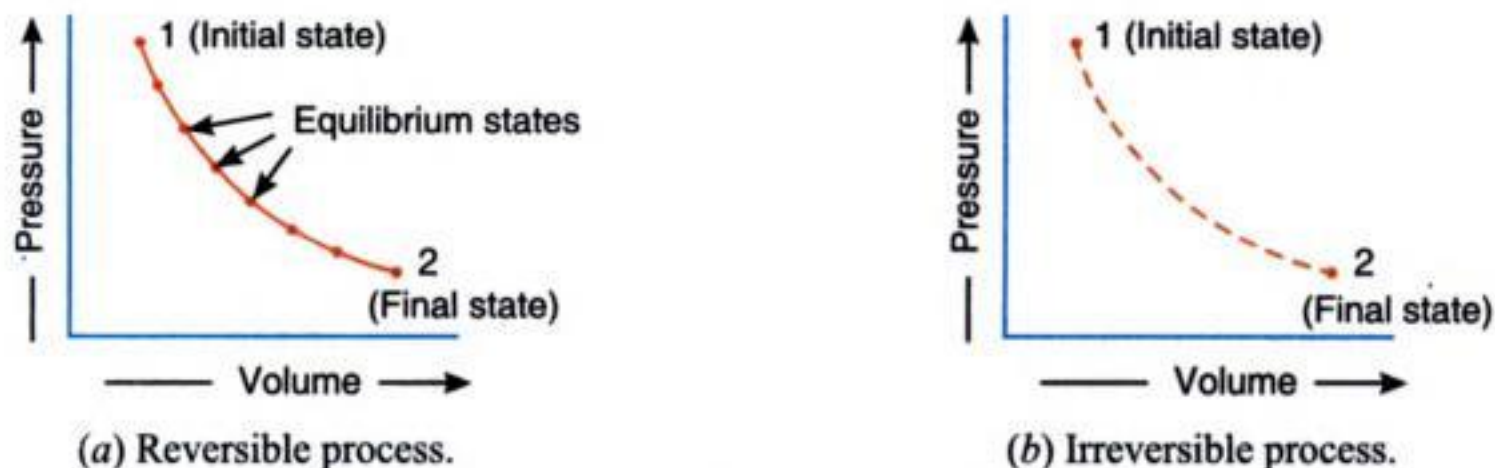
Fig. 1.11. Reversible and irreversible process.

All the thermodynamic processes are classified into the following two groups :

1. Non-flow processes, and 2. Flow processes.

The processes occuring in closed systems which do not permit the transfer of mass across their boundaries, are known as *non-flow processes*. It may be noted that in a non-flow process, the energy crosses the system boundary in the form of heat and work, but there is no mass flow into or out of the system.

The processes occuring in open systems which permit the transfer of mass to and from the system, are known as *flow processes*. It may be noted that in a flow process, the mass enters the system and leaves after enhancing energy. The flow processes may be *steady flow* and *non-steady flow processes*

1.48 Workdone During a Non-flow Process

Consider a system contained in a frictionless piston and cylinder arrangement as shown in Fig. 1.12. As the system expands from its original state 1, it overcomes the external resistance (such as rotation of the flywheel) which opposes the motion of the piston by exerting a force through a distance. The variation of the volume and pressure of the system as it expands to final state 2, is drawn on the pressure-volume diagram (briefly called *p-v* diagram) as shown in Fig. 1.12.

Let at any small section (shown shaded), the pressure (p) of the system is constant. If A is the cross-sectional area of the piston, then force on the piston ($F = pA$) causes the piston to move through a distance dx. Thus, workdone by the system,

$$\delta W = F\,dx = pA\,dx = p\,dv \qquad ...(\because dv = A\,dx)$$

∴ Workdone for non-flow process from state 1 to state 2,

Fig. 1.12. Workdone during a non-flow process.

$$W_{1-2} = \int_{1}^{2} \delta W = \int_{1}^{2} p\,dv$$

From above, we see that the workdone is given by the area under the *p-v* diagram.

Notes: **1.** The workdone by the system is taken as *positive* while the workdone on the system is considered as *negative*.

2. For an irreversible process, $\delta W \neq p\,dv$, because the path of the process is not represented truely on the *p-v* diagram due to its non-equilibrium states in the process.

1.49 Application of First Law of Thermodynamics to a Non-flow Process

We have already discussed in Art. 1.37 that when a system undergoes a change of state or a thermodynamic process, then both the heat transfer and work transfer takes place. The net energy transfer is stored within the system and is known as *stored* or *total energy* of the system. Mathematically,

$$Q_{1-2} - W_{1-2} = dE = E_2 - E_1 \qquad \ldots (i)$$

where
Q_{1-2} = Heat transferred or heat supplied to the system during the process *i.e.* from state 1 to state 2,

W_{1-2} = Workdone by the system on the surrounding during the process *i.e.* from state 1 to state 2,

E_2 = *Total energy or stored energy of the system at the end of the process *i.e.* at state 2, and

E_1 = Total energy or stored energy of the system at the start of the process *i.e.* at state 1.

For a non-flow process, the stored energy is the internal energy only. Thus equation (i) of the First Law of Thermodynamics, when applied to non-flow process or a static system, may be written as

$$Q_{1-2} - W_{1-2} = dU = U_2 - U_1$$

where
dU = Change in internal energy = $U_2 - U_1$

It may be noted that heat and work are not a property of the system, but their difference $(Q_{1-2} - W_{1-2})$ during a process is the numerical equivalent of stored energy. Since the stored energy is a property, therefore $(Q_{1-2} - W_{1-2})$ is also a property.

1.50 Classification of Non-flow Processes

The various non-flow processes which take place in the cycle of a closed system are discussed in the following pages :

1. Constant volume process or Isochoric process

We have already discussed that when a gas is heated at a constant volume, its temperature and pressure will increase. Since there is no change in its volume, therefore no work is done by the gas. All the heat supplied to the gas is stored within the gas in the form of internal energy. Now consider *m* kg of a certain gas being heated at constant volume from initial state 1 to a final state 2.

(a) p-v diagram.　　(b) T-S diagram.

Fig. 1.13. Constant volume process.

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* The total energy of a system is sum of potential energy (PE), kinetic energy (KE) and internal energy (U) plus any other form of the energy such as chemical energy, electrical energy etc.

Let $\qquad p_1, v_1$ and T_1 = Pressure, volume and temperature at the initial state 1, and

$\qquad p_2, v_2$ and T_2 = Pressure, volume and temperature at the final state 2.

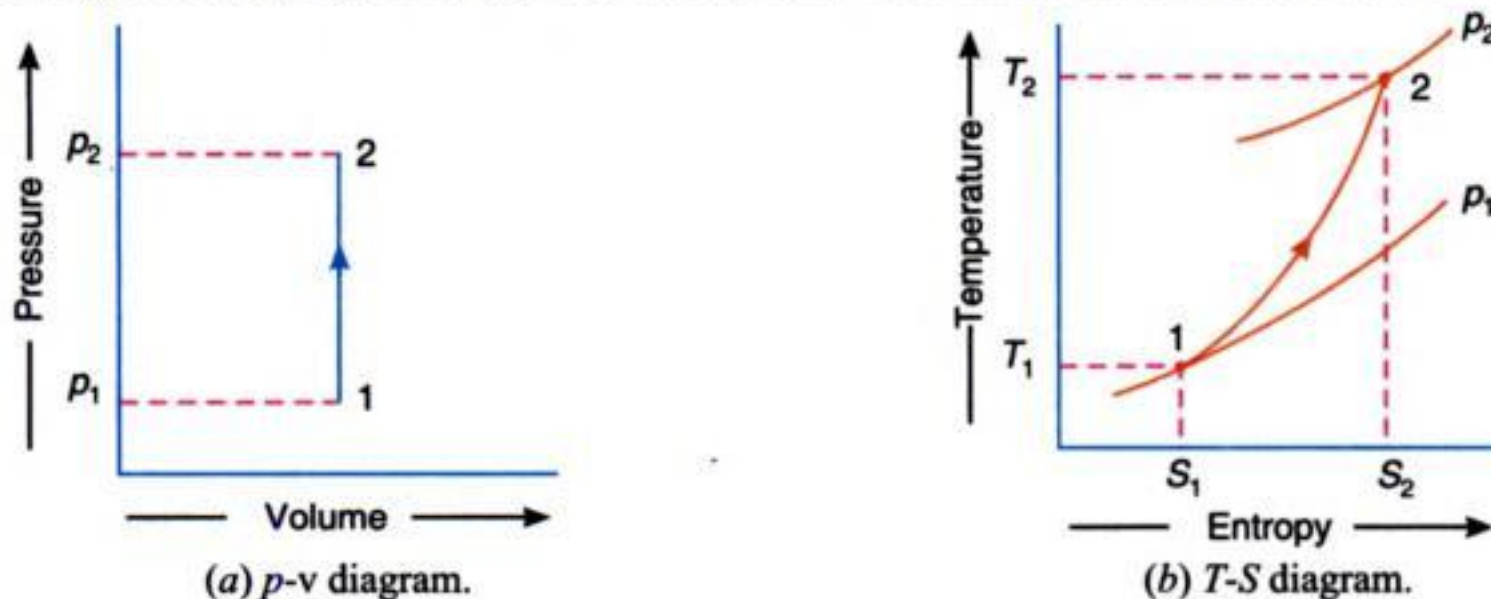The process is shown on the pressure- volume* (p-v) diagram and temperature-entropy** (T-S) diagram in Fig. 1.13 (a) and (b) respectively. It may be noted that the constant volume process is governed by Gay-Lússac law, *i.e.*

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \text{or} \quad \frac{p}{T} = \text{Constant}$$

The following relations are important for the reversible constant volume process.

(a) Heat supplied or heat transfer

We know that $\qquad \delta Q = dU + \delta W \qquad$ ... (First Law of Thermodynamics)

On integrating from state 1 to state 2,

$$\int_1^2 \delta Q = \int_1^2 dU + \int_1^2 \delta W$$

or $\qquad Q_{1-2} = (U_2 - U_1) + W_{1-2}$

Since $W_{1-2} = 0$, therefore heat supplied or heat transfer,

$$Q_{1-2} = U_2 - U_1 = m\,c_v\,(T_2 - T_1)$$

This shows that all the heat supplied to the gas is utilised in increasing the internal energy of the gas.

(b) Change in enthalpy

We know that the change in enthalpy,

$$dH = dU + d\,(pv)$$

On integrating from state 1 to state 2,

$$\int_1^2 dH = \int_1^2 dU + \int_1^2 d\,(pv)$$

or $\qquad H_2 - H_1 = (U_2 - U_1) + (p_2\,v_2 - p_1\,v_1)$

$$= m\,c_v\,(T_2 - T_1) + m\,R\,(T_2 - T_1)$$
$$\qquad\qquad \ldots (\because\ p_1\,v_1 = mR\,T_1 \text{ and } p_2\,v_2 = mR\,T_2)$$
$$= m\,(T_2 - T_1)\,(c_v + R) = m\,c_p\,(T_2 - T_1) \quad \ldots (\because\ c_p - c_v = R)$$

(c) Change in entropy

The change in entropy during constant volume process is given by

$$S_2 - S_1 = m\,c_v\,\log_e\left(\frac{T_2}{T_1}\right) = m\,c_v\,\log_e\left(\frac{p_2}{p_1}\right) \qquad \ldots \left(\because\ \frac{T_2}{T_1} = \frac{p_2}{p_1}\right)$$

Notes: (*i*) The change in internal energy (dU) and the change in enthalpy (dH) have the same expression for each process.

---

* The area below the p-v diagram of any thermodynamic process represents the work done during that process.

** The area below the T-S diagram of a thermodynamic process represents the heat absorbed or rejected during that process.

(*ii*) During expansion or heating process, work is done by the gas (*i.e.* W_{1-2} is + ve); internal energy of the gas increases (*i.e.* dU is +ve) and heat is supplied to the gas (*i.e.* Q_{1-2} is +ve).

(*iii*) During compression or cooling process, work is done on the gas (*i.e.* W_{1-2} is –ve); internal energy of the gas decreases (*i.e.* dU is –ve) and heat is rejected by the gas (*i.e.* Q_{1-2} is –ve).

2. *Constant pressure process or Isobaric process*

We have already discussed that when a gas is heated at a constant pressure, its temperature and volume will increase. Since there is a change in its volume, therefore the heat supplied to the gas is utilised to increase the internal energy of the gas and for doing some external work.

Now consider m kg of a certain gas being heated at a constant pressure from an initial state 1 to a final state 2.

Let
p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and
p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.

(*a*) *p-v* diagram.   (*b*) *T-S* diagram.

Fig. 1.14. Constant pressure process.

The process is shown on the *p-v* and *T-S* diagrams in Fig. 1.14 (*a*) and (*b*) respectively.

It may be noted that the constant pressure process is governed by Charles' law, *i.e.*

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad \frac{v}{T} = \text{Constant}$$

The following relations are important for the reversible constant pressure process.

(a) *Workdone by the gas*

We know that $\delta W = p \, dv$

On integrating from state 1 to state 2,

$$\int_1^2 \delta W = \int_1^2 p \, dv = p \int_1^2 dv$$

or $\quad\quad\quad\quad W_{1-2} = p \, (v_2 - v_1) = m R \, (T_2 - T_1)$

$$\ldots (\because p \, v_1 = m R \, T_1 \text{ and } p \, v_2 = m R \, T_2)$$

(b) *Change in internal energy*

We have discussed above that the change in internal energy (dU) is same for all the processes. Therefore change in internal energy,

$$dU = U_2 - U_1 = m \, c_v \, (T_2 - T_1)$$

(c) *Heat supplied or heat transferred*

We know that $\quad\quad\quad\quad \delta Q = dU + \delta W$

On integrating from state 1 to state 2,

$$\int_1^2 \delta Q = \int_1^2 dU + \int_1^2 \delta W$$

or
$$Q_{1-2} = (U_2 - U_1) + W_{1-2} \qquad \ldots (i)$$
$$= m c_v (T_2 - T_1) + m R (T_2 - T_1) = m (T_2 - T_1).(c_v + R)$$
$$= m c_p (T_2 - T_1) \qquad \ldots (\because c_p - c_v = R)$$

The equation (i) shows that the heat supplied to the gas is utilised in increasing the internal energy of the gas and for doing some external work.

(d) Change in enthalpy

We have already discussed in the previous article that the change in enthalpy (dH) is same for all the processes. Therefore, change in enthalpy,

$$dH = H_2 - H_1 = m c_p (T_2 - T_1)$$

We see that change in enthalpy is equal to the heat supplied or heat transferred.

(e) Change in entropy

The change in entropy during constant pressure process is given by

$$S_2 - S_1 = m c_p \log_e \left(\frac{T_2}{T_1}\right) = m c_p \log_e \left(\frac{v_2}{v_1}\right) \qquad \ldots \left(\because \frac{T_2}{T_1} = \frac{v_2}{v_1}\right)$$

Note: If the gas is cooled at a constant pressure,then there will be a compression. It is thus obvious that, during cooling, the temperature and volume will decrease and work is said to be done on the gas. In this case,

Workdone on the gas, $\qquad W_{1-2} = p (v_1 - v_2) = m R (T_1 - T_2)$

Decrease in internal energy, $\qquad dU = U_1 - U_2 = m c_v (T_1 - T_2)$

and heat rejected by the gas, $\qquad Q_{1-2} = m c_p (T_1 - T_2)$

3. Constant temperature process or Isothermal process
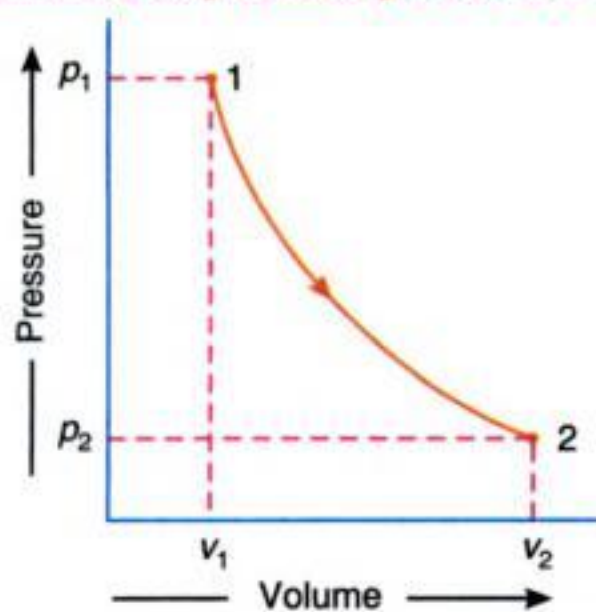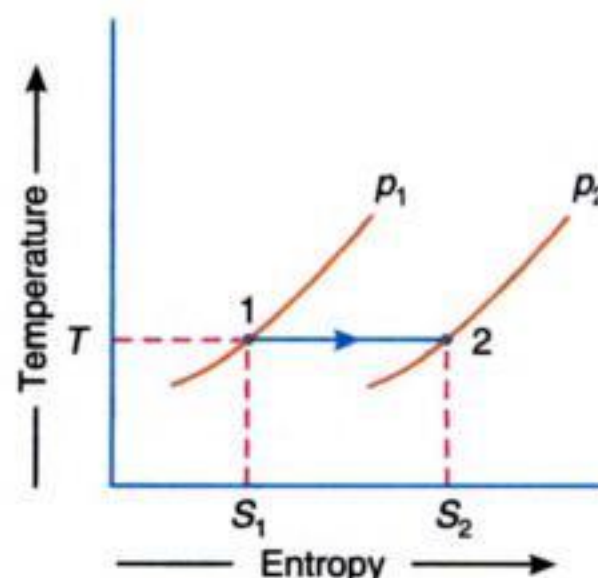
(a) p-v diagram.   (b) T-S diagram.

Fig. 1.15. Constant temperature process.

A process, in which the temperature of the working substance remains constant during its expansion or compression, is called constant temperature process or isothermal process. This will happen when the working substance remains in a perfect thermal contact with the surroundings, so that the heat 'sucked in' or 'squeezed out' is compensated exactly for the work done by the gas or on the gas respectively. It is thus obvious that in an isothermal process :

1. there is no change in temperature,
2. there is no change in internal energy, and
3. there is no change in enthalpy.

Now consider m kg of a certain gas being heated at constant temperature from an initial state 1 to final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.
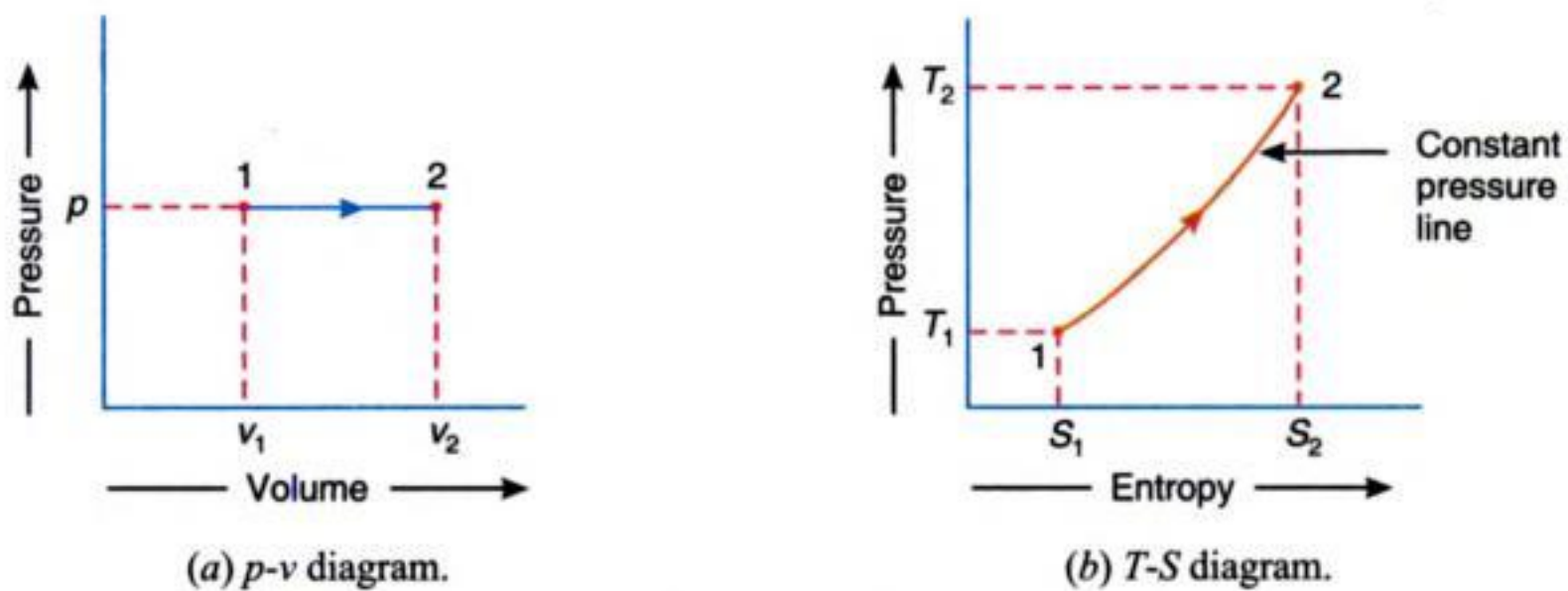
The process is shown on the p-v and T-S diagrams in Fig. 1.15 (a) and (b) respectively. It may be noted that the constant temperature process is governed by Boyle's law, $i.e.$

$$p_1 v_1 = p_2 v_2 \quad \text{or} \quad pv = \text{Constant}$$

The following relations are important for the constant temperature process:

(a) **Workdone by the gas**

The workdone by the gas during isothermal expansion is given by

$$W_{1-2} = p_1 v_1 \log_e \left(\frac{v_2}{v_1} \right) = 2.3\, p_1\, v_1 \log r$$

$$= 2.3\, m\, R\, T \log r \qquad \qquad \dots (\because p_1 v_1 = m\, RT)$$

where r is the expansion ratio or compression ratio if the process is isothermal compression.

Notes: (i) Expansion ratio, $r = \dfrac{\text{Volume at the end of expansion}}{\text{Volume at the beginning of expansion}}$

(ii) Compression ratio, $r = \dfrac{\text{Volume at the beginning of compression}}{\text{Volume at the end of compression}}$

(b) **Change in internal energy**

We know that change in internal energy,

$$dU = U_2 - U_1 = m\, c_v\, (T_2 - T_1)$$

Since it is a constant temperature process, $i.e.$ $T_2 = T_1$, therefore

$$dU = U_2 - U_1 = 0 \quad \text{or} \quad U_1 = U_2$$

(c) **Heat supplied or heat transferred**

We know that heat supplied or heat transferred from state 1 to state 2,

$$Q_{1-2} = dU + W_{1-2} = W_{1-2} \qquad \qquad \dots (\because dU = 0)$$

This shows that all the heat supplied to the gas is equal to the workdone by the gas.

(d) **Change in enthalpy**

We know that change in enthalpy,

$$dH = H_2 - H_1 = m\, c_p\, (T_2 - T_1)$$

Since it is a constant temperature process, $i.e.$ $T_2 = T_1$, therefore

$$dH = H_2 - H_1 = 0 \quad \text{or} \quad H_1 = H_2$$

(e) **Change in entropy**

The change in entropy during isothermal process is given by,

$$S_2 - S_1 = m\, R \log_e \left(\frac{v_2}{v_1} \right) = 2.3\, m\, R \log r$$

4. Reversible adiabatic process or Isentropic process

A process, in which the working substance neither receives nor gives out heat to its surroundings, during its expansion or compression, is called an *adiabatic process*. This will

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* It may be noted that the adiabatic process may be reversible or irreversible. The reversible adiabatic process or frictionless adiabatic process is known as isentropic process (or constant entropy process). But when friction is involved in the process, then the adiabatic process is said to be irreversible, in which case the entropy does not remain constant $i.e.$ the entropy increases.

happen when the working substance remains thermally insulated, so that no heat enters or leaves it during the process. It is thus obvious, that in an adiabatic or isentropic (*i.e.* constant entropy) process

1. No heat leaves or enters the gas,

2. The temperature of the gas changes, as the work is done at the cost of internal energy, and

3. The change in internal energy is equal to the work done.

Now consider *m* kg of a certain gas being heated reversibly and adiabatically from an initial state 1 to a final state 2.

Let p_1, v_1 and T_1 = Pressure, volume and temperature at the initial state 1, and

p_2, v_2 and T_2 = Pressure, volume and temperature at the final state 2.
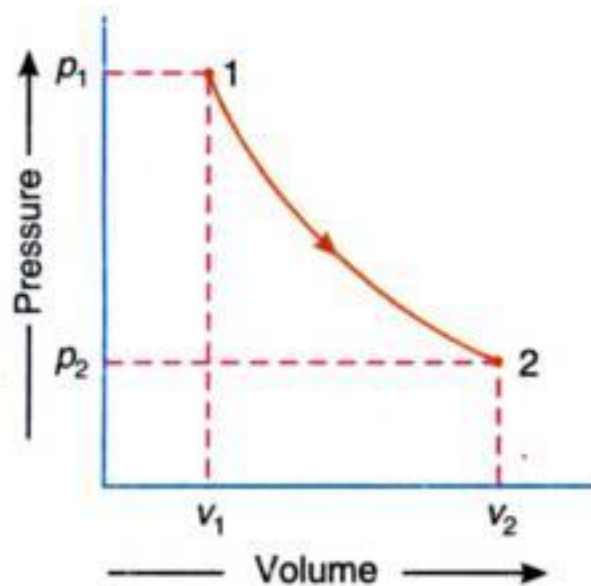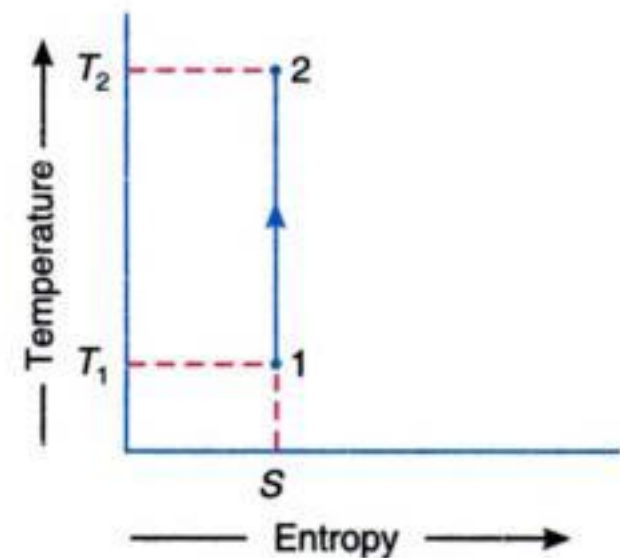
(a) *p-v* diagram. (b) *T-S* diagram.

Fig. 1.16. Reversible adiabatic or Isentropic process.

The process is shown on the *p-v* and *T-S* diagrams in Fig 1.16 (*a*) and (*b*) respectively. It may be noted that the reversible adiabatic or isentropic process follows the law pv^γ = constant, where γ is called the adiabatic or isentropic index.

The following relations are important for the reversible adiabatic or isentropic process.

(a) Work done during expansion

The work done during reversible adiabatic expansion is given by

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1} \quad \dots (\because p_1 v_1 = mRT_1, \text{ and } p_2 v_2 = mRT_2)$$

(b) Change in internal energy

We know that change in internal energy,

$$dU = U_2 - U_1 = m c_v (T_2 - T_1)$$

(c) Heat supplied or heat transferred

We know that heat supplied or heat transferred in case of adiabatic process is zero, therefore

$$Q_{1-2} = 0$$

(d) Change in enthalpy

We know that change in enthalpy,

$$dH = H_2 - H_1 = m c_p (T_2 - T_1)$$

(e) Change in entropy

Since no heat enters or leaves the system during this process, therefore the change in entropy is zero.

Note: Since the reversible adiabatic or isentropic process follows the law

$$p\, v^\gamma = \text{Constant} \quad \text{or} \quad p_1 v_1^\gamma = p_2 v_2^\gamma$$

$$\therefore \qquad \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^\gamma \qquad \qquad \dots (i)$$

We know that

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{p_1}{p_2} = \frac{T_1}{T_2} \times \frac{v_2}{v_1} \qquad \dots (ii)$$

From equations (i) and (ii),

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

The equation (i) may be written as

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \qquad \qquad \dots (iii)$$

and equation (ii) may be written as

$$\frac{v_2}{v_1} = \frac{p_1}{p_2} \times \frac{T_2}{T_1} \qquad \qquad \dots (iv)$$

Now from equations (iii) and (iv),

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad \frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

5. Polytropic process

The general law for the expansion and compression of gases is given by the relation

$$p\, v^n = \text{Constant}$$

where n is called the polytropic index which may have value from zero to infinity, depending upon the manner, in which the expansion or compression has taken place.

The various equations for polytropic process may be expressed by changing the index n for γ in the adiabatic process, i.e.

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} \quad \text{and} \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$$

Also work done during polytropic process,

$$= \frac{m\, R\, (T_1 - T_2)}{n - 1}$$

and heat absorbed or rejected during polytropic process,

$$= \frac{\gamma - n}{\gamma - 1} \times \text{Work done}$$

1.51 Thermodynamic Cycle

A thermodynamic cycle or a cyclic process consists of a series of thermodynamic operations (processes), which take place in a certain order, and the initial conditions are restored at the end

of the process. When the operations or processes of cycle are plotted on *p-v* diagram, they form a closed figure, each operation being represented by its own curve. Since the area under each curve gives the work done to some scale, during each operation, it therefore follows that the net work done during one cycle will be given by the enclosed area of the diagram as shown shaded in Fig. 1.17.

Fig. 1.17. A thermodynamic cycle.

The thermodynamic cycles, in general, may be classified into the following two types :

1. Reversible cycle. A thermodynamically reversible cycle consists of reversible processes only. We have already discussed that a reversible process is one which is performed in such a way that at the end of the process, both the system and the surroundings may be restored to their initial states. For example, consider a process in which the system (gas) is expanded from state 1 to state 2 following the path 1-2 as shown in Fig. 1.18. Let during the thermodynamic process 1-2, the workdone by the system is W_{1-2} and the heat absorbed is Q_{1-2}. Now, if by doing the work (W_{1-2}) on the system (*i.e.* by compressing the gas) and extracting heat (Q_{1-2}) from the system, we can bring the system and the surroundings, back from state 2 to state 1 (*i.e.* initial state), following the same path 2-1, then process is said to be a reversible process.

Fig. 1.18. Reversible process.

In a reversible process, there should not be any loss of heat due to friction, radiation or conduction, etc. A cycle will be reversible if all the processes constituting the cycle are reversible. Thus in a reversible cycle, the initial conditions are restored at the end of the cycle.

A little consideration will show that when the operations are performed in the reversed order, the cycle draws heat from the cold body and rejects it to the hot body. This operation requires an external power to drive the mechanism according to second law of thermodynamics. A machine which operates on a reversed cycle is regarded as a "heat pump", such as a refrigerator, because it pumps heat from the cold body to the hot body. Following are the conditions for reversibility of a cycle :

    (*a*) The pressure and temperature of the working substance must not differ, appreciably, from those of the surroundings at any stage in the process.

    (*b*) All the processes, taking place in the cycle of operation, must be extremely slow.

    (*c*) The working parts of the engine must be friction free.

    (*d*) There should be no loss of energy during the cycle of operation.

Note: A reversible cycle should not be confused with a mechanically reversible engine. Steam engine cranks may be made to revolve in a reversed direction by mechanically altering the valve settings. But this does not reverse the cycle, on which it works. A two-stroke petrol engine may be made to revolve in reverse direction by altering the timing of ignition. But this also does not reverse the actual cycle.

2. Irreversible cycle. We have discussed above that in a reversible process, the heat and work are completely restored back by reversing the process (*i.e.* by compressing the gas). But when the heat and work are not completely restored back by reversing the process, then the process is known as *irreversible process* (also called *natural* or *real process*). In an irreversible process, there is a loss of heat due to friction, radiation or conduction.

In actual practice, most of the processes are irreversible to some degree. The main causes for the irreversibility are :
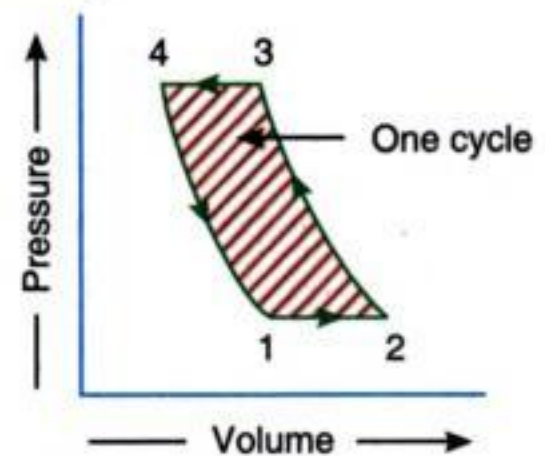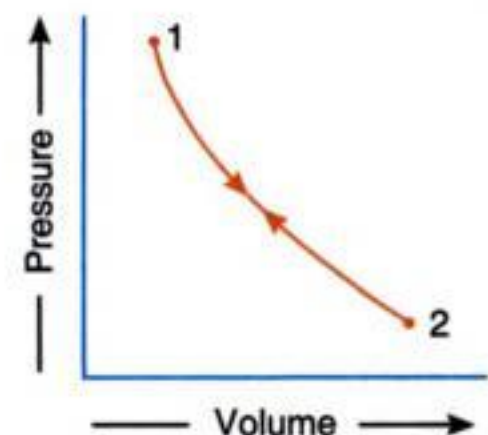
1. mechanical and fluid friction, 2. unrestricted expansion, and 3. heat transfer with a finite temperature difference. Moreover, friction converts the mechanical work into heat. This heat cannot supply back the same amount of mechanical work, which was consumed for its production. Thus, if there is some friction involved in the process, it becomes irreversible. A cycle will be irreversible if any of the processes, constituting the cycle, is irreversible. Thus in an irreversible cycle, the initial conditions are not restored at the end of the cycle.

1.52 Reversibility and Irreversibility of Thermodynamic Processes

We have already discussed the various thermodynamic processes in Art. 1.50. Now we shall discuss their conditions of reversibility and irreversibility.

1. Isothermal and adiabatic processes. It may be noted that a complete process or cycle is only an ideal case. But in actual practice, complete isothermal and adiabatic operations are not achieved. However, they can be approximated. The simple reason for the same is that it is impossible to transfer heat at a constant temperature in case of an isothermal operation. Moreover, it is also impossible to make an absolutely non-conducting cylinder in case of an adiabatic operation. In actual practice, however, an isothermal operation may be approached if the process is so slow that the heat is absorbed or rejected at such a rate that the temperature remains, practically, constant. Similarly, an adiabatic operation may be approached if the process takes place so quickly that no time is given to the heat to enter or leave the gas.

In view of the above, the isothermal and adiabatic processes are taken as reversible processes.

2. Constant volume, constant pressure and constant pv^n processes. We know that when the temperature of the hot body, supplying the heat, remains constant during the process, the temperature of the working substance will vary as the operation proceeds. In view of this, the above three operations are irreversible. But, these can be made to approximate to reversibility by manipulating the temperature of the hot body to vary so that at any stage the temperature of the working substance remains constant.

In this way, the constant volume, constant pressure and constant pv^n processes are regarded as reversible processes.

3. Free expansion and throttling processes. These processes are irreversible, as there is always a loss of heat due to friction when the working substance passes through an orifice.

1.53 Flow Processes

We have already discussed in Art. 1.47, that the processes occuring in open system which permit the transfer of mass to and from the system, are known as *flow processes*. In a flow process, the mass (working substance) enters the system and leaves after doing the work. The flow process may be classified as:

1. Steady flow process, and 2. Unsteady flow process.

In a steady flow process, the following conditions must be satisfied :

(*a*) The rate of mass flow at inlet and outlet is same, *i.e.* the mass flow rate through the system remains constant.

(*b*) The rate of heat transfer is constant.

(*c*) The rate of work transfer is constant.

(*d*) The state of working substance at any point within the system is same at all times.

(*e*) There is no change in the chemical composition of the system. Thus no chemical energy is involved.

If any one of these conditions are not satisfied, then the process is said to be non-steady flow process. In engineering, we are mainly concerned with steady flow processes.

1.54 Application of First Law of Thermodynamics to a Steady Flow Process

Consider an open system through which the working substance flows at a steady rate, as shown in Fig. 1.19. The working substance enters the system at section 1 and leaves the system at section 2.
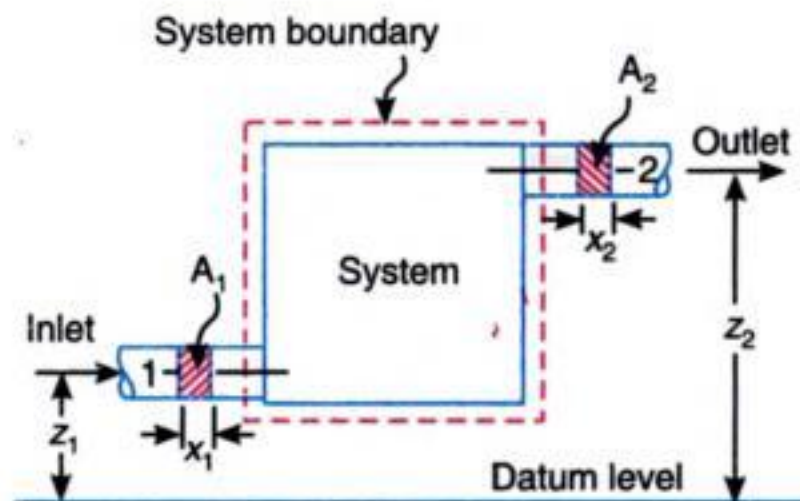

Fig. 1.19. Steady flow process.

Let
p_1 = Pressure of the working substance entering the system in N/m^2,

v_{s1} = Specific volume of the working substance entering the system in m^3/kg.

V_1 = Velocity of the working substance entering the system in m/s,

u_1 = Specific internal energy of the working substance entering the system in J/kg,

z_1 = Height above datum level for inlet in metres,

p_2, v_{s2}, V_2, u_2 and z_2 = Corresponding values for the working substance leaving the system.

q_{1-2} = Heat supplied to the system in J/kg, and

w_{1-2} = Work delivered by the system in J/kg.

Consider 1 kg of mass of the working substance.

We know that total energy entering the system per kg of the working substance,

e_1 = Internal energy + *Flow or displacement energy + Kinetic energy + Potential energy + Heat supplied

$$= u_1 + p_1 v_{s1} + \frac{V_1^2}{2} + g\, z_1 + q_{1-2} \quad \text{(in J/kg)}$$

---

* The *flow* or *displacement energy* is the energy required to flow or move the working substance against its pressure. It is also known as *flow work*.

For example, let the working substance with pressure p_1 (in N/m^2) flows through area A_1 (in m^2) and moves through a distance x_1 (in metres).

∴ Energy or work required to flow the working substance,

FE = Force × Distance = $(p_1 A_1) x_1 = p_1 v_1$ (in joules)   ... ($\because v_1 = A_1 x_1$)

where v_1 = Volume of the working substance in m^3,

For 1 kg mass of the working substance,

$v_1 = v_{s1}$ = Specific volume of the working substance in m^3/kg.

∴ $FE = p_1 v_{s1}$ (in J/kg)

Similarly, total energy leaving the system per kg of the working substance,

$$e_2 = u_2 + p_2 v_{s2} + \frac{V_2^2}{2} + g\, z_2 + w_{1-2} \quad \text{(in J/kg)}$$

Assuming no loss of energy during flow, then according to First Law of Thermodynamics (*i.e.* Law of Conservation of Energy), $e_1 = e_2$.

$$\therefore \; u_1 + p_1 v_{s1} + \frac{V_1^2}{2} + g\, z_1 + q_{1-2} = u_2 + p_2 v_{s2} + \frac{V_2^2}{2} + g\, z_2 + w_{1-2}$$

We know that

$$u_1 + p_1 v_{s1} = h_1 = \text{Enthalpy of the working substance entering the system in J/kg, and}$$

$$u_2 + p_2 v_{s2} = h_2 = \text{Enthalpy of the working substance leaving the system in J/kg.}$$

Thus, the above expression may be written as

$$h_1 + \frac{V_1^2}{2} + g\, z_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + g\, z_2 + w_{1-2} \qquad \ldots (i)$$

or $\qquad h_1 + ke_1 + pe_1 + q_{1-2} = h_2 + ke_2 + pe_2 + w_{1-2}$

It may be noted that all the terms in equation (*i*) represent the energy flow per unit mass of the working substance (*i.e.* in J/kg). When the equation (*i*) is multiplied throughout by the mass of the working substance (*m*) in kg/s, then all the terms will represent the energy flow per unit time (*i.e.* in J/s).

Thus the equation (*i*) may also be written as

$$m\left(h_1 + \frac{V_1^2}{2} + g\, z_1 + q_{1-2} \right) = m\left(h_2 + \frac{V_2^2}{2} + g\, z_2 + w_{1-2} \right) \qquad \ldots (ii)$$

Both the equations (*i*) and (*ii*) are known as *steady flow energy equations*.

Notes: **1.** In a steady flow, the mass flow rate (*m*) of the working substance entering and leaving the system is given by

$$m = \frac{A_1 V_1}{v_{s1}} = \frac{A_2 V_2}{v_{s2}} \quad \text{(in kg/s)}$$

This equation is known as *equation of continuity*.

2. The steady flow energy equation (*i*), for unit mass flow may be written as

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) + (g\, z_2 - g\, z_1) \qquad \ldots (iii)$$

$$= (h_2 - h_1) + (ke_2 - ke_1) + (pe_2 - pe_1)$$

In differential form, this expression is written as

$$\delta q - \delta w = dh + d\,(ke) - d\,(pe)$$

3. In thermodynamics, the effect of gravity is generally neglected. Therefore equation (*iii*) may be written as

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) \qquad \ldots (iv)$$

If $V_1 = V_2$, then equation (*iv*) reduces to
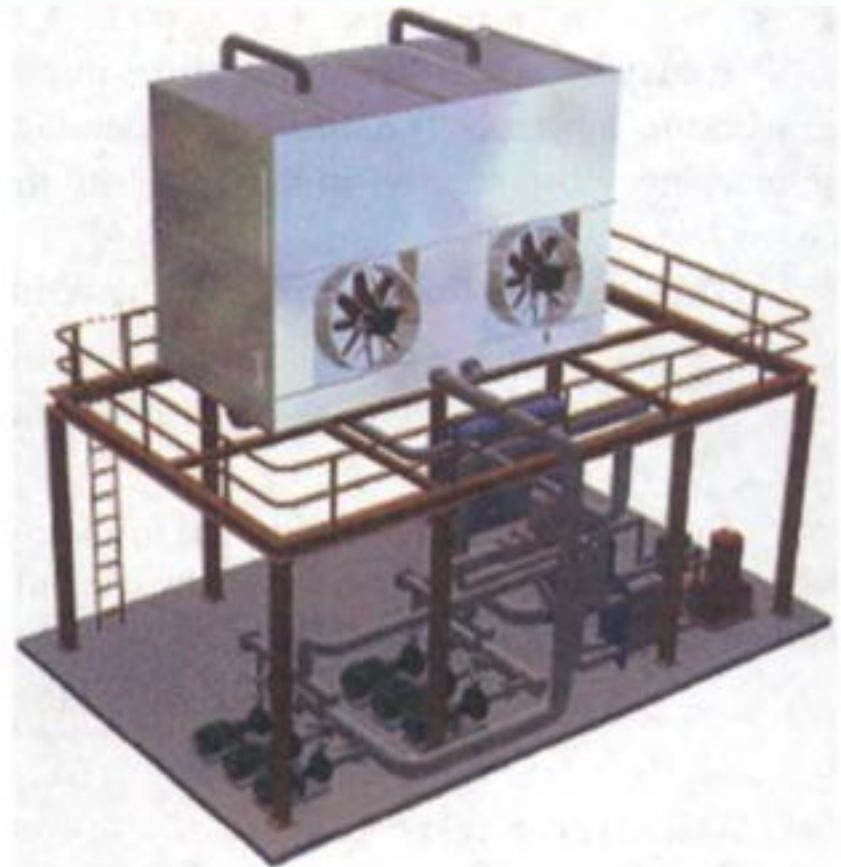
$$q_{1-2} - w_{1-2} = (h_2 - h_1) \qquad \ldots (v)$$

4. In a non-flow process, the flow or displacement energy at inlet and outlet is zero, *i.e.* $p_1 v_{s1} = 0$ and $p_2 v_{s2} = 0$. Therefore $h_2 = u_2$ and $h_1 = u_1$.

Thus the equation (*v*) may be written as $q_{1-2} - w_{1-2} = u_2 - u_1$; which is same as for non-flow process.

2

Air Refrigeration Cycles

2.1 Introduction

In an air refrigeration cycle, the air is used as a refrigerant. In olden days, air was widely used in commercial applications because of its availability at free of cost. Since air does not change its phase *i.e.* remains gaseous throughout the cycle, therefore the heat carrying capacity per kg of air is very small as compared to vapour absorbing systems. The air-cycle refrigeration systems, as originally designed and installed, are now practically obsolete because of their low coefficient of performance and high power requirements. However, this system continues to be favoured for air refrigeration because of the low weight

and volume of the equipment. The basic elements of an air cycle refrigeration system are the compressor, the cooler or heat exchanger, the expander and the refrigerator.

Before discussing the air refrigeration cycles, we should first know about the unit of refrigeration, coefficient of performance of a refrigerator and the difference between the heat engine, a refrigerator and a heat pump.

Warm Air   Cool air

1 tonne of Ice
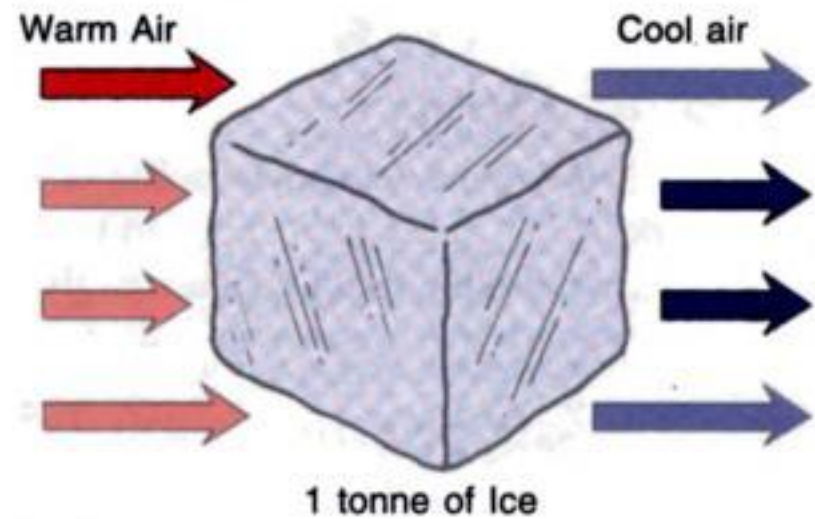
2.2 **Units of Refrigeration**

The practical unit of refrigeration is expressed in terms of 'tonne of refrigeration' (briefly written as TR). A *tonne of refrigeration* is defined as the amount of refrigeration effect produced by the uniform melting of one tonne (1000 kg) of ice from and at 0°C in 24 hours.

One tonne (1000 kg) of ice requires 335 kJ/kg to melt. When this is accomplished in 24 hours, it is known as a heat transfer rate of 1 tonne of refrigeration (1TR).

Since the latent heat of ice is 335 kJ/kg, therefore one tonne of refrigeration,

$$1TR = 1000 \times 335 \text{ kJ in 24 hours}$$

$$= \frac{1000 \times 335}{24 \times 60} = 232.6 \text{ kJ/min}$$

In actual practice, one tonne of refrigeration is taken as equivalent to 210 kJ/min or 3.5 kW (*i.e.* 3.5 kJ/s).

2.3 **Coefficient of Performance of a Refrigerator**

The coefficient of performance (briefly written as C.O.P.) is the ratio of heat extracted in the refrigerator to the work done on the refrigerant. It is also known as theoretical coefficient of performance. Mathematically,

$$\text{Theoretical C.O.P.} = \frac{Q}{W}$$

where

Q = Amount of heat extracted in the refrigerator (or the amount of refrigeration produced, or the capacity of a refrigerator), and

W = Amount of work done.

Notes: **1.** For per unit mass, C.O.P. $= \dfrac{q}{w}$

2. The coefficient of performance is the reciprocal of the efficiency (*i.e.* 1/η) of a heat engine. It is thus obvious, that the value of C.O.P is always greater than unity.

3. The ratio of the actual C.O.P to the theoretical C.O.P. is known as *relative coefficient of performance*. Mathematically,

$$\text{Relative C.O.P.} = \frac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$$

Example 2.1. *Find the C.O.P. of a refrigeration system if the work input is 80 kJ/kg and refrigeration effect produced is 160 kJ/kg of refrigerant flowing.*

Solution. Given : $w = 80$ kJ/kg ; $q = 160$ kJ/kg

We know that C.O.P. of a refrigeration system

$$= \frac{q}{w} = \frac{160}{80} = 2 \text{ Ans.}$$

2.4 Difference Between a Heat Engine, Refrigerator and Heat Pump

In a heat engine, as shown in Fig. 2.1 (*a*), the heat supplied to the engine is converted into useful work. If Q_2 is the heat supplied to the engine and Q_1 is the heat rejected from the engine, then the net work done by the engine is given by
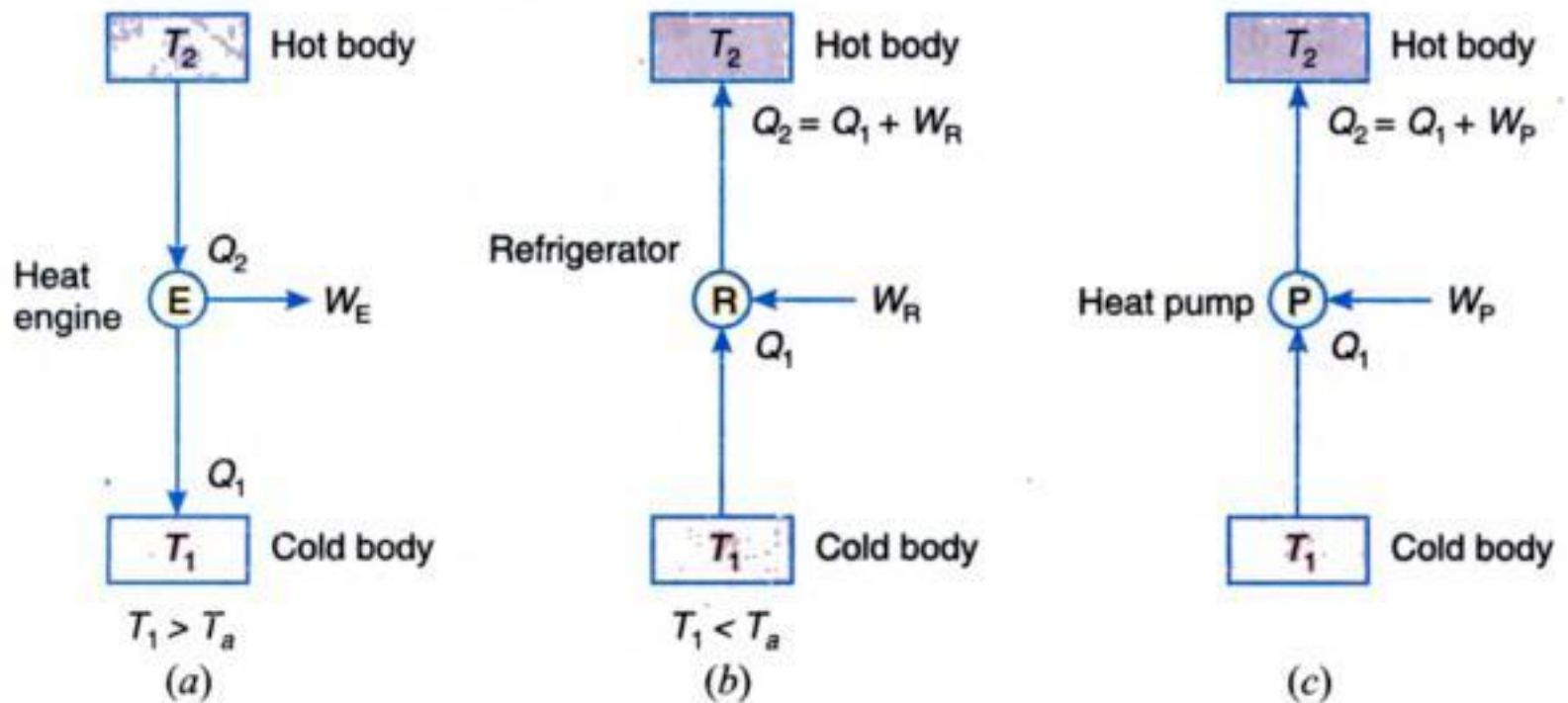
$$W_E = Q_2 - Q_1$$

Fig. 2.1. Difference between a heat engine, refrigerator and heat pump.

The performance of a heat engine is expressed by its efficiency. We know that the efficiency or coefficient of performance of an engine,

$$\eta_E \text{ or } (C.O.P.)_E = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_E}{Q_2} = \frac{Q_2 - Q_1}{Q_2}$$

A refrigerator as shown in Fig. 2.1 (*b*), is a reversed heat engine which either cool or maintain the temperature of a body (T_1) lower than the atmosheric temperature (T_a). This is done by extracting the heat (Q_1) from a cold body and delivering it to a hot body (Q_2). In doing so, work W_R is required to be done on the system. According to First Law of Thermodynamics,

$$W_R = Q_2 - Q_1$$

The performance of a refrigerator is expressed by the ratio of amount of heat taken from the cold body (Q_1) to the amount of work required to be done on the system (W_R). This ratio is called coefficient of performance. Mathematically, coefficient of performance of a refrigerator,

$$(C.O.P.)_R = \frac{Q_1}{W_R} = \frac{Q_1}{Q_2 - Q_1}$$

Any refrigerating system is a heat pump as shown in Fig 2.1 (*c*), which extracts heat (Q_1) from a cold body and delivers it to a hot body. Thus there is no difference between the cycle of operations of a heat pump and a refrigerator. The main difference between the two is in their operating temperatures. A refrigerator works between the cold body temperature (T_1) and the atmospheric temperature (T_a) whereas the heat pump operates between the hot body temperature (T_2) and the atmospheric temperature (T_a). A refrigerator used for cooling in summer can be used as a heat pump for heating in winter.

In the similar way, as discussed for refrigerator, we have

$$W_P = Q_2 - Q_1$$

The performance of a heat pump is expressed by the ratio of the amount of heat delivered to the hot body (Q_2) to the amount of work required to be done on the system (W_P). This ratio is called coefficient of performance or energy performance ratio (E.P.R.) of a heat pump. Mathematically, coefficient of performance or energy performance ratio of a heat pump,

$$(C.O.P.)_P \text{ or E.P.R.} = \frac{Q_2}{W_P} = \frac{Q_2}{Q_2 - Q_1}$$

$$= \frac{Q_1}{Q_2 - Q_1} + 1 = (C.O.P.)_R + 1$$

From above we see that the C.O.P. may be less than one or greater than one depending on the type of refrigeration system used. But the C.O.P. of a heat pump is always greater than one.

2.5 Open Air Refrigeration Cycle

In an open air refrigeration cycle, the air is directly led to the space to be cooled (*i.e.* a refrigerator), allowed to circulate through the cooler and then returned to the compressor to start another cycle. Since the air is supplied to the refrigerator at atmospheric pressure, therefore, volume of air handled by the compressor and expander is large. Thus the size of compressor and expander should be large. Another disadvantage of the open cycle system is that the moisture is regularly carried away by the air circulated through the cooled space. This leads to the formation of frost at the end of expansion process and clog the line. Thus in an open cycle system, a drier should be used.

2.6 Closed or Dense Air Refrigeration Cycle

In a closed or dense air refrigeration cycle, the air is passed through the pipes and component parts of the system at all times. The air, in this system, is used for absorbing heat from the other fluid (say brine) and this cooled brine is circulated into the space to be cooled. The air in the closed system does not come in contact directly with the space to be cooled.

The closed air refrigeration cycle has the following thermodynamic advantages :

1. Since it can work at a suction pressure higher than that of atmospheric pressure, therefore the volume of air handled by the compressor and expander are smaller as compared to an open air refrigeration cycle system.

2. The operating pressure ratio can be reduced, which results in higher coefficient of performance.

2.7 Air Refrigerator Working on Reversed Carnot Cycle

In refrigerating systems, the Carnot cycle considered is the reversed Carnot cycle. We know that a heat engine working on Carnot cycle has the highest possible efficiency. Similarly, a refrigerating system working on the reversed Carnot cycle, will have the maximum possible coefficient of performance. We also know that it is not possible to make an engine working on the Carnot cycle. Similarly, it is also not possible to make a refrigerating machine working on the reversed Carnot cycle. However, it is used as the ultimate standard of comparison.

A reversed Carnot cycle, using air as working medium (or refrigerant) is shown on *p-v* and *T-s* diagrams in Fig. 2.2(*a*) and (*b*) respectively. At point 1, let p_1, v_1, T_1 be the pressure, volume and temperature of air respectively.

(a) p-v diagram.
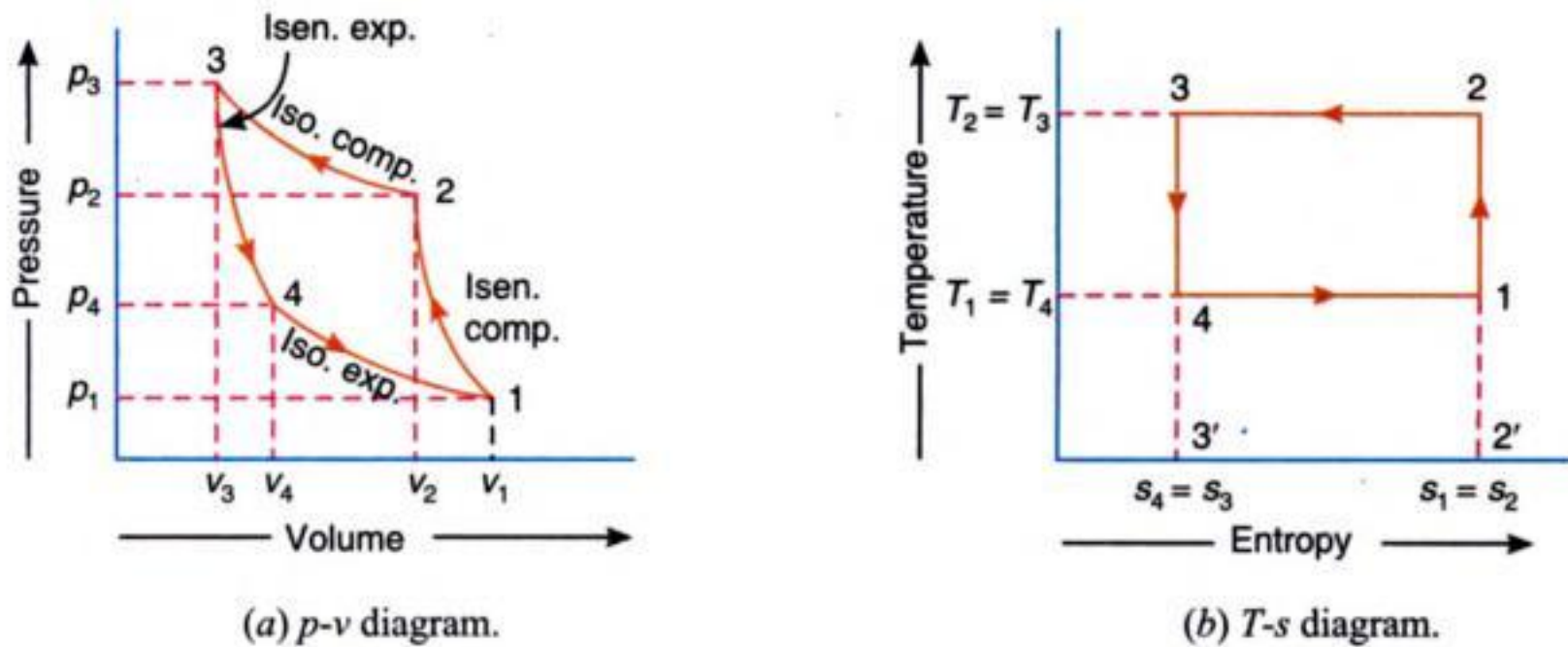
(b) T-s diagram.

Fig. 2.2. Reversed Carnot cycle.

The four processes of the cycle are as follows :

1. *Isentropic compression process.* The air is compressed isentropically as shown by the curve 1-2 on p-v and T-s diagrams. During this process, the pressure of air increases from p_1 to p_2, specific volume decreases from v_1 to v_2 and temperature increases from T_1 to T_2. We know that during isentropic compression, no heat is absorbed or rejected by the air.

2. *Isothermal compression process.* The air is now compressed isothermally (*i.e.* at constant temperature, $T_2 = T_3$) as shown by the curve 2-3 on p-v and T-s diagrams. During this process, the pressure of air increases from p_2 to p_3 and specific volume decreases from v_2 to v_3. We know that the heat rejected by the air during isothermal compression per kg of air,

$$q_{2-3} = \text{Area } 2\text{-}3\text{-}3'\text{-}2'$$
$$= T_3 (s_2 - s_3) = T_2 (s_2 - s_3)$$

3. *Isentropic expansion process.* The air is now expanded isentropically as shown by the curve 3-4 on p-v and T-s diagrams. The pressure of air decreases from p_3 to p_4, specific volume increases from v_3 to v_4 and the temperature decreases from T_3 to T_4, We know that during isentropic expansion, no heat is absorbed or rejected by the air.

4. *Isothermal expansion process.* The air is now expanded isothermally (*i.e.* at constant temperature, $T_4 = T_1$) as shown by the curve 4-1 on p-v and T-s diagrams. The pressure of air decreases from p_4 to p_1, and specific volume increases from v_4 to v_1. We know that the heat absorbed by the air (or heat extracted from the cold body) during isothermal expansion per kg of air,

$$q_{4-1} = \text{Area } 4\text{-}1\text{-}2'\text{-}3'$$
$$= T_4 (s_1 - s_4) = T_4 (s_2 - s_3) = T_1 (s_2 - s_3)$$

We know that work done during the cycle per kg of air

$$= \text{*Heat rejected} - \text{Heat absorbed } = q_{2-3} - q_{4-1}$$
$$= T_2 (s_2 - s_3) - T_1 (s_2 - s_3) = (T_2 - T_1)(s_2 - s_3)$$

∴ Coefficient of performance of the refrigeration system working on reversed Carnot cycle,

$$(\text{C.O.P.})_R = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_{4-1}}{q_{2-3} - q_{4-1}}$$

---

* In a refrigerating machine, heat rejected is more than heat absorbed.

$$= \frac{T_1(s_2 - s_3)}{(T_2 - T_1)(s_2 - s_3)} = \frac{T_1}{T_2 - T_1}$$

Though the reversed Carnot cycle is the most efficient between the fixed temperature limits, yet no refrigerator has been made using this cycle. This is due to the reason that the isentropic processes of the cycle require high speed while the isothermal processes require an extremely low speed. This variation in speed of air is not practicable.

Note : We have already discussed that C.O.P. of a heat pump,

$$(\text{C.O.P.})_P = (\text{C.O.P.})_R + 1 = \frac{T_1}{T_2 - T_1} + 1 = \frac{T_2}{T_2 - T_1}$$

and C.O.P. or efficiency of a heat engine,

$$(\text{C.O.P.})_E = \frac{1}{(\text{C.O.P.})_P} = \frac{T_2 - T_1}{T_2}$$

2.8 Temperature Limitations for Reversed Carnot Cycle

We have seen in the previous article that the C.O.P. of the refrigeration system working on reversed Carnot cycle is given by

$$(\text{C.O.P.})_R = \frac{T_1}{T_2 - T_1}$$

where
T_1 = Lower temperature, and
T_2 = Higher temperature.

The C.O.P. of the reversed Carnot cycle may be improved by

1. decreasing the higher temperature (*i.e.* temperature of hot body, T_2), or

2. increasing the lower temperature (*i.e.* temperature of cold body, T_1).

This applies to all refrigerating machines, both theoretical and practical. It may be noted that temperatures T_1 and T_2 cannot be varied at will, due to certain functional limitations. It should be kept in mind that the higher temperature (T_2) is the temperature of cooling water or air available for rejection of heat and the lower temperature (T_1) is the temperature to be maintained in the refrigerator. The heat transfer will take place in the right direction only when the higher temperature is more than the temperature of cooling water or air to which heat is to be rejected, while the lower temperature must be less than the temperature of substance to be cooled.

Thus, if the temperature of cooling water or air (*i.e.* T_2) available for heat rejection is low, the C.O.P. of the Carnot refrigerator will be high. Since T_2 in winter is less than T_2 in summer, therefore, C.O.P. in winter will be higher than C.O.P. in summer. In other words, the Carnot refrigerators work more efficiently in winter than in summer. Similarly, if the lower temperature fixed by the

Domestic Air Conditioner.

refrigeration application is high, the C.O.P. of the Carnot refrigerator will be high. Thus a Carnot refrigerator used for making ice at 0°C (273 K) will have less C.O.P. than a Carnot refrigerator used for air-conditioned plant in summer at 20°C when the atmospheric temperature is 40°C. In other words, we can say that the Carnot C.O.P. of a domestic refrigerator is less than the Carnot C.O.P. of a domestic air-conditioner.

Example 2.2. *A machine working on a Carnot cycle operates between 305 K and 260 K. Determine the C.O.P. when it is operated as 1. a refrigerating machine; 2. a heat pump; and 3. a heat engine.*

Solution. Given : $T_2 = 305$ K ; $T_1 = 260$ K

1. C.O.P. of a refrigerating machine

We know that C.O.P. of a refrigerating machine,

$$(C.O.P.)_R = \frac{T_1}{T_2 - T_1} = \frac{260}{305 - 260} = 5.78 \text{ Ans.}$$

2. C.O.P. of a heat pump

*We know that C.O.P. of a heat pump,

$$(C.O.P.)_P = \frac{T_2}{T_2 - T_1} = \frac{305}{305 - 260} = 6.78 \text{ Ans.}$$

3. C.O.P. of a heat engine

**We know that C.O.P. of a heat engine,

$$(C.O.P.)_E = \frac{T_2 - T_1}{T_2} = \frac{305 - 260}{305} = 0.147 \text{ Ans.}$$

Example 2.3. *A Carnot refrigeration cycle absorbs heat at 270 K and rejects it at 300 K.*

1. Calculate the coefficient of performance of this refrigeration cycle.

2. If the cycle is absorbing 1130 kJ/min at 270 K, how many kJ of work is required per second ?

3. If the Carnot heat pump operates between the same temperatures as the above refrigeration cycle, what is the coefficient of performance ?

4. How many kJ/min will the heat pump deliver at 300 K if it absorbs 1130 kJ/min at 270 K.

Solution. Given : $T_1 = 270$ K ; $T_2 = 300$ K

1. Coefficient of performance of Carnot refrigeration cycle

We know that coefficient of performance of Carnot refrigeration cycle,

$$(C.O.P.)_R = \frac{T_1}{T_2 - T_1} = \frac{270}{300 - 270} = 9 \text{ Ans.}$$

2. Work required per second

Let W_R = Work required per second.

---

* We know that C.O.P. of a heat pump, $(C.O.P.)_P = (C.O.P.)_R + 1 = 5.78 + 1 = 6.78$ **Ans.**

** We know that C.O.P. of a heat engine, $(C.O.P.)_E = \dfrac{1}{(C.O.P.)_P} = \dfrac{1}{6.78} = 0.147$ **Ans.**

Heat absorbed at 270 K (*i.e.* T_1),

$$Q_1 = 1130 \text{ kJ/min} = 18.83 \text{ kJ/s} \qquad \text{...(Given)}$$

We know that $\quad (\text{C.O.P.})_R = \dfrac{Q_1}{W_R} \quad$ or $\quad 9 = \dfrac{18.83}{W_R}$

∴ $\qquad\qquad\qquad W_R = 2.1 \text{ kJ/s}$ **Ans.**

3. Coefficient of performance of Carnot heat pump

We know that coefficient of performance of a Carnot heat pump,

$$(\text{C.O.P.})_P = \frac{T_2}{T_2 - T_1} = \frac{300}{300 - 270} = 10 \text{ **Ans.**}$$

4. Heat delivered by heat pump at 300 K

Let $\qquad\qquad\qquad Q_2 = $ Heat delivered by heat pump at 300 K.

Heat absorbed at 270 K (*i.e.* T_1),

$$Q_1 = 1130 \text{ kJ/min} \qquad \text{... (Given)}$$

We know that

$$(\text{C.O.P.})_P = \frac{Q_2}{Q_2 - Q_1} \quad \text{or} \quad 10 = \frac{Q_2}{Q_2 - 1130}$$

∴ $\qquad 10 Q_2 - 11\,300 = Q_2 \quad$ or $\quad Q_2 = 1256 \text{ kJ/min}$ **Ans.**

Example 2.4. *A cold storage is to be maintained at –5°C while the surroundings are at 35°C. The heat leakage from the surroundings into the cold storage is estimated to be 29 kW. The actual C.O.P. of the refrigeration plant is one-third of an ideal plant working between the same temperatures. Find the power required to drive the plant.*

Solution. Given : $T_1 = -5°C = -5 + 273 = 268 \text{ K}$;
$T_2 = 35°C = 35 + 273 = 308 \text{ K}$; $Q_1 = 29 \text{ kW}$;

$$(\text{C.O.P.})_{actual} = \frac{1}{3}(\text{C.O.P.})_{ideal}$$

The refrigerating plant operating between the temperatures T_1 and T_2 is shown in Fig. 2.3.

Let W_R = Work or power required to drive the plant.

We know that the coefficient of performance of an ideal refrigeration plant,

$$(\text{C.O.P.})_{ideal} = \frac{T_1}{T_2 - T_1} = \frac{268}{308 - 268} = 6.7$$

∴ Actual coefficient of performance,

$$(\text{C.O.P.})_{actual} = \frac{1}{3}(\text{C.O.P.})_{ideal} = \frac{1}{3} \times 6.7 = 2.233$$

We also know that $\quad (\text{C.O.P.})_{actual} = \dfrac{Q_1}{W_R}$

∴ $\qquad W_R = \dfrac{Q_1}{(\text{C.O.P.})_{actual}} = \dfrac{29}{2.233} = 12.987 \text{ kW}$ **Ans.**
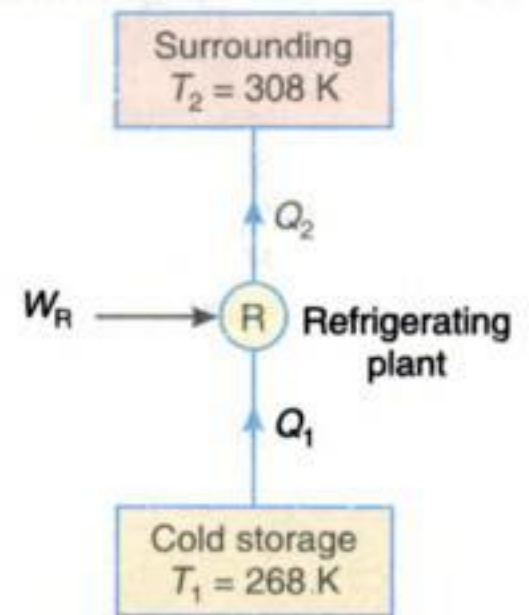
Fig. 2.3

Example 2.5. *1.5 kW per tonne of refrigeration is required to maintain the temperature of* −40°C *in the refrigerator. If the refrigeration cycle works on Carnot cycle, determine the following:*

1. C.O.P. of the cycle; 2. Temperature of the sink ; 3. Heat rejected to the sink per tonne of refrigeration ; and 4. Heat supplied and E.P.R., if the cycle is used as a heat pump.

Solution. Given : $W_R = 1.5$ kW ; $Q_1 = 1$ TR ; $T_1 = -40°C = -40 + 273 = 233$ K

1. C.O.P. of the cycle

The refrigeration cycle working on Carnot cycle is shown in Fig 2.4.

Since 1.5 kW per tonne of refrigeration is required to maintain the temperature in the refrigerator, therefore amount of work required to be done,

$$W_R = 1.5 \text{ kW} = 1.5 \text{ kJ/s} = 1.5 \times 60 = 90 \text{ kJ/min}$$

and heat extracted from the cold body,

$$Q_1 = 1 \text{ TR} = 210 \text{ kJ/min}$$

We know that

$$(\text{C.O.P.})_R = \frac{Q_1}{W_R} = \frac{210}{90} = 2.33 \text{ Ans.}$$

(a)   (b)
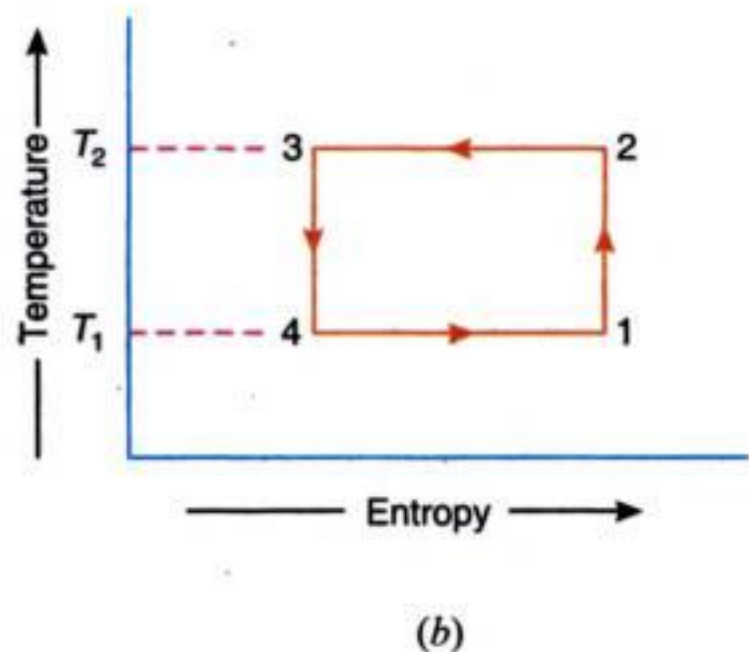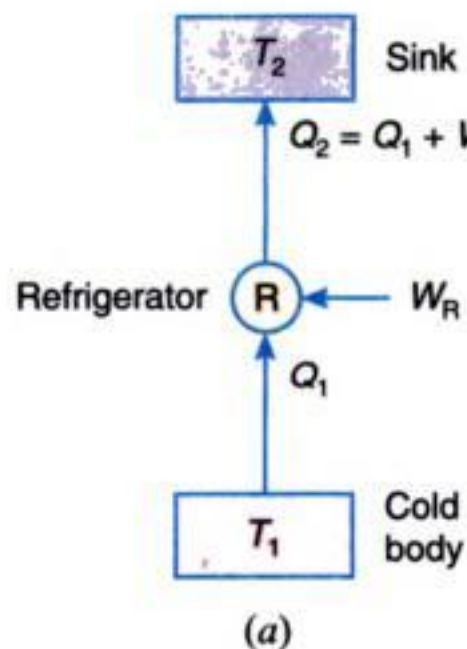
Fig. 2.4

2. Temperature of the sink

Let T_2 = Temperature of the sink.

We know that $(\text{C.O.P.})_R = \dfrac{T_1}{T_2 - T_1}$ or $2.33 = \dfrac{233}{T_2 - 233}$

∴ $T_2 = \dfrac{233}{2.33} + 233 = 333 \text{ K} = 60°C$ **Ans.**

3. Heat rejected to the sink per tonne of refrigeration

We know that heat rejected to the sink,

$$Q_2 = Q_1 + W_R = 210 + 90 = 300 \text{ kJ/min} \text{ Ans.}$$

4. Heat supplied and E.P.R., if the cycle is used as a heat pump

We know that heat supplied when the cycle is used as a heat pump is

$$Q_2 = 300 \text{ kJ/min} \text{ Ans.}$$

and $\text{E.P.R.} = (\text{C.O.P.})_R + 1 = 2.33 + 1 = 3.33$ **Ans.**

Example 2.6. *The capacity of a refrigerator is 200 TR when working between – 6°C and 25°C. Determine the mass of ice produced per day from water at 25°C. Also find the power required to drive the unit. Assume that the cycle operates on reversed Carnot cycle and latent heat of ice is 335 kJ/kg.*

Solution. Given : $Q = 200$ TR ; $T_1 = -6°C = -6 + 273 = 267$ K ; $T_2 = 25°C = 25 + 273 = 298$ K ; $t_w = 25°C$; $h_{fg(ice)} = 335$ kJ/kg

Mass of ice produced per day

We know that heat extraction capacity of the refrigerator

$$= 200 \times 210 = 42\,000 \text{ kJ/min} \qquad \dots (\because 1\text{TR} = 210 \text{ kJ/min})$$

and heat removed from 1 kg of water at 25°C to form ice at 0°C

$$= \text{Mass} \times \text{Sp. heat} \times \text{Rise in temperature} + h_{fg(ice)}$$
$$= 1 \times 4.187(25 - 0) + 335 = 439.7 \text{ kJ/kg}$$

∴ Mass of ice produced per min

$$= \frac{42\,000}{439.7} = 95.52 \text{ kg/min}$$

and mass of ice produced per day $\quad = 95.52 \times 60 \times 24 = 137\,550 \text{ kg} = 137.55 \text{ tonnes}$ **Ans.**

Power required to drive the unit

We know that C.O.P. of the reversed Carnot cycle

$$= \frac{T_1}{T_2 - T_1} = \frac{267}{298 - 267} = 8.6$$

Also $\qquad\qquad$ C.O.P. $= \dfrac{\text{Heat extraction capacity}}{\text{Work done per min}}$

∴ $\qquad\qquad 8.6 = \dfrac{42\,000}{\text{Work done per min}}$

or $\qquad\qquad$ Work done per min $= 42\,000/8.6 = 4884$ kJ/min

∴ Power required to drive the unit

$$= 4884/60 = 81.4 \text{ kW}$$ **Ans.**

Example 2.7. *Five hundred kgs of fruits are supplied to a cold storage at 20°C. The cold storage is maintained at –5°C and the fruits get cooled to the storage temperature in 10 hours. The latent heat of freezing is 105 kJ/kg and specific heat of fruit is 1.256 kJ/kg K. Find the refrigeration capacity of the plant.*

Solution. Given : $m = 500$ kg ; $T_2 = 20°C = 20 + 273 = 293$ K; $T_1 = -5°C = -5 + 273 = 268$ K ; $h_{fg} = 105$ kJ/kg, $c_F = 1.256$ kJ/kg K

We know that heat removed from the fruits in 10 hours,

$$Q_1 = m\, c_F\, (T_2 - T_1)$$
$$= 500 \times 1.256 (293 - 268) = 15\,700 \text{ kJ}$$

and total latent heat of freezing,

$$Q_2 = m \times h_{fg} = 500 \times 105 = 52\,500 \text{ kJ}$$

∴ Total heat removed in 10 hours,

$$Q = Q_1 + Q_2 = 15\,700 + 52\,500 = 68\,200 \text{ kJ}$$

and total heat removed in one minute

$$= 68\ 200/10 \times 60 = 113.7\ \text{kJ/min}$$

∴ Refrigeration capacity of the plant

$$= 113.7/210 = 0.541\ \text{TR}\ \textbf{Ans.} \qquad \dots(\because 1\text{TR} = 210\ \text{kJ/min})$$

Example 2.8. *A cold storage plant is required to store 20 tonnes of fish. The fish is supplied at a temperature of 30°C. The specific heat of fish above freezing point is 2.93 kJ/kg K. The specific heat of fish below freezing point is 1.26 kJ/kg K. The fish is stored in cold storage which is maintained at − 8°C. The freezing point of fish is − 4°C. The latent heat of fish is 235 kJ/kg. If the plant requires 75 kW to drive it, find :*

1. The capacity of the plant, and 2. Time taken to achieve cooling.

Assume actual C.O.P. of the plant as 0.3 of the Carnot C.O.P.

Solution. Given : $m = 20\ \text{t} = 20\ 000\ \text{kg}$; $T_2 = 30°\text{C} = 30 + 273 = 303\ \text{K}$; $c_{AF} = 2.93\ \text{kJ/kg K}$; $c_{BF} = 1.26\ \text{kJ/kgK}$; $T_1 = − 8°\text{C} = − 8 + 273 = 265\ \text{K}$; $T_3 = − 4°\text{C} = − 4 + 273 = 269\ \text{K}$; $h_{fg(Fish)} = 235\ \text{kJ/kg}$; $P = 75\ \text{kW} = 75\ \text{kJ/s}$

1. Capacity of the plant

We know that Carnot C.O.P.

$$= \frac{T_1}{T_2 - T_1} = \frac{265}{303 - 265} = 6.97$$

∴ Actual C.O.P. $= 0.3 \times 6.97 = 2.091$

and heat removed by the plant $= \text{Actual C.O.P.} \times \text{Work required}$

$$= 2.091 \times 75 = 156.8\ \text{kJ/s}$$

$$= 156.8 \times 60\ \text{kJ/min} = 9408\ \text{kJ/min}$$

∴   Capacity of the plant

$$= 9408\ /\ 210 = 44.8\ \text{TR}\ \textbf{Ans.} \qquad \dots (\because 1\text{TR} = 210\ \text{kJ/min})$$

2. Time taken to achieve cooling

We know that heat removed from the fish above freezing point,

$$Q_1 = m \times c_{AF}(T_2 - T_3)$$
$$= 20\ 000 \times 2.93\ (303 - 269) = 1.992 \times 10^6\ \text{kJ}$$

Similarly, heat removed from the fish below freezing point,

$$Q_2 = m \times c_{BF}(T_3 - T_1)$$
$$= 20\ 000 \times 1.26\ (269 - 265) = 0.101 \times 10^6\ \text{kJ}$$

and total latent heat of fish,

$$Q_3 = m \times h_{fg(Fish)} = 20\ 000 \times 235 = 4.7 \times 10^6\ \text{kJ}$$

∴ Total heat removed by the plant

$$= Q_1 + Q_2 + Q_3$$
$$= 1.992 \times 10^6 + 0.101 \times 10^6 + 4.7 \times 10^6 = 6.793 \times 10^6\ \text{kJ}$$

and time taken to achieve cooling

$$= \frac{\text{Total heat removed by the plant}}{\text{Heat removed by the plant per min}}$$

$$= \frac{6.793 \times 10^6}{9408} = 722\ \text{min} = 12.03\ \text{h}\ \textbf{Ans.}$$

2.9 Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

A Bell-Coleman air refrigeration machine was developed by Bell-Coleman and Light Foot by reversing the Joule's air cycle. It was one of the earliest types of refrigerators used in ships carrying frozen meat. Fig. 2.5 shows a schematic diagram of such a machine which consists of a compressor, a cooler, an expander and a refrigerator.
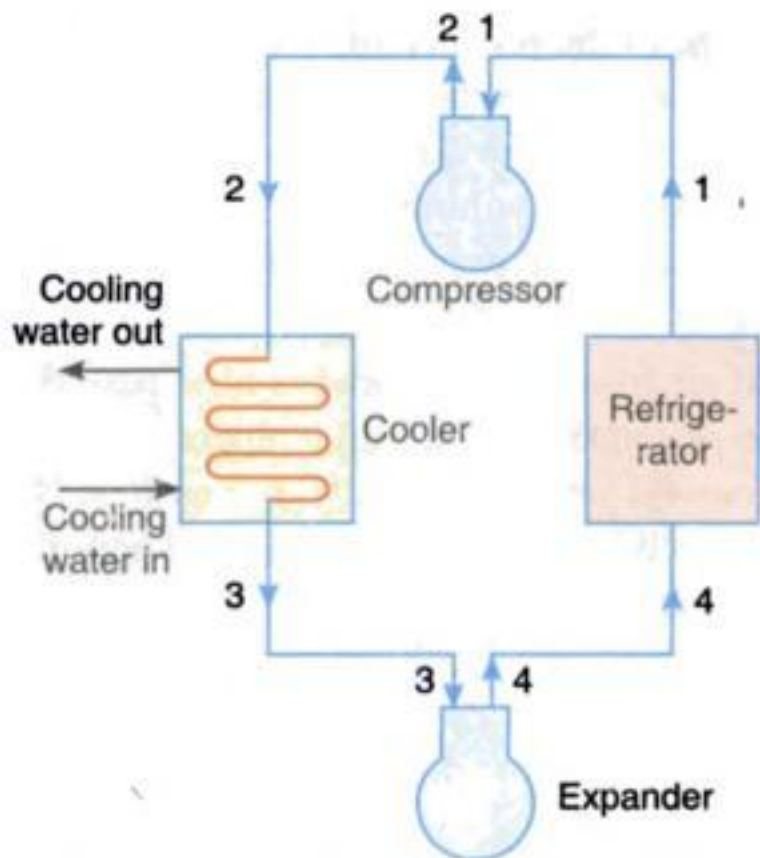
Fig. 2.5. Open cycle air Bell-Coleman Refrigerator.
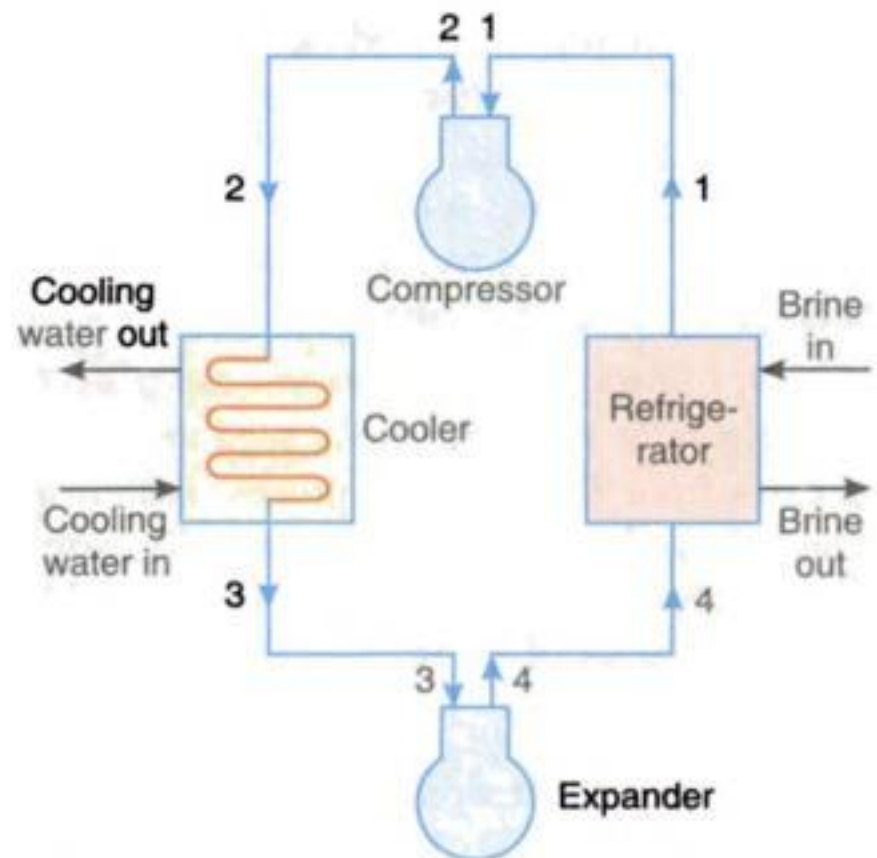
Fig. 2.6. Closed cycle or dense air Bell-Coleman Refrigerator.

The Bell-Coleman cycle (also known as reversed Brayton or Joule cycle) is a modification of reversed Carnot cycle. The cycle is shown on p-v and T-s diagrams in Fig. 2.7 (a) and (b). At point 1, let p_1, v_1 and T_1 be the pressure, volume and temperature of air respectively. The four processes of the cycle are as follows :

1. Isentropic compression process. The cold air from the refrigerator is drawn into the compressor cylinder where it is compressed isentropically in the compressor as shown by the curve 1–2 on p-v and T-s diagrams. During the compression stroke, both the pressure and temperature increases and the specific volume of air at delivery from compressor reduces from v_1 to v_2. We know that during isentropic compression process, no heat is absorbed or rejected by the air.

(a) p-v diagram.

(b) T-s diagram.
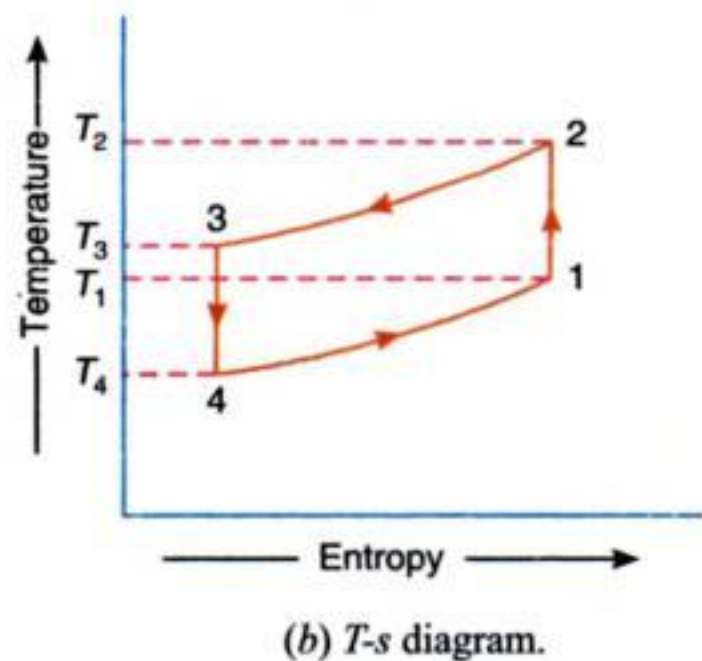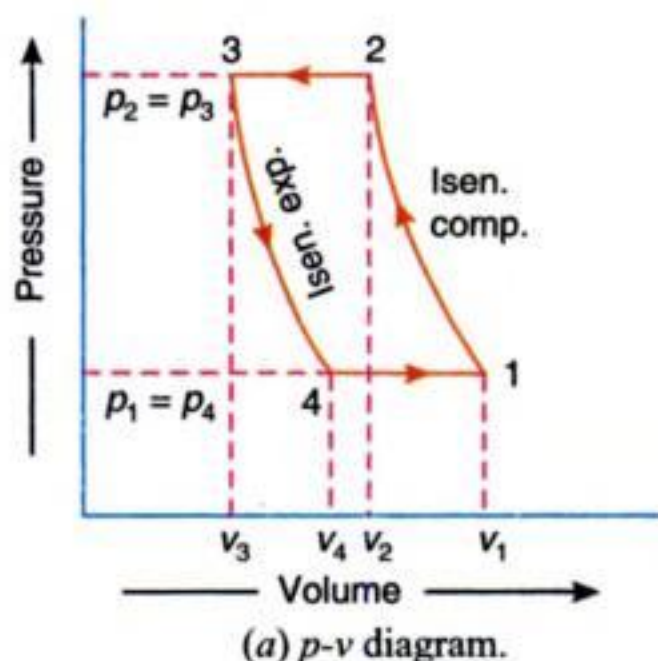
Fig. 2.7. Bell-Coleman cycle.

2. *Constant pressure cooling process.* The warm air from the compressor is now passed into the cooler where it is cooled at constant pressure p_3 (equal to p_2), reducing the temperature from T_2 to T_3 (the temperature of cooling water) as shown by the curve 2-3 on p-v and T-s diagrams. The specific volume also reduces from v_2 to v_3. We know that heat rejected by the air during constant pressure per kg of air,

$$Q_{2-3} = c_p (T_2 - T_3)$$

3. *Isentropic expansion process.* The air from the cooler is now drawn into the expander cylinder where it is expanded isentropically from pressure p_3 to the refrigerator pressure p_4 which is equal to the atmospheric pressure. The temperature of air during expansion falls from T_3 to T_4 (*i.e.* the temperature much below the temperature of cooling water, T_3). The expansion process is shown by the curve 3-4 on the p-v and T-s diagrams. The specific volume of air at entry to the refrigerator increases from v_3 to v_4. We know that during isentropic expansion of air, no heat is absorbed or rejected by the air.

4. *Constant pressure expansion process.* The cold air from the expander is now passed to the refrigerator where it is expanded at constant pressure p_4 (equal to p_1). The temperature of air increases from T_4 to T_1. This process is shown by the curve 4-1 on the p-v and T-s diagrams. Due to heat from the refrigerator, the specific volume of the air changes from v_4 to v_1. We know that the heat absorbed by the air (or heat extracted from the refrigerator) during constant pressure expansion per kg of air is

$$q_{4-1} = c_p (T_1 - T_4)$$

We know that work done during the cycle per kg of air

$$= \text{Heat rejected} - \text{Heat absorbed}$$

$$= c_p (T_2 - T_3) - c_p (T_1 - T_4)$$

∴ Coefficient of performance,

$$\text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{c_p(T_1 - T_4)}{c_p(T_2 - T_3) - c_p(T_1 - T_4)}$$

$$= \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{T_4\left(\dfrac{T_1}{T_4} - 1\right)}{T_3\left(\dfrac{T_2}{T_3} - 1\right) - T_4\left(\dfrac{T_1}{T_4} - 1\right)} \qquad \ldots (i)$$

We know that for isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \ldots (ii)$$

Similarly, for isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \qquad \ldots (iii)$$

Since, $p_2 = p_3$ and $p_1 = p_4$, therefore from equations (*ii*) and (*iii*),

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_2}{T_3} = \frac{T_1}{T_4} \qquad \ldots (iv)$$

Now substituting these values in equation (i), we get

$$\text{C.O.P.} = \frac{T_4}{T_3 - T_4} = \frac{1}{\dfrac{T_3}{T_4} - 1}$$

$$= \frac{1}{\left(\dfrac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1} \qquad \ldots (v)$$

where $\qquad r_p$ = Compression or Expansion ratio = $\dfrac{p_2}{p_1} = \dfrac{p_3}{p_4}$

Sometimes, the compression and expansion processes take place according to the law pv^n = Constant. In such a case, the C.O.P. is obtained from the fundamentals as discussed below :

We know that work done by the compressor during the process 1-2 per kg of air,

$$w_1 = \frac{n}{n-1}(p_2 v_2 - p_1 v_1) = \frac{n}{n-1}(RT_2 - RT_1) \quad \ldots (\because pv = RT)$$

and work done by the expander during the process 3-4 per kg of air,

$$w_2 = \frac{n}{n-1}(p_3 v_3 - p_4 v_4) = \frac{n}{n-1}(RT_3 - RT_4)$$

∴ Net work done during the cycle per kg of air,

$$= w_2 - w_1 = \frac{n}{n-1} \times R\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

We also know that heat absorbed during constant pressure process 4-1,

$$= c_p(T_1 - T_4)$$

∴ $\qquad \text{C.O.P.} = \dfrac{\text{Heat absorbed}}{\text{Work done}} = \dfrac{c_p(T_1 - T_4)}{\dfrac{n}{n-1} \times R\left[(T_2 - T_1) - (T_3 - T_4)\right]} \qquad \ldots (vi)$

We know that $\qquad R = c_p - c_v = c_v(\gamma - 1)$
Substituting the value of R in equation (vi),

$$\text{C.O.P.} = \frac{c_p(T_1 - T_4)}{\dfrac{n}{n-1} \times c_v(\gamma - 1)\left[(T_2 - T_1) - (T_3 - T_4)\right]}$$

$$= \frac{\gamma(T_1 - T_4)}{\dfrac{n}{n-1} \times (\gamma - 1)\left[(T_2 - T_1) - (T_3 - T_4)\right]} \qquad \ldots \left[\because \dfrac{c_p}{c_v} = \gamma\right]$$

$$= \frac{T_1 - T_4}{\dfrac{n}{n-1} \times \dfrac{(\gamma - 1)}{\gamma}\left[(T_2 - T_3) - (T_1 - T_4)\right]} \qquad \ldots (vii)$$

Notes : 1. In this case, the values of T_2 and T_4 are to be obtained from the following relations

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}}$$

2. For isentropic compression or expansion, $n = \gamma$. Therefore, the equation (vii) may be written as

$$\text{C.O.P.} = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} \qquad \text{... (same as before)}$$

3. We have already discussed that the main drawback of the open cycle air refrigerator is freezing of the moisture in the air during expansion stroke which is liable to choke up the valves. Due to this reason, a closed cycle or dense air Bell-Coleman refrigerator as shown in Fig. 2.6 is preferred. In this case, the cold air does not come in direct contact of the refrigerator. The cold air is passed through the pipes and it is used for absorbing heat from the brine and this cooled brine is circulated in the refrigerated space. The term 'dense air system' is derived from the fact that the suction to the compressor is at higher pressure than the open cycle system (which is atmospheric).

Example 2.9. *A refrigerator working on Bell-Coleman cycle operates between pressure limits of 1.05 bar and 8.5 bar. Air is drawn from the cold chamber at 10°C, compressed and then it is cooled to 30°C before entering the expansion cylinder. The expansion and compression follows the law $pv^{1.3} = constant$. Determine the theoretical C.O.P. of the system.*

Solution. Given : $p_1 = p_4 = 1.05$ bar ; $p_2 = p_3 = 8.5$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 30°C = 30 + 273 = 303$ K ; $n = 1.3$

The p-v and T-s diagrams for a refrigerator working on the Bell-Coleman cycle is shown in Fig. 2.8 (a) and (b) respectively.

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively.

Since the compression and expansion follows the law $pv^{1.3} = C$, therefore

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = (8.1)^{0.231} = 1.62$$

$$\therefore \quad T_2 = T_1 \times 1.62 = 283 \times 1.62 = 458.5 \text{ K}$$

Similarly

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = 1.62$$

$$\therefore \quad T_4 = T_3/1.62 = 303/1.62 = 187 \text{ K}$$

Closed cycle refrigerator.
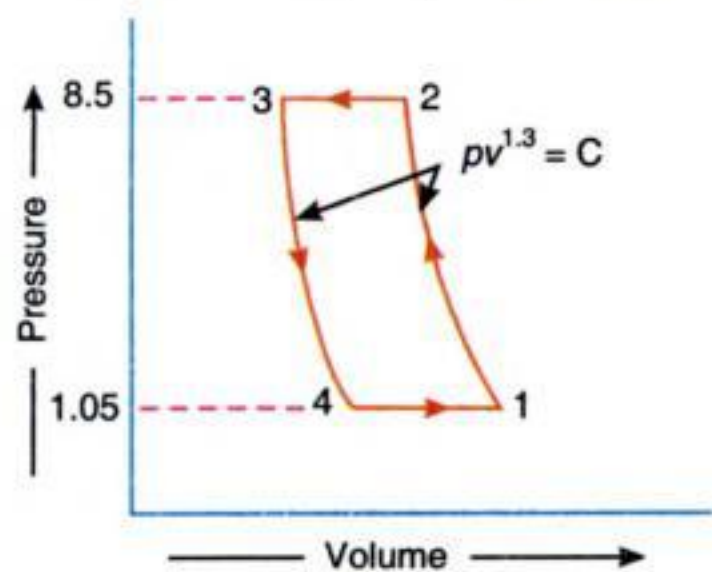
(a) p-v diagram.
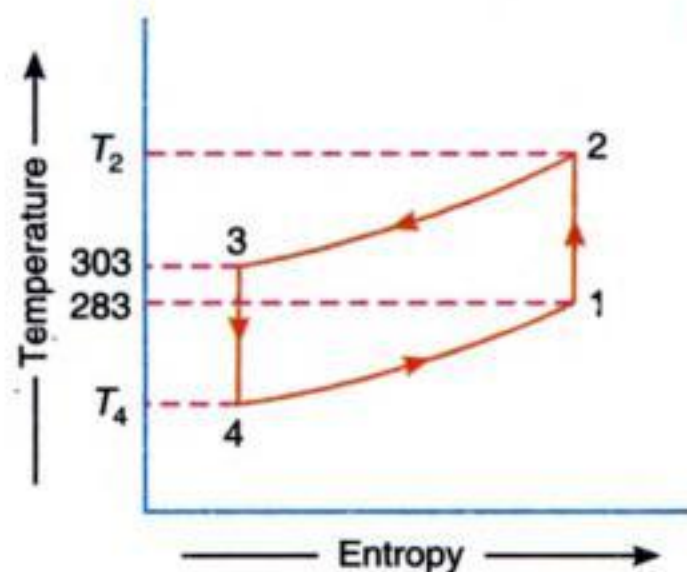
(b) T-s diagram.

Fig. 2.8

We know that theoretical coefficient of performance,

$$\text{C.O.P.} = \cfrac{T_1 - T_4}{\cfrac{n}{n-1} \times \cfrac{(\gamma - 1)}{\gamma} \left[(T_2 - T_3) - (T_1 - T_4) \right]}$$

$$= \cfrac{(283 - 187)}{\cfrac{1.3}{1.3 - 1} \times \cfrac{(1.4 - 1)}{1.4} \left[(458.5 - 303) - (283 - 187) \right]} \quad \text{...(Taking } \gamma = 1.4)$$

$$= \frac{96}{1.24 \times 59.5} = 1.3 \text{ Ans.}$$

Example 2.10. *The atmospheric air at pressure 1 bar and temperature –5°C is drawn in the cylinder of the compressor of a Bell-Coleman refrigerating machine. It is compressed isentropically to a pressure of 5 bar. In the cooler, the compressed air is cooled to 15°C, pressure remaining the same. It is then expanded to a pressure of 1 bar in an expansion cylinder, from where it is passed to the cold chamber. Find : 1. the work done per kg of air, and 2. C.O.P. of the plant.*

For air assume law for expansion, $pv^{1.2} = constant$; law for compression, $pv^{1.4} = constant$ and specific heat of air at constant pressure = 1 kJ/kg K.

Solution. Given : $p_1 = p_4 = 1$ bar ; $T_1 = -5°C + 273 = 268$ K ; $p_2 = p_3 = 5$ bar ; $T_3 = 15°C = 15 + 273 = 288$ K ; $n = 1.2$; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

The *p-v* and *T-s* diagrams for a refrigerating machine working on Bell-Coleman cycle is shown in Fig. 2.9 (*a*) and (*b*) respectively.

1. Work done per kg of air

Let $\qquad T_2$ and T_4 = Temperatures at the end of compression and expansion respectively.

The compression process 1-2 is isentropic and follows the law $pv^{1.4}$ = constant.

$$\therefore \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{5}{1} \right)^{\frac{1.4 - 1}{1.4}} = (5)^{0.386} = 1.585$$

or $\qquad T_2 = T_1 \times 1.585 = 268 \times 1.585 = 424.8$ K
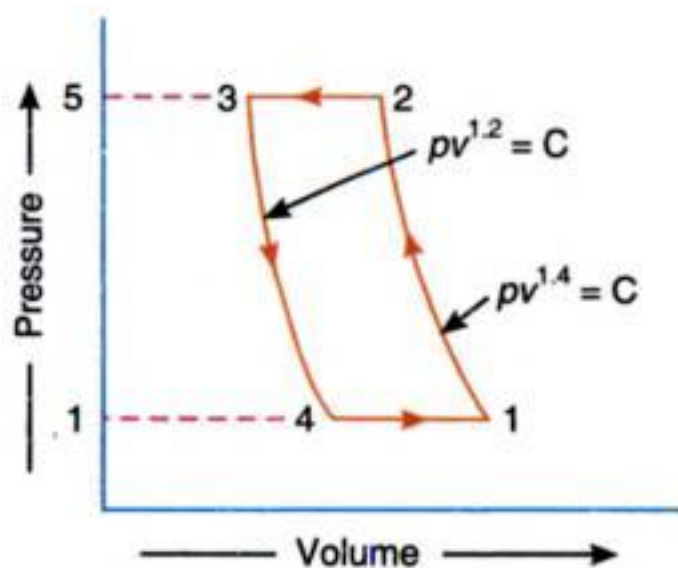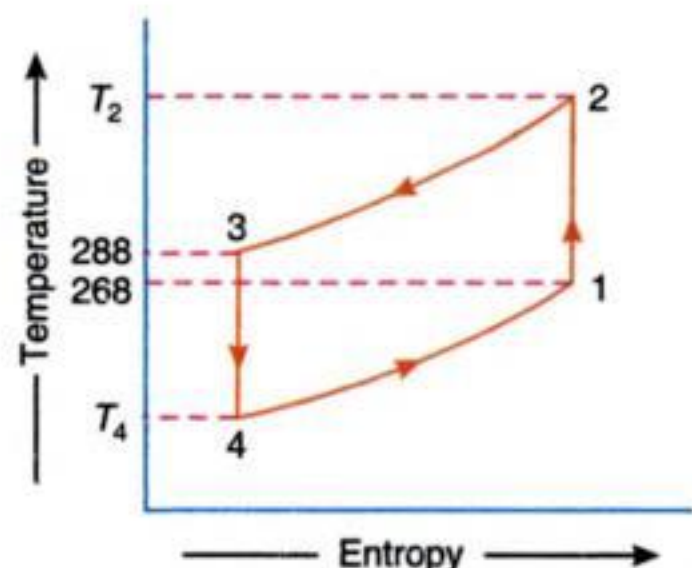
(*a*) *p-v diagram.* (*b*) *T-s diagram.*

Fig. 2.9

Bell-Coleman Refrigeration Machine

The expansion process 3-4 follows the law $pv^{1.2}$ = constant.

$$\therefore \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = (5)^{0.167} = 1.31$$

or
$$T_4 = T_3/1.31 = 288/1.31 = 220 \text{ K}$$

We know that workdone by the compressor during the isentropic process 1-2 per kg of air,

$$w_{1-2} = \frac{\gamma}{\gamma-1} \times R\,(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1} \times 0.287(424.8 - 268) = 159 \text{ kJ/kg}$$

... (Taking R for air = 0.287 kJ/kg K)

and workdone by the expander during the process 3-4 per kg of air,

$$w_{3-4} = \frac{n}{n-1} \times R\,(T_3 - T_4)$$

$$= \frac{1.2}{1.2-1} \times 0.287(288 - 220) = 118.3 \text{ kJ/kg}$$

∴ Net work done per kg of air,

$$w = w_{1-2} - w_{3-4} = 159 - 118.3 = 40.7 \text{ kJ/kg} \textbf{ Ans.}$$

2. C.O.P. of the plant

We know that heat absorbed during constant pressure process 4-1 per kg of air

$$= c_p\,(T_1 - T_4) = 1(268 - 220) = 48 \text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P. of the plant} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{48}{40.7} = 1.18 \textbf{ Ans.}$$

Example 2.11. *An air refrigerator works between the pressure limits of 1 bar and 5 bar. The temperature of the air entering the compressor and expansion cylinder are 10°C and 25°C respectively. The expansion and compression follow the law $pv^{1.3}$ = constant. Find the following :*

1. The theoretical C.O.P. of the refrigerating cycle ;

2. If the load on the refrigerating machine is 10 TR, find the amount of air circulated per minute through the system assuming that the actual C.O.P. is 50% of the theoretical C.O.P.

3. *The stroke length and piston diameter of single acting compressor if the compressor runs at 300 r.p.m. and the volumetric efficiency is 85 %.*

Take L / d = 1.5 ; c_p = 1.005 kJ/kg K ; c_v = 0.71 kJ/ kg K.

Solution. Given : $p_1 = p_4 = 1$ bar ; $p_2 = p_3 = 5$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 25°C = 25 + 273 = 298$ K ; $n = 1.3$; $Q = 10$ TR ; Actual C.O.P. = 50% Theoretical C.O.P.; $N = 300$ r.p.m. ; $\eta_v = 85\% = 085$; $L/d = 1.5$; $c_p = 1.005$ kJ/kg K ; $c_v = 0.71$ kJ/kg K

(*a*) *p-v diagram.*
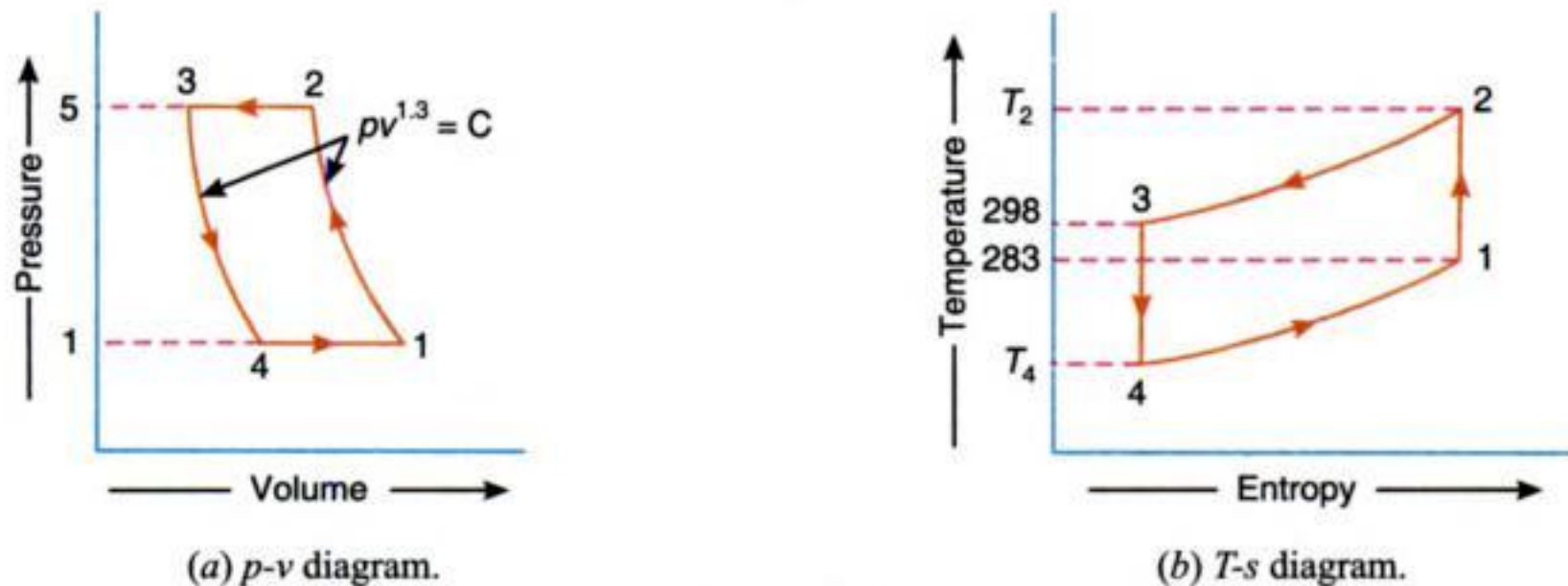
(*b*) *T-s diagram.*

Fig. 2.10

The *p-v* and *T-s* diagrams of the refrigerating cycle are shown in Fig. 2.10 (*a*) and (*b*) respectively.

1. *Theoretical C.O.P. of the refrigerating cycle*

Let $\qquad T_2$ and T_4 = Temperature at the end of compression and expansion respectively.

We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = (5)^{0.23} = 1.45$$

∴ $\qquad T_2 = T_1 \times 1.45 = 283 \times 1.45 = 410.3$ K

Similarly,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = (5)^{0.23} = 1.45$$

∴ $\qquad T_4 = T_3 / 1.45 = 298 / 1.45 = 205.5$ K

We know that heat extracted from the refrigerating system per kg of air

$$= c_p (T_1 - T_4) = 1.005 (283 - 205.5) = 78 \text{ kJ/kg}$$

and characteristic gas constant,

$$R = c_p - c_v = 1.005 - 0.71 = 0.295 \text{ kJ/kg K}$$

Workdone during compression process 1-2 per kg of air,

$$w_{1-2} = \frac{n}{n-1} \times R (T_2 - T_1) = \frac{1.3}{1.3-1} \times 0.295 (410.3 - 283) \text{ kJ/kg}$$

$$= 162.7 \text{ kJ/kg}$$

and workdone during expansion process 3–4 per kg of air,

$$w_{3-4} = \frac{n}{n-1} \times R (T_3 - T_4) = \frac{1.3}{1.3-1} \times 0.295 (298 - 205.5) \text{ kJ/kg}$$

$$= 118.2 \text{ kJ/kg}$$

∴ Net workdone per kg of air supplied,

$$w = w_{1-2} - w_{3-4} = 162.7 - 118.2 = 44.5 \text{ kJ/kg}$$

We know that theoretical C.O.P. of the refrigerating cycle

$$= \frac{\text{Heat extracted}}{\text{Work done}} = \frac{78}{44.5} = 1.75 \text{ Ans.}$$

2. Amount of air circulated per minute

Let m_a = Mass of air circulated per minute.

Since the actual C.O.P. is 50% of the theoretical C.O.P., therefore actual heat extracted or refrigerating capacity of the system per kg of air

$$= 78 \times 0.5 = 39 \text{ kJ/kg}$$

We know that refrigerating capacity of the system

$$= 10 \text{ TR} = 10 \times 210 = 2100 \text{ kJ/min} \qquad \text{...(Given)}$$

∴ Mass of air circulated per minute,

$$m_a = 2100/39 = 53.8 \text{ kg / min Ans.}$$

3. Stroke length and piston diameter of the compressor

Let L = Stroke length, and

d = Piston diameter.

Since the mass of air supplied to the compressor at point 1 is m_a = 53.8 kg / min, therefore its volume,

$$v_1 = \frac{m_a R T_1}{p_1} = \frac{53.8 \times 295 \times 283}{1 \times 10^5} = 45 \text{ m}^3/\text{min}$$

... (R is taken in J/kg K and p is in N/m²)

We also know that volume (v_1),

$$45 = \left(\frac{\pi}{4} \times d^2 \times L \right) N \times \eta_v = \frac{\pi}{4} \times d^2 \times 1.5 \, d \times 300 \times 0.85$$

$$= 300 \, d^3 \qquad \text{... } (\because L/d = 1.5)$$

∴ $\qquad d^3 = 45 / 300 = 0.15 \quad$ or $\quad d = 0.53 \text{ m} = 530 \text{ mm Ans.}$

and $\qquad L = 1.5 \, d = 1.5 \times 530 = 795 \text{ mm Ans.}$

Example 2.12. *A dense closed cycle refrigeration system working between 4 bar and 16 bar extracts 126 MJ of heat per hour. The air enters the compressor at 5°C and into the expander at 20°C. Assuming the unit runs at 300 r.p.m., find out 1. Power required to run the unit; 2. Bore of compressor ; and 3. Refrigerating capacity in tonnes of ice at 0°C per day. Take the following :*

The compressor and expander are double acting and stroke for compressor and expander is 300 mm. The mechanical efficiency of compressor is 80%. The mechanical efficiency of expander is 85%. Assume the compression and expansion are isentropic.

Solution. Given : $p_1 = p_4 = 4$ bar $= 4 \times 10^5$ N/m² ; $p_2 = p_3 = 16$ bar $= 16 \times 10^5$ N/m²; $Q = 126$ MJ/h $= 2100$ kJ/min ; $T_1 = 5°C = 5 + 273 = 278$ K ; $T_3 = 20°C = 20 + 273 = 293$ K ; $N = 300$ r.p.m ; $L = 300$ mm $= 0.3$ m; $\eta_C = 80\% = 0.8$; $\eta_E = 85\% = 0.85$

The cycle on *p-v* and *T-s* diagrams is shown in Fig. 2.11 (*a*) and (*b*) respectively.

1. *Power required to run the unit*

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

The compression and expansion are isentropic (*i.e.* $pv^\gamma = C$) and γ for air = 1.4. We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

∴ $$T_2 = T_1 \times 1.486 = 278 \times 1.486 = 413 \text{ K}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

∴ $$T_4 = T_3 / 1.486 = 293 / 1.486 = 197 \text{ K}$$

(a) *p-v* diagram.   (b) *T-s* diagram.
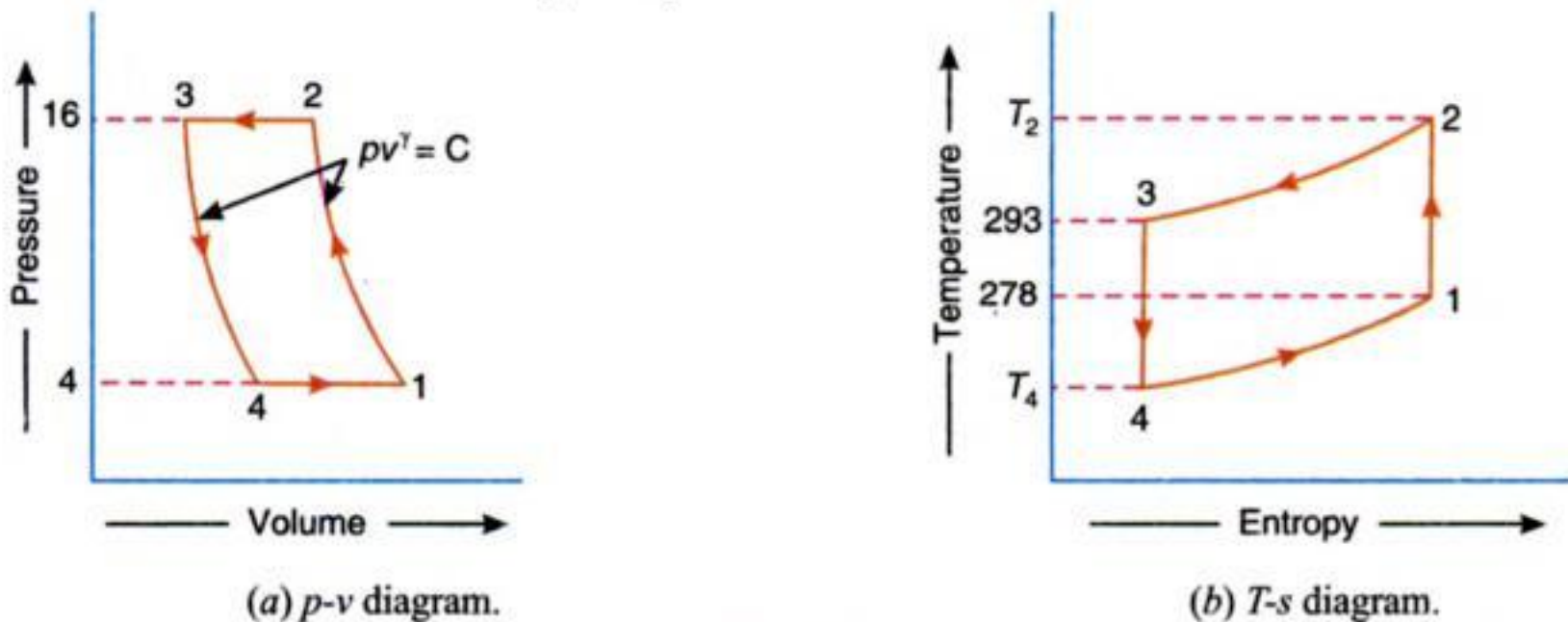
Fig. 2.11

We know that heat extracted from the refrigeration system per kg of air

$$= c_p(T_1 - T_4) = 1(278 - 197) = 81 \text{ kJ/kg}$$

... (Taking c_p for air = 1 kJ/kg K)

∴ Mass of air circulated,

$$m_a = \frac{\text{Heat extracted / min}}{\text{Heat extracted / kg}} = \frac{2100}{81} = 25.9 \text{ kg/min}$$

We know that work done during compression process 1-2 per kg of air,

$$w_{1-2} = \frac{\gamma}{\gamma-1} \times R(T_2 - T_1)\frac{1}{\eta_c}$$

$$= \frac{1.4}{1.4-1} \times 0.287(413 - 278)\frac{1}{0.8} = 169.5 \text{ kJ/kg}$$

and work done during expansion process 3-4 per kg of air,

$$w_{3-4} = \frac{\gamma}{\gamma-1} \times R(T_3 - T_4)\eta_E$$

$$= \frac{1.4}{1.4-1} \times 0.287(293 - 197)0.85 = 82 \text{ kJ/kg}$$

∴ Net work done per kg of air supplied in the system,

$$w = w_{1-2} - w_{3-4} = 169.5 - 82 = 87.5 \text{ kJ/kg}$$

and power required to run the system

$$= \frac{m_a \times w}{60} = \frac{25.9 \times 87.5}{60} = 37.8 \text{ kW Ans.}$$

2. Bore of compressor

Let D = Bore of compressor in metres.

Since the mass of air supplied to the compressor at point 1 is $m_a = 25.9$ kg/min, therefore its volume,

$$v_1 = \frac{m_a R T_1}{p_1} = \frac{25.9 \times 287 \times 278}{4 \times 10^5} = 5.17 \text{ m}^3/\text{min}$$

... ($\because R$ for air = 287 J/kg K)

We also know that volume,

$$v_1 = \left(\frac{\pi}{4} \times D^2 \times L \times 2\right) N \qquad \text{... ($\because$ of double acting)}$$

$$5.17 = \left(\frac{\pi}{4} \times D^2 \times 0.3 \times 2\right) 300 = 141.4\, D^2$$

∴ $D^2 = 5.17/141.4 = 0.037$ or $D = 0.192$ m $= 192$ mm **Ans.**

3. Refrigerating capacity in tonnes of ice at 0°C per day

We know that heat extracted or refrigerating capacity of the system per day

$$= 126 \times 24 = 3024 \text{ MJ} = 3024 \times 10^3 \text{ kJ}$$

Since the latent heat of ice is 335 kJ/kg, therefore ice formation capacity of the system per day

$$= 3024 \times 10^3/335 = 9000 \text{ kg} = 9 \text{ tonnes Ans.}$$

Example 2.13. *In an open cycle air refrigeration machine, air is drawn from a cold chamber at – 2°C and 1 bar and compressed to 11 bar. It is then cooled at this pressure, to the cooler temperature of 20°C and then expanded in expansion cylinder and returned to the cold room. The compression and expansion are isentropic, and follows the law $pv^{1.4} = $ constant. Sketch the p-v and T-s diagrams of the cycle and for a refrigeration of 15 tonnes, find : 1. theoretical C.O.P; 2. rate of circulation of the air in kg/min ; 3. piston displacement per minute in the compressor and expander ; and 4. theoretical power per tonne of refrigeration.*

Solution. Given : $T_1 = -2°C = -2 + 273 = 271$ K ; $p_1 = p_4 = 1$ bar $= 1 \times 10^5$ N/m²; $p_2 = p_3 = 11$ bar ; $T_3 = 20°C = 20 + 273 = 293$ K ; $\gamma = 1.4$; $Q = 15$ TR
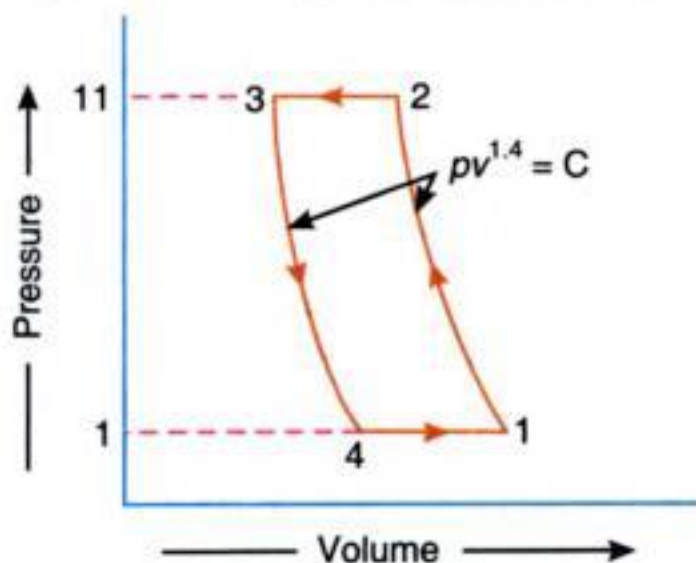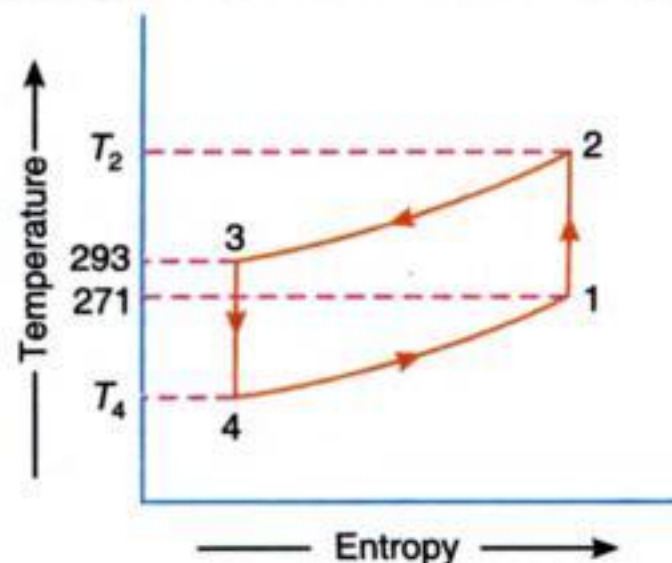
(a) *p-v diagram.*   (b) *T-s diagram.*

Fig. 2.12

The *p-v* and *T-s* diagrams of the cycle are shown in Fig. 2.12 (*a*) and (*b*) respectively.

1. Theoretical C.O.P.

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

We know that
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}} = (11)^{0.286} = 1.985$$

$\therefore$
$$T_2 = T_1 \times 1.985 = 271 \times 1.985 = 538 \text{ K}$$

Similarly,
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}} = (11)^{0.286} = 1.985$$

$\therefore$
$$T_4 = T_3 / 1.985 = 293/1.985 = 147.6 \text{ K}$$

We also know that theoretical C.O.P.

$$= \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} = \frac{271 - 147.6}{(538 - 293) - (271 - 147.6)}$$

$$= \frac{123.4}{121.6} = 1.015 \text{ **Ans.**}$$

2. Rate of circulation of the air in kg/min

Refrigeration capacity = 15 TR ...(Given)

$\therefore$ Heat extracted/min = 15 × 210 = 3150 kJ/min ... ($\because$ 1 TR = 210 kJ/min)

We know that heat extracted from cold chamber per kg of air,

$$= c_p (T_1 - T_4) = 1(271 - 147.6) = 123.4 \text{ kJ/kg}$$

... ($\because$ c_p for air = 1kJ/kg K)

$\therefore$ Rate of circulation of air,

$$m_a = \frac{\text{Heat extracted / min}}{\text{Heat extracted / kg}} = \frac{3150}{123.4} = 25.5 \text{ kg/min} \text{ **Ans.**}$$

3. Piston displacement per minute in the compressor and expander

Let v_1 and v_4 = Piston displacement per minute in the compressor and expander respectively.

We know that
$$p_1 v_1 = m_a R_a T_1$$

$\therefore$
$$v_1 = \frac{m_a R_a T_1}{p_1} = \frac{25.5 \times 287 \times 271}{1 \times 10^5} = 19.8 \text{ m}^3/\text{min} \text{ **Ans.**}$$

...(Taking R_a = 287 J/kg K)

For constant pressure process 4-1,

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

$\therefore$
$$v_4 = v_1 \times \frac{T_4}{T_1} = 19.8 \times \frac{147.6}{271} = 10.8 \text{ m}^3 \text{ **Ans.**}$$

4. Theoretical power per tonne of refrigeration

We know that net work done on the refrigeration machine per minute

$$= m_a (\text{Heat rejected} - \text{Heat extracted})$$

$$= m_a c_p [(T_2 - T_3) - (T_1 - T_4)]$$
$$= 25.5 \times 1[(538 - 293) - (271 - 147.6)] = 3100 \text{ kJ/min}$$

∴ Theoretical power of the refrigerating machine

$$= 3100/60 = 51.67 \text{ kW}$$

and   theoretical power per tonne of refrigeration

$$= 51.67/15 = 3.44 \text{ kW/TR Ans.}$$

Example 2.14. *A dense air machine operates on reversed Brayton cycle and is required for a capacity of 10 TR. The cooler pressure is 4.2 bar and the refrigerator pressure is 1.4 bar. The air is cooled in the cooler at a temperature of 50°C and the temperature of air at inlet to compressor is –20°C. Determine for the ideal cycle : 1. C.O.P.; 2. mass of air circulated per minute ; 3. theoretical piston displacement of compressor ; 4. theoretical piston displacement of expander ; and 5. net power per tonne of refrigeration. Show the cycle on p-v and T-s planes.*

Solution. Given : $Q = 10$ TR; $p_2 = p_3 = 4.2$ bar ; $p_1 = p_4 = 1.4$ bar $= 1.4 \times 10^5$ N/m²; $T_3 = 50°C = 50 + 273 = 323$ K ; $T_1 = -20°C = -20 + 273 = 253$ K

The cycle on *p-v* and *T-s* planes is shown in Fig. 2.13 (*a*) and (*b*) respectively.

(a) p-v diagram.　　　　(b) T-s diagram.
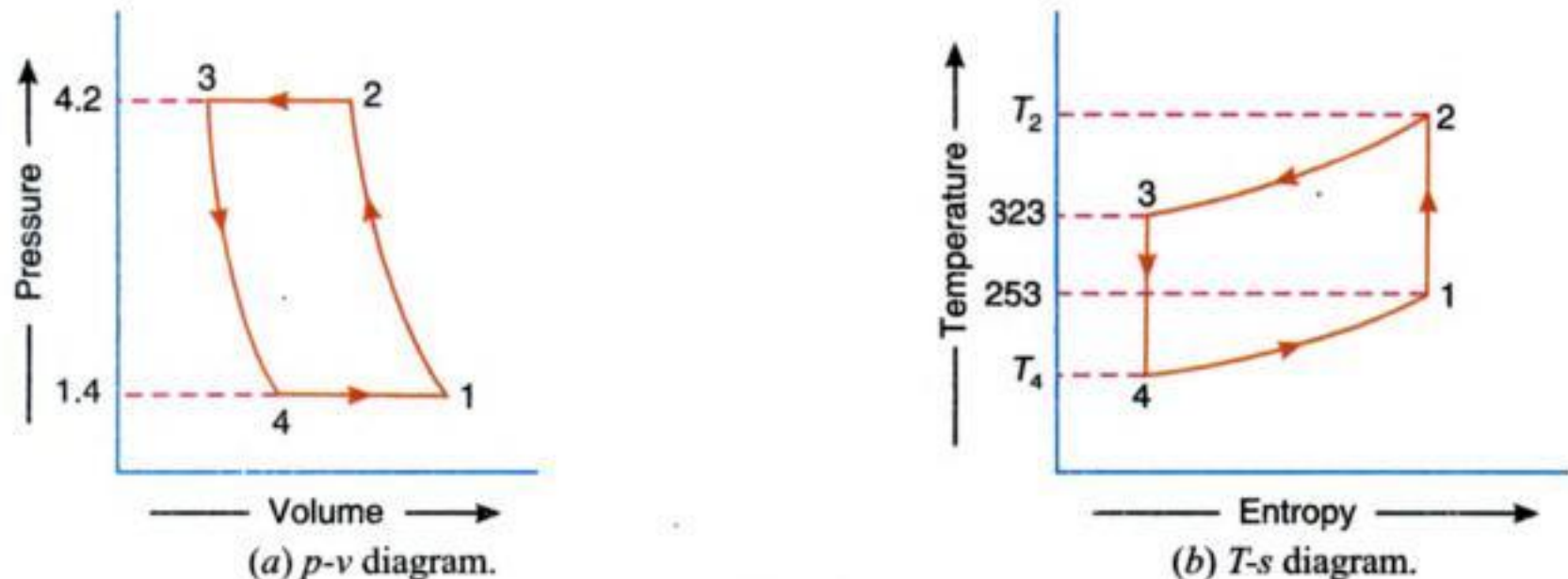
Fig. 2.13

1. Coefficient of performance (C.O.P.)

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

Let us assume the compression and expansion to be isentropic and γ for air as 1.4. We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.2}{1.4}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$$

∴ $T_2 = T_1 \times 1.369 = 253 \times 1.369 = 346$ K

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.2}{1.4}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$$

∴ $T_4 = T_3 / 1.369 = 323 / 1.369 = 236$ K

We know that   C.O.P. $= \dfrac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} = \dfrac{253 - 236}{(346 - 323) - (253 - 236)}$

$$= \frac{17}{6} = 2.83 \text{ Ans.}$$

2. Mass of air circulated per minute

Since the capacity of the machine is 10 TR, therefore heat extracted per min

$$= 10 \times 210 = 2100 \text{ kJ/min} \qquad \dots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

We know that heat extracted from the refrigerator per kg of air

$$= c_p (T_1 - T_4) = 1 (253 - 236) = 17 \text{ kJ/kg}$$

∴ Mass of air circulated per minute.

$$m_a = \frac{\text{Heat extracted / min}}{\text{Heat extracted / kg}} = \frac{2100}{17} = 123.5 \text{ kg/min Ans.}$$

3. Theoretical piston displacement of compressor

Let v_1 = Theoretical piston displacement of compressor per min.

We know that $$v_1 = \frac{m_a R_a T_1}{p_1} = \frac{123.5 \times 287 \times 253}{1.4 \times 10^5} = 64 \text{ m}^3 \text{ Ans.}$$

4. Theoretical displacement of expander

Let v_4 = Theoretical displacement of expander per minute.

We know that for constant pressure process 4-1,

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

or $$v_4 = \frac{v_1 \times T_4}{T_1} = 64 \times \frac{236}{253} = 60 \text{ m}^3 \text{ Ans.}$$

5. Net power per tonne of refrigeration

We know that net work done on the refrigerating machine per minute

$$= m_a \text{ (Heat rejected – Heat extracted)}$$
$$= m_a c_p [(T_2 - T_3) - (T_1 - T_4)]$$
$$= 123.5 \times 1 [(346 - 323) - (253 - 236)] = 741 \text{ kJ/min}$$

∴ Net power of the refrigerating machine

$$= 741/60 = 12.35 \text{ kW}$$

and net power per tonne of refrigeration

$$= 12.35/10 = 1.235 \text{ kW/TR Ans.}$$

Example 2.15. *An air refrigeration used for food storage provides 25 TR. The temperature of air entering the compressor is 7°C and the temperature at exit of cooler is 27°C. Find : 1. C.O.P. of the cycle; and 2. power per tonne of refrigeration required by the compressor. The quantity of air circulated in the system is 3000 kg / h. The compression and expansion both follows the law $pv^{1.3} = constant$ and take $\gamma = 1.4$; and $c_p = 1$ kJ/kg K for air.*

Solution. Given : $Q = 25$ TR ; $T_1 = 7°C = 7 + 273 = 280$ K ; $T_3 = 27°C = 27 + 273 = 300$ K ; $m_a = 3000$ kg / h = 50 kg / min

The refrigeration cycle on p-v and T-s diagram is shown in Fig. 2.14 (a) and (b) respectively.

1. C.O.P. of the cycle

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively.

Since the capacity of the refrigerator is 25 TR, therefore heat extracted from the refrigerator

$$= 25 \times 210 = 5250 \text{ kJ/min} \qquad \ldots (i)$$

Also the heat extracted from the refrigerator

$$= m_a \, c_p \, (T_1 - T_4) = 50 \times 1 \, (280 - T_4)$$
$$= 50 \, (280 - T_4) \text{ kJ/min} \qquad \ldots (ii)$$

From equations (i) and (ii),

$$50 \, (280 - T_4) = 5250$$
$$280 - T_4 = 105 \quad \text{or} \quad T_4 = 280 - 105 = 175 \text{ K}$$

We know that
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \ldots (iii)$$

and
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \ldots (iv)$$

From equations (iii) and (iv),

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad T_2 = \frac{T_1 \times T_3}{T_4} = \frac{280 \times 300}{175} = 480 \text{ K}$$

(a) p-v diagram.   (b) T-s diagram.
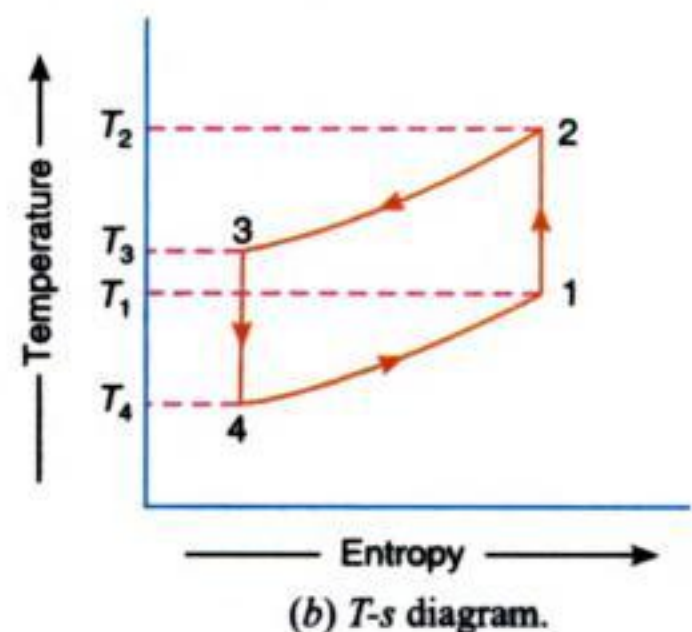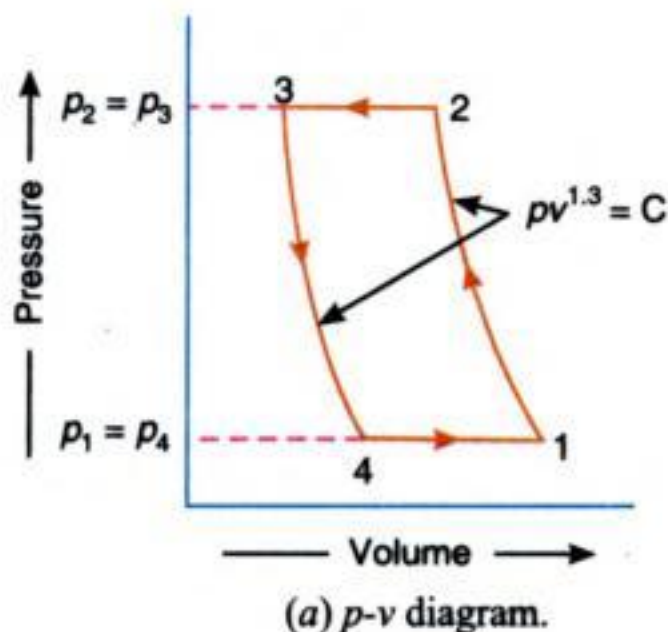
Fig. 2.14

We know that C.O.P. of the cycle

$$= \cfrac{T_1 - T_4}{\cfrac{n}{n-1} \times \cfrac{\gamma - 1}{\gamma}\Big[(T_2 - T_3) - (T_1 - T_4)\Big]}$$

$$= \cfrac{280 - 175}{\cfrac{1.3}{1.3-1} \times \cfrac{1.4-1}{1.4}\Big[(480 - 300) - (280 - 175)\Big]} = 1.13 \text{ Ans.}$$

2. Power per tonne of refrigeration

We know that heat absorbed during the constant pressure process 4–1

$$= m_a \, c_p \, (T_1 - T_4) = 50 \times 1 \, (280 - 175) = 5250 \text{ kJ/min}$$

∴   Work done/min $= \dfrac{\text{Heat absorbed}}{\text{C.O.P.}} = \dfrac{5250}{1.13} = 4646 \text{ kJ/min}$

and power per tonne of refrigeration

$$= \frac{4646}{60 \times 25} = 3.1 \text{ kW/ TR } \textbf{Ans.}$$

Example 2.16. *A dense air refrigeration cycle operates between pressures of 4 bar and 16 bar. The air temperature after heat rejection to surroundings is 37°C and air temperature at exit of refrigerator is 7°C. The isentropic efficiencies of turbine and compressor are 0.85 and 0.8 respectively. Determine compressor and turbine work per TR ; C.O.P.; and power per TR. Take γ = 1.4 and c_p = 1.005 kJ/kg K.*

Solution. Given : $p_1 = p_4 = 4$ bar ; $p_2 = p_3 = 16$ bar ; $T_3 = 37°C = 37 + 273 = 310$ K ; $T_1 = 7°C = 7 + 273 = 280$ K ; $\eta_T = 0.85$; $\eta_C = 0.8$; $\gamma = 1.4$; $c_p = 1.005$ kJ/kg K

The *p-v* and *T-s* diagrams for the cycle are shown in Fig. 2.15 (*a*) and (*b*) respectively.

(*a*) *p-v* diagram.　　　　(*b*) *T-s* diagram.
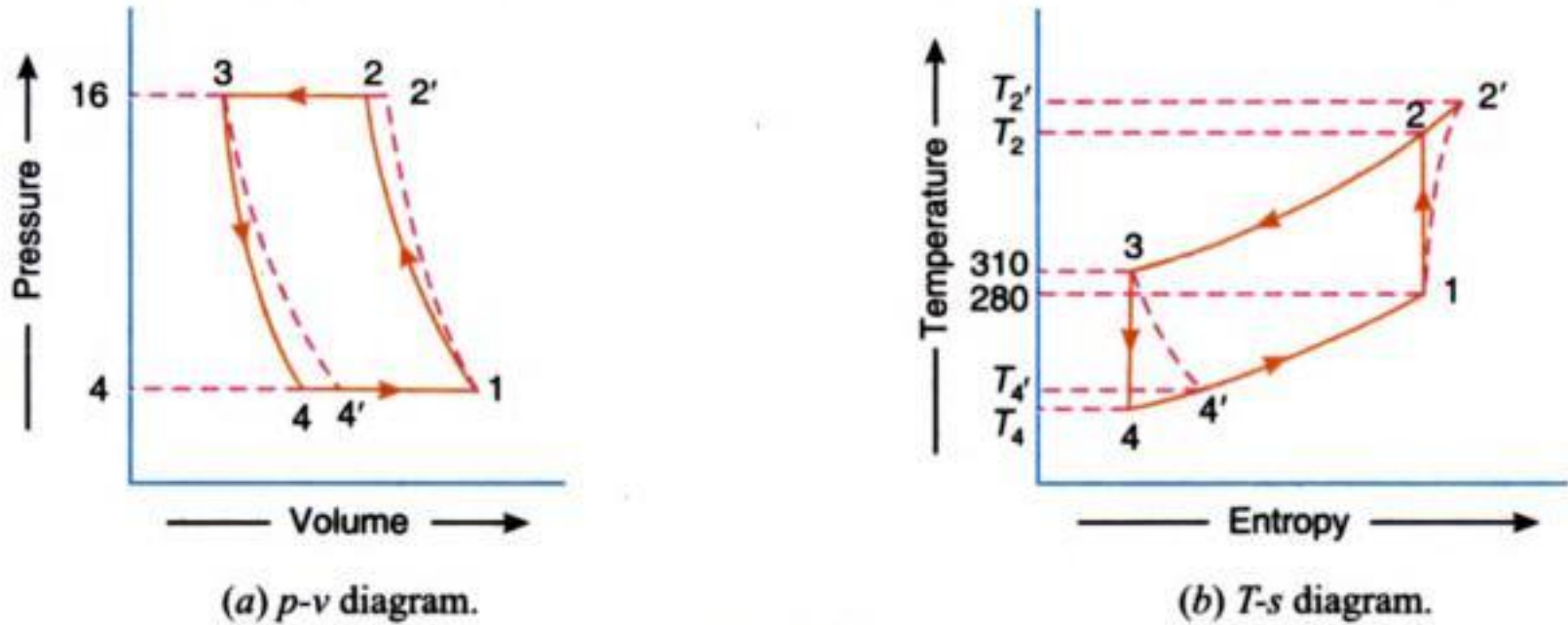
Fig. 2.15

Let　　　　T_2 = Temperature of air at the end of isentropic compression in the compressor,

　　　　$T_{2'}$ = Actual temperature of air leaving the compressor,

　　　　T_4 = Temperature of air at the end of isentropic expansion in the turbine, and

　　　　$T_{4'}$ = Actual temperature of air leaving the turbine.

We know that for isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

∴　　　　$T_2 = T_1 \times 1.486 = 280 \times 1.486 = 416$ K

Similarly, for isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

∴　　　　$T_4 = T_3 / 1.486 = 310 / 1.486 = 208.6$ K

We know that isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic increase in temperature}}{\text{Actual increase in temperature}} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$0.8 = \frac{416 - 280}{T_{2'} - 280} = \frac{136}{T_{2'} - 280}$$

$$\therefore \qquad T_{2'} = \frac{136}{0.8} + 280 = 450 \text{ K}$$

and isentropic efficiency of the turbine,

$$\eta_T = \frac{\text{Actual increase in temperature}}{\text{Isentropic increase in temperature}} = \frac{T_3 - T_{4'}}{T_3 - T_4}$$

$$0.85 = \frac{310 - T_{4'}}{310 - 208.6} = \frac{310 - T_{4'}}{101.4}$$

$$\therefore \qquad T_{4'} = 310 - 0.85 \times 101.4 = 223.8 \text{ K}$$

We know that heat extracted from the refrigerator or refrigerating effect

$$= c_p (T_1 - T_{4'}) = 1.005 (280 - 223.8) = 56.48 \text{ kJ/kg}$$

and mass of air flowing per TR per minute,

$$m_a = \frac{\text{Heat extracted per min}}{\text{Heat extracted per kg}} = \frac{210}{56.48} = 3.72 \text{ kg/min}$$

$$...(\because 1 \text{ TR} = 210 \text{ kJ/min})$$

Compressor work per TR

We know that compressor work,

$$W_C = m_a c_p (T_{2'} - T_1)$$
$$= 3.72 \times 1.005 (450 - 280) = 635.6 \text{ kJ/min } \textbf{Ans.}$$

Turbine work per TR

We know that turbine work,

$$W_T = m_a c_p (T_3 - T_{4'})$$
$$= 3.72 \times 1.005 (310 - 223.8) = 322.3 \text{ kJ/min } \textbf{Ans.}$$

C.O.P.

We know that net work done per TR

$$W_{net} = W_C - W_T = 635.6 - 322.3 = 313.3 \text{ kJ/min}$$

$$\therefore \qquad \text{C.O.P.} = \frac{\text{Heat extracted per min}}{\text{Net work done}} = \frac{210}{313.3} = 0.67 \textbf{ Ans.}$$

Power per TR

We know that net workdone per TR,

$$W_{net} = 313.3 \text{ kJ/min}$$

$$\therefore \qquad \text{Power per TR} = 313.3/60 = 5.22 \text{ kW } \textbf{Ans.}$$

EXERCISES

1. A heat pump is used for heating the interior of a house in a cold climate. The ambient temperature is –5°C and the desired interior temperature is 25°C . The compressor of the heat pump is to be driven by a heat engine working between 1000°C and 25°C. Treating both cycles as reversible, calculate the ratio in which the heat pump and the heat engine share the heating load. **[Ans. 7]**

2. A refrigerating plant is required to produce 2.5 tonnes of ice per day at – 4°C from water at 20°C. If the temperature range in the compressor is between 25°C and – 6°C, calculate power required to drive the compressor. Latent heat of ice = 335 kJ/kg and specific heat of ice = 2.1 kJ/kg K. **[Ans. 1.437 kW]**

3. A refrigerator using Carnot cycle requires 1.25 kW per tonne of refrigeration to maintain a temperature of – 30°C. Find : 1. C.O.P. of the Carnot refrigerator; 2. Temperature at which heat is rejected; and 3. Heat rejected per tonne of refrigeration. **[Ans. 2.8 ; 55.4°C ; 284 kJ/min]**

4. A Carnot cycle machine operates between the temperature limits of 47°C and –30°C. Determine the C.O.P. when it operates as 1. a refrigerating machine ; 2. a heat pump ; and 3. a heat engine. **[Ans. 3.16 ; 4.16 ; 0.24]**

5. Ten tonnes of fish is frozen to –30°C per day. The fish enters the freezing chamber at 30°C and freezing occurs at –3°C. The frozen fish is cooled to –30°C. The specific heats of fresh and frozen fish are 3.77 kJ/kg K and 1.67 kJ/kg K respectively while latent heat of freezing is 251.2 kJ/kg K . Find the tonnage of the plant which runs for 18 hours per day. The evaporator and condensor temperatures are –40°C and 45°C respectively. If the C.O.P. of the plant is 1.8, determine the power consumption of the plant in kW. Also find the refrigerating efficiency of the plant. **[Ans. 18.6 TR ; 36.1 kW ; 65.7%]**

6. A Carnot refrigeration system has working temperature of –30°C and 40°C. What is the maximum C.O.P. possible ? If the actual C.O.P. is 75% of the maximum, calculate the actual refrigerating effect produced per kilowatt hour. **[Ans. 3.47 ; 0.743 TR]**

7. A refrigerator storage is supplied with 30 tonnes of fish at a temperature of 27°C. The fish has to be cooled to – 9°C for preserving it for long period without deterioration . The cooling takes place in 10 hours. The specific heat of fish is 2.93 kJ/kg K above freezing point of fish and 1.26 kJ/kg K below freezing point of fish which is – 3°C. The latent heat of freezing is 232 kJ/kg. What is the capacity of the plant in tonnes of refrigeration for cooling the fish ? What would be the ideal C.O.P. between this temperature range ? If the actual C.O.P. is 40% of the ideal, find the power required to run the cooling plant. **[Ans. 78 TR ; 7.33 ; 93.3 kW]**

8. A refrigerating system working on Bell-Coleman cycle receives air from cold chamber at – 5°C and compresses it from 1 bar to 4.5 bar. The compressed air is then cooled to a temperature of 37°C before it is expanded in the expander. Calculate the C.O.P. of the system when compression and expansion are (i) isentropic ; and (ii) follow the law $pv^{1.25}$ = constant. **[Ans. 1.86 ; 1.98]**

9. A Bell-Coleman refrigerator works between 4 bar and 1 bar pressure limits. After compression, the cooling water reduces the air temperature to 17°C. What is the lowest temperature produced by the ideal machine ? Compare the coefficient of performance of this machine with that of the ideal Carnot cycle machine working between the same pressure limits, the temperature at the beginning of compression being –13°C. **[Ans. –78°C ; 2.07, 1.02]**

10. An air refrigerator working on Bell-Coleman cycle takes air into the compressor at 1 bar and 268 K. It is compressed in the compressor to 5 bar and cooled to 298 K at the same pressure. It is further expanded in the expander to 1 bar and discharged to take the cooling load. The isentropic efficiencies of the compressor and expander are 85% and 90% respectively. Determine ; 1. refrigeration capacity of the system if the air circulated is 40 kg / min ; 2. power required for the compressor ; and 3. C.O.P. of the system. **[Ans. 13.14 TR ; 46 kW; 0.812]**

11. An air refrigeration system having pressure ratio of 5 takes air at 0°C. It is compressed and then cooled to 19°C at constant pressure. If the efficiency of the compressor is 95 % and that of expander is 75%, determine: 1. the refrigeration capacity of the system, if the flow of air is 75 kg/min ;

2. the power of the compressor ; and 3. C.O.P. of the system. Assume compression and expansion processes to be isentropic. Take $\gamma = 1.4$; $c_p = 1$ kJ/kg K ; and $c_v = 0.72$ kJ/kg K.

[**Ans.** 31.68 TR; 106.6 kW; 1.71]

12. A 5 tonne refrigerating machine operating on Bell Coleman cycle has an upper limit of pressure of 12 bar. The pressure and temperature at the start of compression are 1 bar and 17°C respectively. The compressed air cooled at constant pressure to a temperature of 40°C enters the expansion cylinder. Assuming both the expansion and compression processes to be isentropic with $\gamma = 1.4$; Determine : 1. C.O.P.; 2. quantity of air in circulation per minute; 3. piston displacement of compressor and expander; 4. bore of compressor and expansion cylinders. The unit runs at 250 r.p.m. and is double acting. Stroke length is 200 mm ; and 5. power required to drive the unit. Take $c_p = 1$ kJ/kg K ; $c_v = 0.71$ kJ/kg K ; $R = 0.287$ kJ/kg K.

[**Ans.** 0.952 ; 7.65 kg/min ; 6.37 m³/min, 3.35 m³/min ; 284 mm ; 18.4 kW]

13. An air refrigerator used for food storage, provides 50 TR. The temperature of air entering the compressor is 7°C and the temperature before entering into the expander is 27°C. Assuming a 70% mechanical efficiency, find : 1. actual C.O.P; and 2. the power required to run the compressor.

The quantity of air circulated in the system is 100 kg/min. The compression and expansion follow the law $pv^{1.3} = $ constant.

Take $\gamma = 1.4$; $c_p = 1$ kJ/kg K for air.

[**Ans.** 1.13 ; 110.6 kW]

14. A dense air refrigerating system operating between pressures of 17.5 bar and 3.5 bar is to produce 10 tonnes of refrigeration. Air leaves the refrigerating coils at –7°C and it leaves the air cooler at 15.5°C. Neglecting losses and clearance, calculate the net work done per minute and the coefficient of performance. For air $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$.

[**Ans.** 1237 kJ/min ; 1.7]

QUESTIONS

1. How is the effectiveness of a refrigeration system measured ?
2. Explain the term "tonne of refrigeration".
3. Discuss the advantages of the dense air refrigerating system over an open air refrigeration system.
4. What is the difference between a refrigerator and a heat pump ? Derive an expression for the performance factor for both if they are running on reversed Carnot cycle.
5. Prove that the performance factor of a Bell-Coleman cycle refrigeration system is given by

$$\text{C.O.P.} = \frac{T_2}{T_3 - T_2}$$

where T_2 and T_3 are the temperatures of air at the inlet and discharge of compressor respectively. Explain the working of this cycle.

OBJECTIVE TYPE QUESTIONS

1. One tonne of refrigeration is equal to
   (a) 21 kJ/min (b) 210 kJ/min (c) 420 kJ/min (d) 620 kJ/min
2. One tonne refrigerating machine means that
   (a) one tonne is the total mass of the machine
   (b) one tonne of refrigerant is used
   (c) one tonne of water can be converted into ice
   (d) one tonne of ice when melts from and at 0°C in 24 hours, the refrigeration effect produced is equivalent to 210 kJ/min
3. The coefficient of performance is always .... one.
   (a) equal to (b) less than (c) greater than

4. The ratio of heat extracted in the refrigerator to the workdone on the refrigerant is called
   (a) coefficient of performance of refrigeration
   (b) coefficient of performance of heat pump
   (c) relative coefficient of performance
   (d) refrigerating efficiency

5. The relative coefficient of performance is equal to

   (a) $\dfrac{\text{Theoretical C.O.P.}}{\text{Actual C.O.P.}}$
   (b) $\dfrac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$

   (c) Actual C.O.P. × Theoretical C.O.P.

6. In a closed or dense air refrigeration cycle, the operating pressure ratio can be reduced, which results in .... coefficient of performance.
   (a) lower
   (b) higher

7. Air refrigeration cycle is used in
   (a) commercial refrigerators
   (b) domestic refrigerators
   (c) air-conditioning
   (d) gas liquefaction

8. In a refrigerating machine, heat rejected is .... heat absorbed.
   (a) equal to
   (b) less than
   (c) greater than

9. Air refrigerator works on
   (a) Carnot cycle
   (b) Rankine cycle
   (c) reversed Carnot cycle
   (d) Bell-Coleman cycle

10. In air-conditioning of aeroplanes, using air as a refrigerant, the cycle used is
    (a) reversed Carnot cycle
    (b) reversed Joule cycle
    (c) reversed Brayton cycle
    (d) reversed Otto cycle

ANSWERS

1. (b)	2. (d)	3. (c)	4. (a)	5. (b)
6. (b)	7. (d)	8. (c)	9. (c), (d)	10. (c)

3

Air Refrigeration Systems

3.1 Introduction

The advent of high-speed passenger aircraft, jet aircraft and missiles has introduced the need for compact, and simple refrigeration systems, capable of high capacity, with minimum reduction of pay load. When the power requirements, needed to transport the additional weight of the refrigerating system are taken into account, the air cycle systems usually prove to be the most efficient. The cooling demands per unit volume of space, are heavy. An ordinary passenger aircraft requires a cooling system capable of 8 TR capacity and a super constellation requires a cooling system of more than 8 TR capacity. A jet fighter travelling at 950 km/h needs a cooling system capable of

10 to 20 TR capacity. To dissipate the heat load from 10 kW of electronic equipment in a missile or other high speed flight system, approximately 3 TR of cooling capacity are required. The miniaturization of electronic equipment concentrates a heavy cooling load in a small area. It creates difficulty in transferring heat to air at high altitudes. Moreover, low pressure of air further complicates the refrigeration design requirements.

3.2 Methods of Air Refrigeration Systems

The various methods of air refrigeration systems used for aircrafts these days are as follows:

1. Simple air cooling system,
2. Simple air evaporative cooling system,
3. Boot strap air cooling system,
4. Boot strap air evaporative cooling system,
5. Reduced ambient air cooling system, and
6. Regenerative air cooling system.

Now we shall discuss all the above mentioned cooling systems, one by one, in the following pages.

3.3 Simple Air Cooling System

A simple air cooling system for aircrafts is shown in Fig. 3.1. The main components of this system are the main compressor driven by a gas turbine, a heat exchanger, a cooling turbine and a cooling air fan. The air required for refrigeration system is bled off from the main compressor. This high pressure and high temperature air is cooled initially in the heat exchanger where ram air is used for cooling. It is further cooled in the cooling turbine by the process of expansion. The work of this turbine is used to drive the cooling fan which draws cooling air through the heat exchanger. This system is good for ground surface cooling and for low flight speeds.
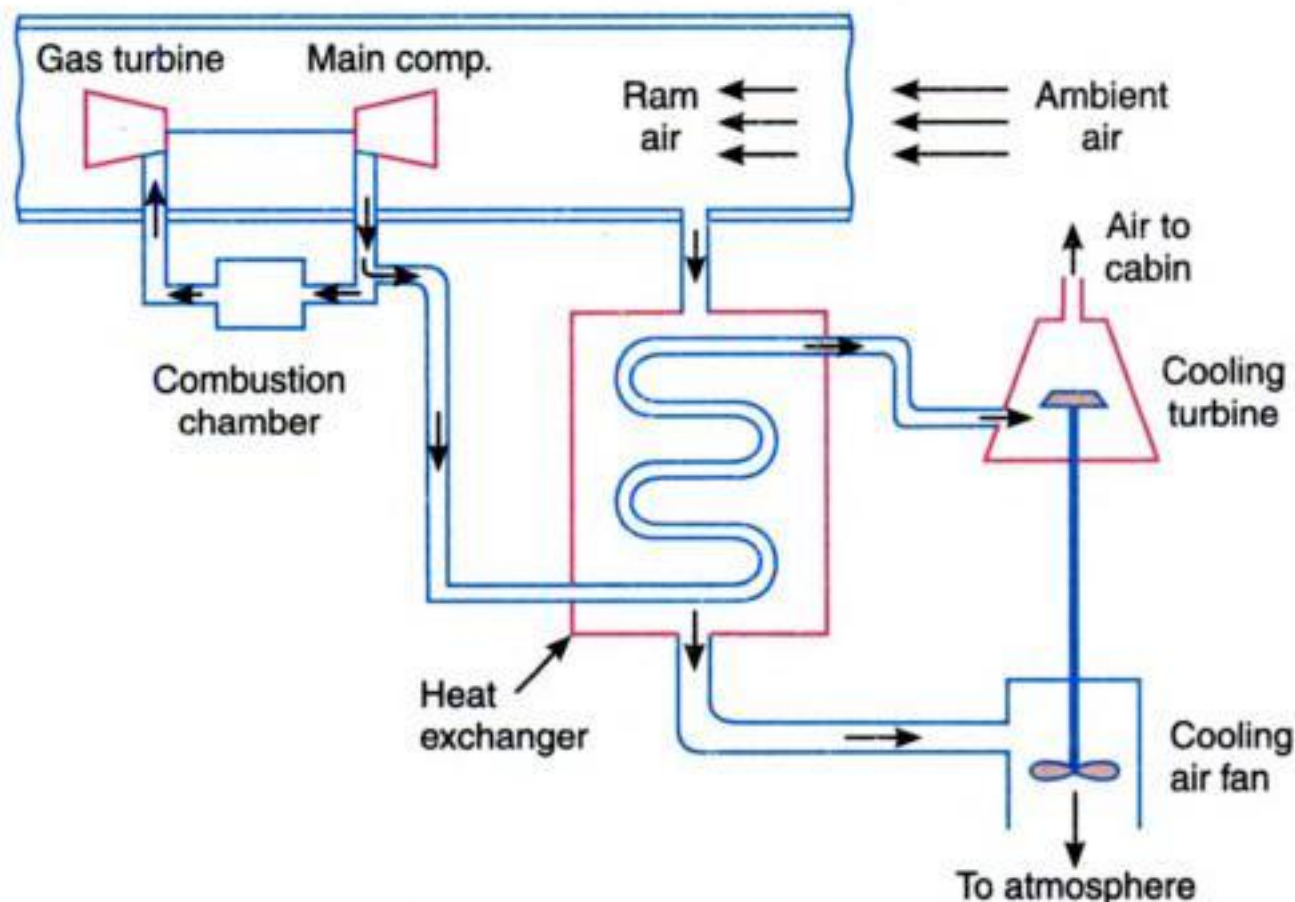
Fig. 3.1. Simple air cooling system.

The T-s diagram for a simple air cooling system is shown in Fig. 3.2.

The various processes are discussed below :

1. *Ramming process.* Let the pressure and temperature of ambient air is p_1 and T_1 respectively. The ambient air is rammed isentropically from pressure p_1 and temperature T_1 to the

pressure p_2 and temperature T_2. This ideal ramming action is shown by the vertical line 1-2 in Fig. 3.2. In actual practice, because of internal friction due to irreversibilities, the temperature of the rammed air is more than T_2. Thus the actual ramming process is shown by the curve 1-2' which is adiabatic but not isentropic due to friction. The pressure and temperature of the rammed air is now p_2' and T_2' respectively. During the ideal or actual ramming process, the total energy or enthalpy remains constant *i.e.* $h_2 = h_2'$ and $T_2 = T_2'$.
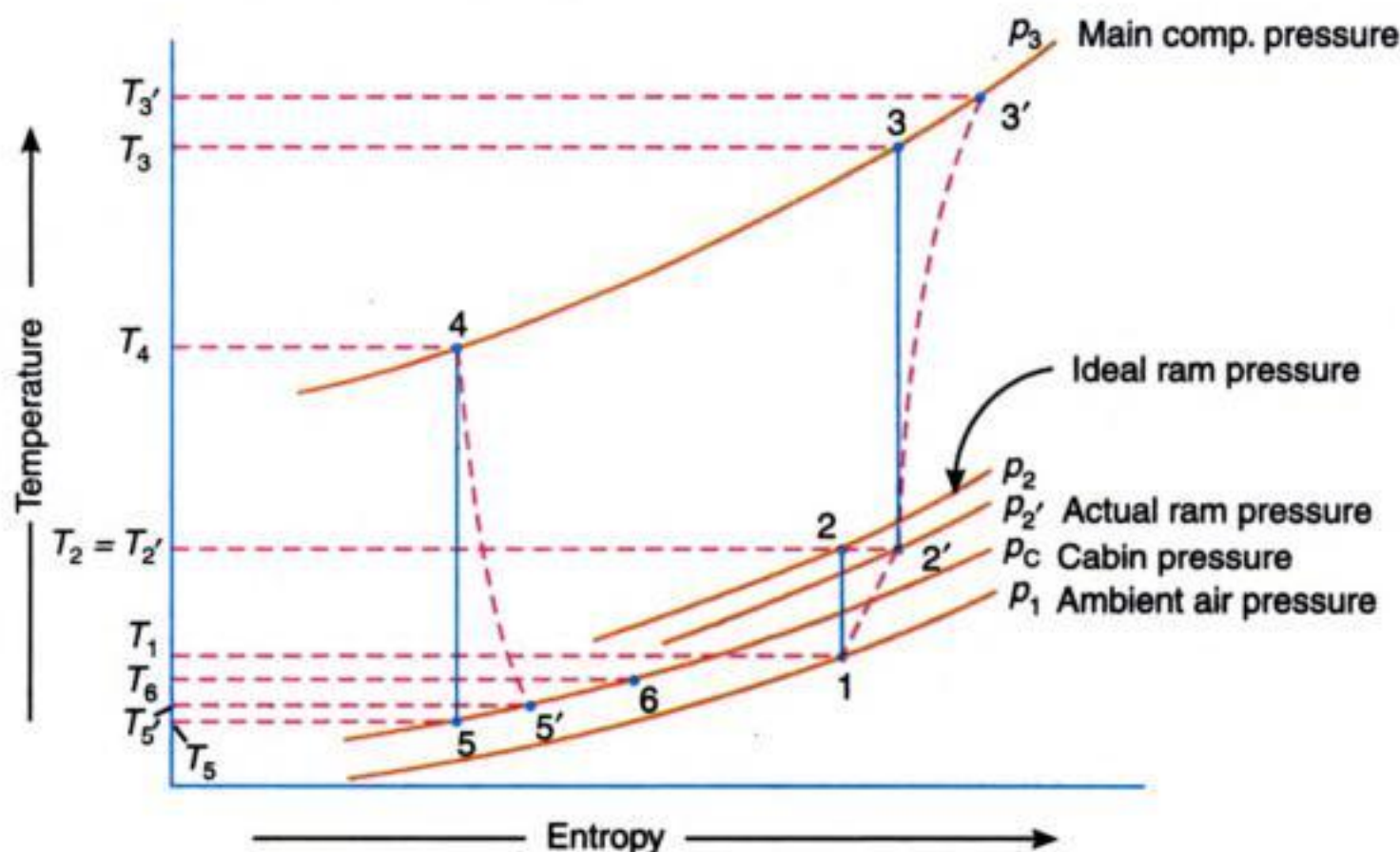
Fig. 3.2. *T-s* diagram for simple air cycle cooling system.

If V is the aircraft velocity or the velocity of air relative to the aircraft in metres per second, then kinetic energy of outside air relative to aircraft,

$$*\text{K.E.} = \frac{V^2}{2000} \text{ kJ/kg} \qquad \qquad \dots (i)$$

From the energy equation, we know that

$$h_2 - h_1 = \frac{V^2}{2000}$$

$$c_p \cdot T_2 - c_p \cdot T_1 = \frac{V^2}{2000}$$

$$\therefore \qquad T_2 = T_1 + \frac{V^2}{2000\, c_p}$$

or

$$\frac{T_2}{T_1} = 1 + \frac{V^2}{2000\, c_p T_1}$$

and

$$\frac{T_2'}{T_1} = 1 + \frac{V^2}{2000\, c_p T_1} \qquad \dots (\because T_2 = T_2') \qquad \dots (ii)$$

We know that, $c_p - c_v = R$

---

* We know that K.E. $= \dfrac{mV^2}{2}$ N-m or J, when m is in kg and V is in m/s

For $m = 1$ kg, K.E. $= \dfrac{V^2}{2}$ J/kg $= \dfrac{V^2}{2000}$ kJ/kg

or $\qquad c_p \left[1 - \dfrac{c_v}{c_p} \right] = R \quad$ or $\quad c_p \left[1 - \dfrac{1}{\gamma} \right] = R \qquad \qquad \dots (\because c_p / c_v = \gamma)$

$\therefore \qquad \qquad c_p = \dfrac{\gamma R}{\gamma - 1}$

Substituting the value of c_p in equation (ii), we have

$$\dfrac{T_2}{T_1} = \dfrac{T_{2'}}{T_1} = 1 + \dfrac{V^2 (\gamma - 1)}{2000 \, \gamma R T_1} \qquad \text{(when } R \text{ is in kJ / kgK)} \qquad \dots (iii)$$

It may be noted that when R is in J/kg K, then equation (iii) may be written as follows:

$$\dfrac{T_2}{T_1} = \dfrac{T_{2'}}{T_1} = 1 + \dfrac{V^2 (\gamma - 1)}{2 \gamma R T_1}$$

$$= 1 + \dfrac{V^2 (\gamma - 1)}{2 a^2} \qquad \dots (iv)$$

where $\qquad \qquad a = $ Local sonic or acoustic velocity at the ambient air conditions.

$\qquad \qquad = \sqrt{\gamma R T_1}$, where R is in J/kg K.

Also $\qquad \qquad \dfrac{T_2}{T_1} = \dfrac{T_{2'}}{T_1} = 1 + \dfrac{\gamma - 1}{2} M^2$

where $\qquad \qquad M = $ Mach number of the flight. It is defined as the ratio of air craft velocity (V) to the local sonic velocity (a).

The temperature $T_2 = T_{2'}$ is called the *stagnation temperature* of the ambient air entering the main compressor. The stagnation pressure after isentropic compression (p_2) is given by

$$\dfrac{p_2}{p_1} = \left(\dfrac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma - 1}}$$

Due to the irreversible compression in the ram, the air reaches point 2′ instead of point 2 at the same stagnation temperature but at a reduced stagnation pressure $p_{2'}$. The pressure $p_{2'}$ may be obtained from the expression of ram efficiency (η_R) which is given as

$$\eta_R = \dfrac{\text{Actual rise in pressure}}{\text{Isentropic rise in pressure}} = \dfrac{p_{2'} - p_1}{p_2 - p_1}$$

2. Compression process. The isentropic compression of air in the main compressor is represented by the line 2′– 3. In actual practice, because of internal friction, due to irreversiblities, the actual compression is represented by the curve 2′– 3′. The work done during this compression process is given by

$$W_C = m_a \, c_p (T_{3'} - T_{2'})$$

where $\qquad \qquad m_a = $ Mass of air bled from the main compressor for refrigeration purposes.

3. Cooling process. The compressed air is cooled by the ram air in the heat exchanger. This process is shown by the curve 3′– 4 in Fig. 3.2. In actual practice, there is a pressure drop in the heat exchanger which is not shown in the figure. The temperature of air decreases from $T_{3'}$ to T_4. The heat rejected in the heat exchanger during the cooling process is given by

$$Q_R = m_a c_p (T_{3'} - T_4)$$

4. *Expansion process.* The cooled air is now expanded isentropically in the cooling turbine as shown by the curve 4 – 5. In actual practice, because of internal friction due to irreversibilities, the actual expansion in the cooling turbine is shown by the curve 4 – 5′. The work done by the cooling turbine during this expansion process is given by

$$W_R = m_a c_p (T_4 - T_{5'})$$

The work of this turbine is used to drive the cooling air fan which draws cooling air from the heat exchanger.

5. *Refrigeration process.* The air from the cooling turbine (*i.e.* after expansion) is sent to the cabin and cock pit where it gets heated by the heat of equipment and occupancy. This process is shown by the curve 5′– 6 in Fig. 3.2. The refrigerating effect produced or heat absorbed is given by

$$R_E = m_a c_p (T_6 - T_{5'})$$

where T_6 = Inside temperature of cabin.

We know that C.O.P. of the air cycle

$$= \frac{\text{Refrigerating effect produced}}{\text{Work done}}$$

$$= \frac{m_a c_p (T_6 - T_{5'})}{m_a c_p (T_{3'} - T_{2'})} = \frac{T_6 - T_{5'}}{T_{3'} - T_{2'}}$$

If Q tonnes of refrigeration is the cooling load in the cabin, then the air required for the refrigeration purpose,

$$m_a = \frac{210 \, Q}{c_p (T_6 - T_{5'})} \text{ kg/min}$$

Power required for the refrigeration system,

$$P = \frac{m_a c_p (T_{3'} - T_{2'})}{60} \text{ kW}$$

and C.O.P. of the refrigerating system

$$= \frac{210 \, Q}{m_a c_p (T_{3'} - T_{2'})} = \frac{210 \, Q}{P \times 60}$$

Note : The value of c_p for air is taken as 1 kJ/kg K.

Example 3.1. *A simple air cooled system is used for an aeroplane having a load of 10 tonnes. The atmospheric pressure and temperature are 0.9 bar and 10°C respectively. The pressure increases to 1.013 bar due to ramming. The temperature of the air is reduced by 50°C in the heat exchanger. The pressure in the cabin is 1.01 bar and the temperature of air leaving the cabin is 25°C. Determine : 1 Power required to take the load of cooling in the cabin; and 2. C.O.P. of the system.*

Assume that all the expansions and compressions are isentropic. The pressure of the compressed air is 3.5 bar.

Solution. Given : $Q = 10$ TR ; $p_1 = 0.9$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $p_2 = 1.013$ bar ; $p_5 = p_6 = 1.01$ bar ; $T_6 = 25°C = 25 + 273 = 298$ K ; $p_3 = 3.5$ bar

1. *Power required to take the load of cooling in the cabin*

First of all, let us find the mass of air (m_a) required for the refrigeration purpose. Since the compressions and expansions are isentropic, therefore the various processes on the T-s diagram are as shown in Fig. 3.3.

Let
T_2 = Temperature of air at the end of ramming or entering the main compressor,

T_3 = Temperature of air leaving the main compressor after isentropic compression,

T_4 = Temperature of air leaving the heat exchanger, and

T_5 = Temperature of air leaving the cooling turbine.
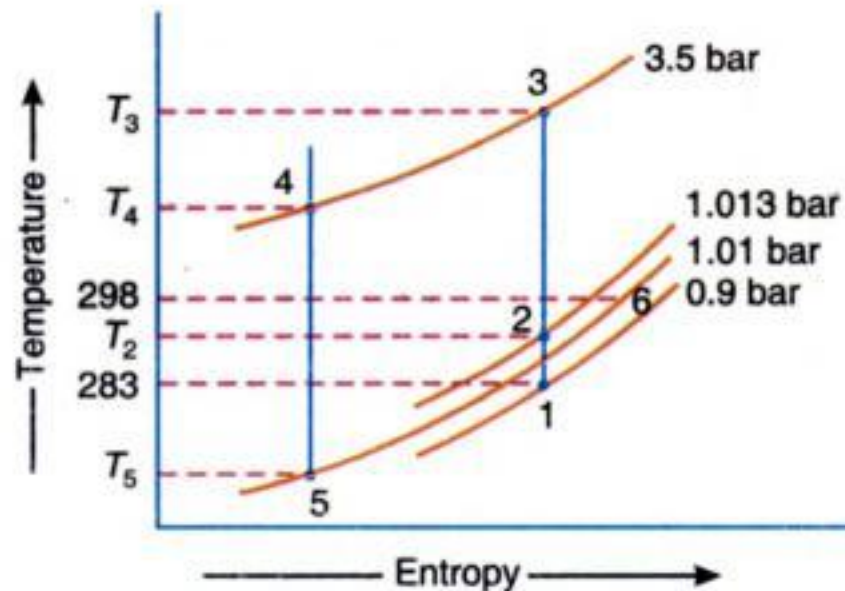
Fig. 3.3

We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.013}{0.9}\right)^{\frac{1.4-1}{1.4}} = (1.125)^{0.286} = 1.034$$

∴

$$T_2 = T_1 \times 1.034 = 283 \times 1.034 = 292.6 \text{ K}$$

Similarly,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.5}{1.013}\right)^{\frac{1.4-1}{1.4}} = (3.45)^{0.286} = 1.425$$

∴

$$T_3 = T_2 \times 1.425 = 292.6 \times 1.425 = 417 \text{ K} = 144°C$$

Since the temperature of air is reduced by 50°C in the heat exchanger, therefore temperature of air leaving the heat exchanger,

$$T_4 = 144 - 50 = 94°C = 367 \text{ K}$$

We know that

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.01}{3.5}\right)^{\frac{1.4-1}{1.4}} = (0.288)^{0.286} = 0.7$$

∴

$$T_5 = T_4 \times 0.7 = 367 \times 0.7 = 257 \text{ K}$$

We know that mass of air required for the refrigeration purpose,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_5)} = \frac{210 \times 10}{1(298 - 257)} = 51.2 \text{ kg/min}$$

... (Taking c_p for air = 1 kJ/kg K)

∴ Power required to take the load of cooling in the cabin,

$$P = \frac{m_a c_p (T_3 - T_2)}{60} = \frac{51.2 \times 1(417 - 292.6)}{60} = 106 \text{ kW Ans.}$$

Rotary engine power generator sets for air-crafts.

2. C.O.P. of the system

We know that C.O.P. of the system

$$= \frac{210\,Q}{P \times 60} = \frac{210 \times 10}{106 \times 60} = 0.33 \text{ Ans.}$$

Example 3.2. *An aircraft refrigeration plant has to handle a cabin load of 30 tonnes. The atmospheric temperature is 17°C. The atmospheric air is compressed to a pressure of 0.95 bar and temperature of 30°C due to ram action. This air is then further compressed in a compressor to 4.75 bar, cooled in a heat exchanger to 67°C, expanded in a turbine to 1 bar pressure and supplied to the cabin. The air leaves the cabin at a temperature of 27°C. The isentropic efficiencies of both compressor and turbine are 0.9. Calculate the mass of air circulated per minute and the C.O.P. For air, $c_p = 1.004$ kJ/kg K and $c_p/c_v = 1.4$*

Solution. Given : $Q = 30$ TR ; $T_1 = 17°C = 17 + 273 = 290$ K ; $p_2 = 0.95$ bar ; $T_2 = 30°C = 30 + 273 = 303$ K ; $p_3 = p_{3'} = 4.75$ bar ; $T_4 = 67°C = 67 + 273 = 340$ K ; $p_5 = p_{5'} = 1$ bar ; $T_6 = 27°C = 27 + 273 = 300$ K ; $\eta_C = \eta_T = 0.9$; $c_p = 1.004$ kJ/kg K ; $c_p/c_v = \gamma = 1.4$

The *T-s* diagram for the simple air refrigeration cycle with the given conditions is shown in Fig. 3.4.
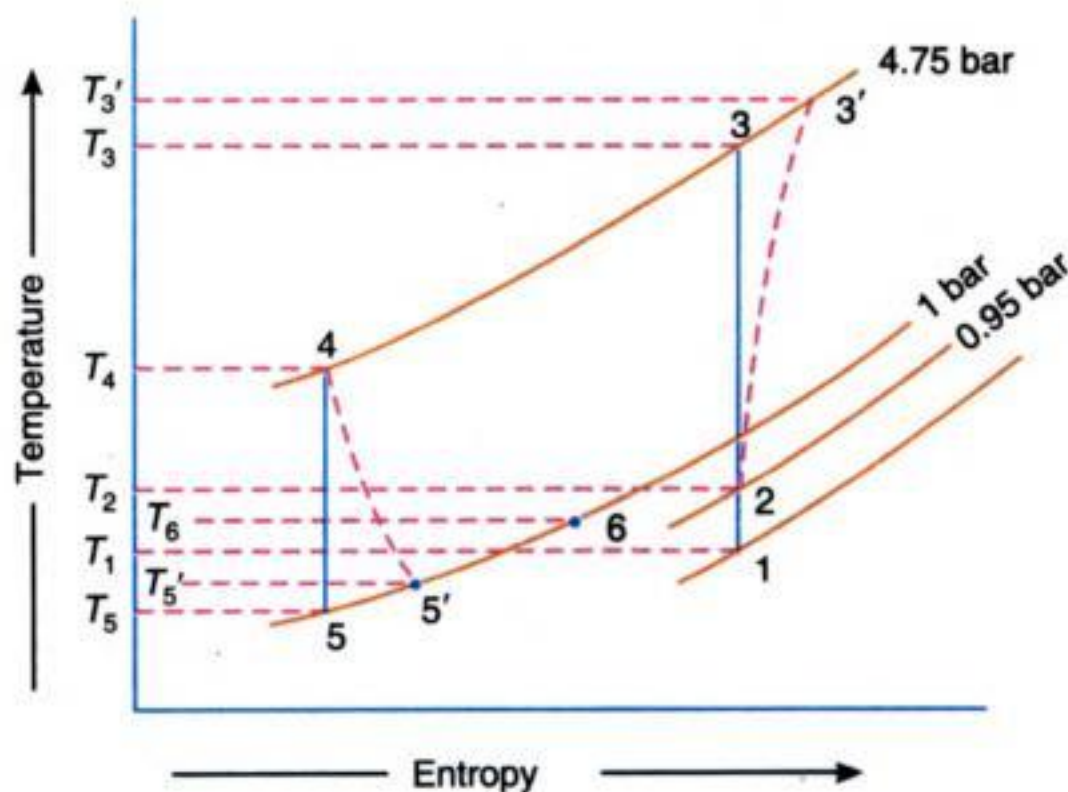
Fig. 3.4

Let

T_3 = Temperature of the air after isentropic compression in the compressor,

$T_{3'}$ = Actual temperature of the air leaving the compressor,

T_5 = Temperature of the air leaving the turbine after isentropic expansion, and

$T_{5'}$ = Actual temperature of the air leaving the turbine.

We know that for isentropic compression process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.75}{0.95}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.584$$

∴ $T_3 = T_2 \times 1.584 = 303 \times 1.584 = 480 \text{ K}$

and isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic increase in temperature}}{\text{Actual increase in temperature}} = \frac{T_3 - T_2}{T_{3'} - T_2}$$

$$0.9 = \frac{480 - 303}{T_{3'} - 303} = \frac{177}{T_{3'} - 303}$$

∴ $T_{3'} - 303 = 177/0.9 = 196.7 \text{ or } T_{3'} = 303 + 196.7 = 499.7 \text{ K}$

Now for the isentropic expansion process 4-5,

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.75}{1}\right)^{\frac{1.4-1}{1.4}} = (4.75)^{0.286} = 1.561$$

∴ $T_5 = T_4 / 1.561 = 340 / 1.561 = 217.8 \text{ K}$

and isentropic efficiency of the turbine,

$$\eta_T = \frac{\text{Actual increase in temperature}}{\text{Isentropic increase in temperature}} = \frac{T_4 - T_{5'}}{T_4 - T_5}$$

$$0.9 = \frac{340 - T_{5'}}{340 - 217.8} = \frac{340 - T_{5'}}{122.2}$$

∴ $T_{5'} = 340 - 0.9 \times 122.2 = 230 \text{ K}$

Mass of air circulated per minute

We know that mass of air circulated per minute,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_{5'})} = \frac{210 \times 30}{1.004\,(300 - 230)} = 89.64 \text{ kg/min } \textbf{Ans.}$$

C.O.P.

We know that C.O.P. $= \dfrac{210\,Q}{m_a c_p (T_{3'} - T_2)} = \dfrac{210 \times 30}{89.64 \times 1.004\,(499.7 - 303)} = 0.356$ **Ans.**

Example 3.3. *An aircraft moving with speed of 1000 km/h uses simple gas refrigeration cycle for air-conditioning. The ambient pressure and temperature are 0.35 bar and –10°C respectively. The pressure ratio of compressor is 4.5. The heat exchanger effectiveness is 0.95. The isentropic efficiencies of compressor and expander are 0.8 each. The cabin pressure and temperature are 1.06 bar and 25°C. Determine temperatures and pressures at all points of the cycle. Also find the volume flow rate through compressor inlet and expander outlet for 100 TR. Take $c_p = 1.005$ kJ/kg K ; R = 0.287 kJ/kg K and $c_p / c_v = 1.4$ for air.*

Solution. Given : $V = 1000$ km / h $= 277.8$ m / s ; $p_1 = 0.35$ bar ; $T_1 = -10°C = -10 + 273$ $= 263$ K ; $p_3 / p_2 = 4.5$; $\eta_E = 0.95$; $\eta_C = \eta_T = 0.8$; $p_5 = p_{5'} = 1.06$ bar ; $T_6 = 25°C = 25 + 273$ $= 298$ K ; $Q = 100$ TR ; $c_p = 1.005$ kJ/kg K ; $R = 0.287$ kJ / kg K $= 287$ J / kg K ; $c_p / c_v = \gamma = 1.4$.
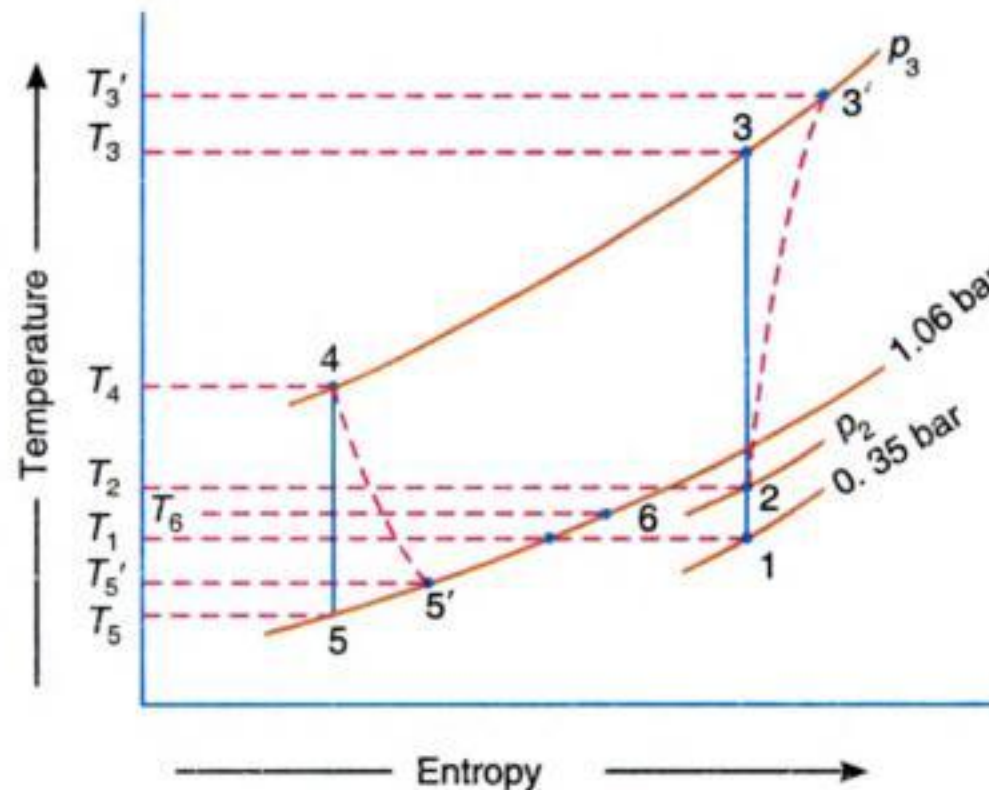
Fig. 3.5

Temperatures and pressures at all points of the cycle

The T-s diagram for the simple gas refrigeration cycle with the given conditions is shown in Fig. 3.5.

Let

T_2 and p_2 = Stagnation temperature and pressure of the ambient air entering the compressor,

T_3 and p_3 = Temperature and pressure of the air leaving the compressor after isentropic compression,

$T_{3'}$ = Actual temperature of the air leaving the compressor,

T_4 = Temperature of the air leaving the heat exchanger or entering the expander,

p_4 = Pressure of the air leaving the heat exchanger or entering the expander $= p_3 = p_{3'}$,

T_5 = Temperature of the air leaving the expander after isentropic expansion,

$T_{5'}$ = Actual temperature of the air leaving the expander.

We know that

$$T_2 = T_1 + \frac{V^2}{2000 \, c_p} = 263 + \frac{(277.8)^2}{2000 \times 1.005}$$

$$= 263 + 38.4 = 301.4 \text{ K } \textbf{Ans.}$$

and

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{301.4}{263}\right)^{\frac{1.4}{1.4-1}} = (1.146)^{3.5} = 1.611$$

∴

$$p_2 = p_1 \times 1.611 = 0.35 \times 1.611 = 0.564 \text{ bar } \textbf{Ans.}$$

Since $p_3 / p_2 = 4.5$ (Given), therefore

$$p_3 = p_2 \times 4.5 = 0.564 \times 4.5 = 2.54 \text{ bar } \textbf{Ans.}$$

We know that for isentropic compression process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = (4.5)^{\frac{1.4-1}{1.4}} = (4.5)^{0.286} = 1.537$$

∴ $\qquad T_3 = T_2 \times 1.537 = 301.4 \times 1.537 = 463.3$ K

We also know that isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}} = \frac{T_3 - T_2}{T_{3'} - T_2}$$

$$0.8 = \frac{463.3 - 301.4}{T_{3'} - 301.4} = \frac{161.9}{T_{3'} - 301.4}$$

$$T_{3'} - 301.4 = 161.9/0.8 = 202.4$$

∴ $\qquad T_{3'} = 301.4 + 202.4 = 503.8$ K **Ans.**

Effectiveness of the heat exchanger (η_H),

$$0.95 = \frac{T_{3'} - T_4}{T_{3'} - T_2} = \frac{503.8 - T_4}{503.8 - 301.4} = \frac{503.8 - T_4}{202.4}$$

∴ $\qquad T_4 = 503.8 - 0.95 \times 202.4 = 311.5$ K **Ans.**

and $\qquad p_4 = p_3 = 2.54$ bar **Ans.**

Now isentropic efficiency of the expander,

$$\eta_E = \frac{\text{Actual temperature rise}}{\text{Isentropic temperature rise}} = \frac{T_4 - T_{5'}}{T_4 - T_5}$$

$$0.8 = \frac{311.5 - T_{5'}}{311.5 - 243} = \frac{311.5 - T_{5'}}{68.5}$$

∴ $\qquad T_{5'} = 311.5 - 0.8 \times 68.5 = 256.7$ K **Ans.**

Volume flow rate

Let $\qquad v_2 =$ Volume flow rate through the compressor inlet, and

$\qquad v_{5'} =$ Volume flow rate through the expander outlet.

We know that mass flow rate of air,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_{5'})} = \frac{210 \times 100}{1.005\,(298 - 256.7)} = 506 \text{ kg/min}$$

and $\qquad p_2 v_2 = m_a R T_2$

∴ $\qquad v_2 = \frac{m_a R T_2}{p_2} = \frac{506 \times 287 \times 301.4}{0.564 \times 10^5} = 776 \text{ m}^3/\text{min}$ **Ans.**

... (*R* is taken in J/kg K and p_2 is taken in N/m²)

Similarly $\qquad p_{5'} v_{5'} = m_a R T_{5'}$

∴ $\qquad v_{5'} = \frac{m_a R T_{5'}}{p_{5'}} = \frac{506 \times 287 \times 256.7}{1.06 \times 10^5} = 351.7 \text{ m}^3/\text{min}$ **Ans.**

Example 3.4. *The cock pit of a jet plane flying at a speed of 1200 km/h is to be cooled by a simple air cooling system. The cock pit is to be maintained at 25°C and the pressure in the cock pit is 1 bar. The ambient air pressure and temperature are 0.85 bar and 30°C. The other data available is as follows :*

Cock-pit cooling load = 10 TR ; Main compressor pressure ratio = 4 ; Ram efficiency = 90% ; Temperature of air leaving the heat exchanger and entering the cooling turbine = 60°C ; Pressure drop in the heat exchanger = 0.5 bar ; Pressure loss between the cooler turbine and cock pit = 0.2 bar.

Assuming the isentropic efficiencies of main compressor and cooler turbine as 80%, find the quantity of air passed through the cooling turbine and C.O.P. of the system. Take γ = 1.4 and c_p = 1 kJ/kg K.

Solution. Given : V = 1200 km / h = 333.3 m / s ; T_6 = 25°C = 25 + 273 = 298 K ; p_6 = 1 bar ; p_1 = 0.85 bar ; T_1 = 30°C = 30 + 273 = 303 K ; Q = 10 TR ; $p_3/p_{2'}$ = 4 ; η_R = 90% = 0.9 ; T_4 = 60°C = 60 + 273 = 333 K ; $p_4 = (p_{3'} - 0.5)$ bar ; $p_5 = p_{5'} = p_6 + 0.2 = 1 + 0.2 = 1.2$ bar ; $\eta_C = \eta_T$ = 80% = 0.8 ; γ = 1.4 ; c_p = 1 kJ/kg K

The *T-s* diagram for the simple air cooling system with the given conditions is shown in Fig. 3.6.
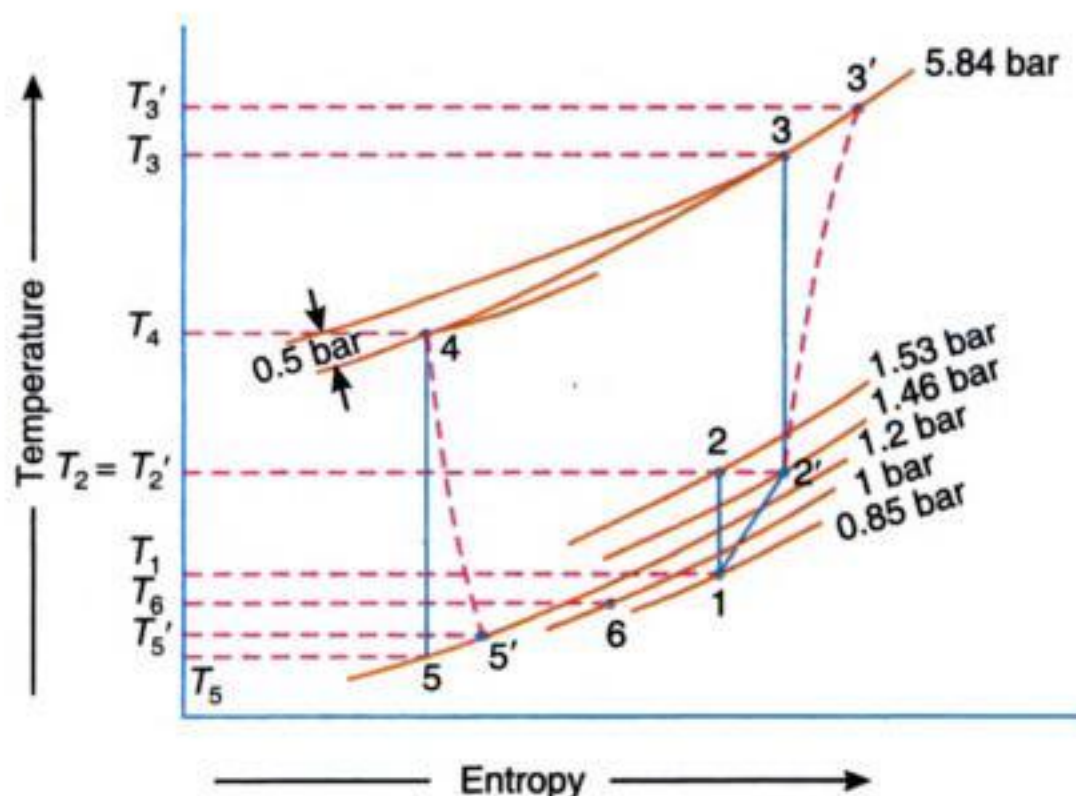
Fig. 3.6

Let $T_{2'}$ = Stagnation temperature of the ambient air entering the main compressor = T_2,

p_2 = Pressure of air after isentropic ramming, and

$p_{2'}$ = Stagnation pressure of air entering the main compressor.

We know that $T_2 = T_{2'} = T_1 + \dfrac{V^2}{2000\, c_p} = 303 + \dfrac{(333.3)^2}{2000 \times 1}$

$$= 303 + 55.5 = 358.5 \text{ K}$$

and $\dfrac{p_2}{p_1} = \left(\dfrac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\dfrac{358.5}{303}\right)^{\frac{1.4}{1.4-1}} = (1.183)^{3.5} = 1.8$

$\therefore$ $p_2 = p_1 \times 1.8 = 0.85 \times 1.8 = 1.53$ bar

We know that ram efficiency,

$$\eta_R = \frac{\text{Actual pressure rise}}{\text{Isentropic pressure rise}} = \frac{p_{2'} - p_1}{p_2 - p_1}$$

$$0.9 = \frac{p_{2'} - 0.85}{1.53 - 0.85} = \frac{p_{2'} - 0.85}{0.68}$$

∴ $\qquad p_{2'} = 0.9 \times 0.68 + 0.85 = 1.46 \text{ bar}$

Now for the isentropic process 2′– 3,

$$\frac{T_3}{T_{2'}} = \left(\frac{p_3}{p_{2'}}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

∴ $\qquad T_3 = T_{2'} \times 1.486 = 358.5 \times 1.486 = 532.7 \text{ K}$

and isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}} = \frac{T_3 - T_{2'}}{T_{3'} - T_{2'}}$$

$$0.8 = \frac{532.7 - 358.5}{T_{3'} - 358.5} = \frac{174.2}{T_{3'} - 358.5}$$

∴ $\qquad T_{3'} = \frac{174.2}{0.8} + 358.5 = 576 \text{ K}$

Since the pressure ratio of the main compressor ($p_3 / p_{2'}$) is 4, therefore pressure of air leaving the main compressor,

$$p_3 = p_{3'} = 4\, p_{2'} = 4 \times 1.46 = 5.84 \text{ bar}$$

Pressure drop in the heat exchanger

$$= 0.5 \text{ bar}$$

∴ Pressure of air after passing through the heat exchanger or at entrance to the cooling turbine,

$$p_4 = p_{3'} - 0.5 = 5.84 - 0.5 = 5.34 \text{ bar}$$

Also there is a pressure loss of 0.2 bar between the cooling turbine and the cock pit. Therefore pressure of air leaving the cooling turbine,

$$p_5 = p_{5'} = p_6 + 0.2 = 1 + 0.2 = 1.2 \text{ bar}$$

Now for the isentropic process 4-5,

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5.34}{1.2}\right)^{\frac{1.4-1}{1.4}} = (4.45)^{0.286} = 1.53$$

∴ $\qquad T_5 = T_4 / 1.53 = 333 / 1.53 = 217.6 \text{ K}$

We know that isentropic efficiency of the cooling turbine,

$$\eta_T = \frac{\text{Actual temperature rise}}{\text{Isentropic temperature rise}} = \frac{T_4 - T_{5'}}{T_4 - T_5}$$

$$0.8 = \frac{333 - T_{5'}}{333 - 217.6} = \frac{333 - T_{5'}}{115.4}$$

∴ $\qquad T_{5'} = 333 - 0.8 \times 115.4 = 240.7 \text{ K}$

Quantity of air passed through the cooling turbine

We know that quantity of air passed through the cooling turbine,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_{5'})} = \frac{210 \times 10}{1\,(298 - 240.7)} = 36.6 \text{ kg / min } \textbf{Ans.}$$

C.O.P. of the system

We know that C.O.P. of the system

$$= \frac{210\,Q}{m_a c_p(T_{3'} - T_{2'})} = \frac{210 \times 10}{36.5 \times 1\,(576 - 358.5)} = 0.264 \text{ \textbf{Ans.}}$$

Example 3.5. *In an aeroplane, a simple air refrigeration is used. The main compressor delivers the air at 5 bar and 200°C. The bled air taken from compressor is passed through a heat exchanger, cooled with the help of ram air so that the temperature of air leaving the heat exchanger is 45°C and the pressure is 4.5 bar. The cooling turbine drives the exhaust fan which is used to force the ram air through the heat exchanger. The air leaving the heat exchanger passes through the cooling turbine and then supplied to cabin at 1 bar. The pressure loss between the cooling turbine and cabin is 0.2 bar. If the rate of flow of air through the cooling turbine is 20 kg/min, determine the following :*

1. The temperature of the air leaving the expander ;

2. The power delivered to the ram air which is passed through the heat exchanger ; and

3. The refrigeration load in tonnes when the temperature of the air leaving the cabin is limited to 25°C.

Assume that the isentropic efficiency of the cooling turbine is 75% and no loss of heat from air between the cooling turbine and cabin. Take γ = 1.4 and c_p = 1 kJ/kg K.

Solution. Given : p_3 = 5 bar ; T_3 = 200°C = 200 + 273 = 473 K ; T_4 = 45°C = 45 + 273 = 318 K ; p_4 = 4.5 bar ; p_6 = 1 bar ; $p_5 = p_{6'} = p_6 + 0.2 = 1 + 0.2 = 1.2$ bar ; m_a = 20 kg / min ; T_6 = 25°C = 25 + 273 = 298 K ; η_T = 75% = 0.75 ; γ = 1.4 ; c_p = 1 kJ/kg K

Cools in Heats in
Summer Winter

Heat Transfer
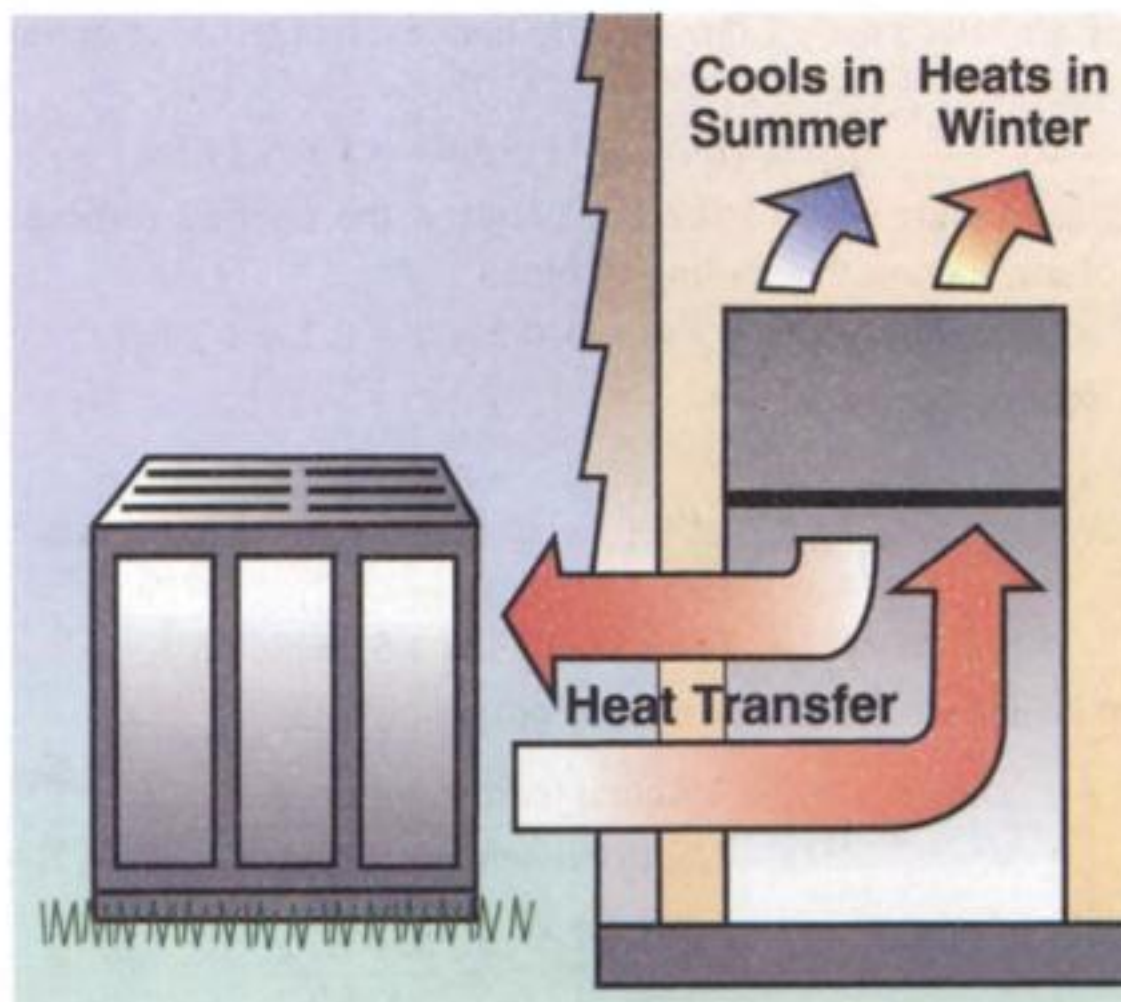
Simple air-refrigeration unit.

The schematic diagram for the simple air refrigeration system is shown in Fig. 3.1. The various processes on the *T-s* diagram are shown in Fig. 3.7. The point 3 represents the air delivered from the compressor to heat exchanger and the point 4 shows the condition of air leaving the heat exchanger. The vertical line 4 – 5 represents the isentropic expansion of air in the cooling turbine and the curve 4 – 5' shows the actual expansion of air in the cooling turbnine due to internal friction. The line 5'– 6 represents the refrigeration process.
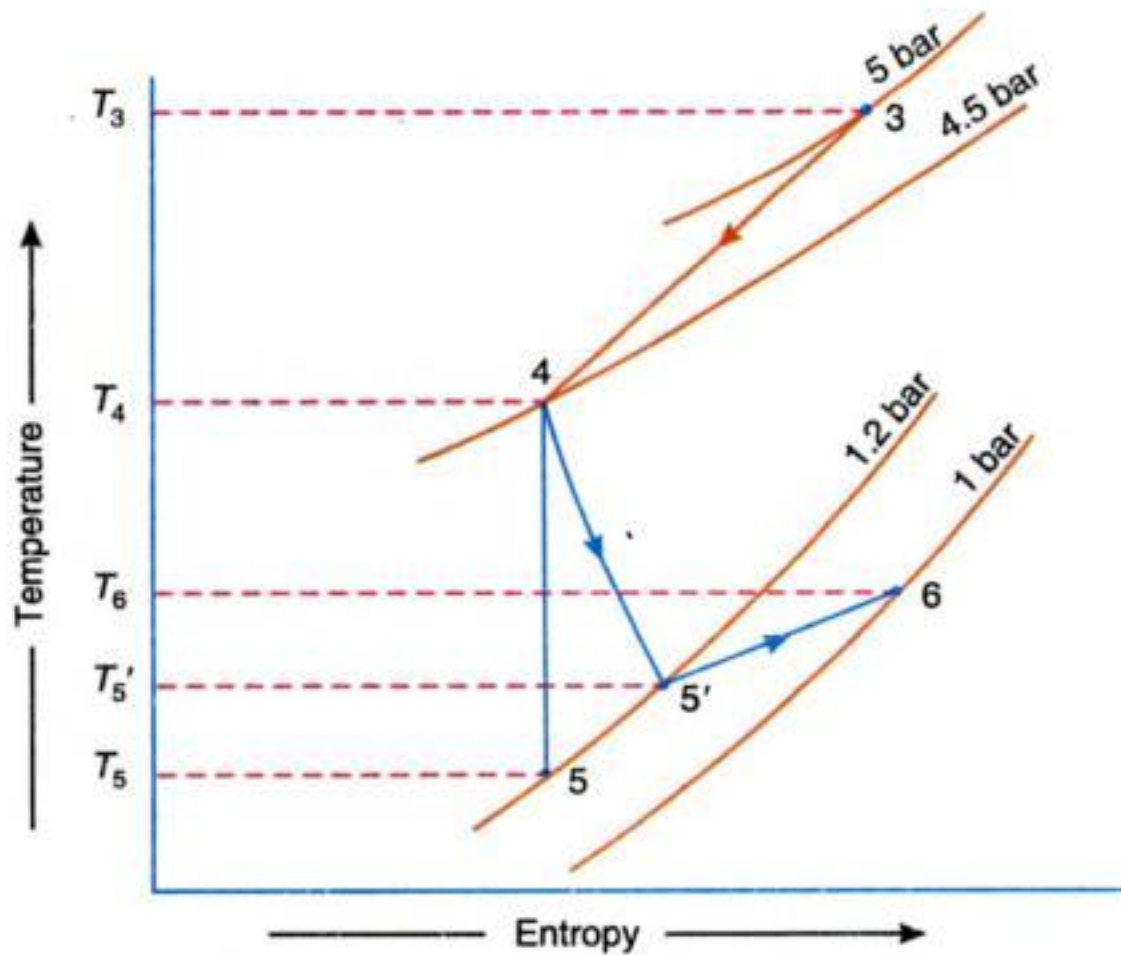
Fig. 3.7

1. *Temperature of air leaving the expander*

Let T_5 = Temperature of air at the end of isentropic expansion in the cooling turbine or expander, and

$T_{5'}$ = Actual temperature of air leaving the cooling turbine or expander.

We know that $\dfrac{T_5}{T_4} = \left(\dfrac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{1.2}{4.5}\right)^{\frac{1.4-1}{1.4}} = (0.267)^{0.286} = 0.685$

∴ $T_5 = T_4 \times 0.685 = 318 \times 0.685 = 217.8$ K

Isentropic efficiency of the cooling turbine,

$$\eta_T = \frac{\text{Actual temperature rise}}{\text{Isentropic temperature rise}} = \frac{T_4 - T_{5'}}{T_4 - T_5}$$

$$0.75 = \frac{318 - T_{5'}}{318 - 217.8} = \frac{318 - T_{5'}}{100.2}$$

∴ $T_{5'} = 318 - 0.75 \times 100.2 = 242.85$ K **Ans.**

2. *Power delivered to the ram air which is passed through the heat exchanger*

We know that work delivered to the ram air which is passed through the heat exchanger,

$$= m_a c_p (T_4 - T_{5'}) = 20 \times 1 (318 - 242.85) = 1503 \text{ kJ/min}$$

∴ Power delivered = $1503/60 = 25.05$ kW **Ans.**

3. Refrigeration load

We know that the refrigeration load taken from the cabin

$$= m_a c_p (T_6 - T_5') = 20 \times 1 (298 - 242.85) = 1103 \text{ kJ/min}$$
$$= 1103/210 = 5.25 \text{ TR } \textbf{Ans.}$$

3.4 Simple Air Evaporative Cooling system

A simple air evaporative cooling system is shown in Fig. 3.8. It is similar to the simple cooling system except that the addition of an evaporator between the heat exchanger and cooling turbine. The evaporator provides an additional cooling effect through evaporation of a refrigerant such as water. At high altitudes, the evaporative cooling may be obtained by using alcohol or ammonia. The water, alcohol and ammonia have different refrigerating effects at different altitudes. At 20 000 metres height, water boils at 40°C, alcohol at 9°C and ammonia at − 70°C.

Air evaporative cooling system
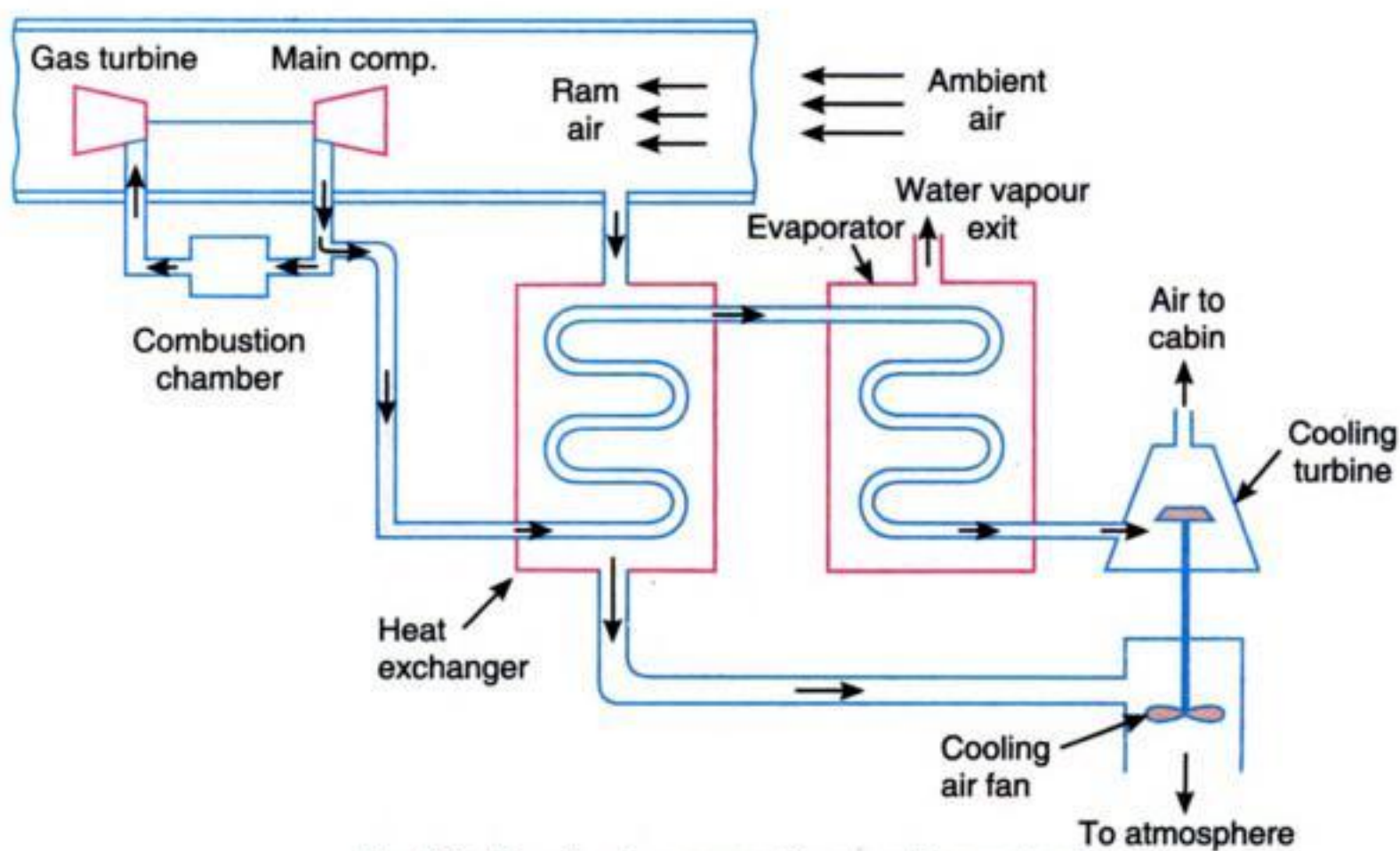
Fig. 3.8. Simple air evaporative cooling system.

The *T-s* diagram for a simple air cycle evaporative cooling system is shown in Fig. 3.9.

The various processes are same as discussed in the previous article, except that the cooling process in the evaporator is shown by $4 - 4'$ in Fig. 3.9.
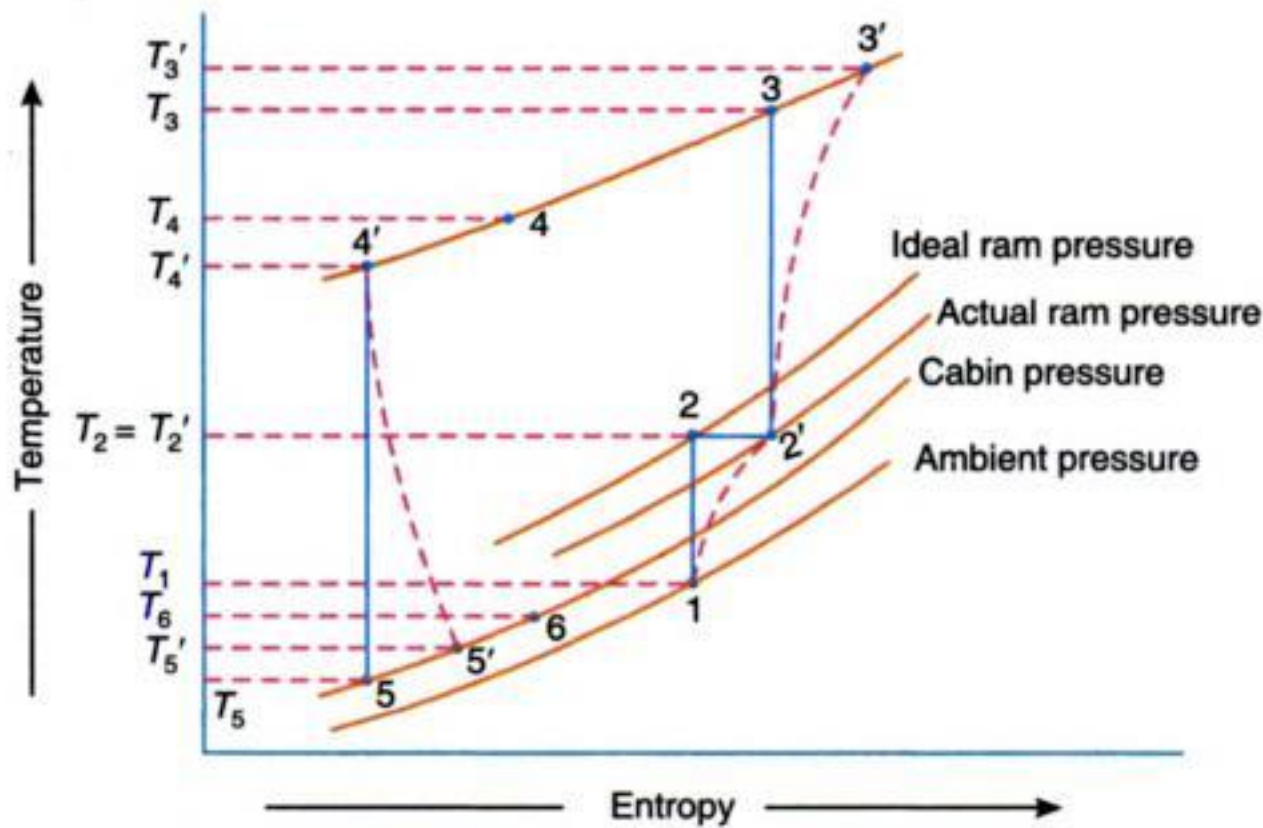
Fig. 3.9. *T-s* diagram for simple evaporative cooling system.

If Q tonnes of refrigeration is the cooling load in the cabin, then the air required for the refrigeration purpose,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_{5'})} \text{ kg / min}$$

Power required for the refrigerating system,

$$P = \frac{m_a c_p(T_{3'} - T_{2'})}{60} \text{ kW}$$

and C.O.P. of the refrigerating system

$$= \frac{210\,Q}{m_a c_p(T_{3'} - T_{2'})} = \frac{210\,Q}{P \times 60}$$

The initial mass of evaporant (m_e) required to be carried for the given flight time is given by

$$m_e = \frac{Q_e \cdot t}{h_{fg}}$$

where
Q_e = Heat to be removed in evaporation in kJ/min,
t = Flight time in minutes, and
h_{fg} = Latent heat of vaporisation of evaporant in kJ/kg.

Notes : 1. In *T-s* diagram as shown in Fig. 3.9, the thick lines show the ideal condition of the process, while the dotted lines show actual conditions of the process.

2. If cooling of 45 minutes duration or less is required, it may be advantageous to use evaporative cooling alone.

Example 3.6. *A simple evaporative air refrigeration system is used for an aeroplane to take 20 tonnes of refrigeration load. The ambient air conditions are 20°C and 0.9 bar. The ambient air is rammed isentropically to a pressure of 1 bar. The air leaving the main compressor at pressure 3.5 bar is first cooled in the heat exchanger having effectiveness of 0.6 and then in the evaporator where its temperature is reduced by 5°C. The air from the evaporator is passed through the cooling turbine and then it is supplied to the cabin which is to be maintained at a temperature of 25°C and at a pressure of 1.05 bar. If the internal efficiency of the compressor is 80% and that of cooling turbine is 75% ,determine :*

1. Mass of air bled off the main compressor; 2. Power required for the refrigerating system; and 3. C.O.P. of the refrigerating system.

Solution. Given : Q = 20 TR ; T_1 = 20°C = 20 + 273 = 293 K ; p_1 = 0.9 bar ; p_2 = 1 bar ; $p_3 = p_{3'}$ = 3.5 bar ; η_H = 0.6 ; T_6 = 25°C = 25 + 273 = 298 K ; p_6 = 1.05 bar ; η_C = 80% = 0.8 ; η_T = 75% = 0.75

The *T-s* diagram for the simple evaporative air refrigeration system with the given conditions is shown in Fig. 3.10.
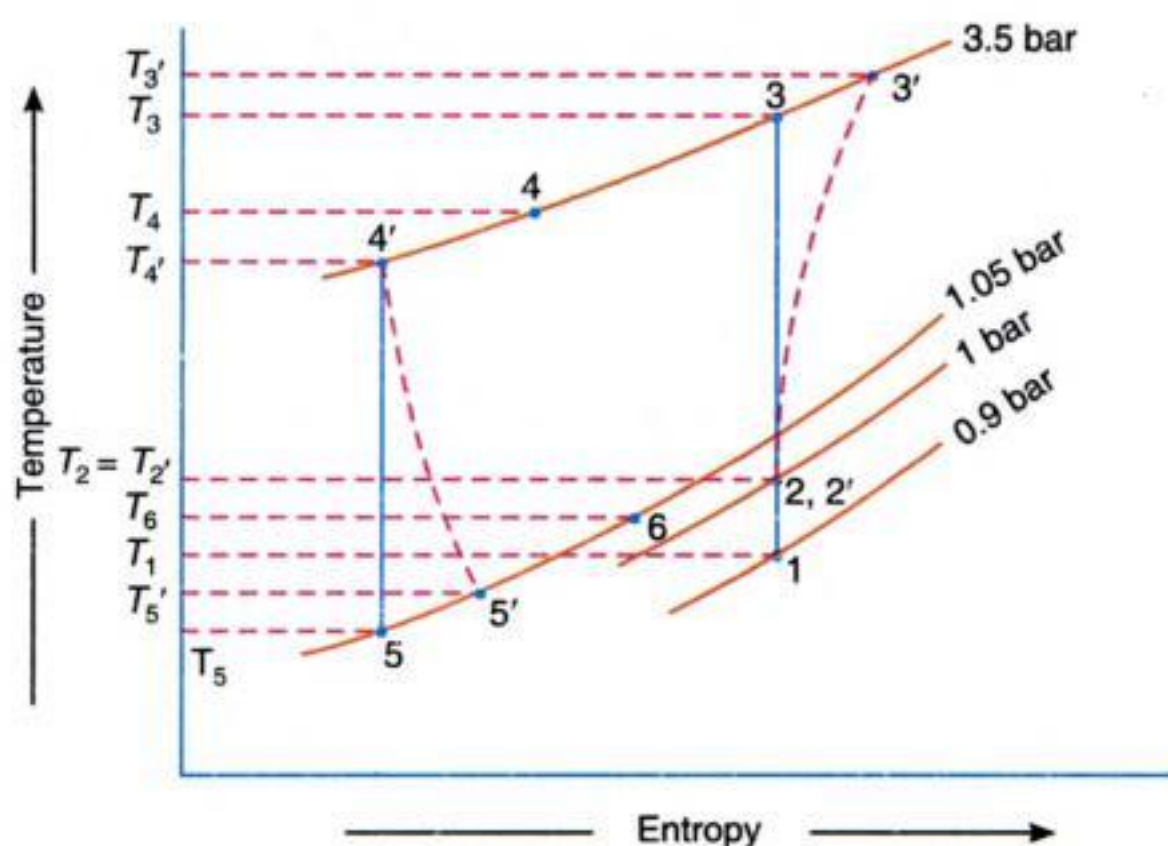
Fig. 3.10

Let $\qquad$ T_2 = Temperature of air entering the main compressor,

$\qquad$ T_3 = Temperature of air after isentropic compression in the main compressor,

$\qquad$ $T_{3'}$ = Actual temperature of air leaving the main compressor, and

$\qquad$ T_4 = Temperature of air entering the evaporator.

We know that for an isentropic ramming process 1-2,,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{0.9}\right)^{\frac{1.4-1}{1.4}} = (1.11)^{0.286} = 1.03$$

$$\qquad \text{... (Taking } \gamma = 1.4)$$

∴ $\qquad$ $T_2 = T_1 \times 1.03 = 293 \times 1.03 = 301.8$ K

Now for the isentropic compression process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.5}{1}\right)^{\frac{1.4-1}{1.4}} = (3.5)^{0.286} = 1.43$$

∴ $\qquad T_3 = T_2 \times 1.43 = 301.8 \times 1.43 = 431.6 \text{ K}$

We know that efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic increase in temperature}}{\text{Actual increase in temperature}} = \frac{T_3 - T_2}{T_{3'} - T_2}$$

$$0.8 = \frac{431.6 - 301.8}{T_{3'} - 301.8} = \frac{129.8}{T_{3'} - 301.8}$$

∴ $\qquad T_{3'} = 301.8 + 129.8/0.8 = 464 \text{ K}$

Effectiveness of the heat exchanger (η_H),

$$0.6 = \frac{T_{3'} - T_4}{T_{3'} - T_{2'}} = \frac{464 - T_4}{464 - 301.8} = \frac{464 - T_4}{162.2} \qquad \dots (\because T_{2'} = T_2)$$

∴ $\qquad T_4 = 464 - 0.6 \times 162.2 = 366.7 \text{ K} = 93.7°C$

Since the temperature of air in the evaporator is reduced by 5°C, therefore the temperature of air leaving the evaporator and entering the cooling turbine,

$$T_{4'} = T_4 - 5 = 93.7 - 5 = 88.7°C = 361.7 \text{ K}$$

Now for the isentropic expansion process 4'– 5,

$$\frac{T_{4'}}{T_5} = \left(\frac{p_3}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.5}{1.05}\right)^{\frac{1.4-1}{1.4}} = (3.33)^{0.286} = 1.41$$

∴ $\qquad T_5 = T_{4'} / 1.41 = 361.7/1.41 = 256.5 \text{ K}$

Efficiency of the cooling turbine,

$$\eta_T = \frac{\text{Actual increase in temperature}}{\text{Isentropic increase in temperature}} = \frac{T_{4'} - T_{5'}}{T_{4'} - T_5}$$

$$0.75 = \frac{361.7 - T_{5'}}{361.7 - 256.5} = \frac{361.7 - T_{5'}}{105.2}$$

∴ $\qquad T_{5'} = 361.7 - 0.75 \times 105.2 = 282.8 \text{ K}$

1. Mass of air bled off the main compressor

We know that mass of air bled off the main compressor,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_{5'})} = \frac{210 \times 20}{1(298 - 282.8)} = 276 \text{ kg / min } \textbf{Ans.}$$

2. Power required for the refrigerating system

We know that power required for the refrigerating system,

$$P = \frac{m_a c_p(T_{3'} - T_{2'})}{60} = \frac{276 \times 1(464 - 301.8)}{60} = 746 \text{ kW } \textbf{Ans.}$$

3. C.O.P. of the refrigerating system

We know that C.O.P. of the refrigerating system

$$= \frac{210\,Q}{P \times 60} = \frac{210 \times 20}{746 \times 60} = 0.094 \textbf{ Ans.}$$

3.5 Boot-strap Air Cooling System

A boot-strap air cooling system is shown in Fig. 3.11. This cooling system has two heat exchangers instead of one and a cooling turbine drives a secondary compressor instead of cooling fan. The air bled from the main compressor is first cooled by the ram air in the first heat exchanger. This cooled air, after compression in the secondary compressor, is led to the second heat exchanger where it is again cooled by the ram air before passing to the cooling turbine. This type of cooling system is mostly used in transport type aircraft.

Fig. 3.11. Boot-strap air cooling system.

The *T-s* diagram for a boot-strap air cycle cooling system is shown in Fig. 3.12. The various processes are as follows :

1. The process 1–2 represents the isentropic ramming of ambient air from pressure p_1 and temperature T_1 to pressure p_2 and temperature T_2. The process 1–2′ represents the actual ramming process because of internal friction due to irreversibilities.

2. The process 2′–3 represents the isentropic compression of air in the main compressor and the process 2′–3′ represents the actual compression of air because of internal friction due to irreversibilities.

3. The process 3′–4 represents the cooling by ram air in the first heat exchanger. The pressure drop in the heat exchanger is neglected.

4. The process 4 – 5 represents the isentropic compression of cooled air, from first heat exchanger, in the secondary compressor. The process 4 – 5′ represents the actual compression process because of internal friction due to irreversibilities.

5. The process 5′– 6 represents the cooling by ram air in the second heat exchanger. The pressure drop in the heat exchanger in neglected.

6. The process 6 – 7 represents the isentropic expansion of cooled air in the cooling turbine upto the cabin pressure. The process 6 – 7′ represents actual expansion of the cooled air in the cooling turbine.

7. The process 7′– 8 represents the heating of air upto the cabin temperature T_8.
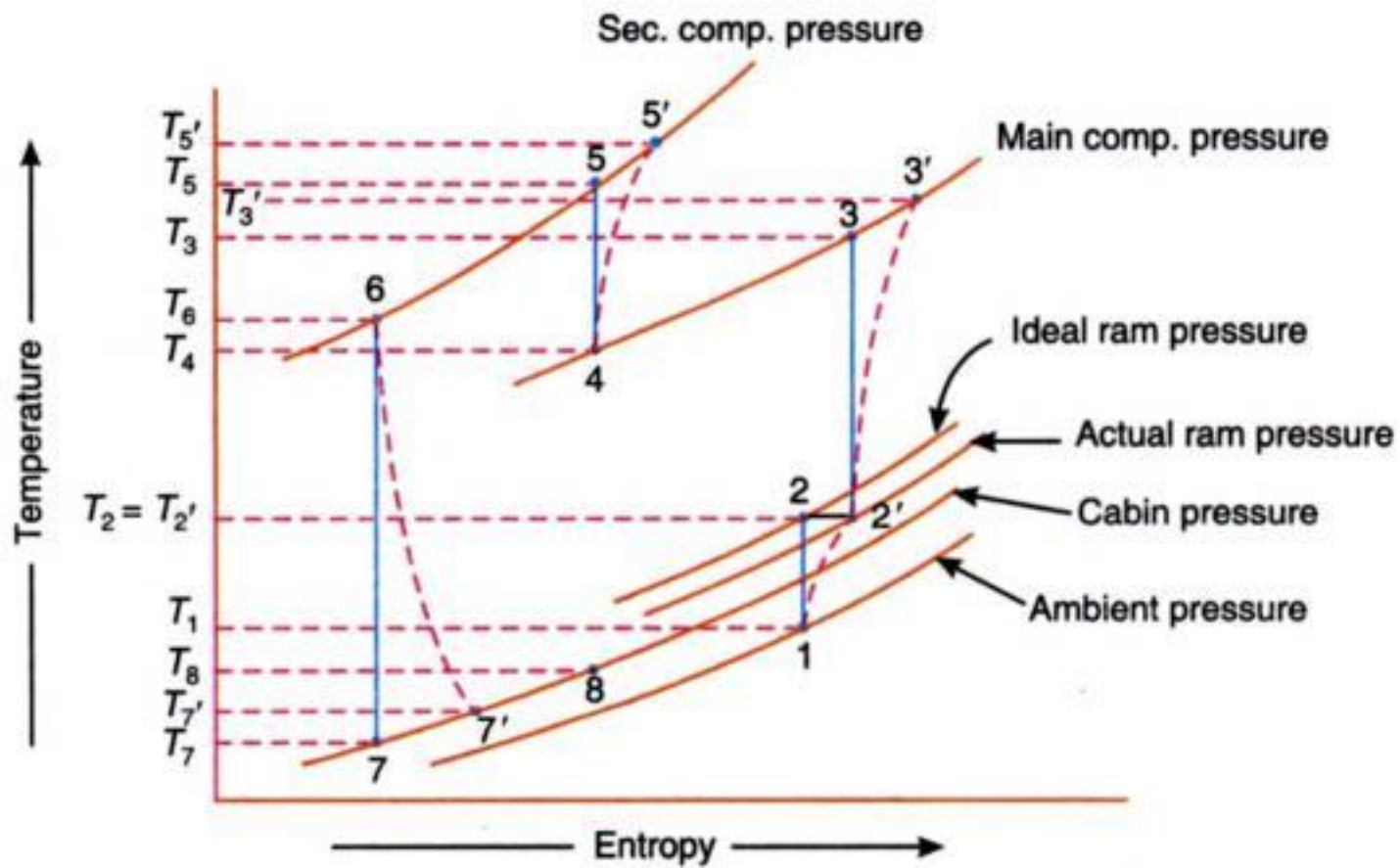
Fig. 3.12. *T-s diagram for boot strap air cooling system.*

If Q tonnes of refrigeration is the cooling load in the cabin, then the quantity of air required for the refrigeration purpose will be

$$m_a = \frac{210\,Q}{c_p(T_8 - T_{7'})} \text{ kg / min}$$

Power required for the refrigerating system,

$$P = \frac{m_a c_p (T_{3'} - T_{2'})}{60} \text{ kW}$$

and C.O.P. of the refrigerating system

$$= \frac{210\,Q}{m_a c_p (T_{3'} - T_{2'})} = \frac{210}{P \times 60}$$

Example 3.7. *A boot-strap cooling system of 10 TR capacity is used in an aeroplane. The ambient air temperature and pressure are 20°C and 0.85 bar respectively. The pressure of air increases from 0.85 bar to 1 bar due to ramming action of air. The pressure of air discharged from the main compressor is 3 bar. The discharge pressure of air from the auxiliary compressor is 4 bar. The isentropic efficiency of each of the compressor is 80%, while that of turbine is 85%. 50% of the enthalpy of air discharged from the main compressor is removed in the first heat exchanger and 30% of the enthalpy of air discharged from the auxiliary compressor is removed in the second heat exchanger using rammed air. Assuming ramming action to be isentropic, the required cabin pressure of 0.9 bar and temperature of the air leaving the cabin not more than 20°C, find : 1. the power required to operate the system; and 2. the C.O.P. of the system. Draw the schematic and temperature -entropy diagram of the system. Take γ = 1.4 and c_p = 1 kJ/kg K.*

Solution. Given : $Q = 10$ TR ; $T_1 = 20°C = 20 + 273 = 293$ K ; $p_1 = 0.85$ bar ; $p_2 = 1$ bar ; $p_3 = p_{3'} = p_4 = 3$ bar ; $p_5 = p_{5'} = p_6 = 4$ bar ; $\eta_{C1} = \eta_{C2} = 80\% = 0.8$; $\eta_T = 85\% = 0.85$; $p_7 = p_{7'} = p_8 = 0.9$ bar ; $T_8 = 20°C = 20 + 273 = 293$ K ; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

The schematic diagram for a boot-strap cooling system is shown in Fig. 3.9. The temperature- entropy (*T-s*) diagram with the given conditions is shown in Fig. 3.13.
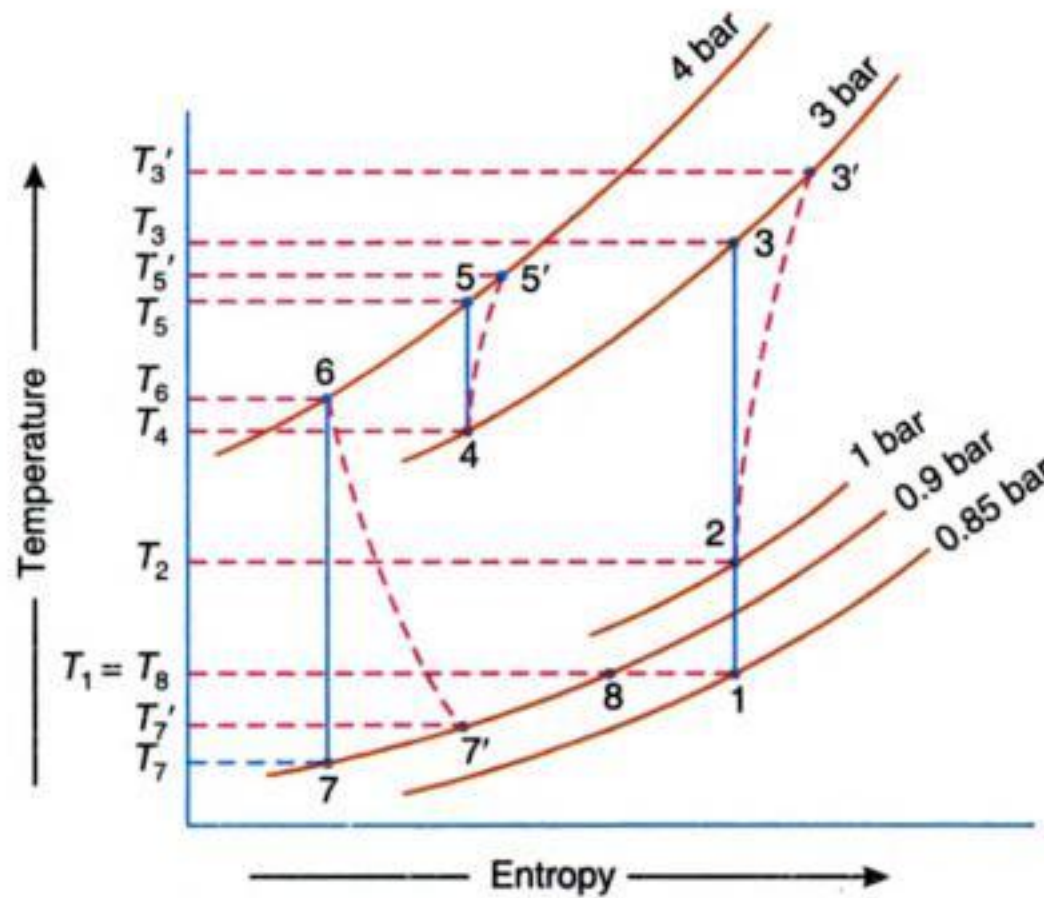
Fig. 3.13

We know that for isentropic ramming process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{0.85}\right)^{\frac{1.4-1}{1.4}} = (1.176)^{0.286} = 1.047$$

∴ $T_2 = T_1 \times 1.047 = 293 \times 1.047 = 306.8 \text{ K} = 33.8°C$

Now for isentropic process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.37$$

∴ $T_3 = T_2 \times 1.37 = 306.8 \times 1.37 = 420.3 \text{ K} = 147.3°C$

We know that isentropic efficiency of the compressor,

$$\eta_{C1} = \frac{\text{Isentropic increase in temperature}}{\text{Actual increase in temperature}} = \frac{T_3 - T_2}{T_{3'} - T_2}$$

$$0.8 = \frac{420.3 - 306.8}{T_{3'} - 306.8} = \frac{113.5}{T_{3'} - 306.8}$$

∴ $T_{3'} = 306.8 + 113.5/0.8 = 448.7 \text{ K} = 175.7°C$

Since 50% of the enthalpy of air discharged from the main compressor is removed in the first heat exchanger (*i.e.* during the process 3′– 4), therefore temperature of air leaving the first heat exchanger,

$$T_4 = 0.5 \times 175.7 = 87.85°C = 360.85 \text{ K}$$

Now for the isentropic process 4–5,

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{3}\right)^{\frac{1.4-1}{1.4}} = (1.33)^{0.286} = 1.085$$

∴ $T_5 = T_4 \times 1.085 = 360.85 \times 1.085 = 391.5 \text{ K} = 118.5°C$

We know that isentropic efficiency of the auxiliary compressor,

$$\eta_{C2} = \frac{T_5 - T_4}{T_{5'} - T_4}$$

$$0.8 = \frac{391.5 - 360.85}{T_{5'} - 360.85} = \frac{30.65}{T_{5'} - 360.85}$$

∴ $T_{5'} = 360.85 + 30.65/0.8 = 399.16 \text{ K} = 126.16°\text{C}$

Since 30% of the enthalpy of air discharged from the auxiliary compressor is removed in the second heat exchanger (*i.e.* during the process 5'– 6), therefore temperature of air leaving the second heat exchanger,

$$T_6 = 0.7 \times 126.16 = 88.3°\text{C} = 361.3 \text{ K}$$

For the isentropic process 6–7,

$$\frac{T_7}{T_6} = \left(\frac{p_7}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{0.9}{4}\right)^{\frac{1.4-1}{1.4}} = (0.225)^{0.286} = 0.653$$

∴ $T_7 = T_6 \times 0.653 = 361.3 \times 0.653 = 236 \text{ K} = -37°\text{C}$

We know that turbine efficiency,

$$\eta_T = \frac{\text{Actual increase in temperature}}{\text{Isentropic increase in temperature}} = \frac{T_6 - T_{7'}}{T_6 - T_7}$$

$$0.85 = \frac{361.3 - T_{7'}}{361.3 - 236} = \frac{361.3 - T_{7'}}{125.3}$$

∴ $T_{7'} = 361.3 - 0.85 \times 125.3 = 254.8 \text{ K} = -18.2°\text{C}$

1. *Power required to operate the system*

We know that amount of air required for cooling the cabin,

$$m_a = \frac{210\,Q}{c_p(T_8 - T_{7'})} = \frac{210 \times 10}{1\,(293 - 254.8)} = 55 \text{ kg / min}$$

and power required to operate the system,

$$P = \frac{m_a c_p(T_{3'} - T_2)}{60} = \frac{55 \times 1\,(448.7 - 306.8)}{60} = 130 \text{ kW Ans.}$$

2. *C.O.P. of the system*

We know that C.O.P. of the system

$$= \frac{210\,Q}{m_a c_p(T_{3'} - T_2)} = \frac{210 \times 10}{55 \times 1\,(448.7 - 306.8)} = 0.27 \text{ Ans.}$$

3.6 Boot-strap Air Evaporative Cooling System

A boot-strap air cycle evaporative cooling system is shown in Fig. 3.14. It is similar to the boot-strap air cycle cooling system except that the addition of an evaporator between the second heat exchanger and the cooling turbine.

The *T-s* diagram for a boot-strap air evaporative cooling system is shown in Fig 3.15. The various processes of this cycle are same as a simple boot-strap system except the process 5'–6 which represents cooling in the evaporator using any suitable evaporant.
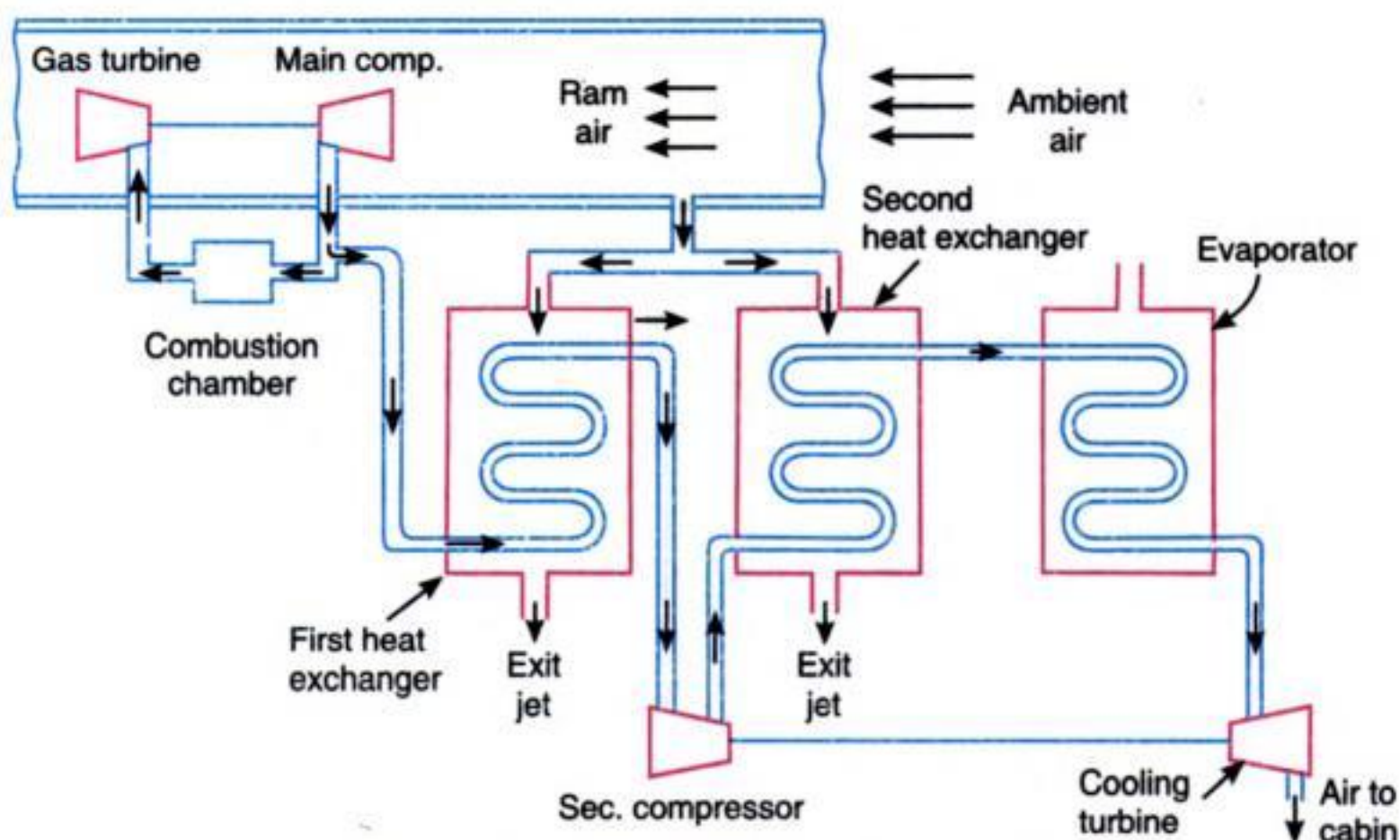
Fig. 3.14. Boot-strap air evaporative cooling system.
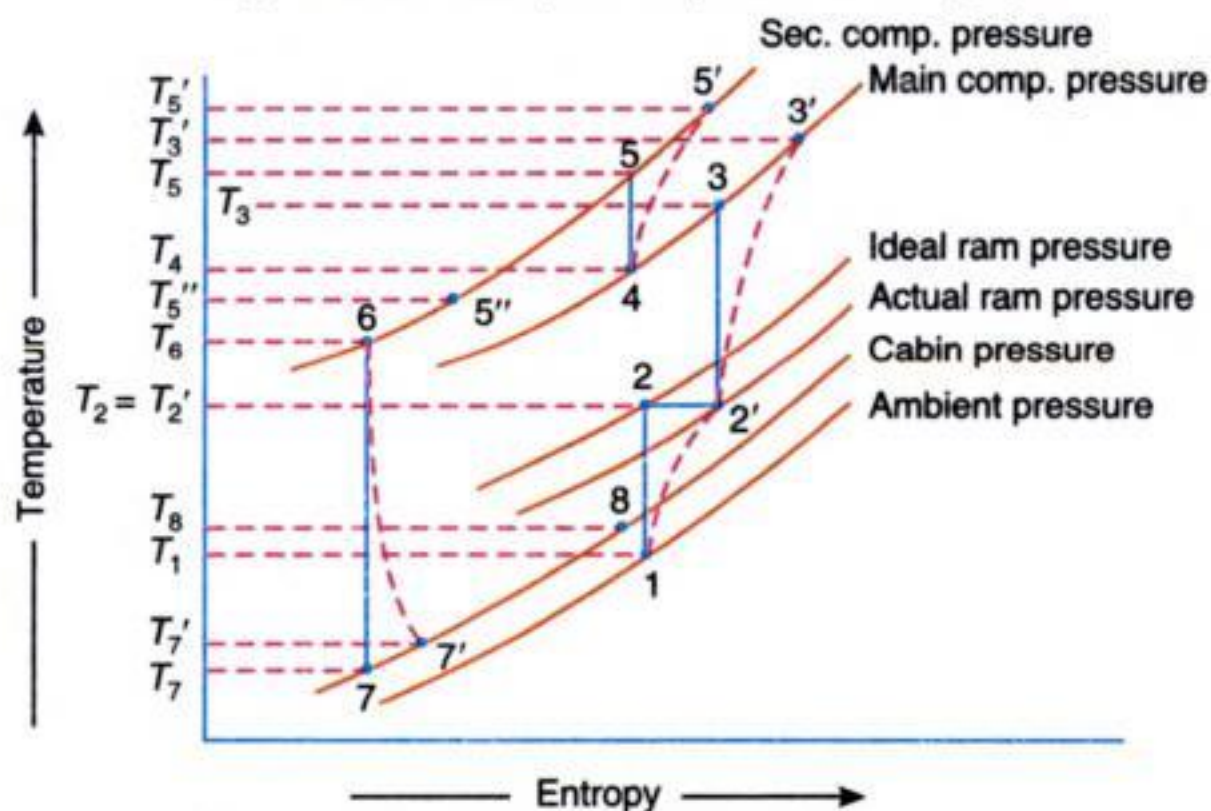
Fig. 3.15. T-s diagram for boot-strap air evaporative cooling system.

If Q tonnes of refrigeration is the cooling load in the cabin, then the quantity of air required for the refrigeration purpose will be

$$m_a = \frac{210 \, Q}{c_p (T_8 - T_{7'})} \text{ kg / min}$$

Power required for the refrigeration system is given by

$$P = \frac{m_a c_p (T_{3'} - T_{2'})}{60} \text{ kW}$$

and C.O.P. of the refrigerating system

$$= \frac{210 \, Q}{m_a c_p (T_{3'} - T_{2'})} = \frac{210 \, Q}{P \times 60}$$

Note: Since the temperature of air leaving the cooling turbine in boot-strap evaporative system is lower than the simple boot-strap system, therefore mass of air (m_a) per tonne of refrigeration will be less in boot-strap evaporative system.

Example 3.8. *The following data refer to a boot strap air cycle evaporative refrigeration system used for an aeroplane to take 20 tonnes of refrigeration load :*

Ambient air temperature	= 15°C
Ambient air pressure	= 0.8 bar
Mach number of the flight	= 1.2
Ram efficiency	= 90%
Pressure of air bled off the main compressor	= 4 bar
Pressure of air in the secondary compressor	= 5 bar
Isentropic efficiency of the main compressor	= 90%
Isentropic efficiency of the secondary compressor	= 80%
Isentropic efficiency of the cooling turbine	= 80%
Temperature of air leaving the first heat exchanger	= 170°C
Temperature of air leaving the second heat exchanger	= 155°C
Temperature of air leaving the evaporator	= 100°C
Cabin temperature	= 25°C
Cabin pressure	= 1 bar

Find : 1. Mass of air required to take the cabin load, 2. Power required for the refrigeration system, and 3. C.O.P. of the system.

Solution. Given : $Q = 20$ TR ; $T_1 = 15°C = 15 + 273 = 288$ K ; $p_1 = 0.8$ bar ; $M = 1.2$; $\eta_R = 90\% = 0.9$; $p_3 = p_3' = p_4 = 4$ bar ; $p_5 = p_5' = p_5'' = p_6 = 5$ bar ; $\eta_{C1} = 90\% = 0.9$; $\eta_{C2} = 80\% = 0.8$; $\eta_T = 80\% = 0.8$; $T_4 = 170°C = 170 + 273 = 443$ K; $T_5'' = 155°C = 155 + 273 = 428$ K ; $T_6 = 100°C = 100 + 273 = 373$ K ; $T_8 = 25°C = 25 + 273 = 298$ K ; $p_8 = p_7 = p_7' = 1$ bar

The T-s diagram for the boot-strap air cycle evaporative refrigeration system, with the given conditions, is shown in Fig. 3.16.
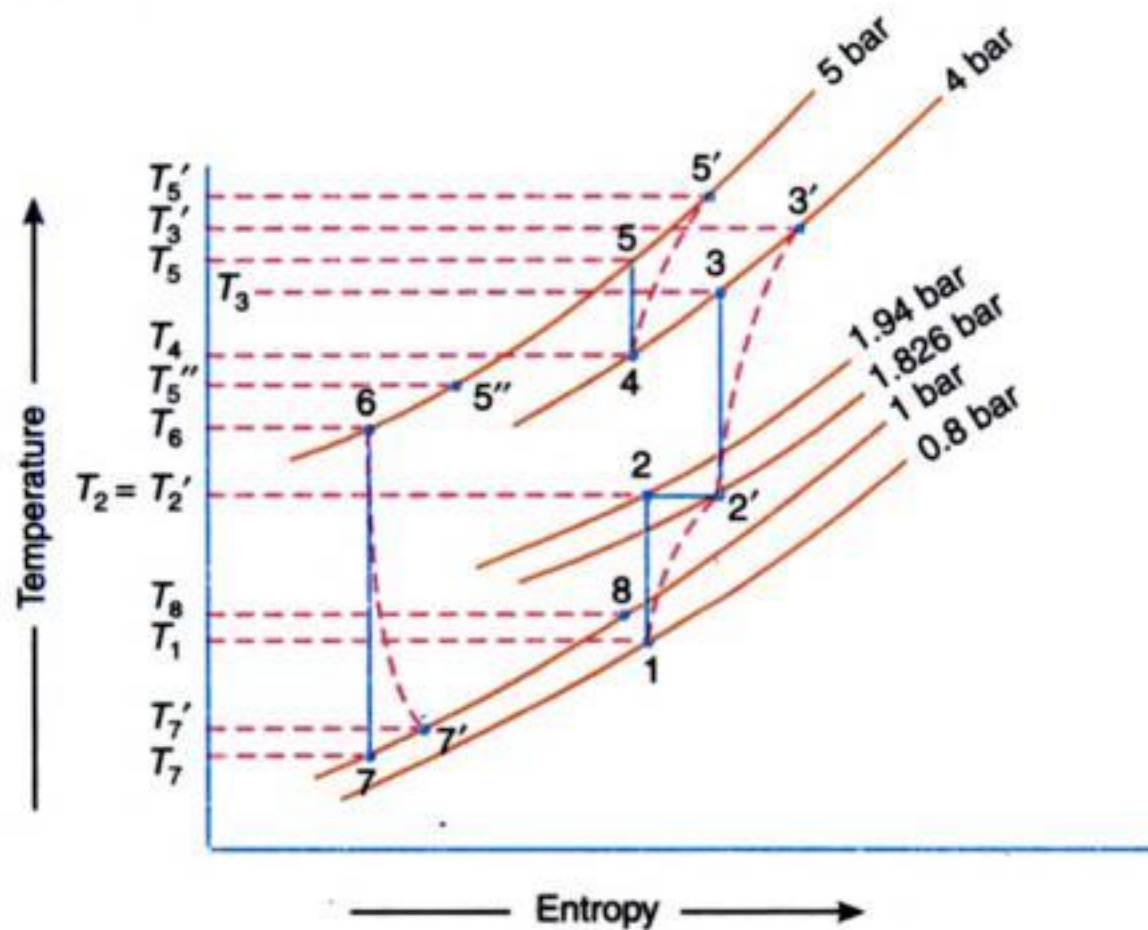
Fig. 3.16

Let T_2'	= Stagnation temperature of ambient air entering the main compressor,
p_2	= Pressure of air at the end of isentropic ramming, and

$p_{2'}$ = Stagnation pressure of ambient air entering the main compressor.

We know that

$$\frac{T_{2'}}{T_1} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{1.4 - 1}{2} (1.2)^2 = 1.288$$

∴ $\quad T_{2'} = T_1 \times 1.288 = 288 \times 1.288 = 371 \text{ K}$

For isentropic process 1–2,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{371}{288}\right)^{\frac{1.4}{1.4 - 1}} = (1.288)^{3.5} = 2.425$$

∴ $\quad p_2 = p_1 \times 2.425 = 0.8 \times 2.425 = 1.94 \text{ bar}$

We know that ram efficiency,

$$\eta_R = \frac{\text{Actual pressure rise}}{\text{Isentropic pressure rise}} = \frac{p_{2'} - p_1}{p_2 - p_1}$$

$$0.9 = \frac{p_{2'} - 0.8}{1.94 - 0.8} = \frac{p_{2'} - 0.8}{1.14}$$

∴ $\quad p_{2'} = 0.9 \times 1.14 + 0.8 = 1.826 \text{ bar}$

Now for the isentropic process 2′–3,

$$\frac{T_3}{T_{2'}} = \left(\frac{p_3}{p_{2'}}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{4}{1.826}\right)^{\frac{1.4 - 1}{1.4}} = (2.19)^{0.286} = 1.25$$

∴ $\quad T_3 = T_{2'} \times 1.25 = 371 \times 1.25 = 463.8 \text{ K}$

We know that isentropic efficiency of the main compressor,

$$\eta_{C1} = \frac{\text{Isentropic increase in temp.}}{\text{Actual increase in temp.}} = \frac{T_3 - T_{2'}}{T_{3'} - T_{2'}}$$

$$0.9 = \frac{463.8 - 371}{T_{3'} - 371} = \frac{92.8}{T_{3'} - 371}$$

∴ $\quad T_{3'} = 371 + 92.8 / 0.9 = 474 \text{ K}$

Temperature of air leaving the first heat exchanger,

$\quad T_4 = 443 \text{ K}$ ... (Given)

For the isentropic process 4–5,

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{5}{4}\right)^{\frac{1.4 - 1}{1.4}} = (1.25)^{0.286} = 1.066$$

∴ $\quad T_5 = T_4 \times 1.066 = 443 \times 1.066 = 472 \text{ K}$

Isentropic efficiency of the secondary compressor,

$$\eta_{C2} = \frac{T_5 - T_4}{T_{5'} - T_4}$$

$$0.8 = \frac{472 - 443}{T_{5'} - 443} = \frac{29}{T_{5'} - 443}$$

∴ $\quad T_{5'} = 443 + 29 / 0.8 = 479 \text{ K}$

Temperature of air leaving the second heat exchanger,

$\quad T_{5''} = 428 \text{ K}$ ... (Given)

Temperature of air leaving the evaporator,

$$T_6 = 373 \text{ K} \qquad \qquad \text{... (Given)}$$

Now for the isentropic process 6–7,

$$\frac{T_6}{T_7} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.584$$

$$\therefore \qquad T_7 = T_6 / 1.584 = 373/1.584 = 235.5 \text{ K}$$

We know that isentropic efficiency of the cooling turbine,

$$\eta_T = \frac{\text{Actual increase in temp.}}{\text{Isentropic increase in temp.}} = \frac{T_6 - T_{7'}}{T_6 - T_7}$$

$$0.8 = \frac{373 - T_{7'}}{373 - 235.5} = \frac{373 - T_{7'}}{137.5}$$

$$\therefore \qquad T_{7'} = 373 - 0.8 \times 137.5 = 263 \text{ K}$$

1. Mass of air required to take the cabin load

We know that mass of air required to take the cabin load,

$$m_a = \frac{210\, Q}{c_p (T_8 - T_7)} = \frac{210 \times 20}{1\,(298 - 263)} = 120 \text{ kg / min } \textbf{Ans.}$$

2. Power required for the refrigeration system

We know that power required for the refrigeration system,

$$P = \frac{m_a c_p (T_{3'} - T_{2'})}{60} = \frac{120 \times 1\,(474 - 371)}{60} = 206 \text{ kW } \textbf{Ans.}$$

3. C.O.P. of the system

We know that C.O.P. of the system $= \dfrac{210\, Q}{P \times 60} = \dfrac{210 \times 20}{206 \times 60} = 0.34$ **Ans.**

3.7 Reduced Ambient Air Cooling System
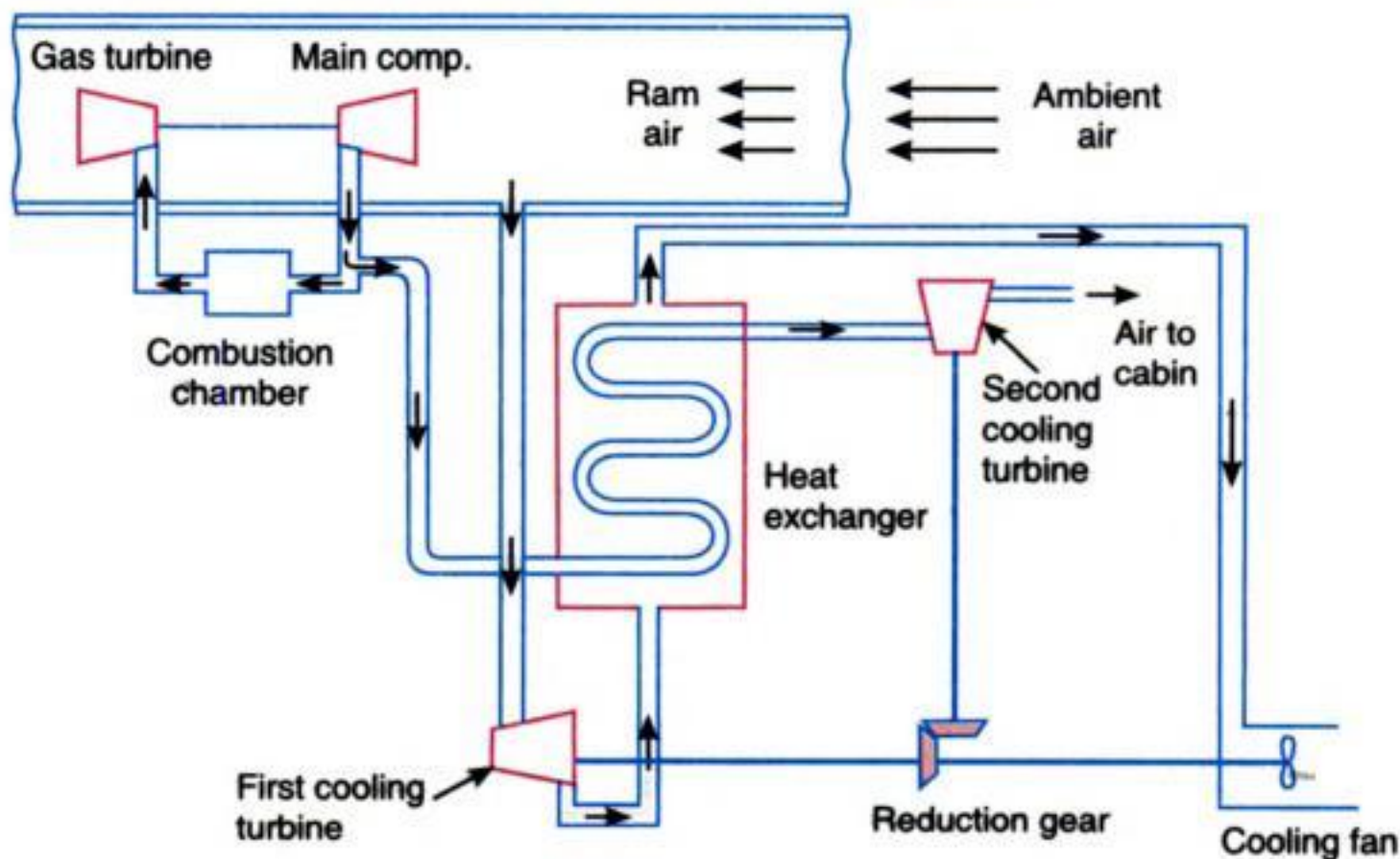
Fig. 3.17. Reduced ambient air cooling system.

The reduced ambient air cooling system is shown in Fig. 3.17. This cooling system includes two cooling turbines and one heat exchanger. The air reduced for the refrigeraion system is bled off from the main compressor. This high pressure and high temperature air is cooled initially in the heat exchanger. The air for cooling is taken from the cooling turbine which lowers the high temperature of rammed air. The cooled air from the heat exchanger is passed through the second cooling turbine from where the air is supplied to the cabin. The work of the cooling turbine is used to drive the cooling fan (through reduction gears) which draws cooling air from the heat exchanger. The reduced ambient air cooling system is used for very high speed aircrafts.
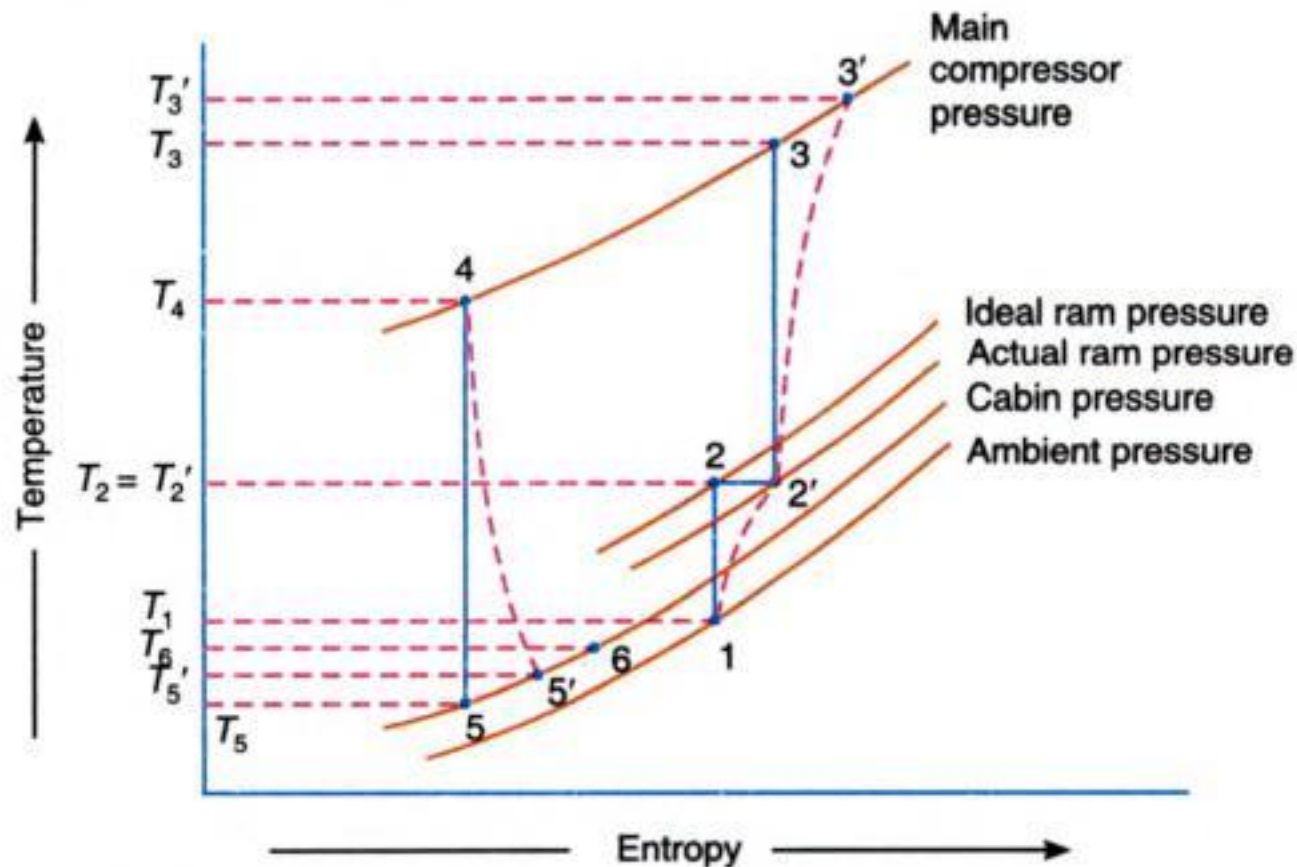
Fig. 3.18. T-s diagram for reduced ambient air cycle cooling system.

The T-s diagram for the reduced ambient air cycle cooling system is shown in Fig. 3.18. The various processes are as follows :

1. The process 1–2 represents isentropic ramming of air and the process 1–2′ represents actual ramming of air because of internal friction due to irreversibilities.

2. The process 2′–3 represents isentropic compression in the main compressor and the process 2′– 3′ represents actual compression of air, because of internal friction due to irreversibilities.

3. The process 3′–4 represents cooling of compressed air by ram air which after passing through the first cooling turbine is led to the heat exchanger. The pressure drop in the heat exchanger is neglected.

4. The process 4–5 represents isentropic expansion of air in the second cooling turbine upto the cabin pressure. The actual expansion of air in the second cooling turbine is represented by the curve 4–5′.

5. The process 5′–6 represents the heating of air upto the cabin temperature T_6.

If Q tonnes of refrigeration is the cooling load in the cabin, then the quantity of air required for the refrigeration purpose will be

$$m_a = \frac{210\,Q}{c_p(T_6 - T_{5'})} \text{ kg / min}$$

Power required for the refrigeration system is given by

$$P = \frac{m_a c_p (T_{3'} - T_{2'})}{60} \text{ kW}$$

and C.O.P. of the system

$$= \frac{210\,Q}{m_a c_p (T_{3'} - T_{2'})} = \frac{210\,Q}{P \times 60}$$

Example 3.9. *The reduced ambient air refrigeration system used for an aircraft consists of two cooling turbines, one heat exchanger and one air cooling fan. The speed of aircraft is 1500 km/h. The ambient air conditions are 0.8 bar and 10°C. The ram efficiency may be taken as 90%. The rammed air used for cooling is expanded in the first cooling turbine and leaves it at a pressure of 0.8 bar. The air bled from the main compressor at 6 bar is cooled in the heat exchanger and leaves it at 100°C. The cabin is to be maintained at 20°C and 1 bar. The pressure loss between the second cooling turbine and cabin is 0.1 bar. If the isentropic efficiency for the main compressor and both of the cooling turbines are 85% and 80% respectively, find :*

1. mass flow rate of air supplied to cabin to take a cabin load of 10 tonnes of refrigeration;

2. quantity of air passing through the heat exchanger if the temperature rise of ram air is limited to 80 K;

3. power used to drive the cooling fan; and

4. C.O.P. of the system.

Solution. Given : $V = 1500$ km / h $= 417$ m / s ; $p_1 = 0.8$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $\eta_R = 90\% = 0.9$; $p_3 = p_4 = 6$ bar ; $T_4 = 100°C = 100 + 273 = 373$ K ; $T_6 = 20°C = 20 + 273 = 293$ K ; $p_6 = 1$ bar ; $\eta_C = 85\% = 0.85$; $\eta_{T1} = \eta_{T2} = 80\% = 0.8$; $Q = 10$ TR

The *T-s* diagram for the reduced ambient air refrigeration system with the given conditions is shown in Fig. 3.19.
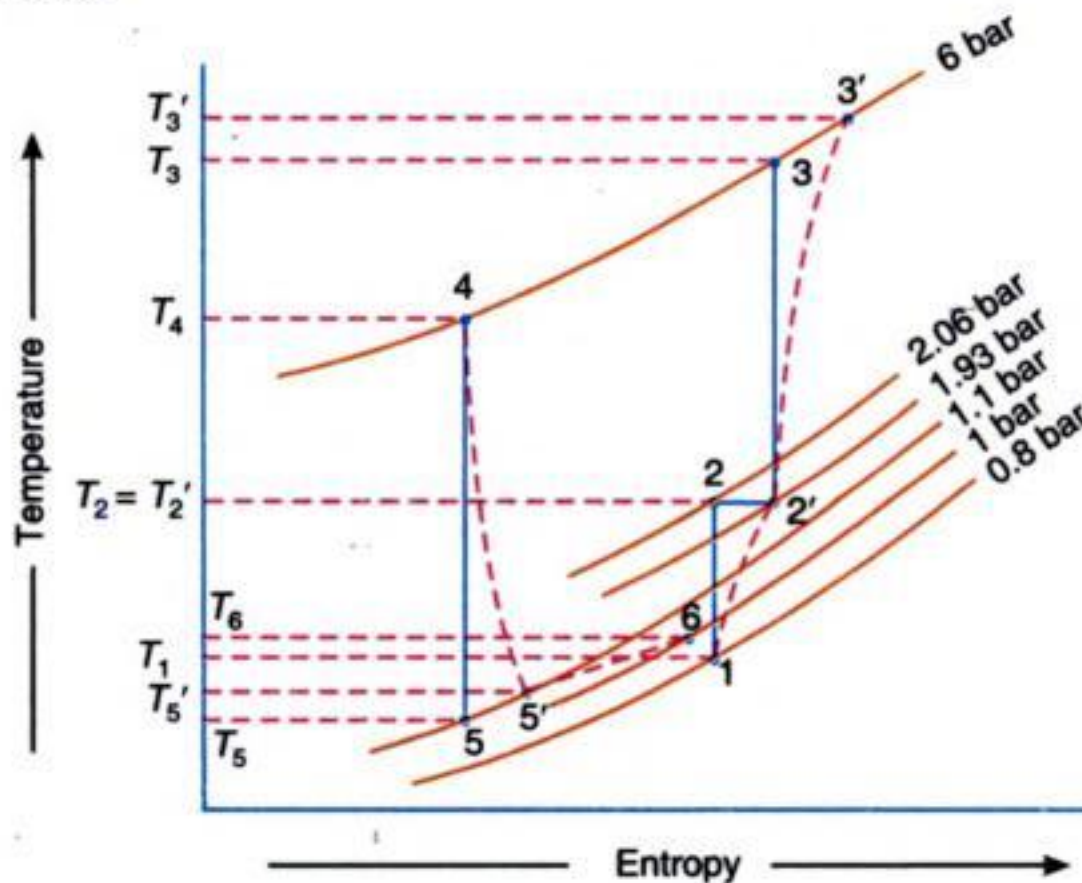
Fig. 3.19

Let $T_{2'}$ = Stagnation temperature of ambient air entering the main compressor,

p_2 = Pressure of air at the end of isentropic ramming, and

$p_{2'}$ = Stagnation pressure of air entering the main compressor.

We know that $T_{2'} = T_1 + \dfrac{V^2}{2000\,c_p} = 283 + \dfrac{(417)^2}{2000 \times 1} = 370$ K

... (Taking $c_p = 1$ kJ/kg K)

For the isentropic ramming process 1-2,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{370}{283}\right)^{\frac{1.4}{1.4-1}} = (1.31)^{3.5} = 2.57$$

$\therefore \qquad p_2 = p_1 \times 2.57 = 0.8 \times 2.57 = 2.06 \text{ bar}$

We know that ram efficiency,

$$\eta_R = \frac{\text{Actual rise in pressure}}{\text{Isentropic rise in pressure}} = \frac{p_2' - p_1}{p_2 - p_1}$$

$$0.9 = \frac{p_2' - 0.8}{2.06 - 0.8} = \frac{p_2' - 0.8}{1.26}$$

$\therefore \qquad p_2' = 0.9 \times 1.26 + 0.8 = 1.93 \text{ bar}$

The *T-s* diagram for the expansion of ram air in the first cooling turbine is shown in Fig. 3.20. The vertical line 2′–1′ represents the isentropic cooling process and the curve 2′–2″ represents the actual cooling process.
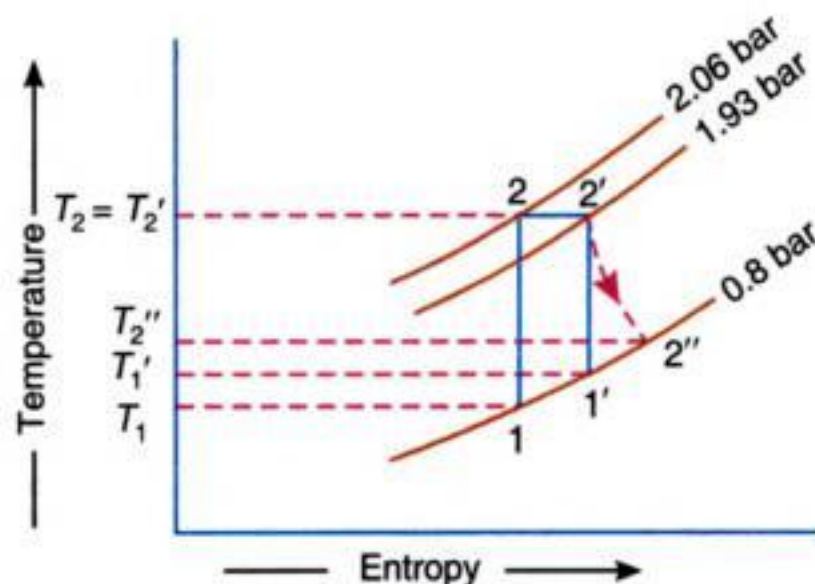
Fig. 3.20. *T-s* diagram for the first cooling turbine.

Now for the isentropic cooling process 2′–1′,

$$\frac{T_{2'}}{T_{1'}} = \left(\frac{p_{2'}}{p_{1'}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.93}{0.8}\right)^{\frac{1.4-1}{1.4}} = (2.4)^{0.286} = 1.284$$

$$\dots (\because p_{1'} = p_1)$$

$\therefore \qquad T_{1'} = T_{2'}/1.284 = 370/1.284 = 288 \text{ K}$

Isentropic efficiency of the first cooling turbine,

$$\eta_{T1} = \frac{\text{Actual increase in temp.}}{\text{Isentropic increase in temp.}} = \frac{T_{2'} - T_{2''}}{T_{2'} - T_{1'}}$$

$$0.8 = \frac{370 - T_{2''}}{370 - 288} = \frac{370 - T_{2''}}{82}$$

$\therefore \qquad T_{2''} = 370 - 0.8 \times 82 = 304 \text{ K}$

For the isentropic compression process 2′–3,

$$\frac{T_3}{T_{2'}} = \left(\frac{p_3}{p_{2'}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{1.93}\right)^{\frac{1.4-1}{1.4}} = (3.11)^{0.286} = 1.38$$

∴ $\qquad T_3 = T_{2'} \times 1.38 = 370 \times 1.38 = 511 \text{ K}$

We know that isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic increase in temp.}}{\text{Actual increase in temp.}} = \frac{T_3 - T_{2'}}{T_{3'} - T_{2'}}$$

$$0.85 = \frac{511 - 370}{T_{3'} - 370} = \frac{141}{T_{3'} - 370}$$

∴ $\qquad T_{3'} = 370 + 141/0.85 = 536 \text{ K}$

Since there is a pressure drop of 0.1 bar between the second cooling turbine and the cabin, therefore pressure of air at exit from the second cooling turbine,

$$p_5 = p_{5'} = p_6 + 0.1 = 1 + 0.1 = 1.1 \text{ bar}$$

Now for the isentropic expansion of air in the second cooling turbine (process 4–5),

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{1.1}\right)^{\frac{1.4-1}{1.4}} = (5.45)^{0.286} = 1.62$$

∴ $\qquad T_5 = T_4 / 1.62 = 373 / 1.62 = 230 \text{ K}$

We know that the isentropic efficiency of the second cooling turbine,

$$\eta_{T2} = \frac{\text{Actual increase in temp.}}{\text{Isentropic increase in temp.}} = \frac{T_4 - T_{5'}}{T_4 - T_5}$$

$$0.8 = \frac{373 - T_{5'}}{373 - 230} = \frac{373 - T_{5'}}{143}$$

∴ $\qquad T_{5'} = 373 - 0.8 \times 143 = 258.6 \text{ K}$

1. Mass flow rate of air supplied to cabin

We know that mass flow rate of air supplied to cabin,

$$m_a = \frac{210 \, Q}{c_p (T_6 - T_{5'})}$$

$$= \frac{210 \times 10}{1 (293 - 258.6)} = 61 \text{ kg / min} \quad \textbf{Ans.}$$

2. Quantity of ram air passing through the heat exchanger

Let $\qquad m_R$ = Quantity of ram air passing through the heat exchanger.

The compressed air bled off at temperature $T_{3'} = 536 \text{ K}$ is cooled in the heat exchanger to a temperature $T_4 = 373 \text{ K}$ by the ram air from the first cooling turbine at a temperature $T_{2''} = 304 \text{ K}$. The temperature rise of ram air in the heat exchanger is limited to 80 K. Considering perfect heat transfer in the heat exchanger,

$$m_R \times c_p \times 80 = m_a \times c_p (T_{3'} - T_4)$$
$$m_R \times 1 \times 80 = 61 \times 1 (536 - 373) = 9943$$

∴ $\qquad m_R = 9943 / 80 = 124.3 \text{ kg / min} \quad \textbf{Ans.}$

3. Power used to drive the cooling fan

Since the work output of both the cooling turbines is used to drive the cooling fan, therefore work output from the first cooling turbine,

$$W_{T1} = m_R \times c_p (T_{2'} - T_{2''})$$
$$= 124.3 \times 1 (370 - 304) = 8204 \text{ kJ/min}$$

and work output from the second cooling turbine,

$$W_{T2} = m_a \times c_p (T_4 - T_5')$$
$$= 61 \times 1 (373 - 258.6) = 6978 \text{ kJ/min}$$

∴ Combined work output from both the cooling turbines,

$$W_T = W_{T1} + W_{T2} = 8204 + 6978 = 15\ 182 \text{ kJ/min}$$

and power used to drive the cooling fan

$$= 15\ 182 / 60 = 253 \text{ kW } \textbf{Ans.}$$

4. C.O.P. of the system

We know that C.O.P. of the system

$$= \frac{210\ Q}{m_a c_p (T_{3'} - T_{2'})} = \frac{210 \times 10}{61 \times 1 (536 - 370)} = 0.21 \textbf{ Ans.}$$

Example 3.10. *The reduced ambient system of air refrigeration for cooling an aircraft cabin consists of two cooling turbines, one heat exchanger and one fan. The first cooling turbine is supplied with the ram air at 1.1 bar and 15° C and delivers after expansion to the heat exchanger at 0.9 bar for cooling the air bled off from the main compressor at 3.5 bar. The cooling air from the heat exchanger is sucked by a fan and discharged to the atmosphere. The cooled air from the heat exchanger is expanded upto 1 bar in the second cooling turbine and discharged into air cabin to be cooled. The air from the cabin is exhausted at 22°C. The refrigerating capacity required is 10 tonnes. If the compression index for the main compressor is 1.5 and the expansion index for both the cooling turbines is 1.35, determine :*

1. Mass flow rate of the cabin air ;

2. Cooling capacity of the heat exchanger and flow rate of the ram air when compressed air is to be cooled to 60°C in the heat exchanger and temperature rise in the heat exchanger for the ram air is not to exceed 30 K ;

3. Combined output of both cooling turbines driving the air fan with transmission efficiency of 60% ; and

4. C.O.P. of the refrigerating system considering only power input to the compressor.

Solution. Given : $p_2 = 1.1$ bar ; $T_2 = 15°C = 15 + 273 = 288$ K ; $p_{2'} = 0.9$ bar ; $p_3 = p_4 = 3.5$ bar ; $p_5 = p_6 = 1$ bar ; $T_6 = 22°C = 22 + 273 = 295$ K ; $Q = 10$ TR ; $\gamma_1 = 1.5$; $\gamma_2 = 1.35$; $T_4 = 60°C = 60 + 273 = 333$ K ; $\eta_T = 60\% = 0.6$

The *T-s* diagram for the reduced ambient system of air refrigeration for cooling an aircraft cabin with the given conditions is shown in Fig. 3.21. The process of cooling the ram air in the first cooling turbine is shown by the curve 2–2′.

Let $T_{2'}$ = Temperature of ram air after expansion in the first cooling turbine, and

T_3 = Temperature of air bled off from the main compressor.

We know that for the expansion in the first cooling turbine (process 2–2′),

$$\frac{T_2}{T_{2'}} = \left(\frac{p_2}{p_{2'}}\right)^{\frac{\gamma_2 - 1}{\gamma_2}} = \left(\frac{1.1}{0.9}\right)^{\frac{1.35 - 1}{1.35}} = (1.22)^{0.26} = 1.053$$

∴ $T_{2'} = T_2 / 1.053 = 288 / 1.053 = 273.5 \text{ K}$

For the process 2–3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma_1-1}{\gamma_1}} = \left(\frac{3.5}{1.1}\right)^{\frac{1.5-1}{1.5}} = (3.18)^{0.333} = 1.47$$

∴ $T_3 = T_2 \times 1.47 = 288 \times 1.47 = 423.4 \text{ K}$

and for the process 4–5,

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma_2-1}{\gamma_2}} = \left(\frac{3.5}{1}\right)^{\frac{1.35-1}{1.35}} = (3.5)^{0.26} = 1.385$$

∴ $T_5 = T_4 / 1.385 = 333 / 1.385 = 240.4 \text{ K}$
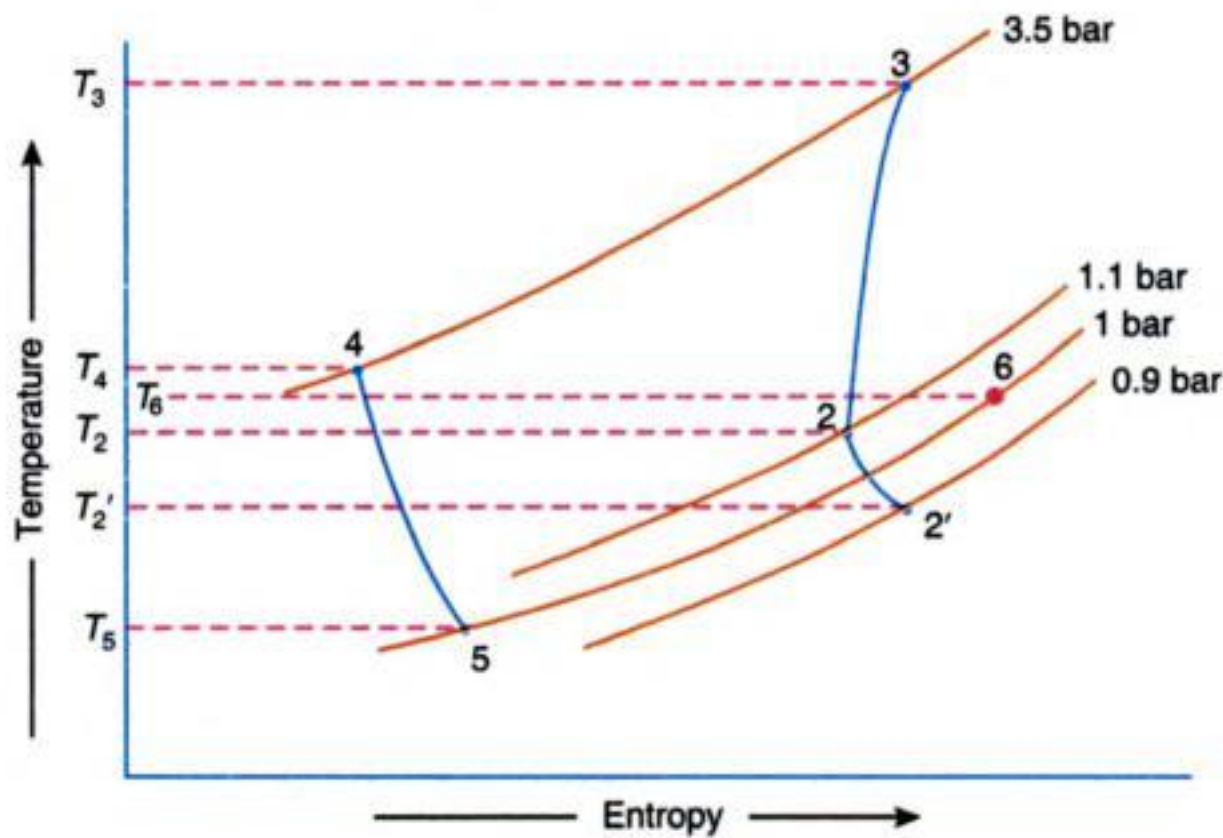
Fig. 3.21

1. Mass flow rate of cabin air

We know that mass flow rate of cabin air,

$$m_a = \frac{210\,Q}{c_p(T_6 - T_5)} = \frac{210 \times 10}{1\,(295 - 240.4)} = 38.5 \text{ kg / min}$$

$$... (c_p \text{ for air} = 1\text{kJ/kg K})$$

2. Cooling capacity of the heat exchanger and flow rate of ram air

We know that cooling capacity of the heat exchanger

$$= m_a c_p(T_3 - T_4) = 38.5 \times 1\,(423.4 - 333) \text{ kJ / min}$$
$$= 3480 \text{ kJ/min} \quad \textbf{Ans.}$$

In order to find the flow rate of ram air (m_R), equate the enthalpy lost by compressed air to the enthalpy gained by ram air.

We know that enthalpy lost by compressed air

$$= m_a c_p(T_3 - T_4) = 38.5 \times 1(423.4 - 333) \text{ kJ / min}$$
$$= 3480 \text{ kJ/min} \qquad ... (i)$$

Since it is given that the temperature rise for the ram air is not to exceed 30 K, therefore enthalpy gained by ram air

$$= m_R \times 1 \times 30 = 30\, m_R \text{ kJ/min} \qquad ... (ii)$$

Equating equations (i) and (ii),

$$m_R = 116 \text{ kg / min } \textbf{Ans.}$$

3. *Combined output of both cooling turbines*

We know that output of first cooling turbine

$$= m_a c_p (T_2 - T_2') = 116 \times 1 (288 - 273.5) = 1682 \text{ kJ/min}$$
$$= 1682 / 60 = 28 \text{ kW}$$

and output of second cooling turbine

$$= m_a c_p (T_4 - T_5) = 38.5 \times 1 (333 - 240.4) = 3565 \text{ kJ/min}$$
$$= 3565 / 60 = 59.4 \text{ kW}$$

Since the transmission efficiency is 60%, therefore combined output of both the cooling turbines

$$= (28 + 59.4) \, 0.6 = 52.44 \text{ kW} \textbf{ Ans.}$$

4. *C.O.P. of the refrigerating system*

We know that C.O.P. of the refrigerating system

$$= \frac{210 \, Q}{m_a c_p (T_3 - T_2)} = \frac{210 \times 10}{38.5 \times 1 (423.4 - 288)} = 0.4 \quad \textbf{Ans.}$$

3.8 Regenerative Air Cooling System
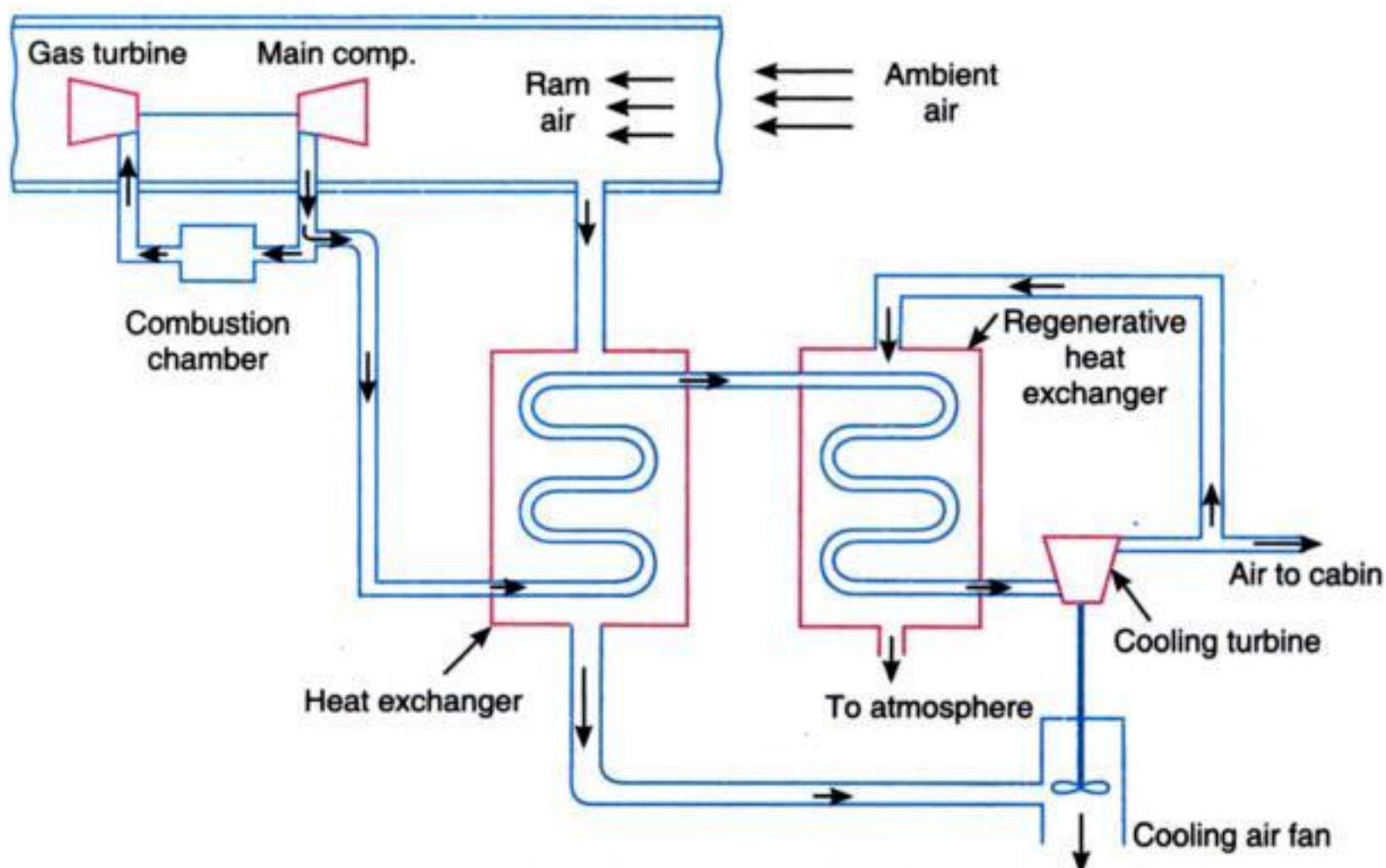
Fig. 3.22. Regenerative air cooling system.

The regenerative air cooling system is shown in Fig. 3.22. It is a modification of a simple air cooling system with the addition of a regenerative heat exchanger. The high pressure and high temperature air from the main compressor is first cooled by the ram air in the heat exchanger. This air is further cooled in the regenerative heat exchanger with a portion of the air bled after expansion in the cooling turbine. This type of cooling system is used for supersonic aircrafts and rockets.

The *T-s* diagram for the regenerative air cooling system is shown in Fig. 3.23. The various processes are as follows :

1. The process 1–2 represents isentropic ramming of air and process 1– 2′ represents actual ramming of air because of internal friction due to irreversibilities.
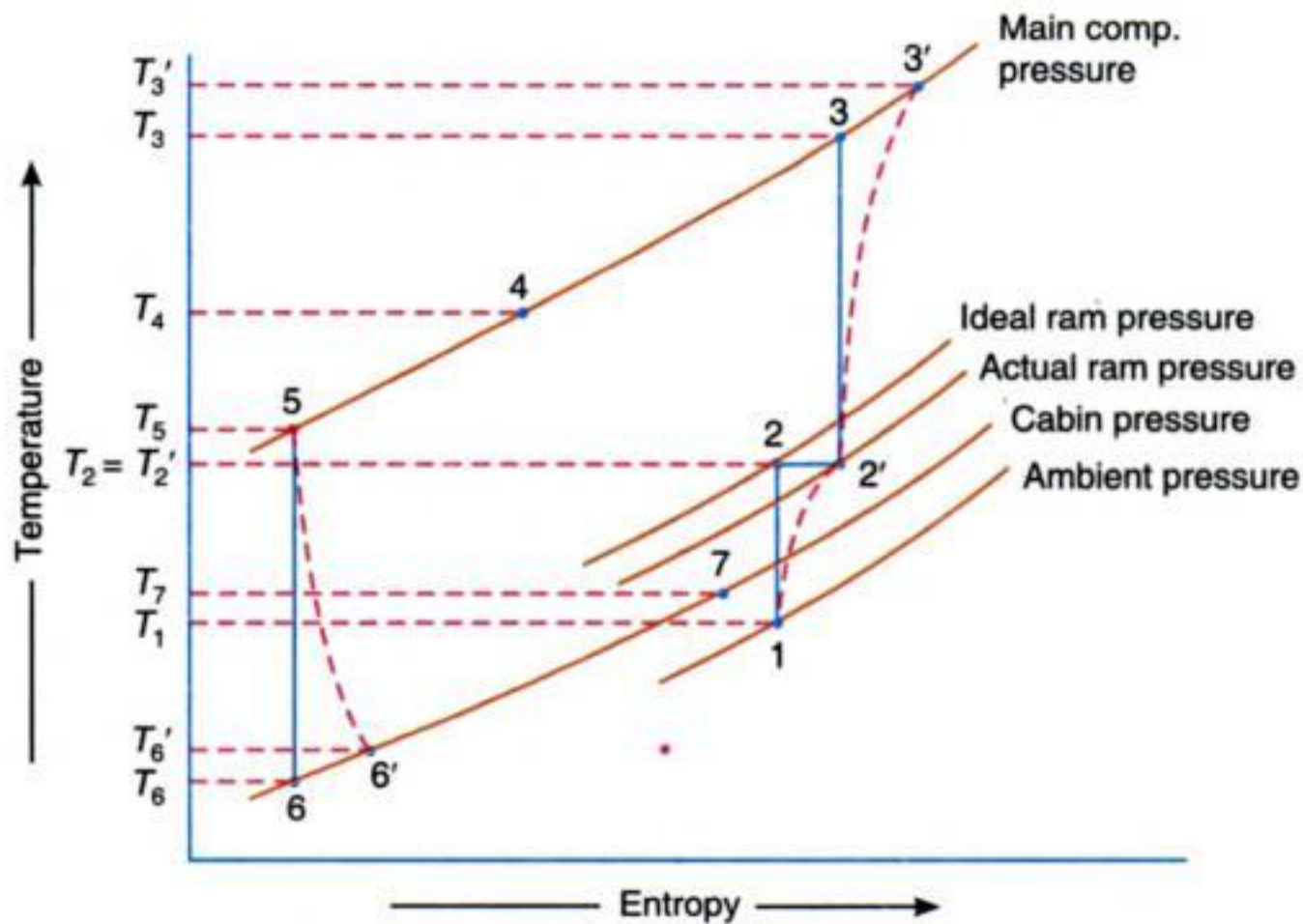
Fig. 3.23. *T-s* diagram for regenerative air cooling system.

2. The process 2′–3 represents isentropic compression of air in the main compressor and the process 2′–3′ represents actual compression of air because of internal friction due to irreversibilities.
3. The process 3′–4 represents cooling of compressed air by ram air in the heat exchanger.
4. The process 4–5 represents cooling of air in the regenerative heat exchanger.
5. The process 5–6 represents isentropic expansion of air in the cooling turbine upto the cabin pressure and the process 5–6′ represents actual expansion of air in the cooling turbine.
6. The process 6′–7 represents heating of air upto the cabin temperature T_7.

If Q tonnes of refrigeration is the cooling load in the cabin, then the quantity of air required for the refrigeration purpose will be

$$m_a = \frac{210\, Q}{c_p\, (T_7 - T_{6'})} \text{ kg / min}$$

Let m_1 = Total mass of air bled from the main compressor, and
  m_2 = Mass of cold air bled from the cooling turbine for regenerative heat exchanger.

For the energy balance of regenerative heat exchanger, we have

$$m_2\, c_p\, (T_8 - T_{6'}) = m_1 c_p\, (T_4 - T_5)$$

∴ $$m_2 = \frac{m_1 (T_4 - T_5)}{(T_8 - T_{6'})}$$

where T_8 = Temperature of air leaving to atmosphere from the regenerative heat exchanger.

Power required for the refrigeration system,

$$P = \frac{m_1 c_p\, (T_{3'} - T_{2'})}{60} \text{ kW}$$

and C.O.P. of the refrigerating system

$$= \frac{210\,Q}{m_1 c_p (T_{3'} - T_{2'})} = \frac{210\,Q}{P \times 60}$$

Example 3.11. *A regenerative air cooling system is used for an air plane to take 20 tonnes of refrigeration load. The ambient air at pressure 0.8 bar and temperature 10°C is rammed isentropically till the pressure rises to 1.2 bar. The air bled off the main compressor at 4.5 bar is cooled by the ram air in the heat exchanger whose effectiveness is 60%. The air from the heat exchanger is further cooled to 60°C in the regenerative heat exchanger with a portion of the air bled after expansion in the cooling turbine. The cabin is to be maintained at a temperature of 25°C and a pressure of 1 bar. If the isentropic efficiencies of the compressor and turbine are 90% and 80% respectively, find :*

1. Mass of the air bled from cooling turbine to be used for regenerative cooling ;

2. Power required for maintaining the cabin at the required condition ; and

3. C.O.P. of the system.

Assume the temperature of air leaving to atmosphere from the regenerative heat exchanger as 100°C.

Solution. Given : $Q = 20$ TR ; $p_1 = 0.8$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $p_2 = 1.2$ bar ; $p_3 = p_4 = p_5 = 4.5$ bar ; $\eta_H = 60\% = 0.6$; $T_5 = 60°C = 60 + 273 = 333$ K ; $T_7 = 25°C = 25 + 273 = 298$ K ; $p_7 = p_6 = p_{6'} = 1$ bar ; $\eta_C = 90\% = 0.9$; $\eta_T = 80\% = 0.8$; $T_8 = 100°C = 100 + 273 = 373$ K

The *T-s* diagram for the regenerative air cooling system with the given conditions is shown in Fig. 3.24.
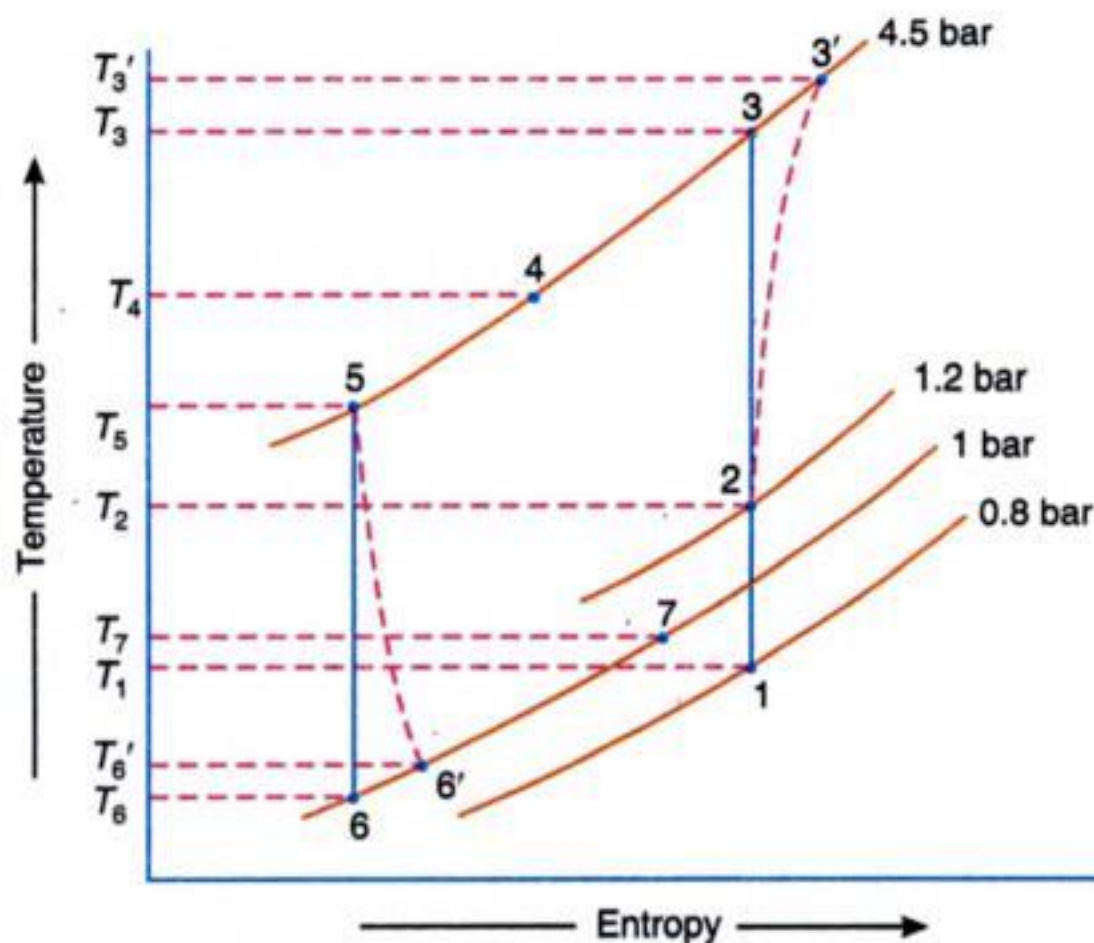
Fig. 3.24

Let T_2 = Temperature of air at the end of ramming and entering to the main compressor,

            T_3 = Temperature of air after isentropic compression in the main compressor, and

            $T_{3'}$ = Actual temperature of air leaving the main compressor.

We know that for the isentropic ramming of air (process 1–2),

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.2}{0.8}\right)^{\frac{1.4-1}{1.4}} = (1.5)^{0.286} = 1.123$$

∴ $T_2 = T_1 \times 1.123 = 283 \times 1.123 = 317.8$ K

and for the isentropic compression process 2–3,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.5}{1.2}\right)^{\frac{1.4-1}{1.4}} = (3.75)^{0.286} = 1.46$$

∴ $T_3 = T_2 \times 1.46 = 317.8 \times 1.46 = 464$ K

Isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic increase in temp.}}{\text{Actual increase in temp.}} = \frac{T_3 - T_2}{T_{3'} - T_2}$$

$$0.9 = \frac{464 - 317.8}{T_{3'} - 317.8} = \frac{146.2}{T_{3'} - 317.8}$$

∴ $T_{3'} = 317.8 + 146.2 / 0.9 = 480$ K

We know that effectiveness of the heat exchanger (η_H),

$$0.6 = \frac{T_{3'} - T_4}{T_{3'} - T_2} = \frac{480 - T_4}{480 - 317.8} = \frac{480 - T_4}{162.2}$$

∴ $T_4 = 480 - 0.6 \times 162.2 = 382.7$ K

Now for the isentropic cooling in the cooling turbine (process 5–6),

$$\frac{T_5}{T_6} = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.5}{1}\right)^{\frac{1.4-1}{1.4}} = (4.5)^{0.286} = 1.54$$

∴ $T_6 = T_5 / 1.54 = 333 / 1.54 = 216$ K

and isentropic efficiency of the cooling turbine,

$$\eta_T = \frac{\text{Actual increase in temp.}}{\text{Isentropic increase in temp.}} = \frac{T_5 - T_{6'}}{T_5 - T_6}$$

$$0.8 = \frac{333 - T_{6'}}{333 - 216} = \frac{333 - T_{6'}}{117}$$

∴ $T_{6'} = 333 - 0.8 \times 117 = 239.4$ K

1. Mass of air bled from the cooling turbine to be used for regenerative cooling

Let m_a = Mass of air bled from the cooling turbine to be used for regenerative cooling,

m_1 = Total mass of air bled from the main compressor, and

m_2 = Mass of cold air bled from the cooling turbine for regenerative heat exchanger.

We know that the mass of air supplied to the cabin,

$$m_a = m_1 - m_2$$

$$= \frac{210\,Q}{c_p(T_7 - T_{6'})} = \frac{210 \times 20}{1\,(298 - 239.4)} = 71.7 \text{ kg / min} \quad \dots (i)$$

and
$$m_2 = \frac{m_1(T_4 - T_5)}{(T_8 - T_{6'})} = \frac{m_1(382.7 - 333)}{(373 - 239.4)} = 0.372 \, m_1 \qquad \ldots (ii)$$

From equation (i), we find that
$$m_1 - m_2 = 71.7 \quad \text{or} \quad m_1 - 0.372 \, m_1 = 71.7$$

∴
$$m_1 = \frac{71.7}{1 - 0.372} = 113.4 \text{ kg / min}$$

and
$$m_2 = 0.372 \, m_1 = 0.372 \times 113.4 = 42.2 \text{ kg / min} \quad \textbf{Ans.}$$

Note: From equation (ii), $m_2 / m_1 = 0.372$. Therefore we can say that the air bled from the cooling turbine for regenerative cooling is 37.2% of the total air bled from the main compressor.

2. Power required for maintaining the cabin at the required condition

We know that the power required for maintaining the cabin at the required condition,
$$P = \frac{m_1 c_p (T_{3'} - T_2)}{60} = \frac{113.4 \times 1 (480 - 317.8)}{60} = 307 \text{ kW} \quad \textbf{Ans.}$$

3. C.O.P. of the system

We know that C.O.P. of the system
$$= \frac{210 \, Q}{m_1 c_p (T_{3'} - T_2)} = \frac{210 \times 20}{113.4 \times 1 (480 - 317.8)} = 0.23 \quad \textbf{Ans.}$$

3.9 Comparison of Various Air Cooling Systems used for Aircraft
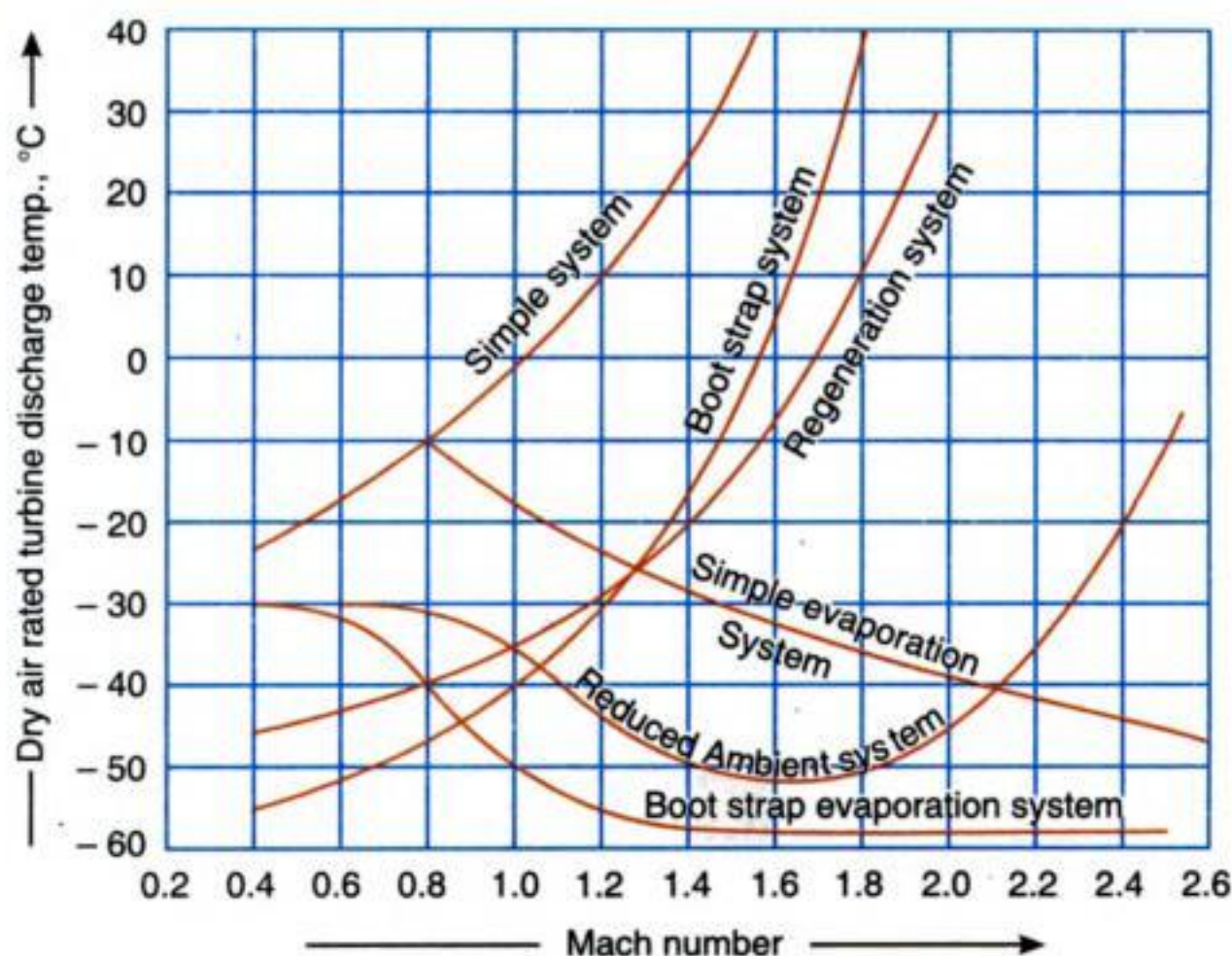
Fig. 3.25. Performance curves for various air cooling systems.

The performance curves for the various air cooling systems used for aircraft are shown in Fig. 3.25. These curves show the dry air rated turbine discharge temperature (DART) against the Mach number. From Fig. 3.25, we see that the simple air cooling system gives maximum cooling effect on the ground surface and decreases as the speed of aircraft increases. The boot strap system, on the other hand, requires the air plane to be in flight so that the ram air can be used for cooling in the heat exchangers. One method of overcoming this drawback of boot strap system is to use

part of work derived from turbine to drive a fan which pulls air over the secondary heat exchanger, thus combining the features of a simple and boot strap system. As the speed of aircraft increases, the temperature of ram cooling air rises and the ram air becomes less effective as a coolant in the heat exchanger. In such cases, a suitable evaporant is used with the ram air so that the cabin temperature does not rise. For high speed aircrafts, the boot strap evaporative or regenerative systems are used because they give lower turbine discharge temperature than the simple cooling system. In some cases, aeroplanes carry an auxiliary gas turbine for cabin pressurisation and air conditioning.

From Fig. 3.25, we see that the turbine discharge temperature of the air is variable. Therefore, in order to maintain the constant temperature of supply air to the cabin, it requires some control system.

EXERCISES

1. A simple air refrigeration system is used for an aircraft to take a load of 20 TR. The ambient pressure and temperature are 0.9 bar and 22°C respectively. The pressure of air is increased to 1 bar due to isentropic ramming action. The air is further compressed in a compressor to 3.5 bar and then cooled in a heat exchanger to 72°C. Finally, the air is passed through the cooling turbine and then it is supplied to the cabin at a pressure of 1.03 bar. The air leaves the cabin at a temperature of 25°C. Assuming the isentropic efficiencies of the compressor and turbine as 80 per cent and 75 percent respectively, find :

   1. Power required to take the load in the cooling cabin; and 2. C.O.P. of the system.

   Take c_p = 1.005 kJ / kg K ; and γ = 1.4.   [**Ans.** 390 kW ; 0.18]

2. The cockpit of a jet plane is to be cooled by a simple air refrigeration system. The data available is as follows :

Cock-pit cooling load	= 20 TR
Speed of the plane	= 1000 km / h
Ambient air pressure	= 0.35 bar
Ambient air temperature	= –15°C
Ram efficiency	= 90%
Pressure ratio in the main compressor	= 3
Pressure drop in the heat exchanger	= 0.1 bar
Isentropic efficiencies of main compressor and cooling turbine	= 80%
Temperature of air entering the cooling turbine	= 30°C
Pressure of air leaving the cooling turbine	= 1.06 bar
Pressure in the cock-pit	= 1 bar

   If the cock-pit is to be maintained at 25°C, find :

   1. Stagnation temperature and pressure of air entering the main compressor ;
   2. Mass flow rate of air to cock-pit ;
   3. Power required to drive he refrigerating system ; and
   4. C.O.P. of system.   [**Ans.** 210 kg / min ; 477 kW ; 0.147]

3. A boot-strap air refrigeration system of 20 TR capacity is used for an aeroplane flying at an altitude of 2000 m. The ambient air pressure and temperature are 0.8 bar and 0°C. The ram air pressure and temperature are 1.05 bar and 17°C. The pressure of air after isentropic compression in the main compressor is 4 bar. This air is now cooled to 27°C in another auxiliary heat exchanger and then expanded isentropically upto the cabin pressure of 1.01 bar. If the air leaves the cabin at 25°C and the efficiencies for the main compressor, auxiliary compressor and the cooling turbine are 80%; 75% and 80% respectively; find :1. Power required to operate the system ; and 2. C.O.P. of the system.

                    [**Ans.** 106.2 kW ; 0.66]

4. A boot strap air refrigeration system is used for an aeroplane to take 10 tonnes of refrigeration load. The ambient air conditions are 15°C and 0.9 bar. This air is rammed isentropically to a pressure of 1.1

bar. The pressure of the air bled off the main compressor is 3.5 bar and this is further compressed in secondary compressor to a pressure of 4.5 bar. The isentropic efficiency of both the compressors is 90% and that of cooling turbine is 85%. The effectiveness of both the heat exchangers is 0.6. If the cabin is to be maintained at 25°C and the pressure in the cabin is 1 bar, find : 1. mass of air passing through the cabin ; 2. power used for the refrigeration system ; and 3. C.O.P. of the system.

[**Ans.** 55.3 kg / min ; 125 kW ; 0.28]

5. The following data refer to a reduced ambient refrigeration system :

Ambient pressure	= 0.8 bar
Pressure of ram air	= 1.1 bar
Temperature of ram air	= 20°C
Pressure at the end of main compressor	= 3.3 bar
Efficiency of main compressor	= 80%
Heat exchanger effectiveness	= 80%
Pressure at the exit of the auxiliary turbine	= 0.8 bar
Efficiency of auxiliary turbine	= 85%
Temperature of air leaving the cabin	= 25°C
Pressure in the cabin	= 1.013 bar
Flow rate of air through cabin	= 60 kg / min

Find : 1.Capacity of the cooling system required ; 2. Power needed to operate the system ; and 3. C.O.P. of the system. [**Ans.** 19.5 TR ; 136 kW ; 0.504]

6. The following data refers to a reduced ambient air refrigeration system used for an aircraft :

Speed of aircraft	= 1500 km / h
Ambient pressure	= 0.8 bar
Ambient temperature	= 5°C
Ram efficiency	= 100 %
Pressure of cooled air leaving the first cooling turbine	= 0.8 bar
Temperature of cooled air leaving the heat exchanger	= 100°C
Pressure ratio of the main compressor	= 3
Pressure loss between the outlet of second cooling turbine and the cabin	= 0.1 bar
Pressure in the cabin	= 1 bar
Temperature in the cabin	= 22°C
Load in the cabin	= 10 TR
Isentropic efficiency of compressor	= 85%
Isentropic efficiency of both cooling turbines	= 80%

Find: 1. mass flow of the air passing through the second cooling turbine ; 2. quantity of ram air passing through the heat exchanger, if the rise in temperature is limited to 80 K ; and 3. C.O.P. of the system. [**Ans.** 54 kg / min ; 79.5 kg / min ; 0.26]

7. A regenerative air refrigeration system for an aeroplane is designed to take a load of 30 TR. The temperature and pressure conditions of the atmosphere are 5°C and 0.85 bar. The pressure of the air is increased from 0.85 bar to 1.2 bar due to ramming action. The pressure of air leaving the main compressor is 4.8 bar. 60% of the total heat of the air leaving the main compressor is removed in the heat exchanger and then it is passed through the cooling turbine. The temperature of the rammed air which is used for cooling purposes in the heat exchanger is reduced to 50°C by mixing the air coming out from the cooling turbine. The isentropic efficiencies of the compressor and turbine are 90% and 80% respectively. The pressure and temperature required in the cabin are 1 bar and 25°C respectively. Assuming isentropic ramming and mass of cooled air passing through the heat exchanger equal to the mass of cooling air, find : 1. the ratio of by-passed air to ram air used for cooling purposes ; and 2. the power required for maintaining the cabin at required condition. [**Ans.** 41.2% ; 445 kW]

QUESTIONS

1. Explain the working of a simple air cycle cooling system used for aircrafts.
2. List the names of three evaporative coolants that can be used in aircraft refrigeration system at an altitude of 15 000 metres. How will you estimate the amount of the coolant required for a given flight of aircraft ?
3. Describe with a sketch a boot-strap cycle of air refrigeration system.
4. Describe with a diagram the reduced ambient air cooling system.
5. Describe with a sketch a regenerative air cooling system.
6. Compare the various air cooling systems used for aircraft.

OBJECTIVE TYPE QUESTIONS

1. An ordinary passenger aircraft requires a cooling system of capacity
   (a) 2 TR   (b) 4 TR   (c) 8 TR   (d) 10 TR
2. A jet fighter travelling at 950 km / h needs a cooling system of capacity
   (a) 2 to 4 TR   (b) 4 to 8 TR   (c) 8 to 10 TR (d) 10 to 20 TR
3. The simple air cooling system is good for ....flight speeds.
   (a) low   (b) high
4. The water, alcohol and ammonia have .... refrigerating effects at different altitudes.
   (a) same   (b) different
5. A boot-strap air cooling system has
   (a) one heat exchanger   (b) two heat exchangers
   (c) three heat exchangers   (d) four heat exchangers
6. The air cooling system mostly used in transport type aircrafts is
   (a) simple air cooling system   (b) simple evaporative air cooling system
   (c) boot-strap air cooling system (d) all of these
7. In a boot strap air evaporative cooling system, the evaporator is provided
   (a) between the combustion chamber and the first heat exchanger
   (b) between the first heat exchanger and the secondary compressor
   (c) between the secondary compressor and the second heat exchanger
   (d) between the second heat exchanger and the cooling turbine
8. The reduced ambient air cooling system has
   (a) one cooling turbine and one heat exchanger
   (b) one cooling turbine and two heat exchangers
   (c) two cooling turbines and one heat exchanger
   (d) two cooling turbines and two heat exchangers
9. The reduced ambient air cooling system is used for very................. speed aircrafts.
   (a) low   (b) high
10. The cooling system used for supersonic aircrafts and rockets is
    (a) simple air cooling system   (b) boot-strap air cooling system
    (c) reduced ambient air cooling system (d) regenerative air cooling system

ANSWERS

1. (c)	2. (d)	3. (a)	4. (b)	5. (b)
6. (c)	7. (d)	8. (c)	9. (b)	10. (d)

4

Simple Vapour Compression Refrigeration Systems

4.1 Introduction

A vapour compression refrigeration system* is an improved type of air refrigeration system in which a suitable working substance, termed as refrigerant, is used. It condenses and evaporates at temperatures and pressures close to the atmospheric conditions. The refrigerants, usually, used for this purpose are ammonia (NH_3), carbon dioxide (CO_2) and sulphur dioxide (SO_2). The refrigerant used, does not

---

* Since low pressure vapour refrigerant from the evaporator is changed into high pressure vapour refrigerant in the compressor, therefore it is named as vapour compression refrigeration system.

In the following pages, we shall draw the *p-h* chart along with the *T-s* diagram of the cycle.
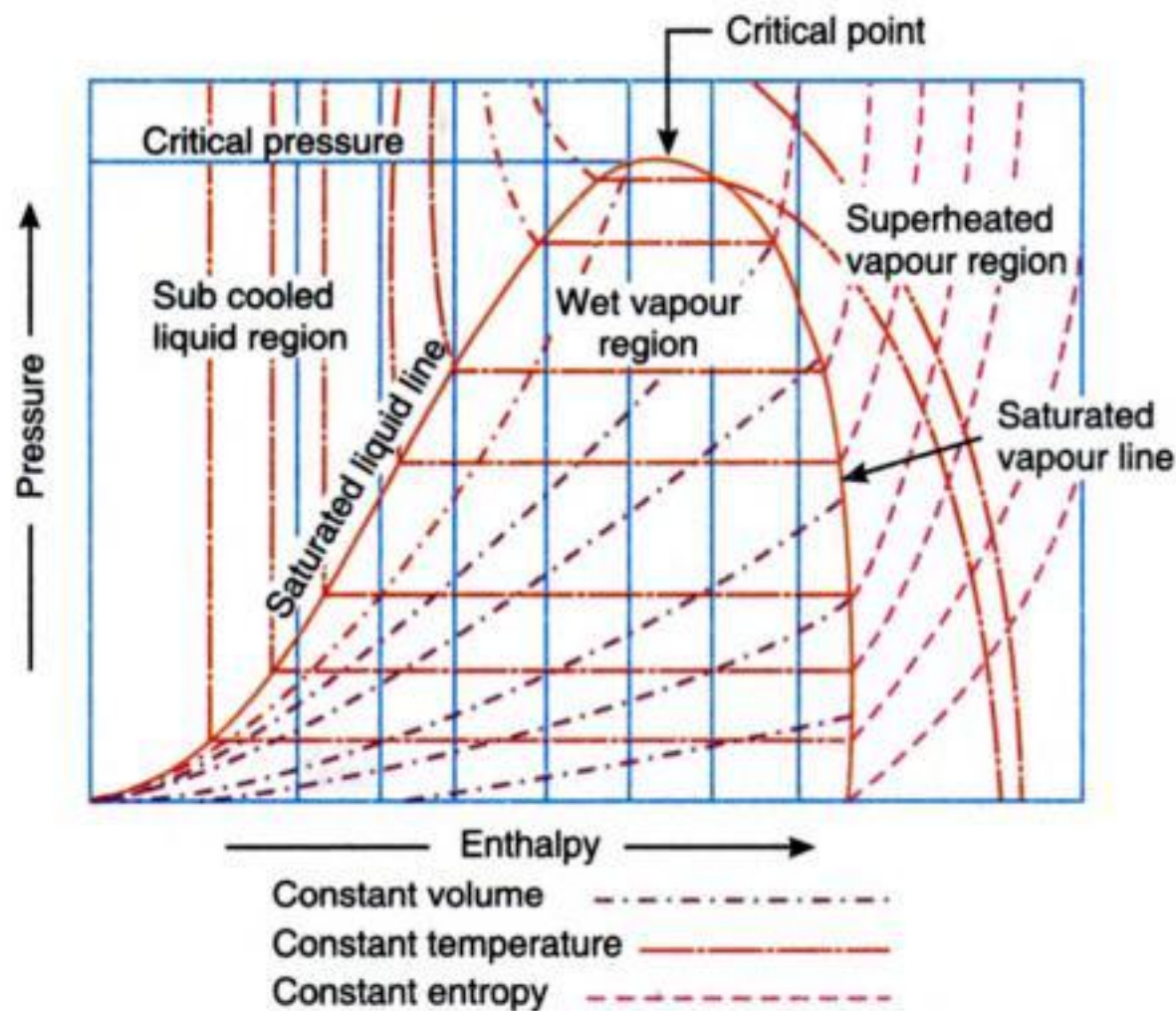
Fig. 4.2. Pressure - enthalpy (*p-h*) chart.

4.5   Types of Vapour Compression Cycles

We have already discussed that vapour compression cycle essentially consists of compression, condensation, throttling and evaporation. Many scientists have focussed their attention to increase the coefficient of performance of the cycle. Though there are many cycles, yet the following are important from the subject point of view :

1. Cycle with dry saturated vapour after compression,
2. Cycle with wet vapour after compression,
3. Cycle with superheated vapour after compression,
4. Cycle with superheated vapour before compression, and
5. Cycle with undercooling or subcooling of refrigerant.

Now we shall discuss all the above mentioned cycles, one by one, in the following pages.

4.6   Theoretical Vapour Compression Cycle with Dry Saturated Vapour after Compression

A vapour compression cycle with dry saturated vapour after compression is shown on *T-s* and *p-h* diagrams in Fig. 4.3 (*a*) and (*b*) respectively. At point 1, let T_1, p_1 and s_1, be the temperature, pressure and entropy of the vapour refrigerant respectively. The four processes of the cycle are as follows :

1. *Compression process*. The vapour refrigerant at low pressure p_1 and temperature T_1 is compressed isentropically to dry saturated vapour as shown by the vertical line 1-2 on *T-s* diagram and by the curve 1-2 on *p-h* diagram. The pressure and temperature rises from p_1 to p_2 and T_1 to T_2 respectively.

The work done during isentropic compression per kg of refrigerant is given by

$$w = h_2 - h_1$$

Solution. Given : $p_2 = p_3 = 60$ bar ; $p_1 = p_4 = 25$ bar ; $T_2 = T_3 = 295$ K ; $T_1 = T_4 = 261$ K ; $h_{f3} = h_4 = 151.96$ kJ/kg ; $h_{f1} = 56.32$ kJ/kg ; $h_{g2} = h_2 = 293.29$ kJ/kg ; $h_{g1} = 322.58$ kJ/kg ; *$s_{f2} = 0.554$ kJ/kg K ; $s_{f1} = 0.226$ kJ/kg K ; $s_{g2} = 1.0332$ kJ/kg K ; $s_{g1} = 1.2464$ kJ/kg K

1. C.O.P. of the cycle

The *T-s* and *p-h* diagrams are shown in Fig. 4.5 (a) and (b) respectively.

(a) *T-s* diagram.	(b) *p-h* diagram.
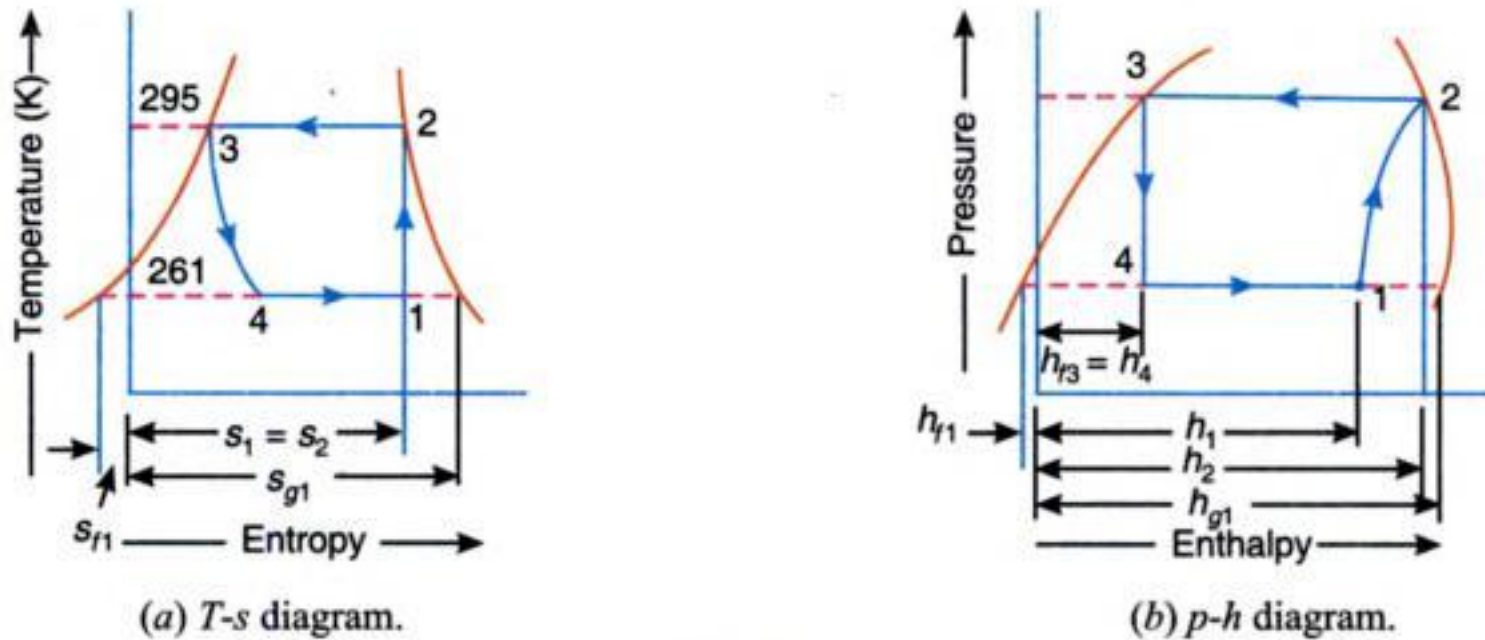
Fig. 4.5

Let x_1 = Dryness fraction of the vapour refrigerant entering the compressor at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1\, s_{fg1} = s_{f1} + x_1(s_{g1} - s_{f1}) \quad \ldots (\because s_{g1} = s_{f1} + s_{fg1})$$
$$= 0.226 + x_1 (1.2464 - 0.226) = 0.226 + 1.0204\, x_1 \quad \ldots (i)$$

and entropy at point 2, $\qquad s_2 = s_{g2} = 1.0332$ kJ/kg K $\qquad$ ... (Given) ... (ii)

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (i) and (ii),

$$0.226 + 1.0204\, x_1 = 1.0332 \qquad \text{or} \qquad x_1 = 0.791$$

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1\, h_{fg1} = h_{f1} + x_1 (h_{g1} - h_{f1}) \ldots (\because h_{g1} = h_{f1} + h_{fg1})$$
$$= 56.32 + 0.791 (322.58 - 56.32) = 266.93 \text{ kJ/kg}$$

∴ C.O.P. of the cycle

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{266.93 - 151.96}{293.29 - 266.93} = 4.36 \text{ Ans.}$$

2. Capacity of the refrigerator

We know that the heat extracted or refrigerating effect produced per kg of refrigerant

$$= h_1 - h_{f3} = 266.93 - 151.96 = 114.97 \text{ kJ/kg}$$

Since the fluid flow is at the rate of 5 kg / min, therefore total heat extracted

$$= 5 \times 114.97 = 574.85 \text{ kJ/min}$$

∴ Capacity of the refrigerator

$$= \frac{574.85}{210} = 2.74 \text{ TR Ans.} \qquad \ldots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

---

* **Superfluous data**

4.8 Theoretical Vapour Compression Cycle with Superheated Vapour after Compression

(a) T-s diagram.

(b) p-h diagram.

Fig. 4.9. Theoretical vapour compression cycle with superheated vapour after compression.

A vapour compression cycle with superheated vapour after compression is shown on T-s and p-h diagrams in Fig. 4.9 (a) and (b) respectively. In this cycle, the enthalpy at point 2 is found out with the help of degree of superheat. The degree of superheat may be found out by equating the entropies at points 1 and 2.

Now the coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

A little consideration will show that the superheating increases the refrigerating effect and the amount of work done in the compressor. Since the increase in refrigerating effect is less as compared to the increase in work done, therefore, the net effect of superheating is to have low coefficient of performance.

Note : In this cycle, the cooling of superheated vapour will take place in two stages. Firstly, it will be condensed to dry saturated stage at constant pressure (shown by graph 2-2′) and secondly, it will be condensed at constant temperature (shown by graph 2′-3). The remaining cycle is same as discussed in the last article.

Example 4.6. *A vapour compression refrigerator uses methyl chloride (R-40) and operates between temperature limits of − 10°C and 45°C. At entry to the compressor, the refrigerant is dry saturated and after compression it acquires a temperature of 60°C. Find the C.O.P. of the refrigerator. The relevant properties of methyl chloride are as follows :*

Saturation	Enthalpy in kJ/kg		Entropy in kJ/kg K	
temperature in °C	Liquid	Vapour	Liquid	Vapour
−10	45.4	460.7	0.183	1.637
45	133.0	483.6	0.485	1.587

Solution. Given : $T_1 = T_4 = - 10°C = - 10 + 273 = 263$ K ; $T_2' = T_3 = 45°C = 45 + 273 = 318$ K ; $T_2 = 60°C = 60 + 273 = 333$ K ; $*h_{f1} = 45.4$ kJ/kg ; $h_{f3} = 133$ kJ/kg ; $h_1 = 460.7$ kJ/kg ; $h_{2'} = 483.6$ kJ/kg ; $*s_{f1} = 0.183$ kJ/kg K ; $*s_{f3} = 0.485$ kJ/kg K ; $s_1 = s_2 = 1.637$ kJ/kg K ; $s_{2'} = 1.587$ kJ/kg K

---

*   Superfluous data.

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$h_{2'} = 198.10$ kJ/kg ; $h_1 = 188.39$ kJ/kg ; $*s_{f3} = 0.2270$ kJ/kg K ; $*s_{f1} = 0.1487$ kJ/kg K ;
$*s_{2'} = 0.6865$ kJ/kg K ; $s_1 = s_2 = 0.6956$ kJ/kg K ; $*c_{p3} = 0.996$ kJ/kg K ; $*c_{p4} = 1.067$ kJ/kg K ;
$*c_{p2'} = 0.674$ kJ/kg K ; $c_{p1} = 0.620$ kJ/kg K ; $*v_{2'} = 0.026$ m^3 / kg ; $v_1 = 0.052$ m^3 / kg

1. Volumetric displacement of the compressor

The T-s and p-h diagrams are shown in Fig. 4.12 (a) and (b) respectively. Since 20% of the useful heat is lost into water cooler, therefore actual heat extracted from the water cooler,

$$h_E = 1.2\, m_w \times c_w\, (T_{w1} - T_{w2})$$
$$= 1.2 \times 100 \times 4.187\, (299 - 279) = 10\,050 \text{ kJ/h} = 167.5 \text{ kJ/min}$$

$$\dots (\because \text{ Sp. heat of water, } c_w = 4.187 \text{ kJ/kg K})$$

We know that heat extracted or the net refrigerating effect per kg of the refrigerant

$$= h_1 - h_{f3} = 188.39 - 60.64 = 127.75 \text{ kJ/kg}$$

$\therefore$ Mass flow of the refrigerant,

$$m_R = \frac{167.5}{127.75} = 1.3 \text{ kg/min}$$

and volumetric displacement of the compressor

$$= \frac{m_R \times v_1}{\eta_v} = \frac{1.3 \times 0.052}{0.80} = 0.085 \text{ m}^3\text{/min Ans.}$$

(a) T-s diagram.

(b) p-h diagram.
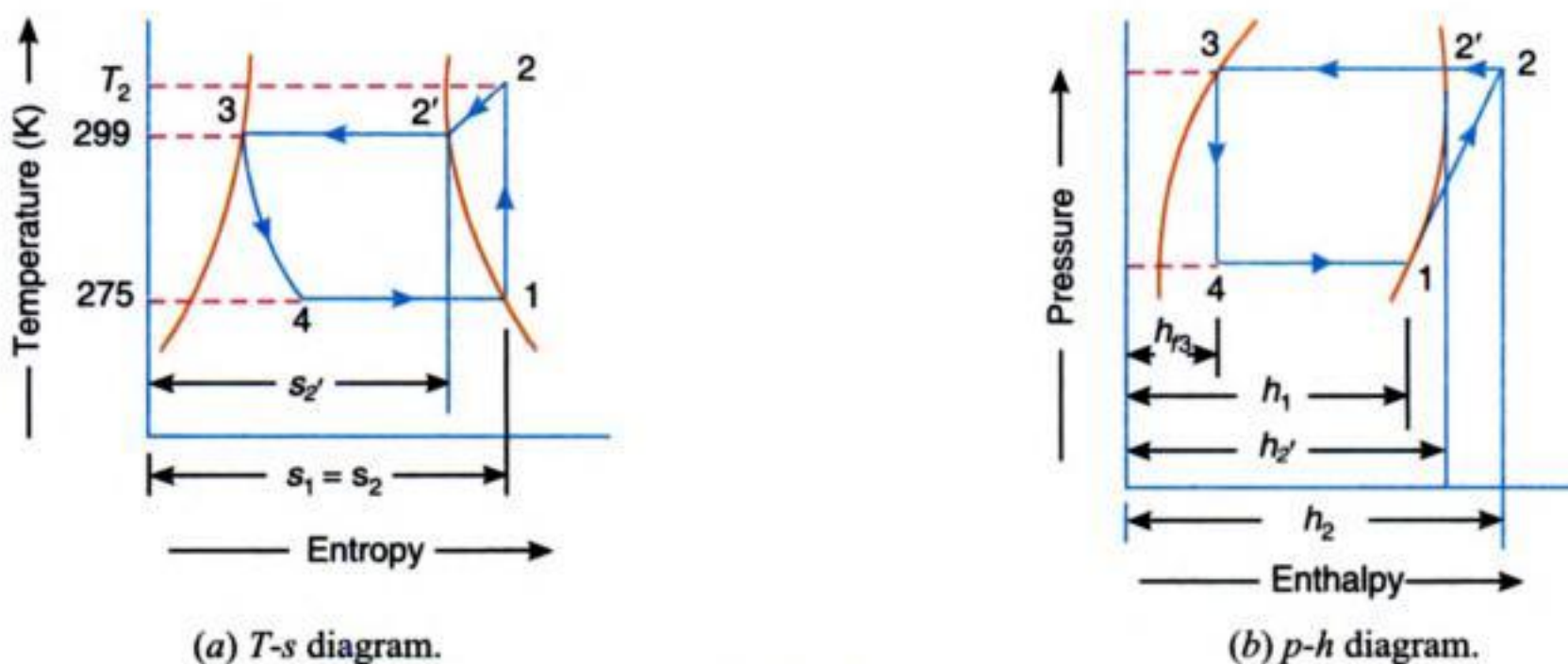
Fig. 4.12

2. Power of the motor

First of all, let us find the temperature at point 2 (T_2). We know that entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{p2'}\, \log\left(\frac{T_2}{T_{2'}}\right)$$

$$0.6956 = 0.6865 + 2.3 \times 0.674\, \log\left(\frac{T_2}{299}\right)$$

$$\log\left(\frac{T_2}{299}\right) = \frac{0.6956 - 0.6865}{2.3 \times 0.674} = 0.005\,87$$

or

$$\frac{T_2}{299} = 1.0136 \qquad \dots \text{(Taking anti-log of 0.005 87)}$$

$\therefore$

$$T_2 = 299 \times 1.0136 = 303 \text{ K}$$

---

* Superfluous data.

and entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{pv} \log\left(\frac{T_2}{T_{2'}}\right)$$

$$= 0.6921 + 2.3 \times 0.64\ \log\left(\frac{288}{283}\right)$$

$$= 0.6921 + 2.3 \times 0.64 \times 0.0077 = 0.7034 \qquad \ldots (ii)$$

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (i) and (ii),

$$0.0904 + 0.6147\, x_1 = 0.7034 \qquad \text{or} \qquad x_1 = 0.997$$
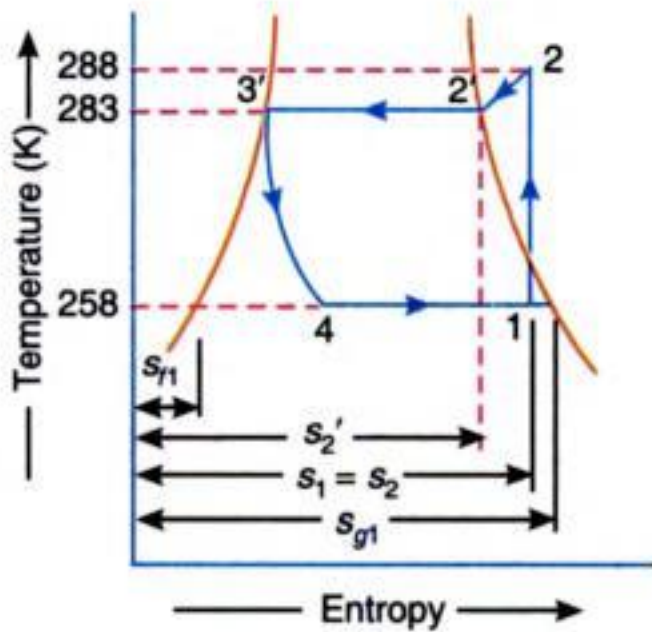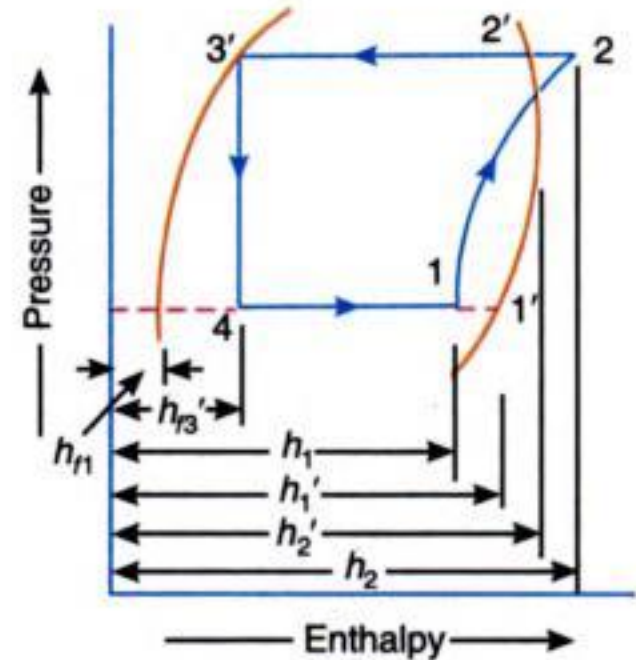
(a) T-s diagram. (b) p-h diagram.

Fig. 4.16

We know that the enthalpy at point 1,

$$h_1 = h_{f1} + x_1\, h_{fg1} = h_{f1} + x_1\,(h_{g1} - h_{f1})$$
$$= 22.3 + 0.997\,(180.88 - 22.3) = 180.4\ \text{kJ/kg}$$

$$\ldots (\because h_{g1} = h_{1'})$$

and enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv}\,(T_2 - T_{2'})$$
$$= 191.76 + 0.64\,(288 - 283) = 194.96\ \text{kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3'}}{h_2 - h_1} = \frac{180.4 - 45.4}{194.96 - 180.4} = 9.27 \ \textbf{Ans.}$$

(ii) Coefficient of performance when there is an undercooling of 5°C

The T-s and p-h diagrams, when there is an undercooling of 5°C, are shown in Fig. 4.17 (a) and (b) respectively.
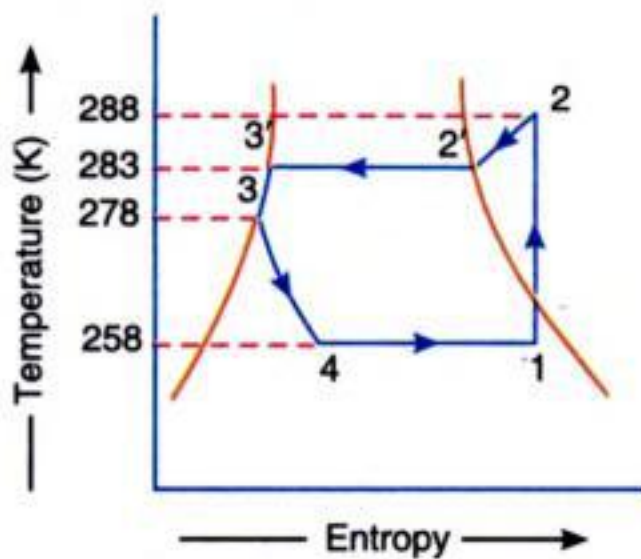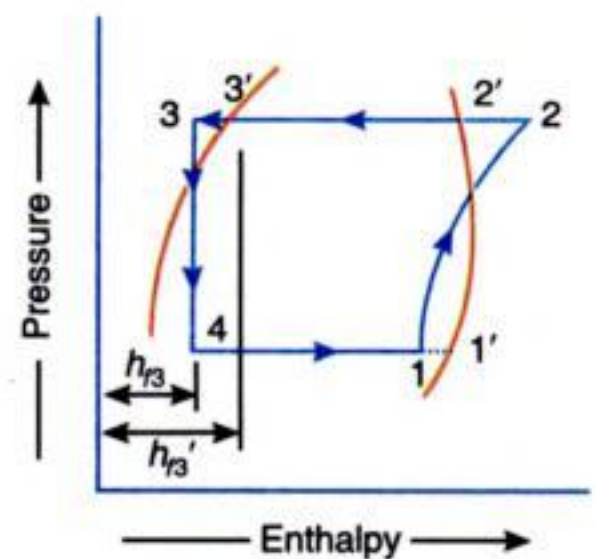
(a) T-s diagram. (b) T-s diagram.

Fig. 4.17

and heat taken by water in the condenser

$$= m_w \times c_{pw} \times \text{Rise in temperature}$$
$$= m_w \times 4.187 \times 12 = 50.244 \, m_w \qquad \qquad \dots (iv)$$

Equating equations (*iii*) and (*iv*),

$$m_w = 252.7/50.244 = 5.03 \text{ kg / min } \textbf{Ans.}$$

Example 4.13. *A commercial refrigerator operates with R-12 between 1.2368 bar and 13.672 bar. The vapour is dry and saturated at the compressor inlet. Assuming isentropic compression, determine the theoretical C.O.P. of the plant. The isentropic discharge temperature is 64.86°C. If the actual C.O.P. of the plant is 80% of the theoretical, calculate the power required to run the compressor to obtain a refrigerating capacity of 1 TR. If the liquid is sub-cooled through 10°C after condensation, calculate the power required. The properties of R-12 are given below :*

Saturation temp. (°C)	Saturation pressure (bar)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
		Liquid	*Vapour*	*Liquid*	*Vapour*
–25	1.2368	13.33	176.48	0.0552	0.7126
55	13.672	90.28	207.95	0.3197	0.6774

Properties of superheated R-12

Temperature (°C)	Pressure (bar)	Enthalpy (kJ/kg)	Entropy (kJ/kg K)
64.86	13.672	220.6	0.7126

Assume specific heat of liquid to be 1.055 kJ/kg K.

Solution. Given : $p_1 = p_4 = 1.2368$ bar ; $p_2 = p_3 = 13.672$ bar ; $T_2 = 64.86°C = 64.86 + 273 = 337.86$ K ; $(C.O.P.)_{actual} = 80\% \, (C.O.P)_{th}$; $Q = 1TR$; $T_1 = -25°C = -25 + 273 = 248$ K ; $T_{2'} = 55°C = 55 + 273 = 328$ K ; *$h_{f1} = 13.33$ kJ/kg ; $h_1 = 176.48$ kJ/kg ; *$s_{f1} = 0.0552$ kJ/kg K ; *$s_1 = s_2 = 0.7126$ kJ/kg K ; $h_{f3'} = 90.28$ kJ/kg ; *$h_{2'} = 207.95$ kJ/kg ; *$s_{f3} = 0.3197$ kJ/kg K ; *$s_{2'} = 0.6774$ kJ/kg K ; $h_2 = 220.6$ kJ/kg ; $c_{pl} = 1.055$ kJ/kg K

(a) T-s diagram.
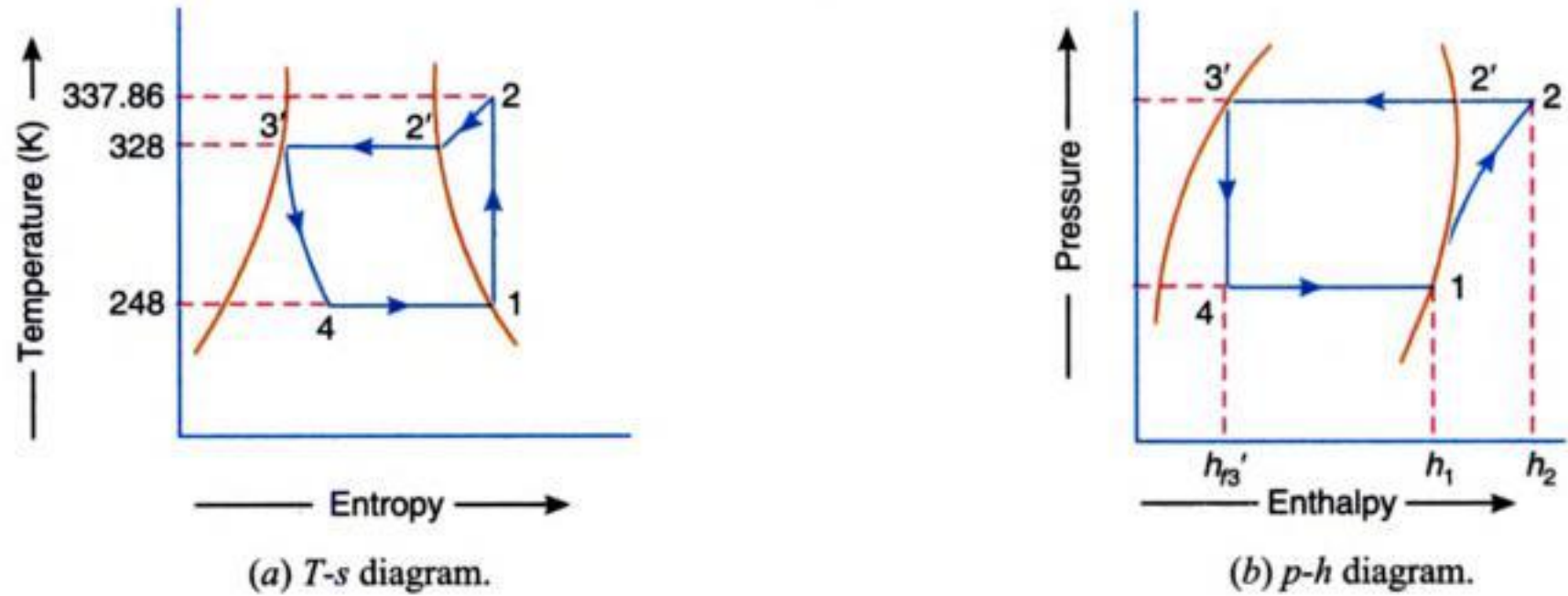
(b) p-h diagram.

Fig. 4.20

The *T-s* and *p-h* diagrams are shown in Fig. 4.20 (*a*) and (*b*) respectively.

---
*   Superfluous data.

Solution. Given : $Q = 15$ TR ; $T_{2'} = T_{3'} = 25°C = 25 + 273 = 298$ K ; $T_1 = T_4 = -10°C$ $= -10 + 273 = 263$ K ; $T_3 = 25 - 5 = 20°C = 20 + 273 = 293$ K ; $x_1 = 0.97$; $h_{f3'} = 298.9$ kJ/kg ; $h_{2'} = 1465.84$ kJ/kg ; $^*s_{f3'} = 1.1242$ kJ/kg K ; $s_{g2'} = s_{2'} = 5.0391$ kJ/kg K ; $c_{pl} = 4.6$ kJ/kg K ; $c_{pv} = 2.8$ kJ/kg K ; $h_{f1} = 135.37$ kJ/kg ; $h_{1'} = 1433.05$ kJ/kg ; $s_{f1} = 0.5443$ kJ/kg ; $s_{g1} = 5.4770$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.23 (*a*) and (*b*) respectively.

(a) *T-s* diagram.

(b) *p-h* diagram.
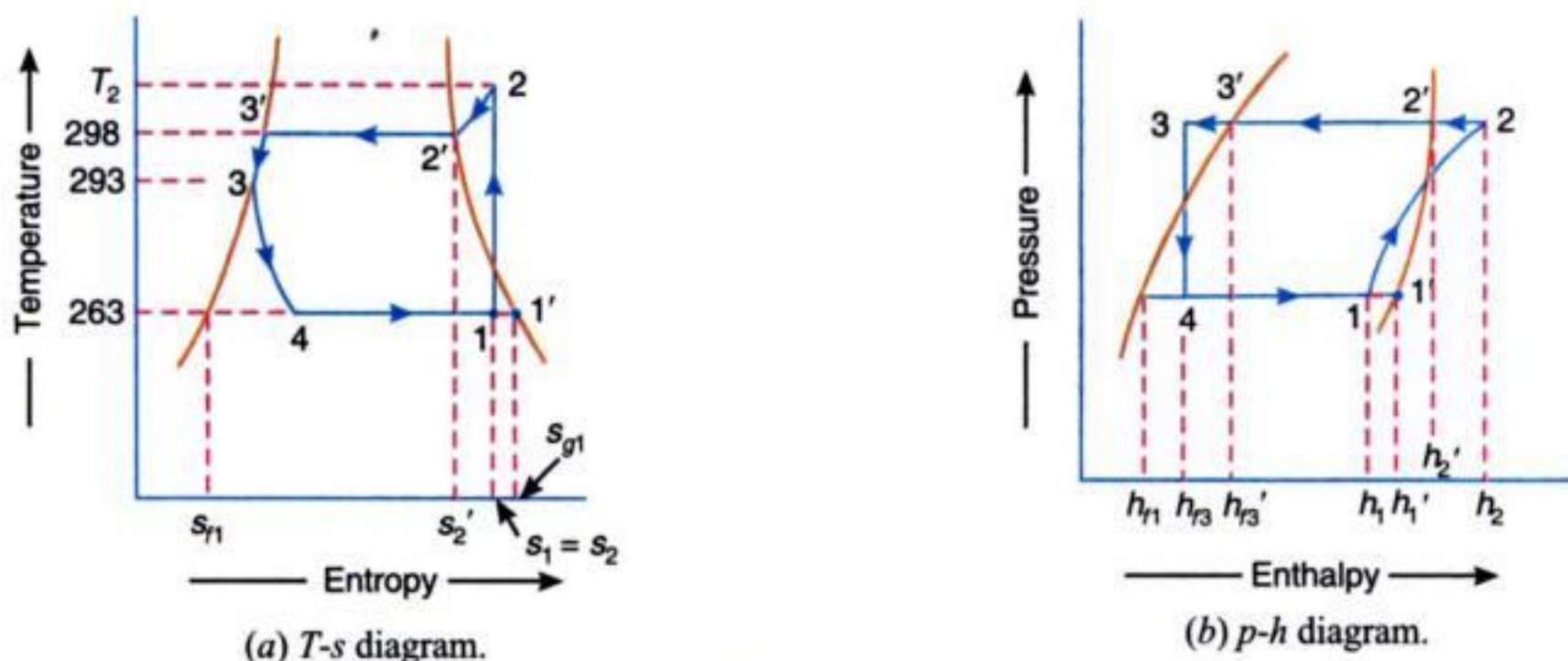
Fig. 4.23

First of all, let us find the temperature of refrigerant at point 2 (T_2).

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1 s_{fg1} = s_{f1} + x_1 (s_{g1} - s_{f1}) \qquad ...(\because s_{g1} = s_{f1} + s_{fg1})$$
$$= 0.5443 + 0.97 (5.4770 - 0.5443) = 5.329 \text{ kJ/kg K} \quad ....(i)$$

and entropy at point 2,

$$s_2 = s_{2'} + 2.3 c_{pv} \log\left(\frac{T_2}{T_{2'}}\right) = 5.0391 + 2.3 \times 2.8 \log\left(\frac{T_2}{298}\right)$$
$$= 5.0391 + 6.44 \log\left(\frac{T_2}{298}\right) \qquad ... (ii)$$

Since entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$5.329 = 5.0391 + 6.44 \log\left(\frac{T_2}{298}\right)$$

$$\log\left(\frac{T_2}{298}\right) = \frac{5.329 - 5.0391}{6.44} = 0.045$$

or

$$\frac{T_2}{298} = 1.109 \qquad ... \text{(Taking antilog of 0.045)}$$

$$\therefore \qquad T_2 = 298 \times 1.109 = 330 \text{ K}$$

1. Coefficient of performance

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1 h_{fg1} = h_{f1} + x_1 (h_{g1} - h_{f1}) \qquad ... (\because h_{g1} = h_{f1} + h_{fg1})$$
$$= 135.37 + 0.97 (1433.05 - 135.37) = 1394.12 \text{ kJ/kg}$$
$$... (\because h_{g1} = h_{1'})$$

* Superfluous data.

Solution. Given : $Q = 12$ TR ; $T_1' = -28°C = -28 + 273 = 245$ K ; $T_2' = T_3' = 26°C$ $= 26 + 273 = 299$ K ; $T_3' - T_3 = 4°C$ or $T_3 = 22°C = 22 + 273 = 295$ K ; $T_1 - T_1' = 5°C$ or $T_1 = -23°C = -23 + 273 = 250$ K ; $L = 1.25D$; Clearance volume = 3% Stroke volume ; $N = 1000$ r.p.m. ; $v_1' = 0.1475$ m³/kg ; $v_2' = 0.0262$ m³/kg ; $h_{f1} = 10.64$ kJ/kg ; $h_{f3'} = 60.67$ kJ/kg ; $h_1' = 175.11$ kJ/kg ; $h_2' = 198.11$ kJ/kg ; $s_{f1} = 0.0444$ kJ/kg K ; $s_{f3} = 0.2271$ kJ/kg K ; $s_1' = 0.7153$ kJ/kg K ; $s_2' = 0.6865$ kJ/kg K ; $c_{pl} = 0.963$ kJ/kg K ; $c_{pv} = 0.615$ kJ/kg K

The T-s and p-h diagrams are shown in Fig. 4.25 (a) and (b) respectively.

1. Theoretical power required

First of all, let us find the temperature of superheated vapour at point 2 (T_2).

We know that entropy at point 1,

$$s_1 = s_1' + 2.3\, c_{pv}\, \log\left(\frac{T_1}{T_1'}\right)$$

$$= 0.7153 + 2.3 \times 0.615\ \log\left(\frac{250}{245}\right) = 0.7277 \qquad \dots (i)$$

and entropy at point 2, $\quad s_2 = s_2' + 2.3\, c_{pv}\, \log\left(\frac{T_2}{T_2'}\right) = 0.6865 + 2.3 \times 0.615\ \log\left(\frac{T_2}{299}\right)$

$$= 0.6865 + 1.4145\ \log\left(\frac{T_2}{299}\right) \qquad \dots (ii)$$

(a) T-s diagram.
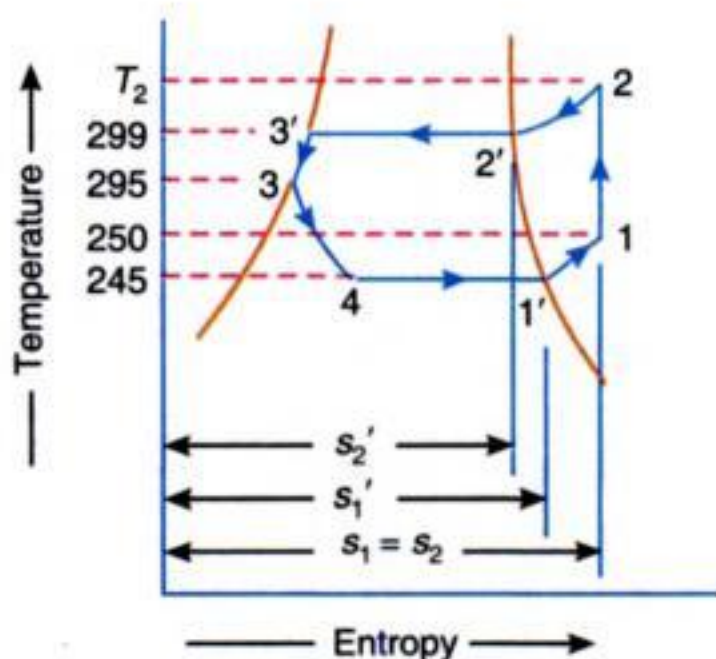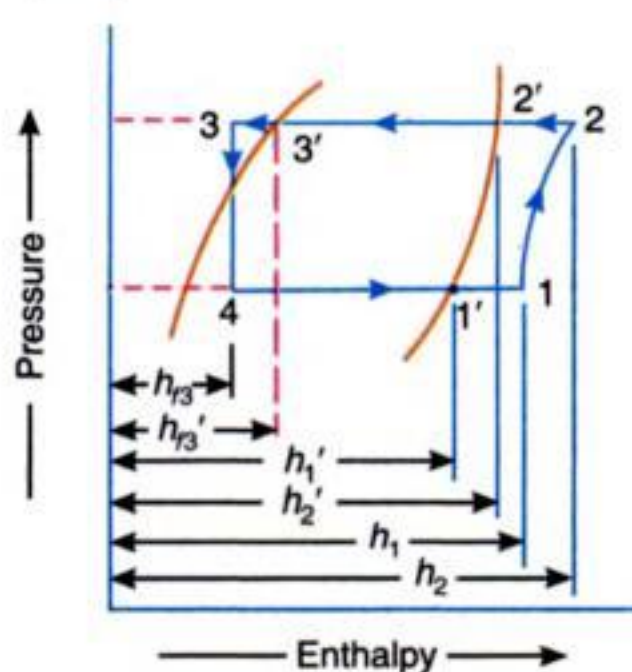
(b) p-h diagram.

Fig. 4.25

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (i) and (ii),

$$0.7277 = 0.6865 + 1.4145\ \log\left(\frac{T_2}{299}\right)$$

$$\log\left(\frac{T_2}{299}\right) = \frac{0.7277 - 0.6865}{1.4145} = 0.0291$$

$$\frac{T_2}{299} = 1.0693 \qquad \dots \text{(Taking antilog of 0.0291)}$$

$$\therefore \qquad T_2 = 299 \times 1.0693 = 319.7 \text{ K}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{188.47 - 58.42}{209.44 - 188.47} = \frac{130.05}{20.97} = 6.2 \text{ Ans.}$$

2. Theoretical power per tonne of refrigeration

We know that the heat extracted or refrigerating effect per kg of the refrigerant,

$$R_E = h_1 - h_{f3} = 188.47 - 58.42 = 130.05 \text{ kJ/kg}$$

and the refrigerating capacity of the system,

$$Q = 12 \text{ TR} = 12 \times 210 = 2520 \text{ kJ/min} \qquad \text{... (Given)}$$

$\therefore$ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{2520}{130.05} = 19.4 \text{ kg/min}$$

Work done during compression of the refrigerant

$$= m_R (h_2 - h_1)$$
$$= 19.4 (209.44 - 188.47) = 406.82 \text{ kJ/min}$$

$\therefore$ Theoretical power per tonne of refrigeration

$$= \frac{406.82}{60 \times 12} = 0.565 \text{ kW/ TR} \text{ Ans.}$$

3. Bore and stroke of compressor

Let $\qquad\qquad D$ = Bore of compressor,

$\qquad\qquad L$ = Stroke of compressor = 1.5 D, and $\qquad$ ... (Given)

$\qquad\qquad N$ = Speed of compressor = 1000 r.p.m. $\qquad$ ... (Given)

First of all, let us find the specific volume at suction to the compressor, *i.e.* at point 1. Applying Charles' law,

$$\frac{v_1}{T_1} = \frac{v_1'}{T_1'}$$

or $\qquad\qquad v_1 = v_1' \times \frac{T_1}{T_1'} = 0.0790 \times \frac{271}{265} = 0.081 \text{ m}^3/\text{kg}$

(a) When there is no clearance

We know that theoretical suction volume or piston displacement per minute

$$= m_R \times v_1 = 19.4 \times 0.081 = 1.57 \text{ m}^3/\text{min}$$

and theoretical suction volume or piston displacement per cylinder per minute

$$= 1.57 / 2 = 0.785 \text{ m}^3/\text{min} \qquad \text{... (iii)}$$

Also theoretical suction volume or piston displacement per minute

$$= \text{Piston area} \times \text{Stroke} \times \text{R.P.M.}$$

$$= \frac{\pi}{4} \times D^2 \times L \times N = \frac{\pi}{4} \times D^2 \times 1.5 D \times 1000$$

$$= 1178.25 \, D^3 \text{ m}^3/\text{min} \qquad \text{...(iv)}$$

Equating equations (iii) and (iv),

$$1178.25 \, D^3 = 0.785$$

or $\qquad\qquad D = 0.785 / 1178.25 = 0.000\ 666$

Solution. Given : $Q = 150$ TR ; $T_2 = T_{2''} = T_{3'} = 35°C = 35 + 273 = 308$ K ; $T_{1''} = T_4$ $= -25°C = -25 + 273 = 248$ K ; $T_1 = -8°C = -8 + 273 = 265$ K ; $n = 1.22$; $\eta_v = 0.75$

The *T-s* and *p-h* diagrams are shown in Fig. 4.30 (*a*) and (*b*) respectively.

From *p-h* diagram, we find that the pressure corresponding to evaporation temperature of $-25°C$,

$$p_1 = p_{1''} = p_4 = 1.518 \text{ bar}$$

Since there is a suction pressure drop of 0.118 bar due to *wire drawing, therefore pressure at point 1',

$$p_{1'} = 1.518 - 0.118 = 1.4 \text{ bar} = 1.4 \times 10^5 \text{ N/m}^2$$

Pressure corresponding to condensation temperature of 35°C

$$= 13.5 \text{ bar}$$

Since there is a discharge pressure drop of 0.23 bar due to wire drawing, therefore pressure at point 2',

$$p_{2'} = 13.5 + 0.23 = 13.73 \text{ bar} = 13.73 \times 10^5 \text{ N/m}^2$$

Note: In Fig. 4.30, point 1 represents the inlet of the suction valve and point 1' is the outlet of the suction valve. The point 2' represents the inlet of discharge valve and point 2 is the outlet of the discharge valve.

(a) *T-s* diagram.　　　　　　　　(b) *p-h* diagram.
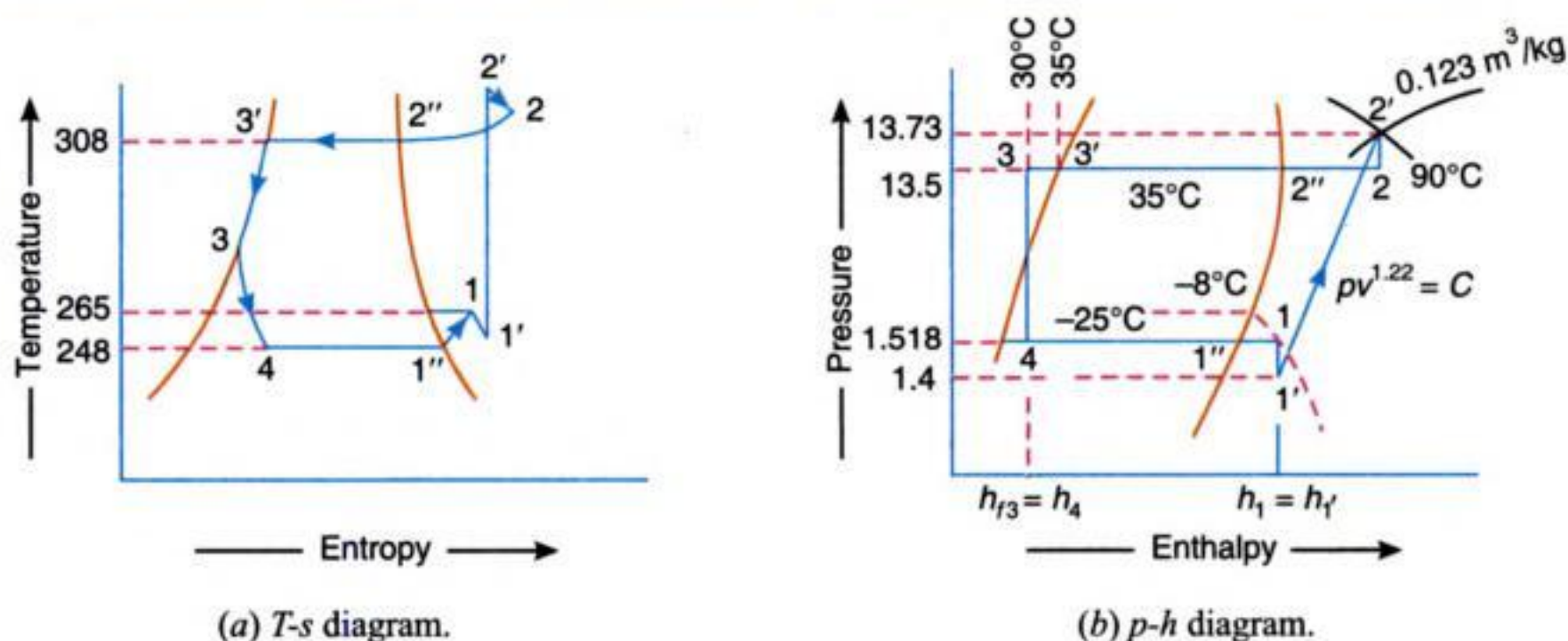
Fig. 4.30

From *p-h* diagram, we also find that enthalpy of superheated ammonia vapours at point 1 or 1',

$$h_1 = h_{1'} = 1440 \text{ kJ/kg}$$

Specific volume at point 1',

$$v_{1'} = 0.8 \text{ m}^3/\text{kg}$$

Temperature at point 1',

$$T_{1'} = -9°C = -9 + 273 = 264 \text{ K}$$

Let　　　　　　$v_{2'}$ = Specific volume at point 2'.

Since the compression is according to $pv^{1.22} = C$, therefore

$$p_{1'}(v_{1'})^n = p_{2'}(v_{2'})^n$$

or

$$v_{2'} = v_{1'}\left(\frac{p_{1'}}{p_{2'}}\right)^{\frac{1}{n}} = 0.8\left(\frac{1.4}{13.73}\right)^{\frac{1}{1.22}} = 0.123 \text{ m}^3/\text{kg}$$

---

*　　Wire drawing is a constant enthalpy process.

and workdone in compressor,

$$W = m_2 (h_2 - h_1)$$

$\therefore$ C.O.P. $= \dfrac{R_E}{W} = \dfrac{m_2(h_1 - h_{f3})}{m_2(h_2 - h_1)} = \dfrac{h_1 - h_{f3}}{h_2 - h_1}$

and power required to drive the compressor,

$$P = \dfrac{m_2(h_2 - h_1)}{60} \text{ kW}$$

From above, we see that the refrigerating effect, coefficient of performance and the power required are same as that of a simple saturation cycle when the flash chamber is not used. Thus the use of flash chamber has no effect on the thermodynamic cycle. The only effect resulting from the use of flash chamber is the reduction in the mass of refrigerant flowing through the evaporator and hence the reduction in size of evaporator.

4.16 Simple Saturation Cycle with Accumulator or Pre-cooler

Sometimes, the liquid refrigerant passing through the evaporator is not completely evaporated. If the compressor is supplied with liquid along with vapour refrigerant, then the compressor has to do an additional work of evaporating and raising the temperature of liquid refrigerant. It will also upset the normal working of the compressor which is meant only for compressing the pure vapour refrigerant.

Fig. 4.33. Simple saturation cycle with accumulator or pre-cooler.

In order to avoid this difficulty, an insulated vessel, known as accumulator or pre-cooler, is used in the system, as shown in Fig. 4.33. The accumulator receives the discharge (a mixture of liquid and vapour refrigerant) from the expansion valve and supplies the liquid refrigerant only to the evaporator, as in the case of flash chamber.

The discharge from the evaporator is sent again to the accumulator which helps to keep off the liquid from entering the compressor. Thus the accumulator supplies dry and saturated vapour to the compressor. A liquid pump is provided in the system in order to maintain circulation of the refrigerant in the evaporator.

Fig. 4.34. p-h diagram of simple saturation cycle with accumulator.

Let

m_1 = Mass of liquid refrigerant circulating through the evaporator, and

m_2 = Mass of refrigerant flowing in the condenser.

sub-cooled by passing it through a heat exchanger which is supplied with liquid refrigerant from the expansion valve, as shown in Fig. 4.37. In the heat exchanger, the liquid refrigerant from the condenser gives heat to the liquid refrigerant from the expansion valve. The p-h diagram of the cycle is shown in Fig. 4.38.
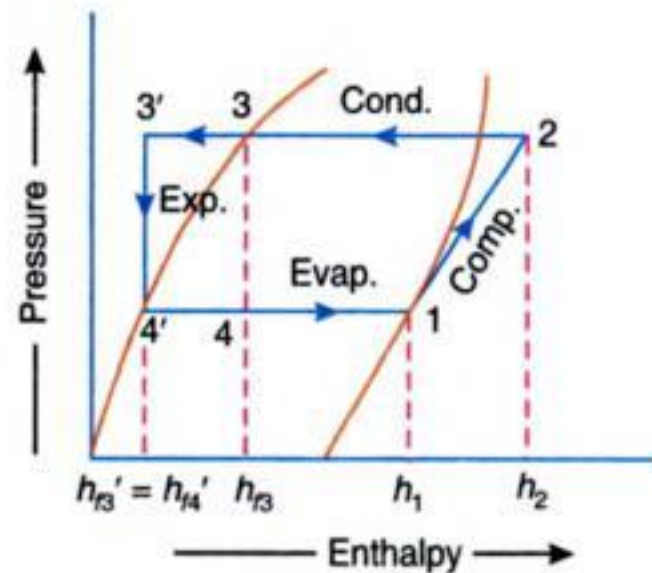
Fig. 4.38. *p-h* diagram of simple saturation cycle with sub-cooling of liquid refrigerant by liquid refrigerant.

Let m_1 = Mass of refrigerant leaving the evaporator,

m_2 = Mass of liquid refrigerant passing through the condenser, and

m_3 = Mass of liquid refrigerant supplied to the heat exchanger, from the expansion valve.

Considering the thermal equilibrium of the heat exchanger,

Heat lost by liquid refrigerant from condenser

= Heat gained by liquid refrigerant from expansion valve

i.e.
$$m_2 (h_{f3} - h_{f3'}) = m_3 (h_1 - h_{f4'})$$

∴
$$m_3 = m_2 \left[\frac{h_{f3} - h_{f3'}}{h_1 - h_{f4'}} \right] = m_2 \left[\frac{h_{f3} - h_{f3'}}{h_1 - h_{f3'}} \right]$$

$$\dots (\because h_{f4'} = h_{f3'}) \dots (i)$$

We know that refrigerating effect,

$$R_E = m_1(h_1 - h_{f4'}) = (m_2 - m_3)(h_1 - h_{f4'}) \quad \dots (\because m_1 = m_2 - m_3)$$

$$= \left[m_2 - m_2 \left(\frac{h_{f3} - h_{f3'}}{h_1 - h_{f4'}} \right) \right] (h_1 - h_{f4'})$$

$$\dots \text{[From equation } (i)\text{]}$$

$$= m_2 (h_1 - h_{f4'}) - m_2 (h_{f3} - h_{f3'})$$
$$= m_2 h_1 - m_2 h_{f4'} - m_2 h_{f3} + m_2 h_{f3'}$$
$$= m_2 h_1 - m_2 h_{f3} = m_2 (h_1 - h_{f3}) \quad \dots (\because m_2 h_{f4'} = m_2 h_{f3'})$$

and work done in compressor,

$$W = m_2 (h_2 - h_1)$$

∴
$$\text{C.O.P.} = \frac{R_E}{W} = \frac{m_2(h_1 - h_{f3})}{m_2(h_2 - h_1)} = \frac{h_1 - h_{f3}}{h_2 - h_1} \quad \dots (ii)$$

If Q tonnes of refrigeration is the load on the evaporator, then the mass of refrigerant (m_1) required to be circulated through the evaporator is given by

$$m_1 = \frac{210\,Q}{h_1 - h_{f4'}}$$

or
$$m_2 - m_3 = \frac{210\,Q}{h_1 - h_{f4'}}$$

11. The following data refer to a single cylinder, single acting compressor of an ammonia refrigeration system :

Bore	= 100 mm
Stroke	= 150 mm
Speed	= 200 r.p.m.
Indicated mean effective pressure	= 3.2 bar
Condenser pressure	= 10 bar
Evaporator pressure	= 3 bar
Temperature of water at entry to condenser	= 55°C
Temperature of water at exit from condenser	= 20°C
Rate of cooling water flowing in the condenser	= 12.5 kg/min
Inlet water temperature	= 12.5°C
Outlet water temperature	= 20.5°C

If the mass of ice produced per hour from water at 15°C is 50 kg and the latent heat of ice is 335 kJ/kg, find ; (a) coefficient of performance ; (b) mass flow of ammonia per minute ; and (c) condition of ammonia entering the compressor.

The relevant properties of ammonia are given below :

Pressure, bar	Saturation temperature, °C	Enthalpy, kJ/kg		Specific heat, kJ/kg K	
		Liquid	Vapour	Liquid	Vapour
12	31	327.9	1469.5	4.6	2.8
2.9	–10	135.4	1433	—	—

[Ans. 4.46 ; 0.326 kg ; 0.86]

12. A freezer of 20 TR capacity has evaporator and condenser temperatures of – 30°C and 25°C respectively. The refrigerant R-12 is sub-cooled by 4°C before it enters the expansion valve and is superheated by 5°C before leaving the evaporator. The compression is isentropic and the valve throttling and clearance are to be neglected. If a six cylinder, single acting compressor with stroke equal to bore running at 1000 r.p.m. is used, determine (a) C.O.P. of the refrigerating system, (b) mass of refrigerant to be circulated per min, (c) theoretical piston displacement per minute, and (d) theoretical bore and stroke of the compressor. The specific heat of liquid R-12 is 1.235 kJ/kg K and of vapour R-12 is 0.733 kJ/kg K.

The properties of R-12 are given below :

Saturation temp., °C	Pressure, bar	Enthalpy, kJ/kg		Entropy, kJ/kg K		Specific volume, m³/kg	
		Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
–30	1.0044	8.86	174.20	0.0371	0.7171	0.006 73	0.1596
25	6.5184	59.7	197.73	0.2239	0.6868	0.007 64	0.0269

[Ans. 3.64 ; 34.12 kg/min ; 5.56 m³/min ; 0.106 m]

13. A refrigeration plant of 8 TR capacity has its evaporation temperature of – 8°C and condenser temperature of 30°C. The refrigerant is sub-cooled by 5°C before entering into the expansion valve and vapour is superheated by 6°C before leaving the refrigerator. The suction pressure drop is 0.2 bar in the suction valve and discharge pressure drop is 0.1 bar in the discharge valve.

If the refrigerant used is R-12, find out the C.O.P. of the plant and theoretical power required for the compressor. Assume compression is isentropic. Use p-h chart for calculation.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

1. Two stage compression with liquid intercooler.
2. Two stage compression with water intercooler.
3. Two stage compression with water intercooler, liquid subcooler and liquid flash chamber.
4. Two stage compression with water intercooler, liquid subcooler and flash intercooler.
5. Three stage compression with flash chambers.
6. Three stage compression with water intercoolers.
7. Three stage compression with flash intercoolers.

The above mentioned types are now discussed, in detail, one by one in the following pages.

5.4 Two Stage Compression with Liquid Intercooler

The arrangement of a two stage compression with liquid intercooler is shown in Fig. 5.1 (*a*). The corresponding *p-h* diagram is shown in Fig. 5.1 (*b*).

(*a*) Two stage compression with liquid intercooler.

(*b*) *p-h* diagram.

Fig. 5.1

Work done in low pressure compressor,

$$W_L = m_1 (h_2 - h_1) = 20 (1527 - 1400) = 2540 \text{ kJ/min}$$

Work done in high pressure compressor,

$$W_H = m_2 (h_4 - h_3) = 21.73 (1550 - 1428) = 2651 \text{ kJ/min}$$

and total work done in both the compressors,

$$W = W_L + W_H = 2540 + 2651 = 5191 \text{ kJ/min}$$

∴ Power needed $= 5191/60 = 86.5$ kW **Ans.**

Power needed when intercooling is not employed

When intercooling is not employed, the compression of refrigerant will follow the path 1-2 in the low pressure compressor and 2–2′ in the high pressure compressor. In such a case,

Work done in the high pressure compressor,

$$W_H = m_1 (h_{2'} - h_2) = 20 (1676 - 1527) = 2980 \text{ kJ /min}$$

... (From p-h diagram, $h_{2'} = 1676$ kJ/kg)

and total workdone in both the compressors,

$$W = W_L + W_H = 2540 + 2980 = 5520 \text{ kJ/min}$$

∴ Power needed $= 5520/60 = 92$ kW **Ans.**

Example 5.2. *Calculate the power needed to compress 20 kg/min of R-12 from saturated vapour at 1.4 bar to a condensing pressure of 10 bar by two-stage compression with intercooling by liquid refrigerant at 4 bar. Assume saturated liquid to leave the condenser and dry saturated vapours to leave the evaporator.*

Use the p-h chart. Sketch the cycle on a skeleton p-h chart and label the values of enthalpy at salient points.

Solution. Given : $m_1 = 20$ kg / min ; $p_E = 1.4$ bar ; $p_C = 10$ bar ; $p_2 = p_3 = 4$ bar

The *p-h* diagram for a two-stage compression with intercooling by liquid refrigerant is shown in Fig. 5.4. The various values for R-12 as read from the *p-h* diagram are as follows :

Enthalpy of saturated vapour refrigerant entering the low pressure compressor at point 1,

$$h_1 = 178 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant entering the low pressure compressor at point 1,

$$s_1 = 0.71 \text{ kJ/kg K}$$

Fig. 5.4

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$$= \frac{53.12 \times 183.19 + 33.58 \times 189.65 + 16.51 \times 191.74}{53.12 + 33.58 + 16.51}$$

$$= \frac{9731.05 + 6368.45 + 3165.63}{103.21} = 186.6 \text{ kJ/kg}$$

Mark point 1 on *p-h* diagram such that h_1 = 186.6 kJ/kg. The specific entropy at this point is s_1 = 0.717 kJ/kg K. Now from point 1, draw a constant entropy line intersecting the horizontal condenser pressure line at point 2 as shown in Fig. 6.4. The enthalpy at point 2 as read from *p-h* diagram is

$$h_2 = 213.4 \text{ kJ/kg}$$

We know that work done in the compressor,

$$W = (m_1 + m_2 + m_3)(h_2 - h_1)$$
$$= (53.12 + 33.58 + 16.51)(213.4 - 186.6) = 2786.7 \text{ kJ/min}$$

∴ Power required to drive the compressor,

$$P = \frac{2786.7}{60} = 46.44 \text{ kW Ans.}$$

(c) *C.O.P of the system*

We know that total refrigerating effect,

$$R_E = 210 (Q_1 + Q_2 + Q_3)$$
$$= 210 (30 + 20 + 10) = 12\,600 \text{ kJ/min}$$

∴ C.O.P. of the system $= \dfrac{R_E}{W} = \dfrac{12\,600}{2786.7} = 4.521$ **Ans.**

6.5 Multiple Evaporators at Different Temperatures with Single Compressor, Multiple Expansion Valves and Back Pressure Valves

The arrangement, as shown in Fig. 6.5 (*a*), consists of three evaporators EP_1, EP_2 and EP_3 operating at different temperatures with single compressor, multiple expansion valves E_1, E_2 and E_3 and back pressure valves. The corresponding *p-h* diagram is shown in Fig. 6.5 (*b*). In this system, the refrigerant flows from the condenser through expansion valve E_3 where its pressure is reduced from the condenser pressure p_C to the pressure of third evaporator (*i.e.* highest temperature evaporator) EP_3 (*i.e.* p_{E3}). All the vapour formed after leaving the expansion valve E_3 plus enough liquid to take care of the load of evaporator EP_3 passes through this evaporator EP_3. The remaining refrigerant then flows through the expansion valve E_2 where its pressure is reduced from p_{E3} to p_{E2}. Again all the vapour formed after leaving the expansion valve E_2 plus enough liquid to take care of the load of evaporator EP_2 passes through the evaporator EP_2. The remaining liquid now flows through the expansion valve E_1 and supplies it to first evaporator (*i.e.* lowest temperature evaporator) EP_1. The vapour refrigerants coming out of the second and third evaporators EP_2 and EP_3 are further expanded through the back pressure valves to reduce their pressures to p_{E1}, as shown by 9-9′ and 6-6′ respectively. Now the refrigerants leaving the back pressure valves at points 6 and 9 are mixed together with the refrigerant leaving the first evaporator at point 11, at pressure p_{E1} which is the suction pressure of the compressor.

Let Q_1, Q_2 and Q_3 = Loads on the evaporators EP_1, EP_2 and EP_3 respectively in tonnes of refrigeration.

We know that the mass of refrigerant required to be circulated (at point 10) through the first evaporator or the lowest temperature evaporator EP_1,

(a) *Mass of refrigerant flowing through each evaporator*

We know that mass of refrigerant flowing through the first evaporator or the lowest temperature evaporator,

$$m_{e1} = m_1 = \frac{210\,Q_1}{h_{11} - h_{10}} = \frac{210 \times 30}{183.19 - 40.69} = 44.21 \text{ kg/min} \quad \textbf{Ans.}$$

Mass of refrigerant (at point 7) required to be circulated through the second evaporator,

$$m_2 = \frac{210\,Q_2}{h_9 - h_7} = \frac{210 \times 20}{189.65 - 45.37} = 29.11 \text{ kg/min}$$

In addition to this, the second evaporator is also supplied with vapours formed by $m_{e1} = 44.21$ kg/min refrigerant while passing through the second expansion valve.

∴ Total mass of refrigerant flowing through the second evaporator (*i.e.* at point 8),

$$m_{e2} = m_2 + m_{e1} \left(\frac{x_7}{1 - x_7} \right)$$

$$= 29.11 + 44.21 \left(\frac{0.04}{1 - 0.04} \right) = 30.95 \text{ kg/min} \quad \textbf{Ans.}$$

Mass of refrigerant (at point 4) required to be circulated through the third evaporator or the highest temperature evaporator,

$$m_3 = \frac{210\,Q_3}{h_6 - h_4} = \frac{210 \times 10}{191.74 - 64.59} = 16.51 \text{ kg/min}$$

In addition to this, the third evaporator is also supplied with vapours formed by $(m_{e1} + m_{e2})$ kg/min refrigerant while passing through the third expansion valve.

∴ Total mass of refrigerant flowing through the third evaporator (*i.e.* at point 5),

$$m_{e3} = m_3 + (m_{e1} + m_{e2}) \left(\frac{x_4}{1 - x_4} \right)$$

$$= 16.51 + (44.21 + 30.95) \left(\frac{0.13}{1 - 0.13} \right) = 27.78 \text{ kg/min} \quad \textbf{Ans.}$$

(b) *Power required to drive the compressor*

The refrigerant coming out of the three evaporators is mixed together before entering into the compressor. The condition of mixed refrigerant entering into the compressor is shown by point 1 on the *p-h* diagram. The enthalpy at point 1 is given by

$$h_1 = \frac{m_{e1} \times h_{11} + m_{e2} \times h_9 + m_{e3} \times h_6}{m_{e1} + m_{e2} + m_{e3}}$$

$$= \frac{44.21 \times 183.19 + 30.95 \times 189.65 + 27.78 \times 191.74}{44.21 + 30.95 + 27.78}$$

$$= \frac{8098.8 + 5869.7 + 5326.5}{103.12} = 187 \text{ kJ/kg}$$

Mark point 1 on *p-h* diagram such that $h_1 = 187$ kJ/kg. The specific entropy at this point is $s_1 = 0.718$ kJ/kg K. Now from point 1, draw a constant entropy line intersecting the horizontal condenser pressure line at point 2, as shown in Fig. 6.6. The enthalpy at point 2 as read from *p-h* diagram is

$$h_2 = 214.2 \text{ kJ/kg}$$

Enthalpy of saturated vapour refrigerant leaving the first evaporator at – 10°C or entering the first compressor at point 1,

$$h_1 = 183.19 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 1,

$$s_1 = 0.7019 \text{ kJ/kg K}$$

Enthalpy of saturated vapour refrigerant leaving the second evaporator at 5°C or entering the second compressor at point 3,

$$h_3 = 189.65 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 3,

$$s_3 = 0.6943 \text{ kJ/kg K}$$

Enthalpy of saturated vapour refrigerant leaving the third evaporator at 10°C or entering the third compressor at point 5,

$$h_5 = 191.74 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 5,

$$s_5 = 0.6921 \text{ kJ/kg K}$$

From points 1, 3 and 5, draw the constant entropy lines of s_1, s_2 and s_3 intersecting the horizontal condenser pressure line (corresponding to 40°C) at points 2, 4 and 6 respectively. From *p-h* diagram, we find that,

Enthalpy of superheated vapour refrigerant leaving the first compressor at point 2,

$$h_2 = 210 \text{ kJ/kg}$$

Enthalpy of superheated vapour refrigerant leaving the second compressor at point 4,

$$h_4 = 208 \text{ kJ/kg}$$

Enthalpy of superheated vapour refrigerant leaving the third compressor at point 6,

$$h_6 = 206 \text{ kJ/kg}$$

Enthalpy of sub-cooled liquid refrigerant at 30°C at point 7,

$$h_{f7} = h_8 = h_9 = h_{10} = 65 \text{ kJ/kg}$$

(a) Mass of the refrigerant flowing through each evaporator

We know that mass of refrigerant flowing through the first evaporator at – 10°C,

$$m_1 = \frac{210 Q_1}{h_1 - h_{10}} = \frac{210 \times 20}{183.19 - 65} = 35.5 \text{ kg/min } \textbf{Ans.}$$

Similarly, mass of refrigerant flowing through the second evaporator at 5°C,

$$m_2 = \frac{210 Q_2}{h_3 - h_9} = \frac{210 \times 30}{189.65 - 65} = 50.5 \text{ kg/min } \textbf{Ans.}$$

and mass of refrigerant flowing through the third evaporator at 10°C,

$$m_3 = \frac{210 Q_3}{h_5 - h_8} = \frac{210 \times 10}{191.74 - 65} = 16.57 \text{ kg/min } \textbf{Ans.}$$

(b) Power required to drive the system

We know that power required to drive the first compressor,

$$P_1 = \frac{m_1 (h_2 - h_1)}{60} = \frac{35.5 (210 - 183.19)}{60} = 15.86 \text{ kW}$$

Power required to drive the second compressor,

$$P_2 = \frac{m_2 (h_4 - h_3)}{60} = \frac{50.5 (208 - 189.65)}{60} = 15.44 \text{ kW}$$

Example 6.5. *A refrigeration system using R-12 as refrigerant consists of three evaporators of capacities 20 TR at – 10°C, 30 TR at 5°C and 10 TR at 10°C . The vapours leaving the three evaporators are dry and saturated. The system is provided with individual compressors and multiple expansion valves. The condenser temperature is 40°C and the liquid refrigerant leaving the condenser is subcooled to 30°C. Assuming isentropic compression in each compressor, find (a) the mass of refrigerant flowing through each evaporator ; (b) the power required to drive the system ; and (c) the C.O.P. of the system.*

Solution. Given : $Q_1 = 20$ TR ; $t_1 = t_{12} = -10°C$; $Q_2 = 30$ TR ; $t_3 = t_{10} = t_{11} = 5°C$; $Q_3 = 10$ TR ; $t_5 = t_8 = t_9 = 10°C$; $t_c = 40°C$; $t_7 = 30°C$
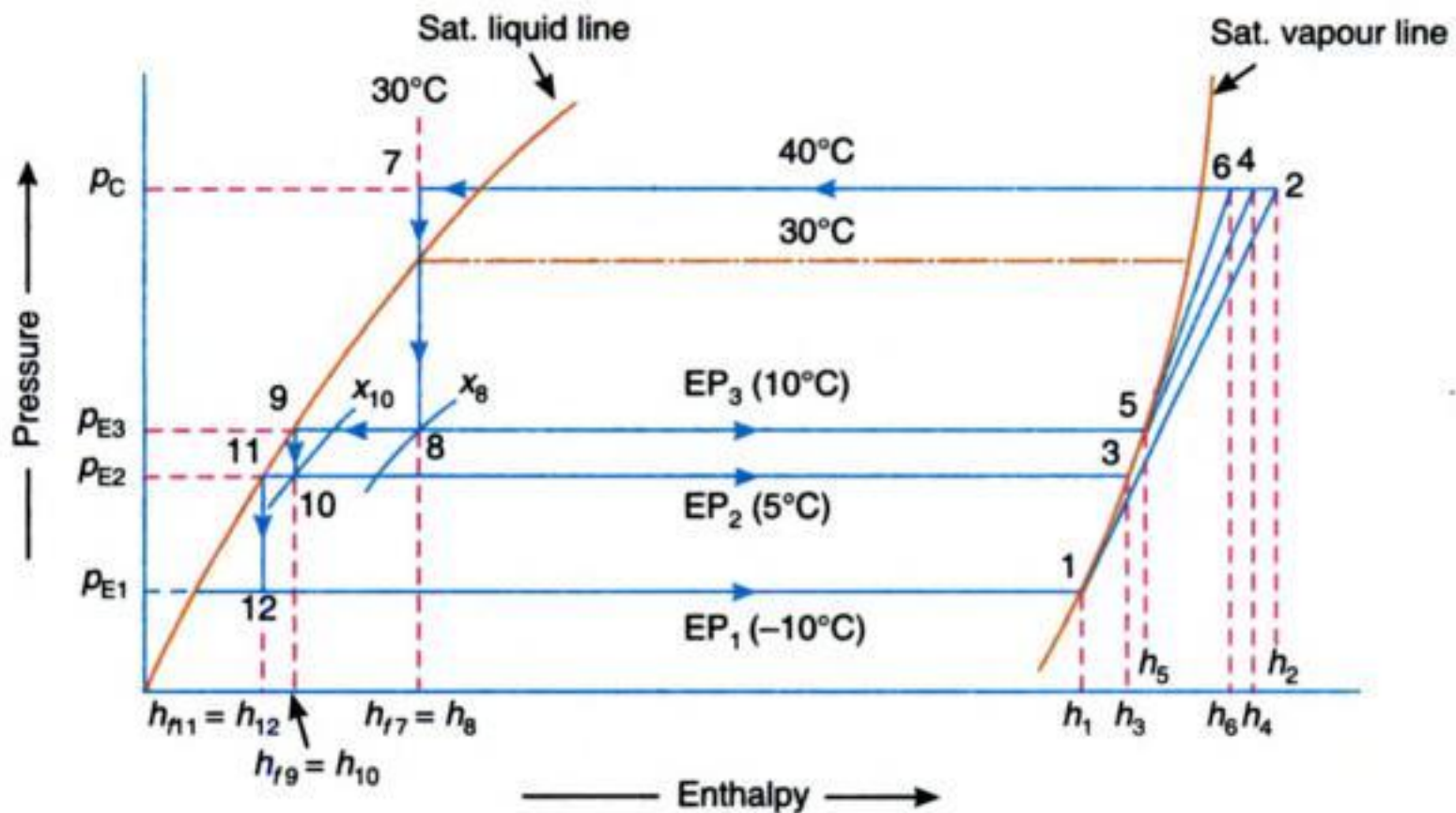
Fig. 6.10

The *p-h* diagram of a refrigeration system consisting of three evaporators with individual compressors and multiple expansion valves is shown in Fig. 6.10. The various values as read from *p-h* diagram for *R*-12 are as follows :

Enthalpy of saturated vapour refrigerant leaving the first evaporator at – 10°C or entering the first compressor at point 1,

$$h_1 = 183.19 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 1,

$$s_1 = 0.702 \text{ kJ/kg K}$$

Enthalpy of saturated vapour refrigerant leaving the second evaporator at 5°C or entering the second compressor at point 3,

$$h_3 = 189.65 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 3,

$$s_3 = 0.6943 \text{ kJ/kg K}$$

Enthalpy of saturated vapour refrigerant leaving the third evaporator at 10°C or entering the third compressor at point 5,

$$h_5 = 191.74 \text{ kJ/kg}$$

Similarly, the refrigerant coming out from the second compressor C_2 (i.e. $m_1 + m_2$) is mixed with the refrigerant coming out from the third evaporator EP_3 (i.e. m_3), before entering into the third compressor C_3. The condition of the mixed refrigerant entering into the third compressor C_3 is shown by point 7 on the p-h diagram. The enthalpy at point 7 is given by

$$(m_1 + m_2 + m_3) h_7 = (m_1 + m_2) h_5 + m_3 h_6$$

∴ $$h_7 = \frac{(m_1 + m_2) h_5 + m_3 h_6}{m_1 + m_2 + m_3}$$

We know that power required to drive the first compressor C_1,

$$P_1 = \frac{m_1 (h_2 - h_1)}{60} \text{ kW}$$

Similarly, power required to drive the second compressor C_2,

$$P_2 = \frac{(m_1 + m_2)(h_5 - h_4)}{60} \text{ kW}$$

and power required to drive the third compressor C_3,

$$P_3 = \frac{(m_1 + m_2 + m_3)(h_8 - h_7)}{60} \text{ kW}$$

∴ Total power required to drive the system,

$$P = P_1 + P_2 + P_3$$

We know that refrigerating effect of the system,

$$R_E = 210 (Q_1 + Q_2 + Q_3) \text{ kJ/min}$$

and total work done in the three compressors,

$$W = P \times 60 \text{ kJ/min}$$

∴ C.O.P. of the system $$= \frac{R_E}{W} = \frac{210 (Q_1 + Q_2 + Q_3)}{P \times 60}$$

Example 6.6. *A compound refrigeration system is used for multi- load purposes, as shown in Fig. 6.12. R-12 is used as refrigerant.*

Find (a) the power required to run the system; and (b) the coefficient of performance of the combined system.

Use p-h chart. There is no undercooling.
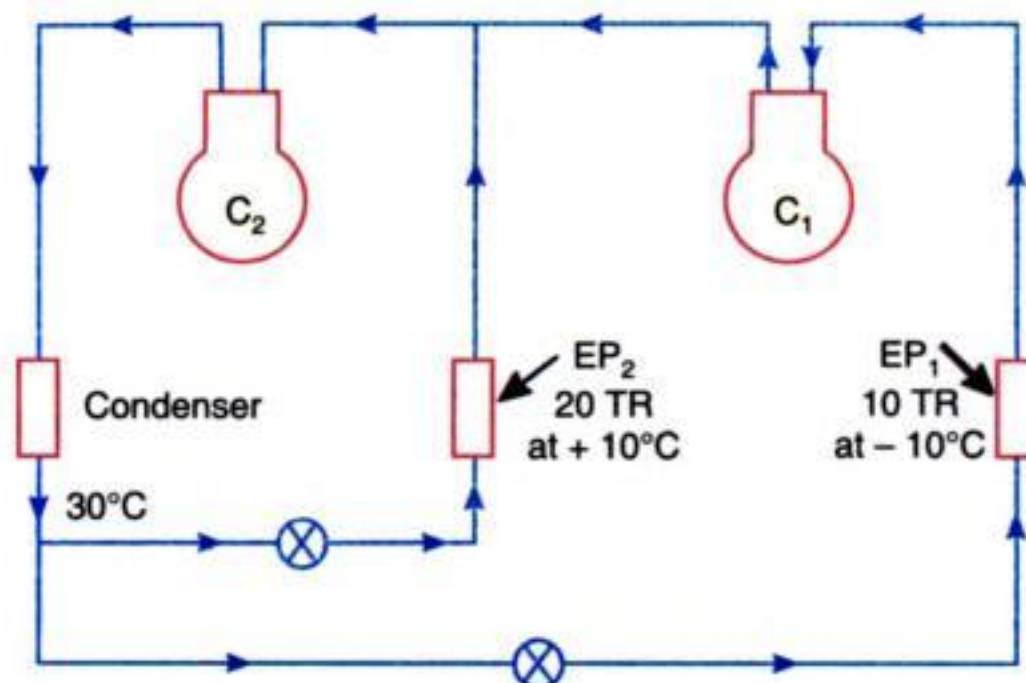
Fig. 6.12

∴ Total mass of refrigerant passing through the third compressor C_3,

$$m_{c3} = m_{c2} + m_3 + m_{3'}$$

We know that power required to drive the first compressor C_1,

$$P_1 = \frac{m_{c1}(h_2 - h_1)}{60} \text{ kW}$$

Similarly, power required to drive the second compressor C_2,

$$P_2 = \frac{m_{c2}(h_4 - h_3)}{60} \text{ kW}$$

and power required to drive the third compressor C_3,

$$P_3 = \frac{m_{c3}(h_6 - h_5)}{60} \text{ kW}$$

∴ Total power required to drive the system,

$$P = P_1 + P_2 + P_3$$

We know that refrigerating effect of the system,

$$R_E = 210 (Q_1 + Q_2 + Q_3) \text{ kJ/min}$$

and total work done in the three compressors

$$W = P \times 60 \text{ kJ/min}$$

∴ C.O.P. of the system $= \dfrac{R_E}{W} = \dfrac{210(Q_1 + Q_2 + Q_3)}{P \times 60}$

Example 6.7. *A compound refrigeration system using R-12 as refrigerant consists of three evaporators of capacities 20 TR at – 5°C, 30 TR at 0°C and 10 TR at 5°C. The vapours leaving the evaporators are dry and saturated. The system is provided with individual expansion valves and flash intercoolers, as shown in Fig. 6.14 (a). The condenser temperature is 40°C and the liquid refrigerant leaving the condenser is sub-cooled to 30°C. Assuming isentropic compression at each stage, find (a) the mass of refrigerant passing through each compressor ; (b) the power required to drive the system ; and (c) C.O.P. of the system.*

Solution. Given : $Q_1 = 20$ TR ; $t_1 = t_{10} = -5°C$; $Q_2 = 30$ TR ; $t_3 = t_9 = 0°C$; $Q_3 = 10$ TR ; $t_5 = t_8 = 5°C$; $t_c = 40°C$; $t_7 = 30°C$

The *p-h* diagram of a refrigeration system consisting of three evaporators with compound compression, individual expansion valves and flash intercoolers is shown in Fig. 6.15. The various values as read from the *p-h* diagram for *R*-12 are as follows :

Enthalpy of saturated vapour refrigerant leaving the first evaporator at – 5°C or entering the first compressor at point 1,

$$h_1 = 185.4 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 1,

$$s_1 = 0.6991 \text{ kJ/kg K}$$

Enthalpy of saturated vapour refrigerant leaving the second evaporator at 0°C or entering the second compressor at point 3,

$$h_3 = 187.5 \text{ kJ/kg}$$

Let $\qquad Q_1$, Q_2 and Q_3 = Loads on the evaporators EP_1, EP_2 and EP_3 respectively in tonnes of refrigeration.

∴ Mass of refrigerant required to be circulated through the first evaporator EP_1 or passing through the first compressor C_1,

$$m_{c1} = m_1 = \frac{210 Q_1}{h_1 - h_{12}} \text{ kg/min}$$

Mass of refrigerant required to be circulated through the second evaporator EP_2,

$$m_2 = \frac{210 Q_2}{h_3 - h_{10}} \text{ kg/min}$$

Mass of refrigerant required in the flash intercooler F_1 (at point 10) for desuperheating the superheated vapour refrigerant (m_{c1}) coming from first compressor C_1 to the dry saturated condition as at point 3 is given by

$$m_{2'} = \frac{m_{c1}(h_2 - h_3)}{h_3 - h_{10}} \text{ kg/min}$$

It may be noted that the second evaporator EP_2 is also supplied with the vapours formed during expansion of m_{c1} kg/min refrigerant while passing through the expansion valve E_2. If x_{10} is the dryness fraction of the refrigerant leaving the expansion valve E_2 at point 10, then the mass of vapours formed by m_{c1} while passing through expansion valve E_2 is given by

$$m_{2''} = m_{c1}\left(\frac{x_{10}}{1 - x_{10}}\right)$$

∴ Total mass of refrigerant flowing through the second compressor C_2,

$$m_{c2} = m_{c1} + m_2 + m_{2'} + m_{2''}$$

Similarly, mass of refrigerant required to be circulated through the third evaporator EP_3,

$$m_3 = \frac{210 Q_3}{h_5 - h_8} \text{ kg/min}$$

Mass of refrigerant required in the flash intercooler F_2 (at point 8) for desuperheating the superheated vapour refrigerant (m_{c2}) coming from second compressor C_2 to the dry saturated condition as at point 5 is given by

$$m_{3'} = \frac{m_{c2}(h_4 - h_5)}{h_5 - h_8} \text{ kg/min}$$

The third evaporator EP_3 is also supplied with the vapours formed by the expansion of m_{c2} kg/min refrigerant while passing through the expansion valve E_3. If x_8 is the dryness fraction of refrigerant leaving the expansion valve E_3, then the mass of vapours formed by m_{c2} while passing through expansion valve E_3 is given by

$$m_{3''} = m_{c2}\left(\frac{x_8}{1 - x_8}\right)$$

∴ Total mass of refrigerant passing through the third compressor C_3,

$$m_{c3} = m_{c2} + m_3 + m_{3'} + m_{3''}$$

We know that power required to drive the first compressor C_1,

$$P_1 = \frac{m_{c1}(h_2 - h_1)}{60} \text{ kW}$$

Power required to drive the second compressor,

$$P_2 = \frac{m_{c2}(h_4 - h_3)}{60} = \frac{76.5(195.3 - 189.65)}{60} = 7.2 \text{ kW}$$

and power required to drive the third compressor,

$$P_3 = \frac{m_{c3}(h_6 - h_5)}{60} = \frac{106.61(201.18 - 191.74)}{60} = 27.4 \text{ kW}$$

∴ Total power required to drive the system,

$$P = P_1 + P_2 + P_3$$
$$= 8.04 + 7.2 + 27.4 = 42.64 \text{ kW Ans.}$$

(c) C.O.P. of the system

We know that refrigerating effect of the system,

$$R_E = 210(Q_1 + Q_2 + Q_3)$$
$$= 210(30 + 20 + 10) = 12\,600 \text{ kJ/min}$$

and total work done in the three compressors,

$$W = P \times 60 = 42.64 \times 60 = 2558 \text{ kJ/min}$$

∴ C.O.P. of the system
$$= \frac{R_E}{W} = \frac{12\,600}{2558.4} = 4.9 \text{ Ans.}$$

EXERCISES

1. A single compressor, using R-12 as refrigerant, has three evaporators of capacities 10 TR, 30 TR and 20 TR. The temperatures in all the three evaporators is to be maintained at −5°C. The vapours leaving the evaporators are dry and saturated. The condenser temperature is 40°C. The liquid refrigerant leaving the condenser is sub-cooled to 30°C. Assuming isentropic compression, find the power required to drive the compressor and C.O.P. of the system. [Ans. 41.4 kW ; 5.07]

2. A single compressor using R-12 as refrigerant has three evaporators of capacities 10 TR, 20 TR, and 10 TR as shown in Fig. 6.18.
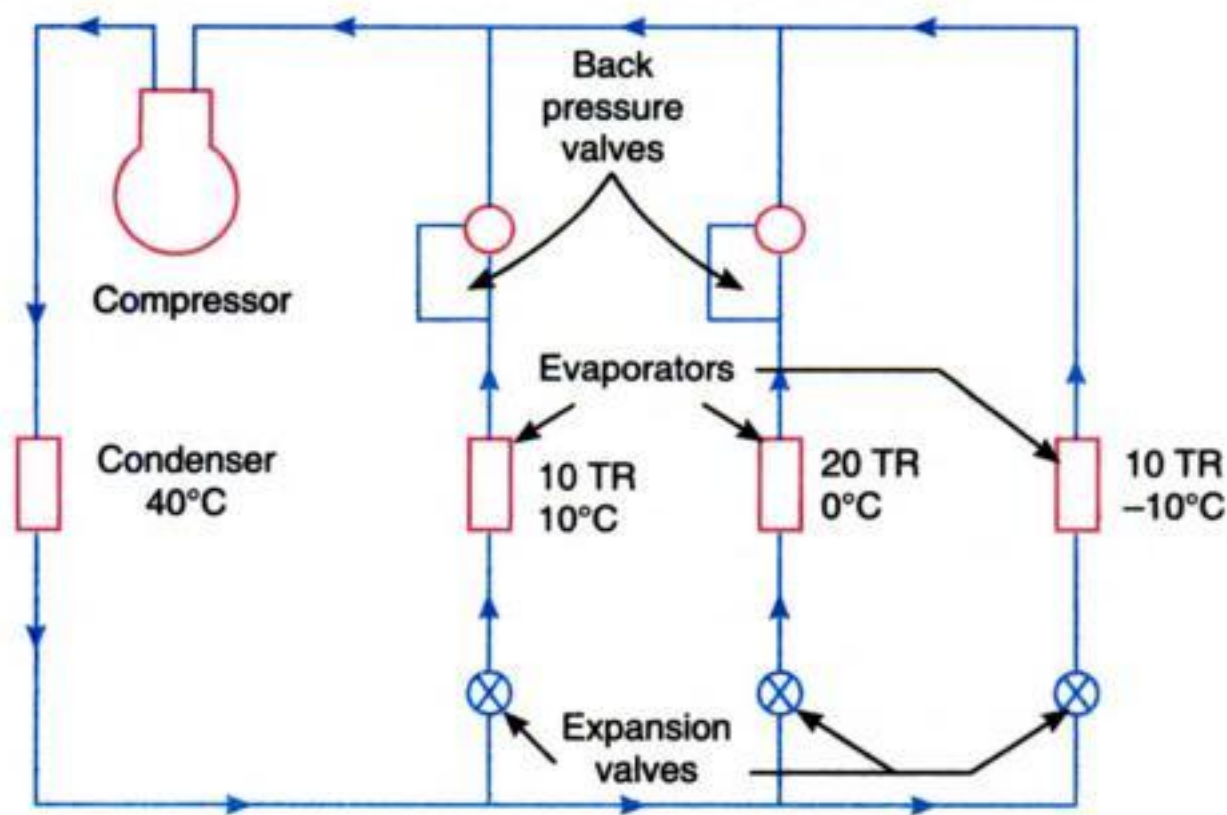
Fig. 6.18

the conditions of the refrigerant required for the operation of the refrigeration cycle. We have discussed in the previous chapters that the function of a compressor, in a vapour compression system, is to withdraw the vapour refrigerant from the evaporator. It then raises its temperature and pressure higher than the cooling agent in the condenser so that the higher pressure vapours can reject heat in the condenser. The liquid refrigerant leaving the condenser is now ready to expand to the evaporator conditions again.

In the vapour absorption system, the compressor is replaced by an absorber, a pump, a generator and a pressure reducing valve. These components in vapour absorption system, perform the same function as that of a compressor in vapour compression system. In this system, the vapour refrigerant from the evaporator is drawn into an absorber where it is absorbed by the weak solution of the refrigerant forming a strong solution. This strong solution is pumped to the generator where it is heated by some external source. During the heating process, the vapour refrigerant is driven off by the solution and enters into the condenser where it is liquified. The liquid refrigerant then flows into the evaporator and thus the cycle is completed.

7.2 Simple Vapour Absorption System

The simple vapour absorption system, as shown in Fig. 7.1, consists of an absorber, a pump, a generator and a pressure reducing valve to replace the compressor of vapour compression system. The other components of the system are condenser, receiver, expansion valve and evaporator as in the vapour compression system.
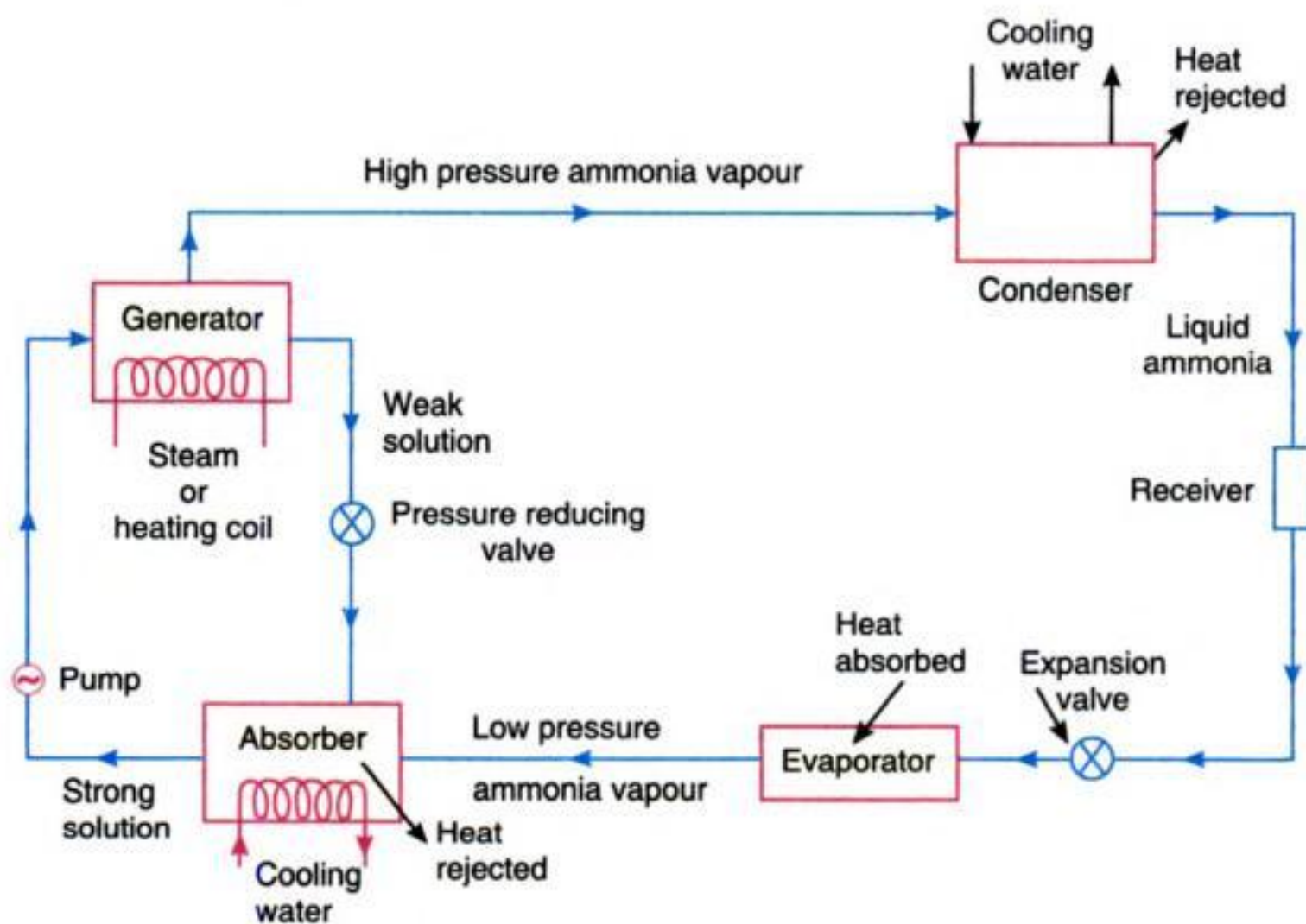
Fig. 7.1. Simple vapour absorption system.

In this system, the low pressure ammonia vapour leaving the evaporator, enters the absorber where it is absorbed by the cold water in the absorber. The water has the ability to absorb very large quantities of ammonia vapour and the solution thus formed, is known as *aqua-ammonia*. The absorption of ammonia vapour in water lowers the pressure in the absorber which in turn draws more ammonia vapour from the evaporator and thus raises the temperature of solution. Some form of cooling arrangement (usually water cooling) is employed in the absorber to remove the heat of solution evolved there. This is necessary in order to increase the absorption capacity of water,

The refrigerant water vapours formed in the generator due to heating of solution are passed to the condenser where they are cooled and condensed by the cooling water flowing through the condenser water tubes. The cooling water for condensing is pumped from the cooling water pond or tower. This cooling water first enters the absorber where it takes away the heat of condensation and dilution. The condensate from the condenser is supplied to the evaporator to compensate the water vapour formed in the evaporator. The pressure reducing valve reduces the pressure of condensate from the condenser pressure to the evaporator pressure. The cooled water from the evaporator is pumped and sprayed in the evaporator in order to cool the water for air conditioning flowing through the chilled tubes. This completes the cycle.

Note : The pressure difference between the generator and the absorber and the gravity due to the height difference of the two shells is utilised to create the pressure for the spray.

EXERCISES

1. In a vapour absorption system, heat is supplied to the generator at a temperature of 90°C. The cooling in condenser and refrigeration evaporator takes place at 20°C and –10°C respectively. Find the maximum C.O.P. of system. **[Ans. 1.69]**

2. In a vapour absorption system, the heat is supplied to the generator by condensing steam at 3 bar and 85% dry. The temperature in the evaporator is to be maintained at – 10°C. If the cooling water rejects heat at 30°C in the condenser, find the maximum C.O.P. of the system.

   When the refrigeration load is 10 tonnes and the actual C.O.P. is 40% of the maximum C.O.P., find the mass of steam required per hour. **[Ans. 1.674 ; 102.3 kg/h]**

QUESTIONS

1. What is the basic function of a compressor in vapour compression refrigeration system ? How this function is achieved in vapour absorption refrigeration system ?

2. Draw a neat sketch of a practical vapour absorption refrigeration cycle. Indicate thereon the phases of various fluids and the name of the equipments. Also indicate the direction of the external energy flow to or from the equipments.

3. What is the function of the following components in an absorption system :

   (i) Absorber,   (ii) Rectifier,   (iii) Analyser, and   (iv) Heat exchangers.

4. Discuss the advantages of vapour absorption refrigeration system over vapour compression refrigeration system.

5. Derive an expression for the C.O.P. of an ideal vapour absorption system in terms of temperature T_G at which heat is supplied to the generator, the temperature T_E at which heat is absorbed in the evaporator and the temperature T_C at which heat is discharged from the condenser and absorber.

6. Draw a neat diagram of three fluid system of refrigeration (electrolux refrigeration system) and explain its working.

7. Mention the function of each fluid in a three-fluid vapour absorption system.

8. Draw a neat diagram of lithium bromide water absorption system and explain its working. List the major field of applications of this system.

OBJECTIVE TYPE QUESTIONS

1. The refrigerant, commonly used in vapour absorption system, is

   (a) water   (b) ammonia   (c) freon   (d) aqua-ammonia

2. A vapour absorption system

   (a) gives noisy operation   (b) gives quiet operation

   (c) requires little power consumption   (d) cools below 0°C

point of sulphur dioxide is – 10°C at atmospheric pressure. The condensing pressure varies between 4.1 bar and 6.2 bar under normal operating conditions. The latent heat of sulphur dioxide at – 15°C is 396 kJ/kg. It is a very stable refrigerant with a high critical temperature and it is non-flammable and non-explosive. It has a very unpleasant and irritating odour. This refrigerant is not injurious to food and is used commercially as a ripener and preservative of foods. It is however, extremely injurious to flowers, plants and shrubbery. The sulphur dioxide in its pure state is not corrosive, but when there is moisture present, the mixture forms sulphurous acid which is corrosive to steel. Thus it is very important that the moisture in the refrigerating system be held to a minimum.

The sulphur dioxide does not mix readily with oil. Therefore, an oil lighter than that used with other refrigerants may be used in the compressors. The refrigerant in the evaporator with oil floating on the top has a tendency to have a higher boiling point than that corresponding to its pressure. The modern evaporators overcome this by having the liquid introduced in such a way that the refrigerant is kept agitated while the unit is in operation. The leaks in the system with sulphur dioxide may be easily detected by means of soap solution or ammonia swab. A dense white smoke forms when sulphur dioxide and ammonia fumes come in contact.

5. R-118 (Water). The principal refrigeration use of water is as ice. The high freezing temperature of water limits its use in vapour compression systems. It is used as the refrigerant vapour in some absorption systems and in systems with steam jet compressors.

8.7 Hydro-carbon Refrigerants

Most of the hydro-carbon refrigerants are successfully used in industrial and commercial installations. They possess satisfactory thermodynamic properties but are highly flammable and explosive. The various hydro-carbon refrigerants are given in the following table :

Table 8.4. **Hydro-carbon refrigerants.**

Refrigerant number	Chemical name	Chemical formula
R-170	Ethane	C_2H_6
R-290	Propane	C_3H_3
R-600	Butane	C_4H_{10}
R-600a	Isobutane	C_4H_{10}
R-1120	Trichloroethylene	$C_2H_4Cl_3$
R-1130	Dichloroethylene	$C_2H_4Cl_2$
R-1150	Ethylene	C_2H_4
R-1270	Propylene	C_3H_6

Since the hydro-carbon refrigerants are not commonly used now-a-days, therefore, they are not discussed in detail.

Manufacture of Hydrocarbon refrigerants.

5. *Coefficient of performance and power requirements.* For an ideal refrigerant operating between – 15°C evaporator temperature and 30°C condenser temperature, the theoretical coefficient of performance for the reversed Carnot cycle is 5.74. The following table shows the values of theoretical coefficient of performance and power per tonne of refrigeration for some common refrigerants operating between – 15°C evaporator temperature and 30°C condenser temperature.

Table 8.9. Coefficient of performance and power per TR.

Refrigerant	Coefficient of performance	kW / TR
R-11	5.09	0.694
R-12	4.70	0.746
R-22	4.66	0.753
R-30	4.90	0.716
R-40	4.90	0.716
R-113	4.92	0.716
R-717	4.76	0.738
R-729	5.74	0.619
R-744	2.56	1.372
R-764	4.87	0.724

From the above table, we see that R-11 has the coefficient of performance equal to 5.09 which is closest to the Carnot value of 5.74. The other refrigerants have also quite high values of coefficient of performance except R-744 (carbon dioxide) which has the value of coefficient of performance as 2.56 with a power requirement of 1.372 kW per tonne of refrigeration. This is due to its low critical point (31°C) and the condensing temperature is very close to it which is 30°C. Practically, all common refrigerants have approximately the same coefficient of performance and power requirement.

6. *Latent heat of vaporisation.* A refrigerant should have a high latent heat of vaporisation at the evaporator temperature. The high latent heat results in high refrigerating effect per kg of refrigerant circulated which reduces the mass of refrigerant to be circulated per tonne of refrigeration. Table 8.10 shows the refrigerating effect for the common refrigerants operating between – 15°C evaporator temperature and 30°C condenser temperature. It also shows the latent heat, mass of refrigerant circulated per tonne of refrigeration and the volume of the liquid refrigerant per tonne of refrigeration.

7. *Specific volume.* The specific volume of the refrigerant vapour at evaporator temperature (*i.e.* volume of suction vapour to the compressor) indicates the theoretical displacement of the compressor. The reciprocating compressors are used with refrigerants having high pressures and low volumes of the suction vapour. The centrifugal or turbo compressors are used with refrigerants having low pressures and high volumes of the suction vapour. The rotary compressors are used with refrigerants having intermediate pressures and volumes of the suction vapour. Table 8.11 shows the specific volume of the refrigerant vapour and theoretical piston displacements for various refrigerants.

Table 8.14. Thermal conductivities.

Refrigerant	Temperature (° C)	Thermal conductivity (W/mK)
Liquid		
R-11	40	0.1022
R-12	40	0.0814
R-22	40	0.0970
R-30	30	0.1664
R-40	20	0.1612
R-113	40	0.0971
R-717	– 10 to + 20	0.5026
R-744	20	0.2080
R-764	20	0.3466
Vapour		
R-11	30	8.318×10^{-3}
R-12	30	9.705×10^{-3}
R-22	30	11.784×10^{-3}
R-30	0	6.759×10^{-3}
R-40	0	8.492×10^{-3}
R-113	30	7.798×10^{-3}
R-717	0	22.182×10^{-3}
R-744	0	14.037×10^{-3}
R-764	0	8.665×10^{-3}

Table 8.15. Dielectric strengths.

Refrigerant	R-11	R-12	R-22	R-30	R-40	R-113	R-717	R-744	R-764
Relative dielectric strength	3	2.4	1.31	1.11	1.06	2.6	0.82	0.88	1.9

6. Leakage tendency. The leakage tendency of a refrigerant should be low. If there is a leakage of refrigerant, it should be easily detectable. The leakage occurs due to opening in the joints or flaws in material used for construction. Since the fluorocarbon refrigerants are colourless, therefore, their leakage will increase the operating cost. The ammonia leakage is easily detected due to its pungent odour.

The leakage of fluorocabon refrigerants may be detected by soap solution, a halide torch or an electronic leak detector. The latter is generally used in big refrigerating plants. The ammonia leakage is detected by using burning sulphur candle which in the presence of ammonia forms white fumes of ammonium sulphite.

7. Cost. The cost of refrigerant is not so important in small refrigerating units but it is very important in high capacity refrigerating systems like industrial and commercial. The ammonia, being the cheapest, is widely used in large industrial plants such as cold storages and ice plants. The refrigerant R-22 is costlier than refrigerant R-12. The cost of losses due to leakage is also important.

8.13 Secondary Refrigerants - Brines

Brines are secondary refrigerants and are generally used where temperatures are required to be maintained below the freezing point of water *i.e.* 0°C. In case the temperature involved is above

9

Refrigerant Compressors

9.1 Introduction

A refrigerant compressor, as the name indicates, is a machine used to compress the vapour refrigerant from the evaporator and to raise its pressure so that the corresponding saturation temperature is higher than that of the cooling medium. It also continually circulates the refrigerant through the refrigerating system. Since the compression of refrigerant requires some work to be done on it, therefore, a compressor must be driven by some prime mover.

Note : Since the compressor virtually takes the heat at a low temperature from the evaporator and pumps it at the high temperature to the condenser, therefore it is often referred to as a heat pump.

$$p_2, v_2 \text{ and } T_2 = \text{Corresponding values for the refrigerant at the discharge point } i.e. \text{ after compression, and}$$

$$r = \text{Compression ratio or pressure ratio, } p_2 / p_1.$$

As a matter of fact, the compression of refrigerant may be isothermal, polytropic, or isentropic (reversible adiabatic). Now we shall find out the amount of work done in compressing the refrigerant in all the above mentioned three cases.

1. *Work done during isothermal compression*

We have already discussed that when the piston moves from the top dead centre (point A), the refrigerant is admitted into the compressor cylinder and it continues till the piston reaches at its bottom dead centre (point B). Thus the line A B represents suction stroke and the area below this line (*i.e.* area AB B′ A′) represents the work done during suction stroke. From the figure, we find that work done during suction stroke,

$$W_1 = \text{Area } A\,B\,B'\,A' = p_1 v_1$$

The refrigerant is compressed during the return stroke (or compression stroke BC_1) of the piston at constant temperature. The compression continues till the pressure (p_2) in the cylinder is sufficient to force open the discharge valve at C_1. After that no compression takes place with the inward movement of the piston. Now during the remaining part of the compression stroke, the compressed refrigerant is discharged to condenser till the piston reaches its top dead centre. The volume of refrigerant delivered (v_2) is represented by the line C_1D.
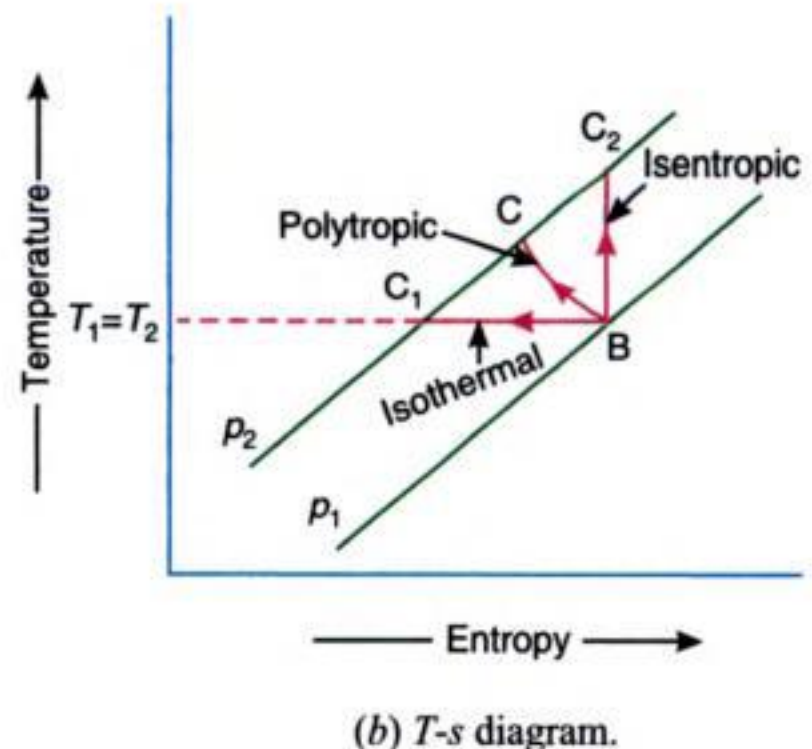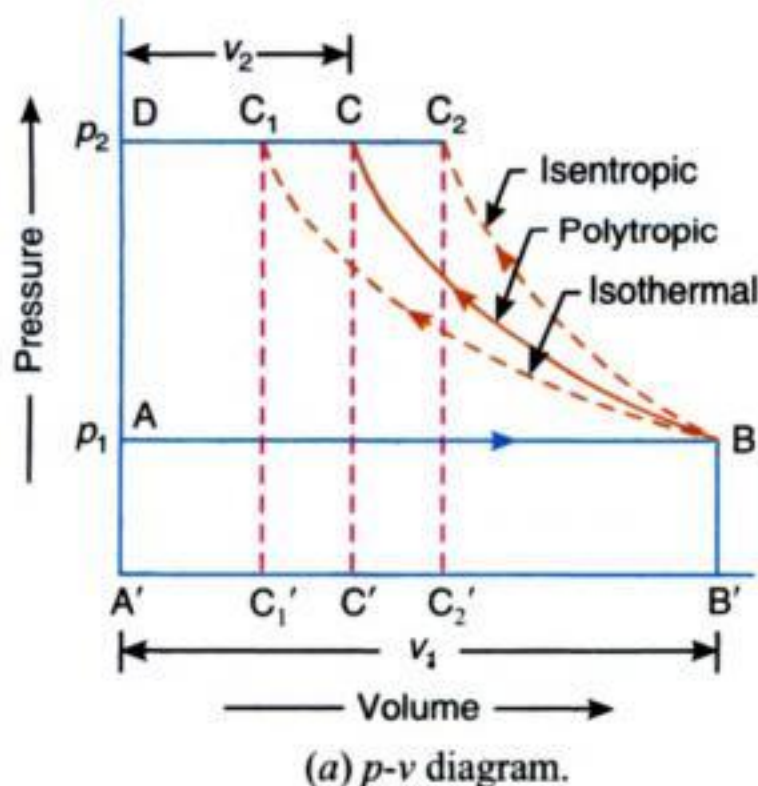
Fig. 9.2. *p-v* and *T-s* diagrams for a single stage reciprocating compressor.

From the figure, we find that

Work done during compression,

$$W_2 = \text{Area } B\,C_1\,C_1'\,B' = p_1 v_1 \log_e\left(\frac{v_1}{v_2}\right)$$

and work done during discharge,

$$W_3 = \text{Area } C_1\,D\,A'\,C_1' = p_2 v_2$$

∴ Work done by the compressor per cycle,

$$W = \text{Area } ABC_1D$$

$$= \text{Area } C_1DA'C_1' + \text{Area } BC_1C_1'B' - \text{Area } ABB'A'$$

$$= W_3 + W_2 - W_1$$

∴ Power required to drive the compressor

$$= 3.12 \times 10^5/60 = 5200 \text{ W} = 5.2 \text{ kW Ans.}$$

2. Polytropic compression

We know that work done by the compressor

$$= \frac{n}{n-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.12}{1.12-1} \times 1 \times 10^5 \times 1.5 \left[\left(\frac{8}{1} \right)^{\frac{1.12-1}{1.12}} - 1 \right] \text{N-m/min}$$

$$= 14 \times 10^5 (1.25 - 1) = 3.5 \times 10^5 \text{ N-m/min}$$

∴ Power required to drive the compressor

$$= 3.5 \times 10^5/60 = 5833 \text{ W} = 5.833 \text{ kW Ans.}$$

3. Isentropic compression

We know that work done by the compressor,

$$= \frac{\gamma}{\gamma-1} \times p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{1.31}{1.31-1} \times 1 \times 10^5 \times 1.5 \left[\left(\frac{8}{1} \right)^{\frac{1.31-1}{1.31}} - 1 \right] \text{N-m/min}$$

$$= 6.34 \times 10^5 (1.63 - 1) = 4 \times 10^5 \text{ N-m/min}$$

∴ Power required to drive the compressor

$$= 4 \times 10^5/60 = 6667 \text{ W} = 6.667 \text{ kW Ans.}$$

9.8 Work Done by Reciprocating Compressor with Clearance Volume

In the previous articles, we have assumed that there is no clearance volume in the compressor cylinder. In other words, the entire volume of the refrigerant, in the compressor cylinder, is compressed by the inward stroke of the piston. But in actual practice, it is not possible to reduce the clearance volume to zero, for mechanical reasons. Moreover, it is not desirable to allow the piston head to come in contact with the cylinder head. In addition to this, the passages leading to the suction and discharge valves always contribute to clearance volume. In general, the clearance volume is expressed as some percentage of the piston displacement.

Now consider a single stage, single acting horizontal reciprocating compressor with clearance volume as shown by the *p-v* diagram in Fig. 9.3.

Let p_1 = Suction pressure of refrigerant (before compression),

v_1 = Total volume of refrigerant in the compressor cylinder (before compression),

T_1 = Suction temperature of refrigerant (before compression),

p_2, v_2 and T_2 = Corresponding values at the discharge point 2 (*i.e.* after compression),

∴ Power rquired to drive the compressor,

$$P = \frac{W \times N_W}{60} = \frac{1695 \times 500}{60} = 141\ 20\ W = 14.12\ kW\ \textbf{Ans.}$$

2. Volumetric efficiency of the compressor

We know that volumetric efficiency of the compressor,

$$\eta_v = \frac{v_1 - v_4}{v_1 - v_c} = \frac{0.009\ 87 - 0.001\ 74}{0.00987 - 0.000\ 47} = 0.865\ \text{or}\ 86.5\%\ \textbf{Ans.}$$

9.10 Factors Effecting Volumetric Efficiency of a Reciprocating Compressor

We have discussed in the previous article that the volumetric efficiency of a reciprocating compressor,

$$\eta_v = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \qquad \qquad \dots (i)$$

where

C = Clearance factor,

p_1 = Suction pressure,

p_2 = Discharge pressure, and

n = Polytropic index of expansion.

The various factors which effect the volumetric efficiency are discussed below:

1. Effect of clearance factor

The equation (i) may be written as

$$\eta_v = 1 + C\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]$$

We know that the discharge pressure (p_2) is always more than the suction pressure (p_1).

Therefore, the ratio $\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$ will be more than unity and the expression $\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]$ will be

negative. Thus, the increase in clearance factor (C) will increase the quantity to be substracted from 1 to give volumetric efficiency. Hence the volumetric efficiency decreases as the clearance factor increases.

2. Effect of valve pressure drops

The p-v diagram of a reciprocating compressor considering the pressure drop at the suction and discharge valves is shown in Fig. 9.5. In actual practice, the compression will start from pressure p_s at point 1' and expansion from pressure p_d at point 3'. The pressure $p_1 = p_4$ is the pressure at the flange on suction side and $p_2 = p_3$ is the pressure at the flange on discharge side.

Fig. 9.5. p-v diagram with valve pressure drops.

3. The compression in both the low pressure (L.P.) and high pressure (H.P.) cylinders is polytropic *i.e.* $pv^n = C$.

9.16 Intercooling of Refrigerant in a Two Stage Reciprocating Compressor

In Art. 9.14, we have discussed the working of a two stage reciprocating compressor with an intercooler between the stages. As a matter of fact, the efficiency of an intercooler plays an important role in the working of a two stage reciprocating compressor. Following two types of intercooling are important from the subject point of view :

1. *Incomplete or imperfect intercooling.* When the temperature of refrigerant leaving the intercooler (*i.e.* T_3) is more than the suction temperature of refrigerant (*i.e.* T_1), the intercooling is said to be *incomplete* or *imperfect intercooling*. In this case, the point 3 lies on the right side of the isothermal curve, as shown in Fig. 9.7 (*a*) and (*b*).

(a) p-v diagram.	(b) T-s diagram.
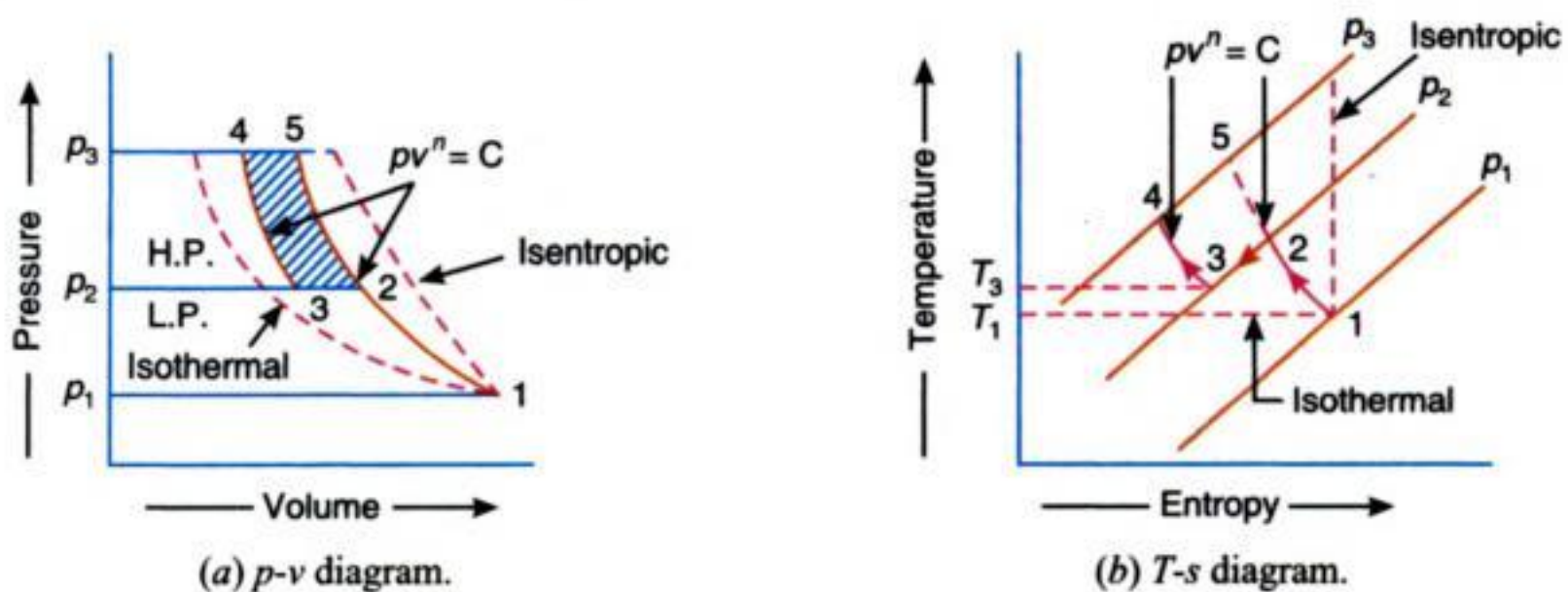
Fig. 9.7. Incomplete or imperfect intercooling.

2. *Complete or perfect intercooling.* When the temperature of refrigerant leaving the intercooler (*i.e.* T_3), is equal to the suction temperature of refrigerant (*i.e.* T_1), the intercooling is said to be *complete* or *perfect intercooling*. In this case, the point 3 lies on the isothermal curve as shown in Fig. 9.8 (*a*) and (*b*).

(a) p-v diagram.	(b) T-s diagram.
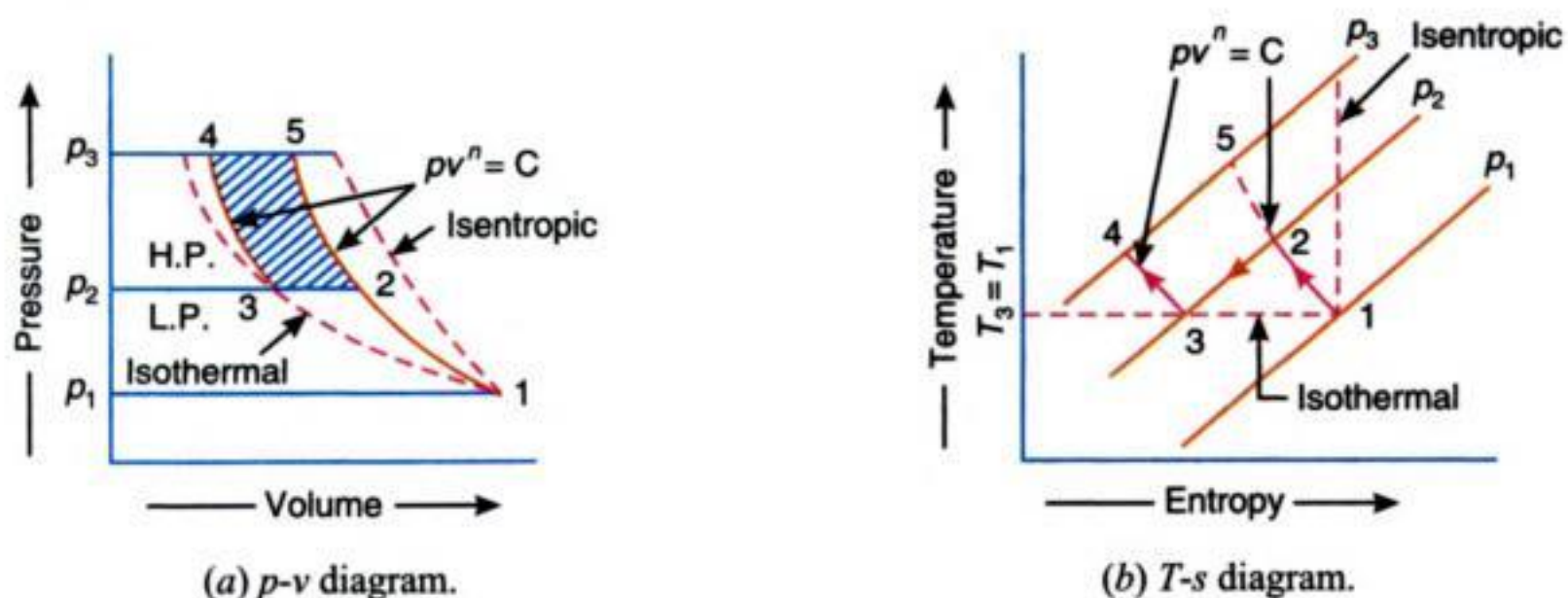
Fig. 9.8. Complete or perfect intercooling.

Note : The amount of work saved due to intercooling is shown by the shaded area 2-3-4-5 in both the cases, to some scale.The amount of work saved with incomplete intercooling is less than that in case of complete intercooling.

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10.6 Air Cooled Condensers

An air-cooled condenser is one in which the removal of heat is done by air. It consists of steel or copper tubing through which the refrigerant flows. The size of tube usually ranges from 6 mm to 18 mm outside diameter, depending upon the size of condenser. Generally copper tubes are used because of its excellent heat transfer ability. The condensers with steel tubes are used in ammonia refrigerating systems. The tubes are usually provided with plate type fins to increase the surface area for heat transfer, as shown in Fig. 10.2. The fins are usually made from aluminium because of its light weight. The fin spacing is quite wide to reduce dust clogging.

Air cooled condenser.

The condensers with single row of tubing provides the most efficient heat transfer. This is because the air temperature rises at it passes through each row of tubing. The temperature difference between the air and the vapour refrigerant decreases in each row of tubing and therefore each row becomes less effective. However, single row condensers require more space than multi-row condensers. The single row condensers are usually used in small capacity refrigeration systems such as domestic refrigerators, freezers, water coolers and room air conditioners.

The air cooled condensers may have two or more rows of tubing, but the condensers with upto six rows of tubing are common. Some condensers have seven or eight rows. However more than eight rows of tubing are usually not efficient. This is because the air temperature will be too close to the condenser temperature to absorb any more heat after passing through eight rows of tubing.
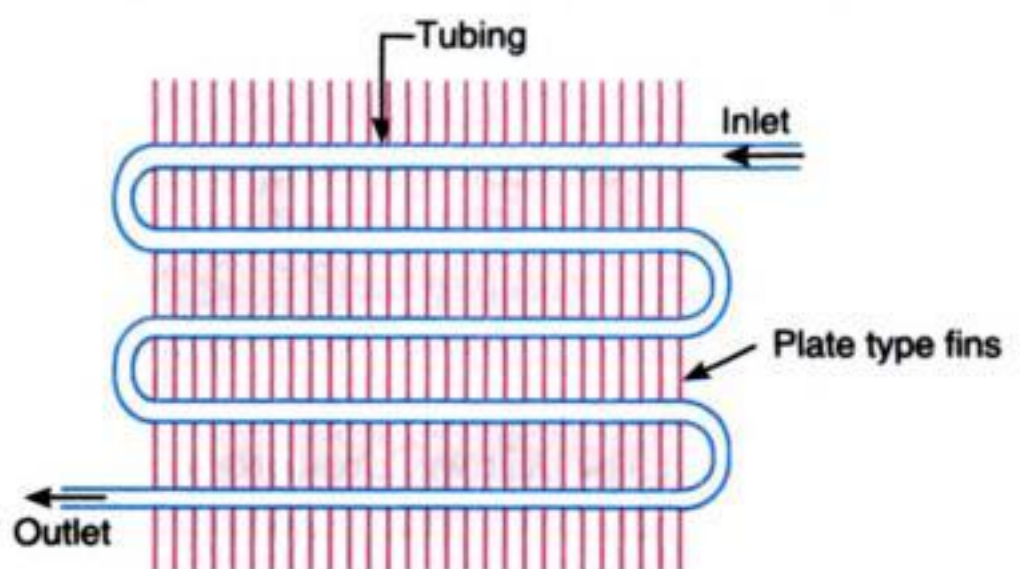
Fig. 10.2. Air cooled condenser.

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15.4 Working of Steam Jet Refrigeration System
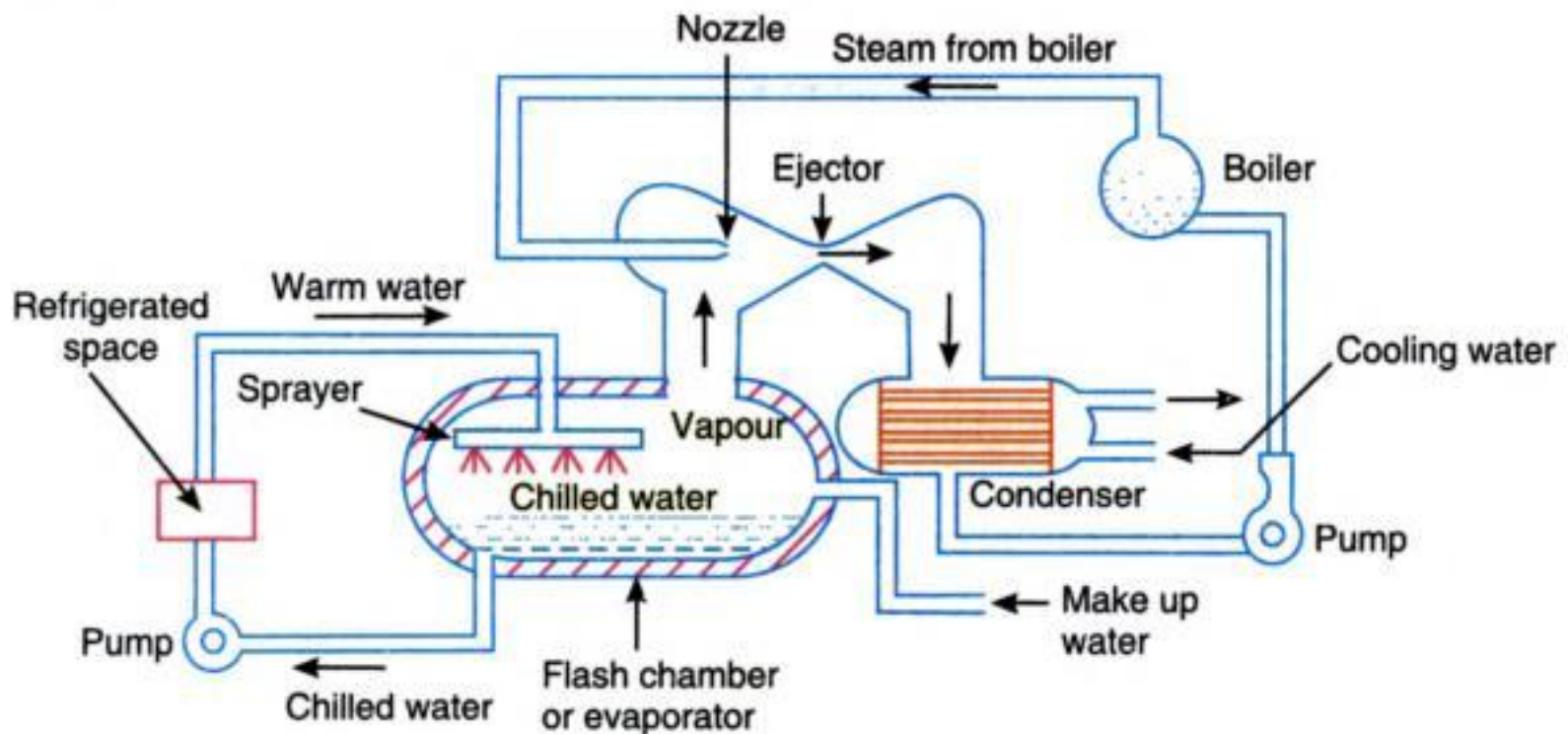
Fig. 15.1. Steam jet refrigeration system.

The main components of the steam jet refrigeration system, as shown in Fig. 15.1, are the flash chamber or evaporator, steam nozzles, ejector and condenser.

The flash chamber or evaporator is a large vessel and is heavily insulated to avoid the rise in temperature of water due to high ambient temperature. It is fitted with perforated pipes for spraying water. The warm water coming out of the refrigerated space is sprayed into the flash water chamber where some of which is converted into vapours after absorbing the latent heat, thereby cooling the rest of water.

The high pressure steam from the boiler is passed through the steam nozzles thereby increasing its velocity. This high velocity steam in the ejector would entrain the water vapours from the flash chamber which would result in further formation of vapours. The mixture of steam and water vapour passes through the venturi-tube of the ejector and gets compressed. The temperature and pressure of the mixture rises considerably and fed to the water cooled condenser where it gets condensed. The condensate is again fed to the boiler as feed water. A constant water level is maintained in the flash chamber and any loss of water due to evaporation is made up from the make-up water line.

Notes : 1. We have already discussed that the steam jet refrigeration system is based on the reduced boiling point at reduced pressure theory. In order to maintain the required reduced pressure in the flash chamber, the water vapours produced should be removed as early as possible. The removed vapour should then be compressed to the point where it condenses at a temperature above the temperature of the medium available for such purpose.

2. When the chilled water from the flash chamber is directly used in the form of spray for cooling the air, then it is known as *open cooling system*.

3. When the chilled water is passed through the coils and it does not come in contact with the air to be cooled, then it is known as *closed cooling system*.

15.5 Steam Ejector

The steam ejector is one of the important component of a steam jet refrigeration system. It is used to compress the water vapours coming out of the flash chamber. It uses the energy of fast moving jet of steam to entrain the vapours from the flash chamber and then compress it. The essential components of a steam ejector are shown in Fig. 15.2.

First of all, let us find the enthalpy at point C. We know that

$$m_v h_C + m_s h_D = (m_s + m_v) h_E$$

$$h_C + \frac{m_s}{m_v} \times h_D = \left(\frac{m_s}{m_v} + 1\right) h_E$$

$$h_C + 1.523 \times 2247.3 = (1.523 + 1) \, 2310.5$$

$$h_C + 3422.6 = 5829.4$$

∴ $\quad\quad\quad\quad\quad h_C = 2406.8 \text{ kJ/kg}$

We also know that enthalpy at point C (h_C),

$$2406.8 = h_{fC} + x_C \times h_{fB} = 18.9 + x_C \times 2490.9$$

$$\dots (\because h_{fC} = h_{fB} \text{ and } h_{fgC} = h_{fgB})$$

∴ $\quad\quad\quad\quad x_C = \dfrac{2406.8 - 18.9}{2490.9} = 0.96$ **Ans.**

3. Refrigerating effect per kg of flash vapour

We know that refrigerating effect per kg of flash vapour,

$$R_E = h_C - h_{fG} = 2406.8 - 75.5 = 2331.3 \text{ kJ/kg}$$

$$\dots (\because \text{From steam tables, } h_{fG} \text{ at } 18°C = 75.5 \text{ kJ/kg})$$

4. Mass of motive steam required per hour per tonne of refrigeration

We know that mass of motive steam required per hour per tonne of refrigeration

$$= \frac{210 \, Q}{h_C - h_{fG}} \times \frac{m_s}{m_v} = \frac{210 \times 1}{2406.8 - 75.5} \times 1.523$$

$$= 0.133 \text{ kg/min/TR} \quad\quad\quad \dots (\because Q = 1 \text{ TR})$$

$$= 0.133 \times 60 = 7.98 \text{ kg/h/TR} \textbf{ Ans.}$$

5. Volume of vapour removed from the flash chamber per hour per tonne of refrigeration

We know that volume of vapour (per kg) removed from the flash chamber,

$$v_C = \text{Volume of liquid at } C + x_C \text{ (Volume of saturated vapour}$$
$$- \text{ Volume of liquid)}$$

$$= 1 + 0.95 (152.22 - 1) = 144.66 \text{ m}^3/\text{kg}$$

. . . (From steam tables, volume of saturated vapour corresponding to 45°C = 152.22 m³/kg)

∴ Volume of vapour removed from the flash chamber per hour per tonne of refrigeration

$$= v_C \times \frac{210 \, Q}{h_C - h_{fG}} \times 60$$

$$= 144.66 \times \frac{210 \times 1}{2406.8 - 75.5} \times 60 = 782 \text{ m}^3/\text{h/TR} \textbf{ Ans.}$$

6. Coefficient of performance of the system

From steam tables, corresponding to a condenser pressure of 0.058 bar, we find that enthalpy of liquid at point G',

$$h_{fG'} = 148.8 \text{ kJ/kg}$$

We know that coefficient of performance of the system,

$$\text{C.O.P.} = \frac{m_v (h_C - h_{fG})}{m_s (h_A - h_{fG'})} = \frac{1 \, (2406.8 - 75.5)}{1.523 \, (2762 - 148.8)} = 0.586 \textbf{ Ans.}$$

QUESTIONS

1. What is the principle of a steam jet refrigeration system ?
2. Explain, with the help of a neat sketch, the working of a steam jet refrigeration system.
3. Draw the temperature-entropy and enthalpy-entropy diagram of a steam jet refrigeration system and write the expressions for the following efficiencies:
   (a) Nozzle efficiency ;
   (b) Entrainment efficiency ; and
   (c) Compression efficiency.
4. Derive an expression for finding out the mass of motive steam required per kg of water vapour produced.
5. What are the advantages and disadvantages of steam jet refrigeration system over other types of refrigeration system ?

OBJECTIVE TYPE QUESTIONS

1. In a steam jet refrigeration system, the motive steam expands in
   (a) convergent nozzle
   (b) divergent nozzle
   (c) convergent - divergent nozzle
   (d) any nozzle
2. The velocity of steam at the exit of the nozzle is
   (a) supersonic
   (b) sonic
   (c) sub-sonic
   (d) none of these
3. The compression device used in a steam jet refrigeration system is a
   (a) vapour compressor
   (b) steam ejector
   (c) diffuser
   (d) liquid pump
4. The ratio of isentropic enthalpy increase to the actual enthalpy increase required for the compression of the motive steam and the water vapours, is known as
   (a) nozzle efficiency
   (b) boiler efficiency
   (c) entrainment efficiency
   (d) compression efficiency
5. The coefficient of performance of the steam jet refrigeration system varies from
   (a) 0.5 to 0.8
   (b) 2 to 4
   (c) 5 to 10
   (d) none of these

ANSWERS

1. (c)　　2. (a)　　3. (b)　　4. (d)　　5. (a)

12. *Dew point depression.* It is the difference between the dry bulb temperature and dew point temperature of air.

13. *Psychrometer.* There are many types of psychrometers, but the sling psychrometer as shown in Fig. 16.2, is widely used. It consists of a dry bulb thermometer and a wet bulb thermometer mounted side by side in a protective case that is attached to a handle by a swivel connection so that the case can be easily rotated. The dry bulb thermometer is directly exposed to air and measures the actual temperature of the air. The bulb of the wet bulb thermometer is covered by a wick thoroughly wetted by distilled water. The temperature measured by this wick covered bulb of a thermometer is the temperature of liquid water in the wick and is called wet bulb temperature.

The sling psychrometer is rotated in the air for approximately one minute after which the readings from both the thermometers are taken. This process is repeated several times to assure that the lowest possible wet bulb temperature is recorded.
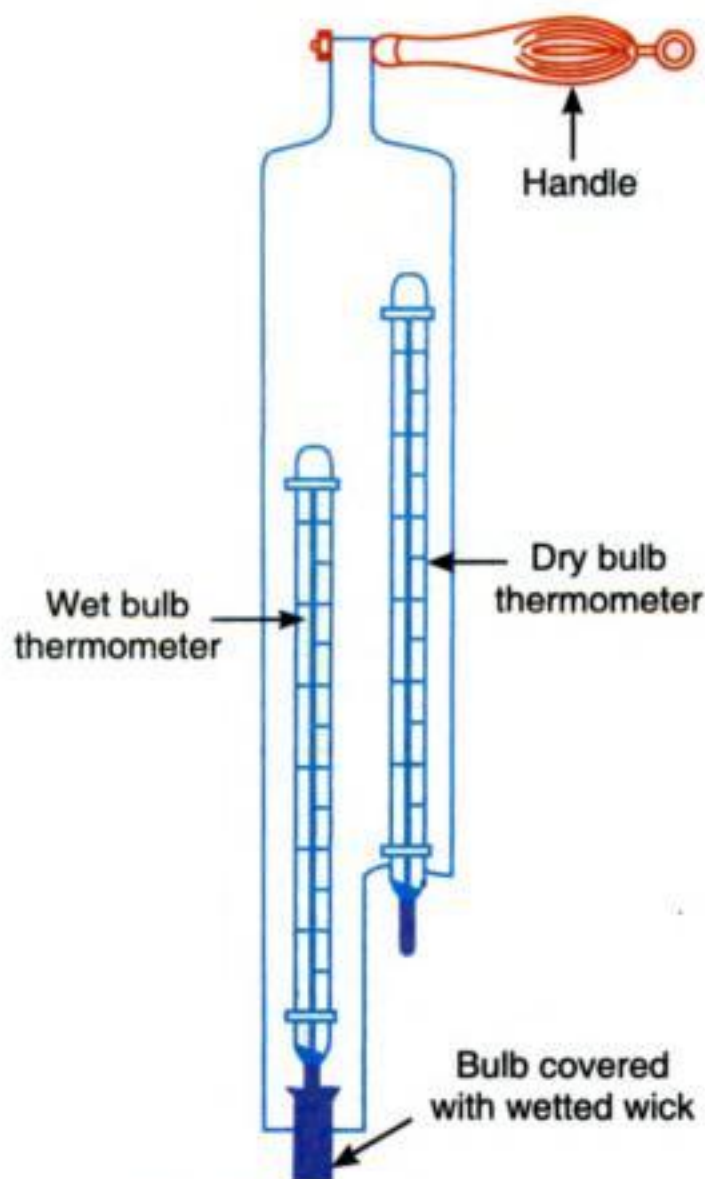
Fig. 16.2. Sling psychrometer.
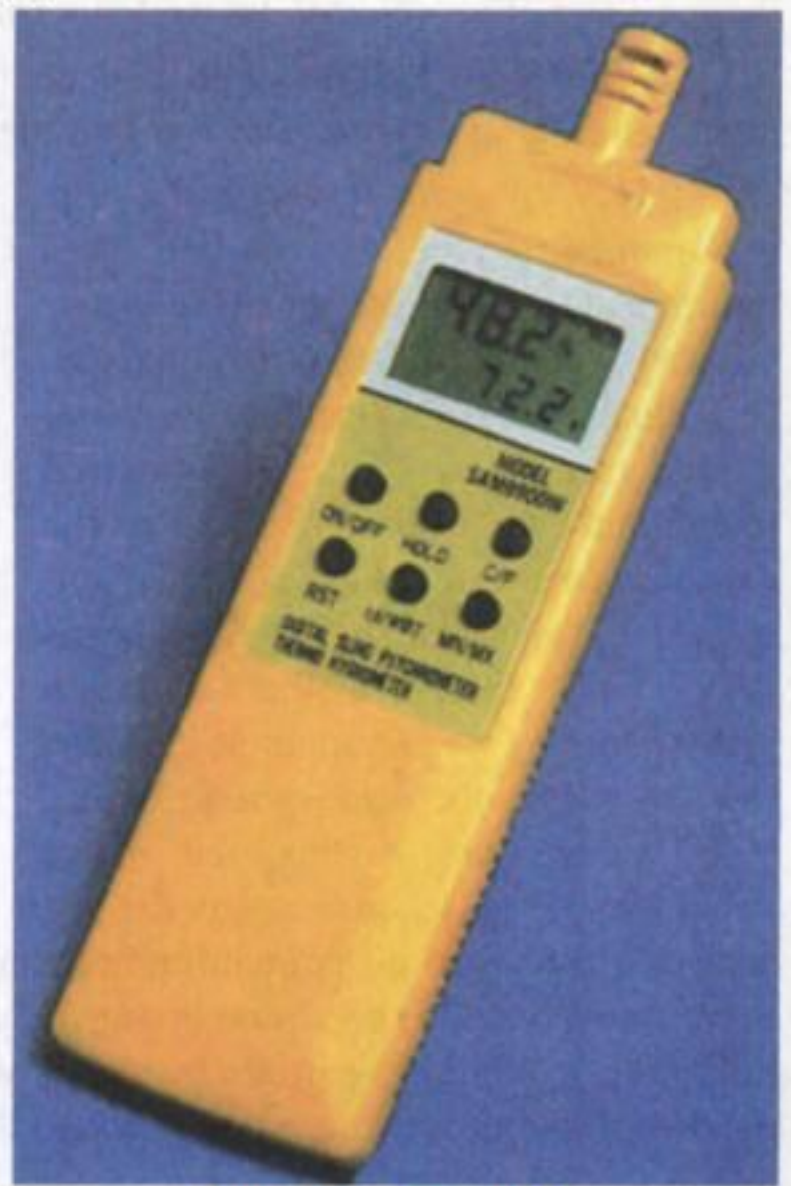
Digital psychrometer.

16.3 Dalton's Law of Partial Pressures

It states, "*The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent would exert, if it occupied the same space by itself*". In other words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Mathematically, barometric pressure of the mixture,

$$p_b = p_a + p_v$$

where p_a = Partial pressure of dry air, and
            p_v = Partial pressure of water vapour.

16.4 Psychrometric Relations

We have already discussed some psychrometric terms in Art. 16.2. These terms have some relations between one another. The following psychrometric relations are important from the subject point of view :

∴ Partial pressure of water vapour,

$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\, t_w}$$

$$= 0.023\,37 - \frac{(0.986\,42 - 0.02337)(30 - 20)}{1544 - 1.44 \times 20}$$

$$= 0.023\,37 - 0.006\,36 = 0.017\,01 \text{ bar}$$

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (p_v), therefore from steam tables, we find that corresponding to a pressure of 0.017 01 bar, the dew point temperature is

$$t_{dp} = 15° \text{C} \text{ **Ans.**}$$

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

$$p_s = 0.042\,42 \text{ bar}$$

We know that relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{0.01701}{0.04242} = 0.40 \text{ or } 40\% \text{ **Ans.**}$$

3. Specific humidity

We know that specific humidity,

$$W = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.01701}{0.986\,42 - 0.01701}$$

$$= \frac{0.01058}{0.96941} = 0.010\,914 \text{ kg/kg of dry air}$$

$$= 10.914 \text{ g/kg of dry air } \text{ **Ans.**}$$

4. Degree of saturation

We know that specific humidity of saturated air,

$$W_s = \frac{0.622\, p_s}{p_b - p_s} = \frac{0.622 \times 0.042\,42}{0.986\,42 - 0.042\,42}$$

$$= \frac{0.02638}{0.944} = 0.027\,945 \text{ kg/kg of dry air}$$

We know that degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{0.010\,914}{0.027\,945} = 0.391 \text{ or } 39.1\% \text{ **Ans.**}$$

Note : The degree of saturation (μ) may also be calculated from the following relation :

$$\mu = \frac{p_v}{p_s}\left(\frac{p_b - p_s}{p_b - p_v}\right)$$

$$= \frac{0.01701}{0.04242}\left[\frac{0.986\,42 - 0.042\,42}{0.986\,42 - 0.017\,01}\right]$$

$$= 0.391 \text{ or } 39.1\% \text{ **Ans.**}$$

Amount of water vapour condensed

If the temperature is 10° C, the air will be saturated before some water condenses out. From steam tables, we find that saturation pressure of vapour corresponding to 10° C is

$$p_s = p_v = 0.012\ 27 \text{ bar} \qquad \dots (\because \text{ Pressure is constant})$$

We know that humidity ratio,

$$W = \frac{0.622\ p_v}{p_b - p_v} = \frac{0.622 \times 0.012\ 27}{1 - 0.012\ 27} = \frac{0.007\ 632}{0.987\ 73}$$

$$= 0.007\ 73 \text{ kg/kg of dry air} = 7.73 \text{ g/kg of dry air}$$

We know that pressure of dry air,

$$p_a = p_b - p_v = 1 - 0.012\ 27 = 0.987\ 73 \text{ bar}$$

$$= 0.987\ 73 \times 10^5 = 987\ 73 \text{ N/m}^2$$

$\therefore$ Mass of dry air,

$$m_a = \frac{p_a\ v}{R_a\ T} = \frac{987\ 73 \times 112}{287\ (10 + 273)} = 136.2 \text{ kg}$$

and mass of water vapour,

$$m_v = W \times m_a = 0.007\ 73 \times 136.2 = 1.053 \text{ kg}$$

$\therefore$ Amount of water vapour condensed

$$= 3.61 - 1.053 = 2.557 \text{ kg} \quad \textbf{Ans.}$$

16.6 Thermodynamic Wet Bulb Temperature or Adiabatic Saturation Temperature

The thermodynamic wet bulb temperature or adiabatic saturation temperature is the temperature at which the air can be brought to saturation state, adiabatically, by the evaporation of water into the flowing air.

The equipment used for the adiabatic saturation of air, in its simplest form, consists of an insulated chamber containing adequate quantity of water. There is also an arrangement for extra water (known as make-up water) to flow into the chamber from its top, as shown in Fig. 16.4.
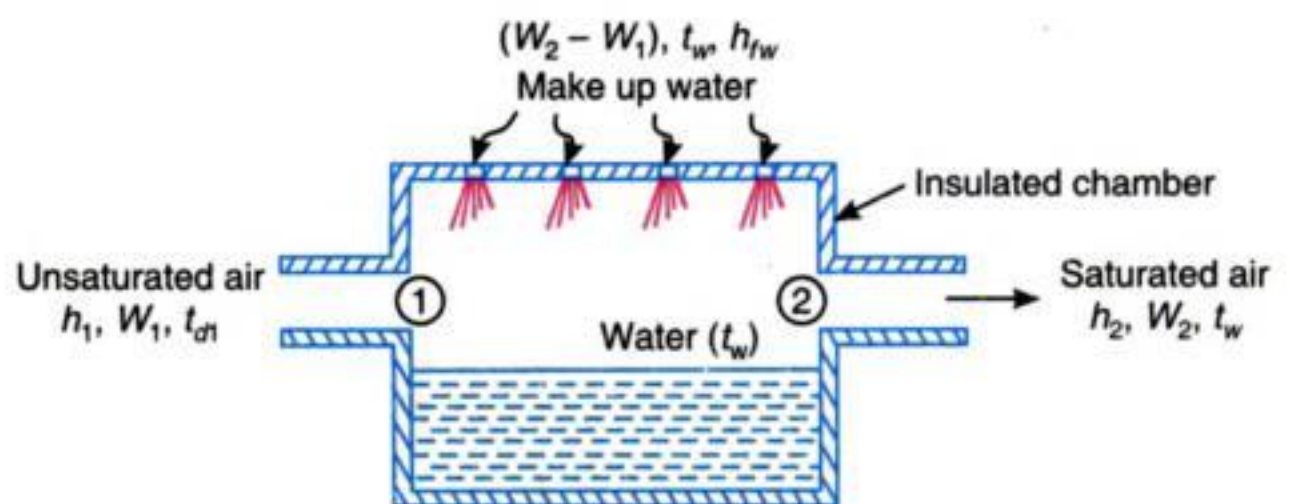
Fig. 16.4. Adiabatic saturation of air.

Let the unsaturated air enters the chamber at section 1. As the air passes through the chamber over a long sheet of water, the water evaporates which is carried with the flowing stream of air, and the specific humidity of the air increases. The make up water is added to the chamber at this temperature to make the water level constant. Both the air and water are cooled as the evaporation takes place. This process continues until the energy transfered from the air to the water is equal to the energy required to vaporise the water. When steady conditions are reached, the air flowing at section 2 is saturated with water vapour. The temperature of the saturated air at section 2 is known as *thermodynamic wet bulb temperature* or *adiabatic saturation temperature*.

Though the psychrometric chart has a number of details, yet the following lines are important from the subject point of view :

1. *Dry bulb temperature lines*. The dry bulb temperature lines are vertical *i.e.* parallel to the ordinate and uniformly spaced as shown in Fig. 16.7. Generally the temperature range of these lines on psychrometric chart is from − 6° C to 45° C. The dry bulb temperature lines are drawn with difference of every 5°C and up to the saturation curve as shown in the figure. The values of dry bulb temperatures are also shown on the saturation curve.

2. *Specific humidity or moisture content lines*. The specific humidity (moisture content) lines are horizontal *i.e.* parallel to the abscissa and are also uniformly spaced as shown in Fig. 16.8. Generally, moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry air (or from 0 to 0.030 kg / kg of dry air). The moisture content lines are drawn with a difference of every 1 g (or 0.001 kg) and up to the saturation curve as shown in the figure.
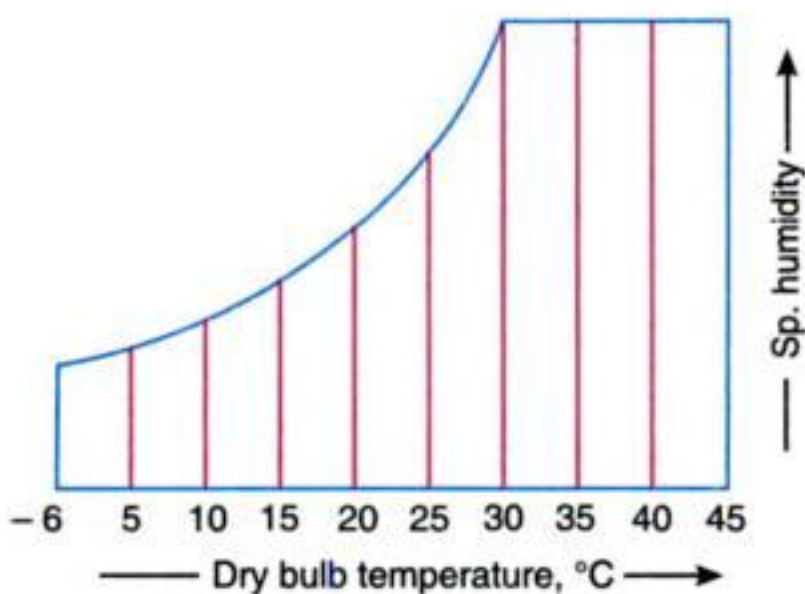
Fig. 16.7. Dry bulb temperature lines.
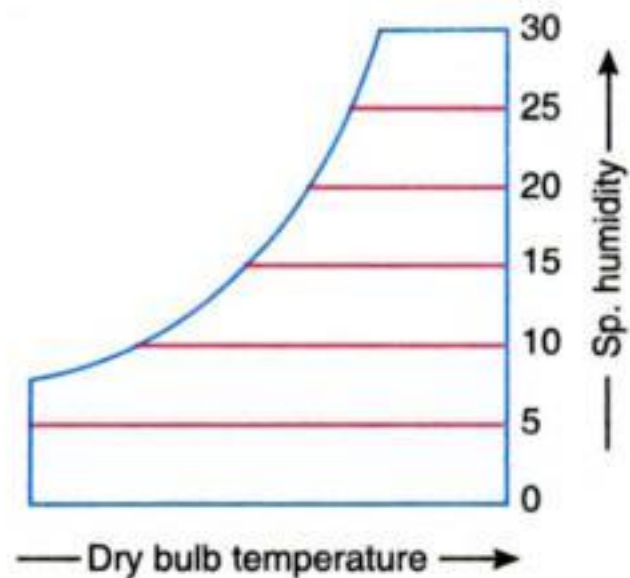
Fig. 16.8. Specific humidity lines.

3. *Dew point temperature lines*. The dew point temperature lines are horizontal *i.e.* parallel to the abscissa and non-uniformly spaced as shown in Fig. 16.9. At any point on the saturation curve, the dry bulb and dew point temperatures are equal.

The values of dew point temperatures are generally given along the saturation curve of the chart as shown in the figure.
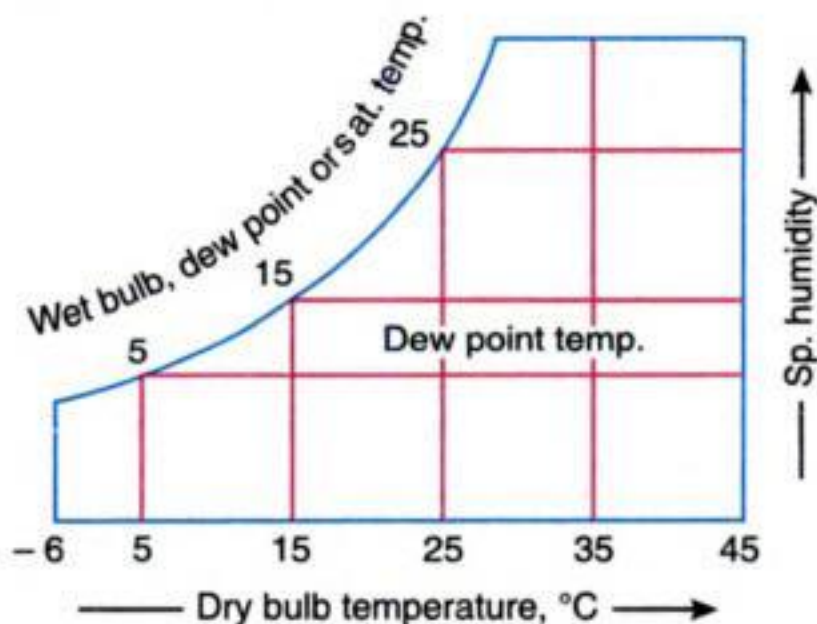
Fig. 16.9. Dew point temperature lines.
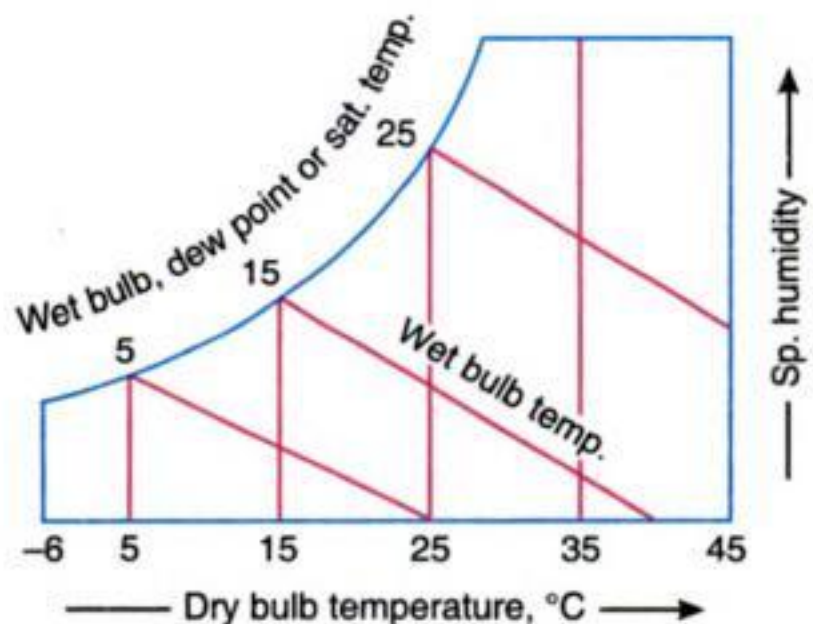
Fig. 16.10. Wet bulb temperature lines.

4. *Wet bulb temperature lines*. The wet bulb temperature lines are inclined straight lines and non-uniformly spaced as shown in Fig. 16.10. At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.

The values of wet bulb temperatures are generally given along the saturation curve of the chart as shown in the figure.

Heat rejected, $q = h_1 - h_2$

$$= c_{pa}(t_{d1} - t_{d2}) + W c_{ps}(t_{d1} - t_{d2})$$

$$= (c_{pa} + W c_{ps})(t_{d1} - t_{d2}) = c_{pm}(t_{d1} - t_{d2})$$

The term $(c_{pa} + W c_{ps})$ is called *humid specific heat* (c_{pm}) and its value is taken as 1.022 kJ /kg K.

∴ Heat rejected, $q = 1.022 (t_{d1} - t_{d2})$ kJ/kg

For air conditioning purposes, the sensible heat per minute is given as

$$SH = m_a c_{pm} \Delta t = v \rho c_{pm} \Delta t \text{ kJ/min} \qquad \dots (\because \ m = v \rho)$$

where

v = Rate of dry air flowing in m^3/min,

ρ = Density of moist air at 20° C and 50% relative humidity
= 1.2 kg / m^3 of dry air,

c_{pm} = Humid specific heat = 1.022 kJ /kg K, and

$\Delta t = t_{d1} - t_{d2}$ = Difference of dry bulb temperatures between the entering and leaving conditions of air in ° C.

Substituting the values of ρ and c_{pm} in the above expression, we get

$$SH = v \times 1.2 \times 1.022 \times \Delta t = 1.2264 \ v \times \Delta t \text{ kJ/min}$$

$$= \frac{1.2264 \ v \times \Delta t}{60} = 0.020 \ 44 \ v \times \Delta t \text{ kJ/s or kW}$$

$$\dots (\because \ 1 \text{ kJ/s} = 1 \text{ kW})$$

Notes : 1. For sensible cooling, the cooling coil may have refrigerant, cooling water or cool gas flowing through it.

2. The sensible cooling can be done only upto the dew point temperature (t_{dp}) as shown in Fig. 16.17 (b). The cooling below this temperature will result in the condensation of moisture.

16.11 By-pass Factor of Heating and Cooling Coil

We have already discussed that the temperature of the air coming out of the apparatus (t_{d2}) will be less than *t_{d3} in case the coil is a heating coil and more than t_{d3} in case the coil is a cooling coil.

Let 1 kg of air at temperature t_{d1} is passed over the coil having its temperature (*i.e.* coil surface temperature) t_{d3} as shown in Fig. 16.18.

A little consideration will show that when air passes over a coil, some of it (say x kg) just by-passes unaffected while the remaining $(1 - x)$ kg comes in direct contact with the coil. This by-pass process of air is measured in terms of a by-pass factor. The amount of air that by-passes or the by-pass factor depends upon the following factors :

1. The number of fins provided in a unit length *i.e.* the pitch of the cooling coil fins ;

2. The number of rows in a coil in the direction of flow; and

3. The velocity of flow of air.

It may be noted that the by-pass factor of a cooling coil decreases with decrease in fin spacing and increase in number of rows.

---

* Under ideal conditions, the dry bulb temperature of the air leaving the apparatus (t_{d2}) should be equal to that of the coil (t_{d3}). But it is not so, because of the inefficiency of the coil. This phenomenon is known as *by-pass factor*

and enthalpy of air at point 2,
$$h_2 = 46 \text{ kJ/kg of dry air}$$
We know that amount of air supplied,
$$*m_a = \frac{v_1}{v_{s1}} = \frac{150}{0.81} = 185.2 \text{ m}^3/\text{min}$$

∴   Rate of heat transfer
$$= m_a (h_2 - h_1) = 185.2 (46 - 26) = 3704 \text{ kJ/min} \text{ Ans.}$$

Example 16.10. *Atmospheric air with dry bulb temperature of 28° C and a wet bulb temperature of 17° C is cooled to 15° C without changing its moisture content. Find : 1. Original relative humidity ; 2. Final relative humidity ; and 3. Final wet bulb temperature.*

Solution. Given : $t_{d1} = 28° \text{ C}$; $t_{w1} = 17° \text{ C}$; $t_{d2} = 15° \text{ C}$

The initial condition of air, *i.e.* 28° C dry bulb temperature and 17° C wet bulb temperature is marked on the psychrometric chart at point 1, as shown in Fig. 16.23. Now mark the final condition of air by drawing a horizontal line through point 1 (because there is no change in moisture content of the air) to meet the 15° C dry bulb temperature line at point 2, as shown in Fig. 16.23.

Fig. 16.23

1. Original relative humidity

From the psychrometric chart, we find that the original relative humidity at point 1,
$$\phi_1 = 34\% \text{ Ans.}$$

2. Final relative humidity

From psychrometric chart, we find that the final relative humidity at point 2,
$$\phi_2 = 73\% \text{ Ans.}$$

3. Final wet bulb temperature

From the psychrometric chart, we find that the final wet bulb temperature at point 2,
$$t_{w2} = 12.2° \text{ C} \text{ Ans.}$$

Example 16.11. *The moist air is heated by steam condensing inside the tubes of a heating coil as shown in Fig. 16.24. The part of the air passes through the coil and part is by-passed around the coil. The barometric pressure is 1 bar. Determine : 1. The air per minute (in 3') which by-pass the coil ; and 2. The heat added by the coil.*

---

*   The amount of air supplied (m_a) may also be obtained as discussed below :

From steam tables, we find that saturation pressure of vapour corresponding to dry bulb temperature of 10° C is
$$p_{s1} = 0.012\ 27 \text{ bar}$$

We know that partial pressure of vapour,
$$p_{v1} = \phi_1 \times p_{s1} = 0.8 \times 0.012\ 27 = 0.009\ 82 \text{ bar} \qquad \dots (\because \ \phi_1 = p_{v1}/p_{s1})$$

∴
$$m_a = \frac{(p_b - p_{v1}) v_1}{R_a T_{d1}} = \frac{(1.013 - 0.009\ 82)\ 10^5 \times 150}{287 (273 + 10)} = 185.2 \text{ m}^3/\text{min}$$
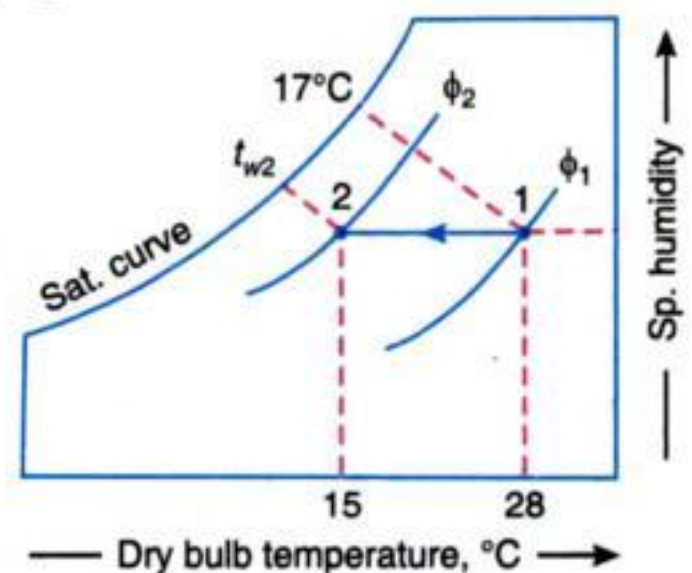
16.15 Sensible Heat Factor

As a matter of fact, the heat added during a psychrometric process may be split up into sensible heat and latent heat. The ratio of the *sensible heat to the total heat is known as *sensible heat factor* (briefly written as *SHF*) or *sensible heat ratio* (briefly written as *SHR*). Mathematically,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH}$$

where SH = Sensible heat, and

LH = Latent heat.

The sensible heat factor scale is shown on the right hand side of the psychrometric chart.

16.16 Cooling and Dehumidification

This process is generally used in summer air conditioning to cool and dehumidify the air. The air is passed over a cooling coil or through a cold water spray. In this process, the dry bulb temperature as well as the specific humidity of air decreases. The final relative humidity of the air is generally higher than that of the entering air. The dehumidification of air is only possible when the effective surface temperature of the cooling coil (*i.e.* t_{d4}) is less than the dew point temperature of the air entering the coil (*i.e.* t_{dp1}). The effective surface temperature of the coil is known as *apparatus dew point* (briefly written as *ADP*). The cooling and dehumidification process is shown in Fig. 16.29.
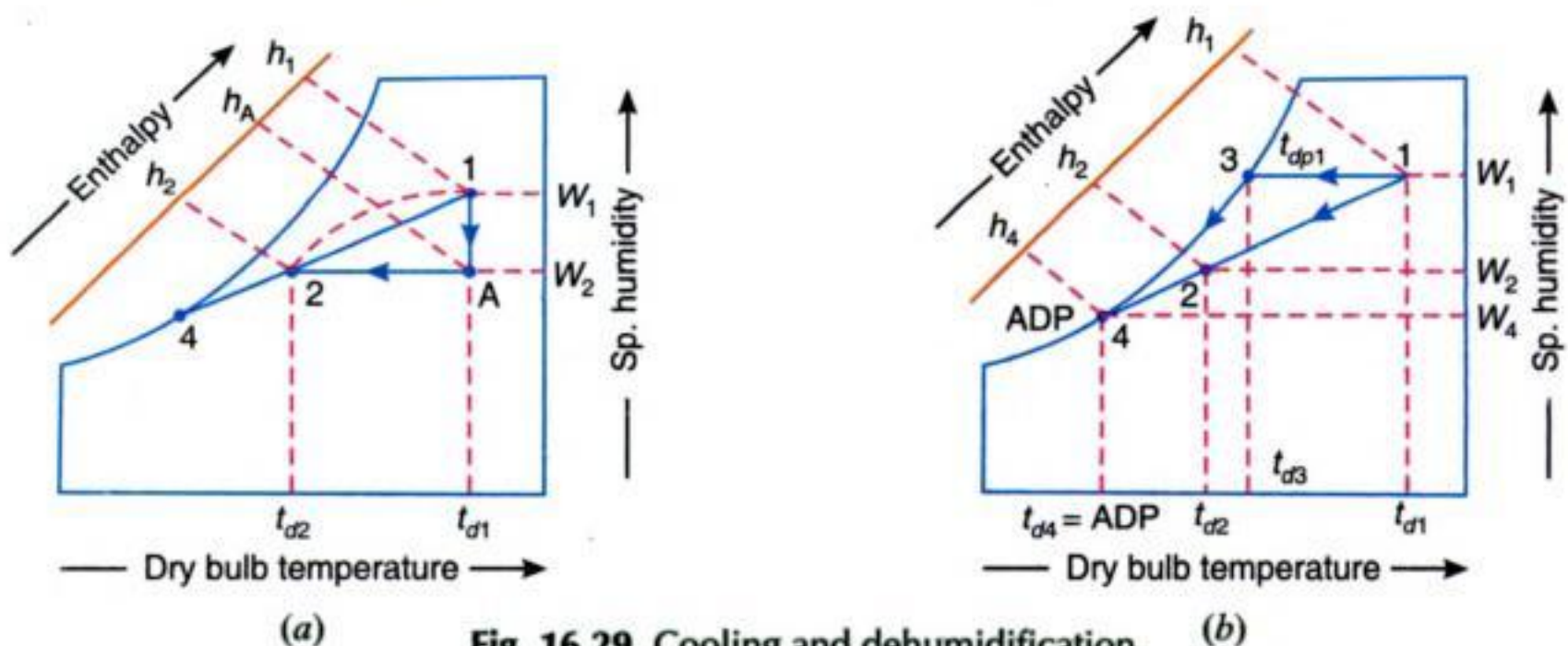
Fig. 16.29. Cooling and dehumidification.

Let t_{d1} = Dry bulb temperature of air entering the coil,

t_{dp1} = Dew point temperature of the entering air = t_{d3}, and

t_{d4} = Effective surface temperature or *ADP* of the coil.

Under ideal conditions, the dry bulb temperature of the air leaving the cooling coil (*i.e.* t_{d4}) should be equal to the surface temperature of the cooling coil (*i.e.* *ADP*), but it is never possible due to inefficiency of the cooling coil. Therefore, the resulting condition of air coming out of the coil is shown by a point 2 on the straight line joining the points 1 and 4. The by-pass factor in this case is given by

$$BPF = \frac{t_{d2} - t_{d4}}{t_{d1} - t_{d4}} = \frac{t_{d2} - ADP}{t_{d1} - ADP}$$

Also $$BPF = \frac{W_2 - W_4}{W_1 - W_4} = \frac{h_4 - h_2}{h_1 - h_2}$$

* Refer also Chapter 18, Art 18.13.

Now from the psychrometric chart, we find that corresponding to 15°C dry bulb temperature and 0.006 75 kg/kg of dry air of specific humidity, the relative humidity at point 1, is

$$\phi_1 = 65\% \quad \text{Ans.}$$

Example 16.18. *Determine the final dry bulb temperature and relative humidity of air washed with recirculated spray water if the air is initially at dry bulb temperature 35° C and 50% relative humidity as it enters an air washer which has humidifying efficiency of 85 per cent.*

Solution. Given : $t_{d1} = 35°$ C ; $\phi_1 = 50\%$; $\eta_H = 85\% = 0.85$

First of all, mark the initial condition of air at 35° C dry bulb temperature and 50% relative humidity on the psychrometric chart at point 1, as shown in Fig. 16.38. The wet bulb temperature of the entering air as read from the psychrometric chart is

$$t_{w1} = 26.1° \text{ C} = t_{d3}$$

Fig. 16.38

Final dry bulb temperature

Let t_{d2} = Final dry bulb temperature of the air leaving the air washer.

We know that humidifying efficiency of an air washer (η_H),

$$0.85 = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}} = \frac{35 - t_{d2}}{35 - 26.1} = \frac{35 - t_{d2}}{8.9}$$

∴ $$t_{d2} = 35 - 0.85 \times 8.9 = 27.435° \text{ C} \quad \text{Ans.}$$

Final relative humidity

On the constant wet bulb temperature line 1-3, mark point 2 such that $t_{d2} = 27.435°$C. Now the relative humidity of the air leaving the air washer (corresponding to point 2) as read from the psychrometric chart is

$$\phi_2 = 90\% \quad \text{Ans.}$$

Example 16.19. *At a certain locality, the dry bulb temperature of air is 30° C and the relative humidity is 40%. Determine the specific humidity and the dew point and wet bulb temperatures of air. If this air is cooled in an air washer using recirculated spray water and having a humidifying efficiency of 0.9, what are dry bulb temperature and dew point temperature of air leaving the air washer ?*

Solution. Given : $t_{d1} = 30°$ C ; $\phi_1 = 40\%$; $\eta_H = 0.9$

Specific humidity, dew point and wet bulb temperature of air.

First of all, mark the initial condition of air at 30° C dry bulb temperature and 40% relative humidity, on the psychrometric chart at point 1, as shown in Fig. 16.39. From the psychrometric chart, we find that

Specific humidity of air,
$$W_1 = 0.0106 \text{ kg/kg of dry air} \quad \text{Ans.}$$

Dew point temperature of air,
$$t_{dp1} = 15° \text{ C} \quad \text{Ans.}$$

Fig. 16.39

2. Total heating required

From the psychrometric chart, we find that enthalpy of air at point 1,

$$h_1 = 27.2 \text{ kJ/kg of dry air}$$

Enthalpy of air at point A,

$$h_A = 51 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 2,

$$h_2 = 68 \text{ kJ/kg of dry air}$$

We know that heat required for preheating of air

$$= h_A - h_1 = 51 - 27.2 = 23.8 \text{ kJ/kg of dry air}$$

and heat required for reheating of air

$$= h_2 - h_B = 68 - 51 = 17 \text{ kJ/kg of dry air} \qquad \dots (\because h_B = h_A)$$

∴ Total heat required

$$= 23.8 + 17 = 40.8 \text{ kJ/kg of dry air} \textbf{ Ans.}$$

3. Make up water required in the air washer

From the psychrometric chart, we find that specific humidity of entering air,

$$W_1 = 0.0068 \text{ kg/kg of dry air}$$

and specific humidity of leaving air,

$$W_2 = 0.0122 \text{ kg/kg of dry air}$$

∴ Make up water required in the air washer

$$= W_B - W_A = W_2 - W_1$$
$$= 0.0122 - 0.0068 = 0.0054 \text{ kg/kg of dry air} \textbf{ Ans.}$$

4. Humidifying efficiency of the air washer

From the psychrometric chart, we find that

$$t_{dB} = 19.1° \text{C} \quad \text{and} \quad t_{dB'} = 18° \text{C}$$

We know that humidifying efficiency of the air washer,

$$\eta_H = \frac{\text{Actual drop in DBT}}{\text{Ideal drop in DBT}} = \frac{t_{dA} - t_{dB}}{t_{dA} - t_{dB'}}$$

$$= \frac{32.6 - 19.1}{32.6 - 18} = \frac{13.5}{14.6} = 0.924 \text{ or } 92.4\% \textbf{ Ans.}$$

16.21 Heating and Dehumidification -Adiabatic Chemical Dehumidification

This process is mainly used in industrial air conditioning and can also be used for some comfort air conditioning installations requiring either a low relative humidity or low dew point temperature in the room.

In this process, the air is passed over chemicals which have an affinity for moisture. As the air comes in contact with these chemicals, the moisture gets condensed out of the air and gives up its latent heat. Due to the condensation, the specific humidity decreases and the heat of condensation supplies sensible heat for heating the air and thus

Fig. 16.46. Heating and dehumidification.

7. Prove that the enthalpy of the humid air remains constant along a wet bulb temperature line on the psychrometric chart.

8. When is dehumidification of air necessary and how it is achieved.

9. Write a short note on by-pass factor for cooling coils.

10. Define sensible heat factor.

11. Show the following processes on the skeleton psychrometric chart :

   (a) Dehumidification of moist air by cooling ; and

   (b) Adiabatic mixing of two air streams.

12. What is fog ? Show on the psychrometric chart when two air streams yield fogged state of air.

OBJECTIVE TYPE QUESTIONS

1. A mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it, is called

   (a) dry air   (b) moist air   (c) saturated air   (d) specific humidity

2. The temperature of air recorded by a thermometer, when it is not effected by the moisture present in it, is called

   (a) wet bulb temperature   (b) dry bulb temperature

   (c) dew point temperature   (d) none of these

3. For unsaturated air, the dew point temperature is .... wet bulb temperature.

   (a) equal to   (b) less than   (c) more than

4. The difference between dry bulb temperature and wet bulb temperature, is called

   (a) dry bulb depression   (b) wet bulb depression

   (c) dew point depression   (d) degree of saturation

5. The wet bulb depression is zero, when relative humidity is equal to

   (a) zero   (b) 0.5   (c) 0.75   (d) 1.0

6. The relative humidity of air is defined as the ratio of

   (a) mass of water vapour in a given volume to the total mass of the mixture of air and water vapour

   (b) mass of water vapour in a given volume to the mass of water vapour, if air is saturated at the same temperature

   (c) mass of water vapour in a given volume to the mass of air

   (d) mass of air to the mass of water vapour in the mixture of air and water vapour

7. The vertical and uniformly spaced lines on a psychrometric chart indicates

   (a) dry bulb temperature   (b) wet bulb temperature

   (c) dew point temperature   (d) specific humidity

8. The curved lines on a psychrometric chart indicates

   (a) dry bulb temperature   (b) wet bulb temperature

   (c) specific humidity   (d) relative humidity

9. During sensible cooling of air, the specific humidity

   (a) increases   (b) decreases   (c) remains constant

10. During sensible cooling of air, the dry bulb temperature

   (a) increases   (b) decreases   (c) remains constant

11. During sensible cooling of air, the wet bulb temperature

   (a) increases   (b) decreases   (c) remains constant

12. The minimum temperature to which moist air can be cooled under ideal conditions in a spray washer is

   (a) dew point temperature of inlet air   (b) wet bulb temperature of inlet air

   (c) water inlet temperature   (d) water outlet temperature

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3. According to the arrangement of equipment

    (a) Unitary air conditioning system, and

    (b) Central air conditioning system.

In this chapter, we shall discuss all the above mentioned air conditioning systems, one by one.

18.6 Comfort Air Conditioning System

In comfort air conditioning, the air is brought to the required dry bulb temperature and relative humidity for the human health, comfort and efficiency. If sufficient data of the required condition is not given, then it is assumed to be 21°C dry bulb temperature and 50% relative humidity. The sensible heat factor is, generally, kept as following :

For residence or private office	= 0.9
For restaurant or busy office	= 0.8
Auditorium or cinema hall	= 0.7
Ball room dance hall etc.	= 0.6

The comfort air conditioning may be adopted for homes, offices, shops, restaurants, theaters, hospitals, schools etc.

Example 18.1. *An air conditioning plant is required to supply 60 m^3 of air per minute at a DBT of 21°C and 55% RH. The outside air is at DBT of 28°C and 60% RH. Determine the mass of water drained and capacity of the cooling coil. Assume the air conditioning plant first to dehumidify and then to cool the air.*

Solution. Given : $v_2 = 60$ m³/min ; $t_{d2} = 21°C$; $\phi_2 = 55\%$; $t_{d1} = 28°C$; $\phi_1 = 60\%$

Mass of water drained

First of all, mark the initial condition of air at 28°C dry bulb temperature and 60% relative humidity on the psychrometric chart as point 1, as shown in Fig. 18.1. Now mark the final condition of air at 21°C dry bulb temperature and 55% relative humidity as point 2. From the psychrometric chart, we find that

Fig. 18.1

    Specific humidity of air at point 1,

$$W_1 = 0.0142 \text{ kg / kg of dry air}$$

    Specific humidity of air at point 2,

$$W_2 = 0.0084 \text{ kg / kg of dry air}$$

and specific volume of air at point 2,

$$v_{s2} = 0.845 \text{ m}^3 \text{/ kg of dry air}$$

We know that mass of air circulated,

$$m_a = \frac{v_2}{v_{s2}} = \frac{60}{0.845} = 71 \text{ kg / min}$$

∴ Mass of water drained

$$= m_a (W_1 - W_2) = 71(0.0142 - 0.0084) = 0.412 \text{ kg / min}$$
$$= 0.412 \times 60 = 24.72 \text{ kg / h } \textbf{Ans.}$$

Capacity of the cooling coil

From the psychrometric chart, we find that

Enthalpy of air at point 1,

$$h_1 = 64.8 \text{ kJ / kg of dry air}$$
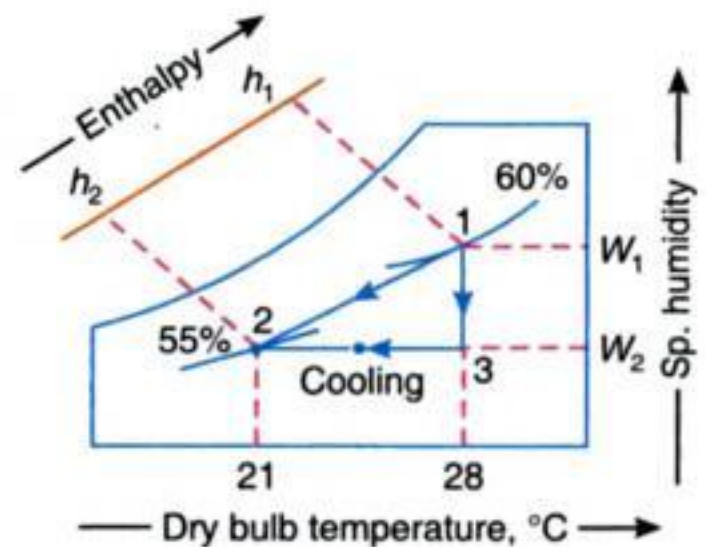
1. The capacity of the cooling coil and surface temperature of the coil if the by-pass factor is 0.25 ; and

2. The capacity of the humidifier and its efficiency.

Solution. Given : t_{d1} = 40°C ; t_{w1} = 20°C ; t_{d2} = 20°C ; ϕ_2 = 50% ; Seating capacity = 1000 ; v_1' = 0.3 m³/min/person = 0.3 × 1000 = 300 m³/min ; BPF = 0.25

First of all, mark the outdoor conditions of air *i.e.* at 40°C dry bulb temperature and 20°C wet bulb temperature on the psychrometric chart as point 1, as shown in Fig. 18.8. Now mark the required comfort conditions of air *i.e.* at 20°C dry bulb temperature and 50% relative humidity, as point 2. From point 1, draw a constant wet bulb temperature line and from point 2 draw a constant specific humidity line. Let these two lines intersect at point 3. The line 1-3 represents adiabatic humidification and the line 3-2 represents sensible cooling.
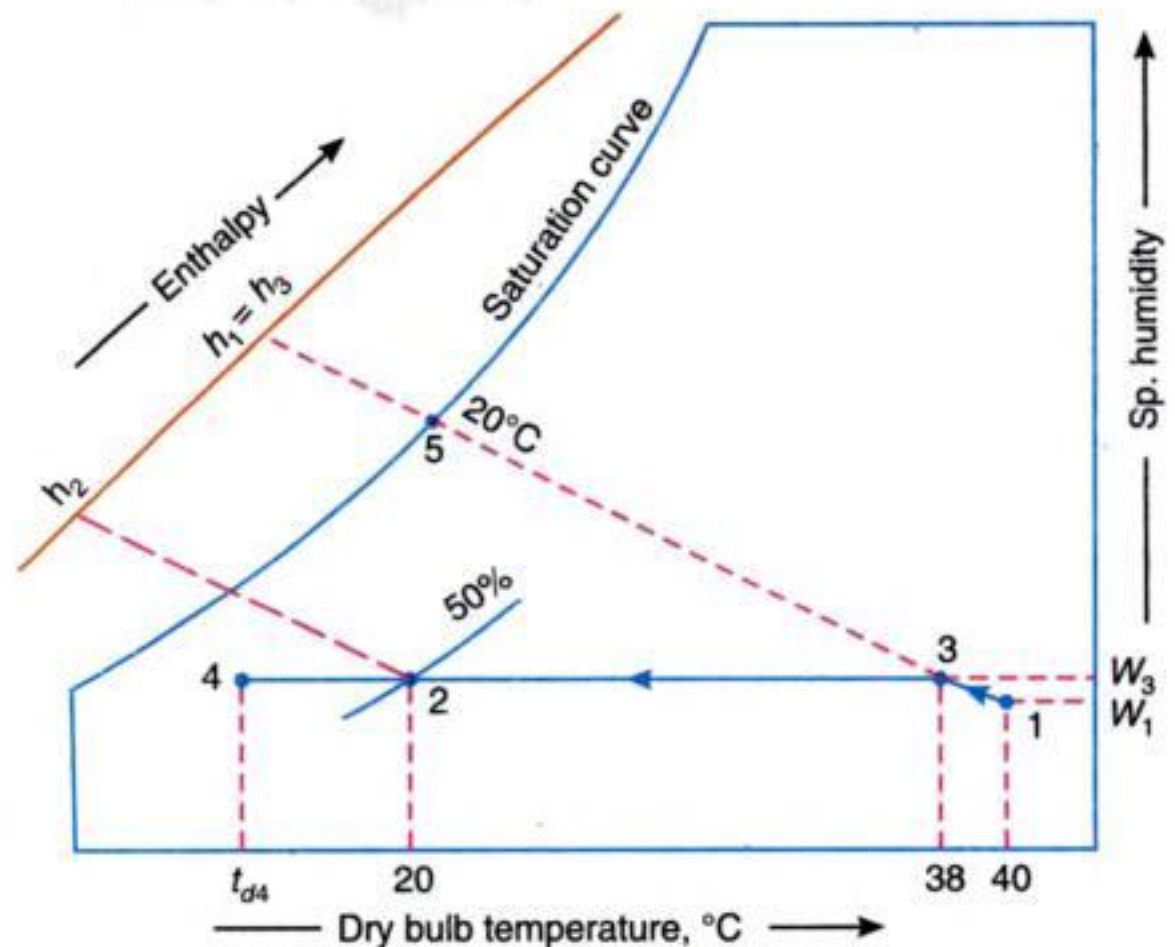
Fig. 18.8

From the psychrometric chart, we find that specific volume of air at point 1,

$$v_{s1} = 0.896 \text{ m}^3/\text{kg of dry air}$$

∴ Mass of air supplied,

$$m_a = \frac{v_1}{v_{s1}} = \frac{300}{0.896} = 334.8 \text{ kg/min}$$

1. Capacity of the cooling coil and surface temperature of the coil

From the psychrometric chart, we find that enthalpy of air at point 3,

$$h_3 = 57.6 \text{ kJ/kg of dry air}$$

Enthalpy of air at point 2,

$$h_2 = 39 \text{ kJ/kg of dry air}$$

Dry bulb temperature of air after humidification *i.e.* at point 3,

$$t_{d3} = 38°C$$

We know that capacity of the cooling coil

$$= m_a (h_3 - h_2) = 334.8 (57.6 - 39) = 6227 \text{ kJ/min}$$
$$= 6227 / 210 = 29.6 \text{ TR } \textbf{Ans.}$$

Let t_{d4} = Surface temperature of the coil

We know that by-pass factor (*BPF*),

$$0.25 = \frac{t_{d2} - t_{d4}}{t_{d3} - t_{d4}} = \frac{20 - t_{d4}}{38 - t_{d4}}$$

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and
$$ERTH = 0.02 \, v_d \, (h_2 - h_{ADP}) \, (1 - BPF) \text{ kW}$$

where
- v_d = Volume of dehumidified air to room or space in m³/min,
- ADP = Apparatus dew point in °C,
- W_{ADP} = Specific humidity at apparatus dew point in kg / kg of dry air, and
- h_{ADP} = Enthalpy at apparatus dew point in kJ / kg of dry air.

2. The mass of dehumidified air is given by

$$m_d = \frac{\text{Room total heat}}{h_2 - h_4}$$

where
- h_2 = Enthalpy of air at room condition, and
- h_4 = Enthalpy of supply air to room from the cooling coil.

Example 18.10. *A room has a sensible heat gain of 24 kW and a latent heat gain of 5.2 kW and it has to be maintained at 26°C DBT and 50% RH. 180 m³/min of air is delivered to the room. Determine the state of supply air.*

Solution. Given : $RSH = 24$ kW ; $RLH = 5.2$ kW ; $t_{d1} = 26°C$; $\phi_1 = 50\%$; $v = 180$ m³/min

Let t_{d2} = Dry bulb temperature of the supply air.

We know that room sensible heat load,

$$RSH = 0.020\,44 \, v \, (t_{d1} - t_{d2})$$
$$24 = 0.020\,44 \times 180 \, (26 - t_{d2})$$
$$= 3.68 \, (26 - t_{d2})$$
$$\therefore \quad t_{d2} = 19.5°C$$

We also know that room sensible heat factor,

$$RSHF = \frac{RSH}{RSH + RLH}$$
$$= \frac{24}{24 + 5.2} = 0.822$$
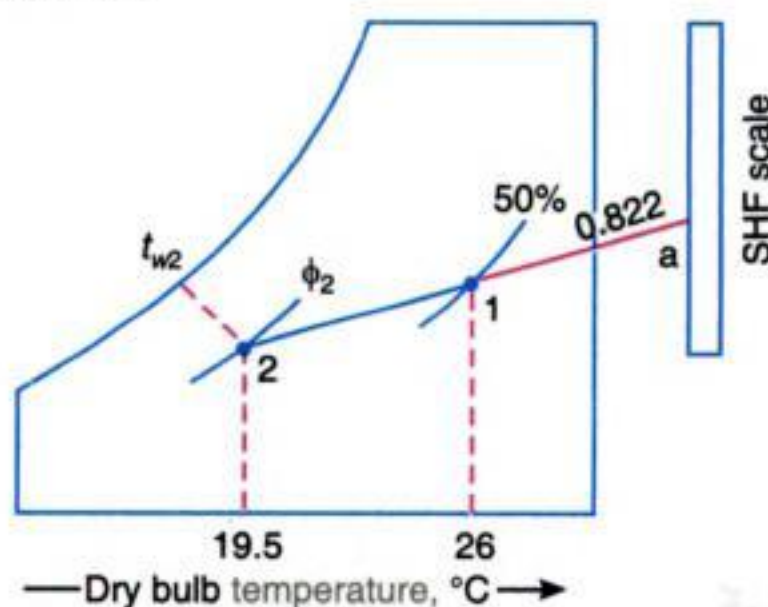
Fig. 18.17

First of all, mark room condition of the air *i.e.* 26°C dry bulb temperature and 50% relative humidity, as point 1 on the psychrometric chart, as shown in Fig. 18.17. The point 1 also represents the alignment circle (*i.e.* 26°C *DBT* and 50% *RH*). Now mark the calculated value of *RSHF* = 0.822 on the sensible heat factor scale as point *a* and join with the alignment circle *i.e.* point 1.

Produce this line upto point 2 such that $t_{d2} = 19.5°C$. The point 2 represents the supply condition of air. From the psychrometric chart, we find that wet bulb temperature of supply air,

$$t_{w2} = 16° \text{ C \textbf{Ans.}}$$

and relative humidity,
$$\phi_2 = 71\% \text{ \textbf{Ans.}}$$

Example 18.11. *An air conditioned auditorium is to be maintained at 27°C dry bulb temperature and 60% relative humidity. The ambient condition is 40°C dry bulb temperature and 30°C wet bulb temperature. The total sensible heat load is 100 000 kJ/h and the total latent heat load is 40 000 kJ/h. 60% of the return air is recirculated and mixed with 40% of make-up air after the cooling coil. The condition of air leaving the cooling coil is at 18°C.*

Determine : 1. Room sensible heat factor ; 2. The condition of air entering the auditorium ; 3. The amount of make-up air ; 4. Apparatus dew point ; and 5. By-pass factor of the cooling coil. Show the processes on the psychrometric chart.

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and enthalpy of air at point A,

$$h_A = 72.8 \text{ kJ/kg of dry air}$$

Also specific volume of air at point 1,

$$v_{s1} = 0.907 \text{ m}^3/\text{kg of dry air}$$

∴ Mass of air infiltrated into the hall,

$$m_a = \frac{v_1}{v_{s1}} = \frac{25}{0.907} = 27.56 \text{ kg/min}$$

Sensible heat load due to the infiltrated air,

$$Q_{S1} = m_a (h_A - h_4) = 27.56 (72.8 - 61) = 325.21 \text{ kJ/min}$$

$$= 325.21/60 = 5.42 \text{ kW}$$

and latent heat load due to the infiltrated air,

$$Q_{L1} = m_a (h_1 - h_A) = 27.56 (85 - 72.8) = 336.23 \text{ kJ/min}$$

$$= 336.23/60 = 5.6 \text{ kW}$$

∴ Total room sensible heat load,

$$RSH = Q_{S4} + Q_{S1} = 46.5 + 5.42 = 51.92 \text{ kW}$$

and total room latent heat load

$$RLH = Q_{L4} + Q_{L1} = 17.5 + 5.6 = 23.1 \text{ kW}$$

We know that room sensible heat factor,

$$RSHF = \frac{RSH}{RSH + RLH} = \frac{51.92}{51.92 + 23.1} = 0.692$$

Now mark this calculated value of *RSHF* on the sensible heat factor scale and join with the alignment circle (*i.e.* 26°C *DBT* and 50% *RH*) as shown in Fig. 18.21. From point 4, draw a line 4-5 (known as *RSHF* line) parallel to this line. Since the outside air marked at point 1 is passed through the cooling coil whose *ADP* = 15°C, therefore join point 1 with *ADP* = 15°C on the saturation curve. This line is the *GSHF* line and intersects the *RSHF* line at point 2, which represents the condition of air leaving the cooling coil. Also 60% of the air from the hall is recirculated and mixed with the conditioned air after the cooling coil. The mixing condition of air is shown at point 3 such that

$$\frac{\text{Length 2-3}}{\text{Length 2-4}} = 0.6$$

1. *Condition of air after the coil and before the recirculated air mixes with it*

The condition of air after the coil and before the recirculated air mixes with it, is shown by point 2 on the psychrometric chart, as shown in Fig. 18.21. At point 2, we find that

Dry bulb temperature, $\quad t_{d2} = 19°C$ **Ans.**

Wet bulb temperature, $\quad t_{w2} = 17.5°C$ **Ans.**

2. *Condition of air entering the hall, i.e. after mixing with recirculated air*

The condition of air entering the hall, *i.e.* after mixing with recirculated air is shown by point 3 on the psychrometric chart, as shown in Fig. 18.21. At point 3, we find that

Dry bulb temperature, $\quad t_{d3} = 24°C$ **Ans.**

Wet bulb temperature, $\quad t_{w3} = 19.8°C$ **Ans.**

∴ Effective room sensible heat factor,

$$ERSHF = \frac{ERSH}{ERSH + ERLH} = \frac{26}{26 + 7.25} = 0.782 \text{ Ans.}$$

The line 2-6 represents the effective room sensible heat factor (*ERSHF*) line.

Example 18.14. *The following data refer to summer air conditioning of a building :*

Outside design conditions	= 43°C DBT, 27°C WBT
Inside design conditions	= 25°C DBT, 50% RH
Room sensible heat gain	= 84 000 kJ / h
Room latent heat gain	= 21 000 kJ / h
By-pass factor of the cooling coil used	= 0.2

The return air from the room is mixed with the outside air before entry to cooling coil in the ratio of 4 : 1 by mass. Determine : (a) Apparatus dew point of the cooling coil ; (b) Entry and exit conditions of air for cooling coil ; (c) Fresh air mass flow rate; and (d) Refrigeration load on the cooling coil.

Solution. Given : $t_{d1} = 43°C$; $t_{w1} = 27°C$; $t_{d2} = 25°C$; $\phi_2 = 50\%$; $RSH = 84\,000$ kJ / h ; $RLH = 21\,000$ kJ / h ; $BPF = 0.2$

The flow diagram for the conditioned space is shown in Fig. 18.24 and it is represented on the psychrometric chart as discussed below :
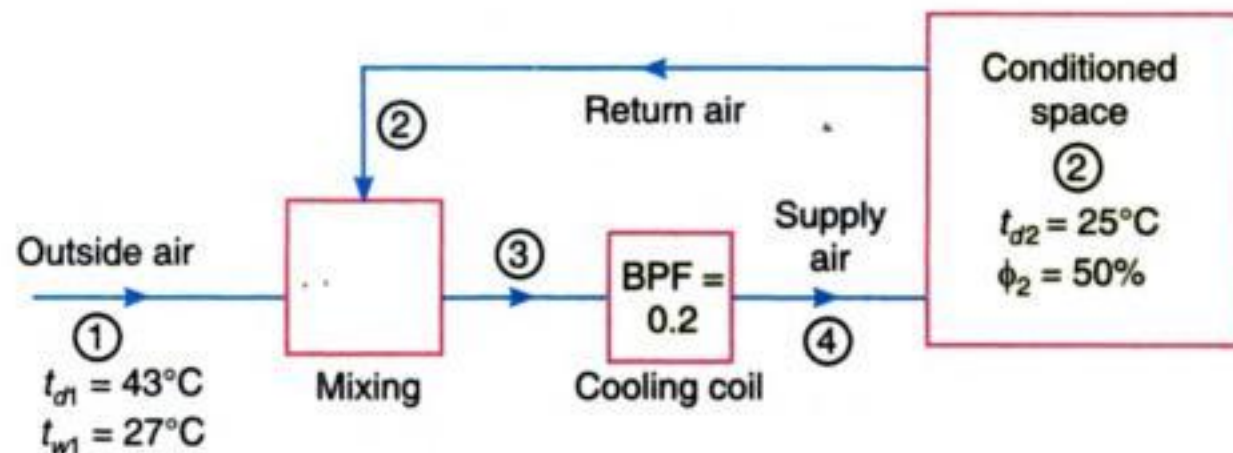
Fig. 18.24

First of all, mark the outside condition of air at 43°C dry bulb temperature and 27°C wet bulb temperature, on the psychrometric chart as point 1, as shown in Fig. 18.25. Now mark the inside conditions of air at 25°C dry bulb temperature and 50% relative humidity as point 2. We know that room sensible heat factor,

$$RSHF = \frac{RSH}{RSH + RLH} = \frac{84\,000}{84\,000 + 21\,000} = 0.8$$

Now mark this calculated value of *RSHF* on the sensible heat factor scale and join with the alignment circle (*i.e.* 26°C *DBT* and 50% *RH*). From point 2, draw a line 2-5 parallel to this line. This line 2-5 is called *RSHF* line. Since the return air from the conditioned space is mixed with outside air before entry to the cooling coil in the ratio of 4 : 1, therefore the condition of air entering the cooling coil after mixing process is marked on the line 1-2 by point 3, such that

$$\text{Length 2-3} = \frac{\text{Length 1-2}}{5}$$

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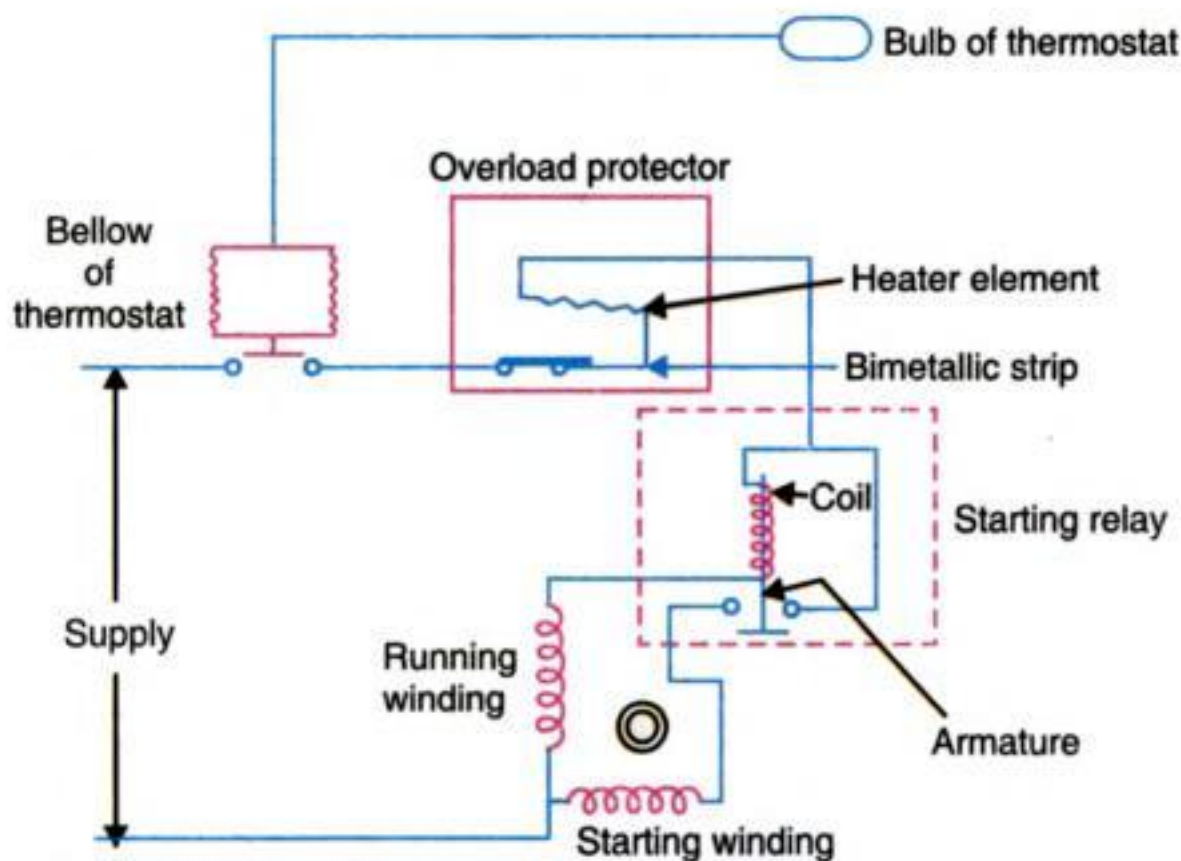
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Fig. 22.2. Electrical connection diagram of a domestic refrigerator.

(c) Thermostat. A thermostat is used to control the temperature in the refrigerator. The bulb of the thermostat is clamped to the evaporator or freezer. The thermostat bulb is charged with few drops of refrigerant. The thermostat can be set to maintain different temperatures at a time. When the desired temperature is obtained, the bulb of the thermostat senses it, the liquid in it compresses and operate bellows of the thermostat and opens the compressor motor contacts. The temperature at which compressor motor stops is called *cut-out temperature.* When the temperature increases, the liquid in the bulb expands thereby closing the bellows contact of the compressor motor. The temperature at which the compressor motor starts, is called *cut-in temperature.* A thermostat is very crucial in the operation of a refrigerator as the running time of the compressor is reduced considerably thereby cutting the operation cost as well as enhancing the compressor life due to non-continuous working.

22.5 Room Air Conditioner

A room air conditioner is a compact, self contained air-conditioning unit which is normally installed in a window or wall opening of the room and is widely known as window type air conditioner. It works on vapour compression cycle. A complete unit of a room air conditioner consists of the refrigeration system, the control system (thermostat and selector switch), electrical protection system (motor overload switches and winding protection thermostat on the compressor motor), air circulation system (fan motor, centrifugal evaporator blower), ventilation (fresh air damper) and exhaust system.

The refrigeration system consists of a hermetic type compressor, forced air-cooled finned condenser coil, finned cooling coil, capillary tube as the throttling device and a refrigerant drier. The refrigerant used is R-12 or R-22. In hermetic compressors, a winding thermostat is embedded in the compressor motor windings. It puts off the compressor if the winding temperature exceeds the safe limit, thus protecting the winding against high temperature.

The condenser is a continuous coil made of copper tubing with aluminium fins attached to it to increase the heat transfer rate (rejecting heat to atmosphere). A propeller type fan provides the necessary air to cool the refrigerant in the condenser and also exhausts air from the air-conditioned space when the exhaust damper is opened. The evaporator is a cooling coil also made of copper with aluminium fins attached to it to increase the heat transfer rate (taking in heat from the room air).

The room air-conditioner is installed in such a way that the evaporator faces the room. A centrifugal blower is installed behind the cooling coil which sends cool air in the room. A filter is installed on the fresh air entering side of the evaporator to remove any dirt from the air. A damper

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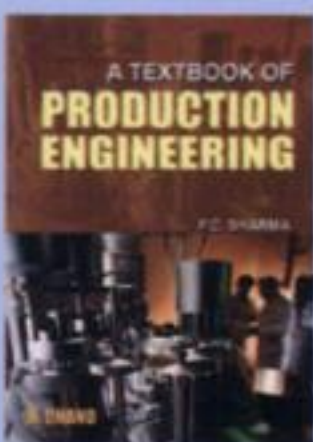
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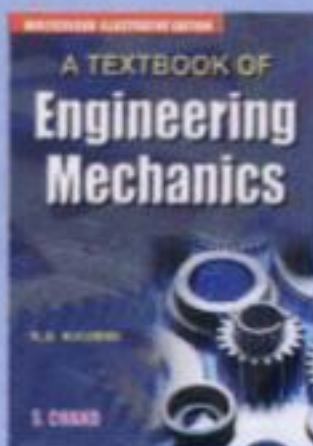
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